

& GETAL | B RUIMTE



Noordhoff

Getal & Ruimte

Uitwerkingen **vwo B** deel 3

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9 Exponentiële en logaritmische functies

Voorkennis De logaritme

Bladzijde 9

- 1** a ${}^2\log(64) = {}^2\log(2^6) = 6$ d ${}^{\frac{1}{3}}\log(1) = {}^{\frac{1}{3}}\log((\frac{1}{3})^0) = 0$
b ${}^3\log(\frac{1}{27}) = {}^3\log(3^{-3}) = -3$ e ${}^{\frac{1}{2}}\log(\frac{1}{8}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^3) = 3$
c ${}^6\log(6\sqrt{6}) = {}^6\log(6^{1\frac{1}{2}}) = 1\frac{1}{2}$ f ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$
- 2** a ${}^3\log(x) = 5$ d ${}^{\frac{1}{2}}\log(3x+4) = -2$
 $x = 3^5 = 243$ $3x+4 = (\frac{1}{2})^{-2}$
b ${}^2\log(3x+7) = 4$ $3x+4 = 4$
 $3x+7 = 2^4$ $3x = 0$
 $3x+7 = 16$ $x = 0$
 $3x = 9$ e $3 + {}^2\log(5 - 1\frac{1}{2}x) = 0$
 $x = 3$ ${}^2\log(5 - 1\frac{1}{2}x) = -3$
c $5 + {}^5\log(x+3) = 7$ $5 - 1\frac{1}{2}x = 2^{-3}$
 ${}^5\log(x+3) = 2$ $5 - 1\frac{1}{2}x = \frac{1}{8}$
 $x+3 = 5^2$ $40 - 12x = 1$
 $x+3 = 25$ $-12x = -39$
 $x = 22$ $x = 3\frac{1}{4}$
f $4 - 2 \cdot {}^3\log(x) = 3$
 $-2 \cdot {}^3\log(x) = -1$
 ${}^3\log(x) = \frac{1}{2}$
 $x = 3^{\frac{1}{2}} = \sqrt{3}$

9.1 Rekenregels voor logaritmen

Bladzijde 10

- 1** a ${}^2\log(a)$ is de exponent van een macht met grondtal 2 waarmee de macht gelijk is aan a , dus $2^{^2\log(a)} = a$. Omdat een macht met grondtal 2 altijd groter is dan 0, geldt dit alleen voor $a > 0$.
b ${}^g\log(x) = y$ oftewel $y = {}^g\log(x)$ substitueren in $x = g^y$ geeft $x = g^{^g\log(x)}$, dus $g^{^g\log(x)} = x$.

Bladzijde 11

- 2** $2 \cdot {}^2\log(800) = 2 \cdot {}^2\log(32 \cdot 25) = 2 \cdot ({}^2\log(32) + {}^2\log(25)) = 2 \cdot ({}^2\log(2^5) + {}^2\log(5^2)) = 2 \cdot (5 + 2 \cdot {}^2\log(5)) = 10 + 4 \cdot {}^2\log(5)$
- 3** a ${}^2\log(6) + {}^2\log(10) = {}^2\log(6 \cdot 10) = {}^2\log(60)$
b ${}^3\log(30) - {}^3\log(6) = {}^3\log(\frac{30}{6}) = {}^3\log(5)$
c $2 \cdot {}^5\log(3) + {}^5\log(\frac{1}{2}) = {}^5\log(3^2) + {}^5\log(\frac{1}{2}) = {}^5\log(9) + {}^5\log(\frac{1}{2}) = {}^5\log(9 \cdot \frac{1}{2}) = {}^5\log(4\frac{1}{2})$
d $\frac{1}{2}\log(15) - 4 \cdot \frac{1}{2}\log(3) = \frac{1}{2}\log(15) - \frac{1}{2}\log(3^4) = \frac{1}{2}\log(15) - \frac{1}{2}\log(81) = \frac{1}{2}\log(\frac{15}{81}) = \frac{1}{2}\log(\frac{5}{27})$
e $-2 \cdot {}^4\log(6) + {}^4\log(12) = {}^4\log(6^{-2}) + {}^4\log(12) = {}^4\log(\frac{1}{36}) + {}^4\log(12) = {}^4\log(\frac{1}{36} \cdot 12) = {}^4\log(\frac{1}{3})$
f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(50) - \log(25) = \log(\frac{50}{25}) = \log(2)$

- 4 a** $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16) + {}^2\log(3) = {}^2\log(16 \cdot 3) = {}^2\log(48)$
- b** $3 - \frac{1}{2}\log(10) = \frac{1}{2}\log\left(\left(\frac{1}{2}\right)^3\right) - \frac{1}{2}\log(10) = \frac{1}{2}\log\left(\frac{1}{8}\right) - \frac{1}{2}\log(10) = \frac{1}{2}\log\left(\frac{\frac{1}{8}}{10}\right) = \frac{1}{2}\log\left(\frac{1}{80}\right)$
- c** $2 - \log(5) = \log(10^2) - \log(5) = \log(100) - \log(5) = \log\left(\frac{100}{5}\right) = \log(20)$
- d** ${}^2\log(12) - {}^3\log(9) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(12) - {}^2\log(4) = {}^2\log\left(\frac{12}{4}\right) = {}^2\log(3)$
- e** $\frac{1}{2} \cdot {}^3\log(16) + \frac{1}{2}\log(8) = {}^3\log(16^{\frac{1}{2}}) - 3 = {}^3\log(4) - {}^3\log(3^3) = {}^3\log(4) - {}^3\log(27) = {}^3\log\left(\frac{4}{27}\right)$
- f** $\log(500) - {}^5\log(125) = \log(500) - 3 = \log(500) - \log(10^3) = \log(500) - \log(1000) = \log\left(\frac{500}{1000}\right) = \log\left(\frac{1}{2}\right)$
- 5 a** ${}^3\log(6) + {}^3\log\left(1\frac{1}{2}\right) = {}^3\log\left(6 \cdot 1\frac{1}{2}\right) = {}^3\log(9) = 2$
- b** ${}^5\log(2) - {}^5\log(50) = {}^5\log\left(\frac{2}{50}\right) = {}^5\log\left(\frac{1}{25}\right) = {}^5\log(5^{-2}) = -2$
- c** ${}^2\log(27) + 3 \cdot {}^2\log\left(\frac{1}{6}\right) = {}^2\log(27) + {}^2\log\left(\left(\frac{1}{6}\right)^3\right) = {}^2\log(27) + {}^2\log\left(\frac{1}{216}\right) = {}^2\log\left(\frac{27}{216}\right) = {}^2\log\left(\frac{1}{8}\right) = {}^2\log(2^{-3}) = -3$
- d** $2 \cdot {}^4\log(6) - 2 \cdot {}^4\log(3) = {}^4\log(6^2) - {}^4\log(3^2) = {}^4\log(36) - {}^4\log(9) = {}^4\log\left(\frac{36}{9}\right) = {}^4\log(4) = 1$
- 6 a** $g^{{}^g\log(a) - {}^g\log(b)} = \frac{g^{{}^g\log(a)}}{g^{{}^g\log(b)}} = \frac{a}{b} = g^{{}^g\log\left(\frac{a}{b}\right)}$, dus ${}^g\log(a) - {}^g\log(b) = {}^g\log\left(\frac{a}{b}\right)$.
- b** $g^{p \cdot {}^g\log(a)} = (g^{{}^g\log(a)})^p = a^p = g^{{}^g\log(a^p)}$, dus $p \cdot {}^g\log(a) = {}^g\log(a^p)$.
- 7 a** ${}^2\log(a) + 3 \cdot {}^2\log(b) = {}^2\log(a) + {}^2\log(b^3) = {}^2\log(ab^3)$
- b** $5 \cdot {}^3\log(a) - 2 \cdot {}^3\log(b) = {}^3\log(a^5) - {}^3\log(b^2) = {}^3\log\left(\frac{a^5}{b^2}\right)$
- c** $2 + {}^5\log(a) = {}^5\log(5^2) + {}^5\log(a) = {}^5\log(25) + {}^5\log(a) = {}^5\log(25a)$
- d** $2 - {}^3\log(a) = {}^3\log(3^2) - {}^3\log(a) = {}^3\log(9) - {}^3\log(a) = {}^3\log\left(\frac{9}{a}\right)$
- e** ${}^6\log(a) - 1 = {}^6\log(a) - {}^6\log(6) = {}^6\log\left(\frac{a}{6}\right) = {}^6\log\left(\frac{1}{6}a\right)$
- f** $2 \cdot {}^5\log(b) + \frac{1}{2} \cdot {}^5\log(a) = {}^5\log(b^2) + {}^5\log(a^{\frac{1}{2}}) = {}^5\log(b^2) + {}^5\log(\sqrt{a}) = {}^5\log(b^2 \cdot \sqrt{a})$

Bladzijde 12

- 8 a** $\log(600) = \log(100 \cdot 6) = \log(100) + \log(6) = 2 + \log(6)$
- b** ${}^2\log(24) = {}^2\log(8 \cdot 3) = {}^2\log(8) + {}^2\log(3) = 3 + {}^2\log(3)$
- c** $4 \cdot {}^3\log(54) = 4 \cdot {}^3\log(27 \cdot 2) = 4 \cdot ({}^3\log(27) + {}^3\log(2)) = 4 \cdot (3 + {}^3\log(2)) = 12 + 4 \cdot {}^3\log(2) = 12 + {}^3\log(2^4) = 12 + {}^3\log(16)$
- d** $2 \cdot {}^5\log(1250) = 2 \cdot {}^5\log(625 \cdot 2) = 2 \cdot ({}^5\log(625) + {}^5\log(2)) = 2 \cdot (4 + {}^5\log(2)) = 8 + 2 \cdot {}^5\log(2) = 8 + {}^5\log(2^2) = 8 + {}^5\log(4)$
- e** $\log(1600) - 1 = \log(100 \cdot 16) - 1 = \log(100) + \log(16) - 1 = 2 + \log(16) - 1 = 1 + \log(16)$
- f** $7 + {}^2\log(1600) = 7 + {}^2\log(64 \cdot 25) = 7 + {}^2\log(64) + {}^2\log(25) = 7 + 6 + {}^2\log(25) = 13 + {}^2\log(25)$
- 9 a** $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as, } 81} y = {}^3\log\left(\frac{1}{81}x\right)$
 Er geldt ${}^3\log\left(\frac{1}{81}x\right) = {}^3\log\left(\frac{1}{81}\right) + {}^3\log(x) = -4 + {}^3\log(x)$.
 Dus $a = -4$.
- b** Je had de grafiek van g ook kunnen krijgen met de translatie $(0, -4)$.
- 10 a** $y = {}^2\log(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{32}} y = {}^2\log(32x)$
 Er geldt ${}^2\log(32x) = {}^2\log(32) + {}^2\log(x) = 5 + {}^2\log(x)$.
 Dus de translatie $(0, 5)$ levert dezelfde beeldgrafiek op.
- b** $y = {}^2\log(x) \xrightarrow{\text{translatie } (0, 3)} y = {}^2\log(x) + 3$
 Er geldt ${}^2\log(x) + 3 = {}^2\log(x) + {}^2\log(2^3) = {}^2\log(x) + {}^2\log(8) = {}^2\log(8x)$
 Dus de vermenigvuldiging ten opzichte van de y -as met $\frac{1}{8}$ levert dezelfde beeldgrafiek op.

$$11 \quad y = {}^2\log(x) \xrightarrow{\text{translatie } (3, 0)} y = {}^2\log(x-3) \xrightarrow{\text{verm. } y \sim a, \frac{1}{4}} y = {}^2\log(4x-3)$$

$$\text{Er geldt } {}^2\log(4x-3) = {}^2\log(4(x-\frac{3}{4})) = {}^2\log(4) + {}^2\log(x-\frac{3}{4}) = 2 + {}^2\log(x-\frac{3}{4}).$$

$$\text{Dus } p = 2 \text{ en } q = -\frac{3}{4}.$$

$$12 \quad y = 3^x \xrightarrow{\text{translatie } (a, 0)} y = 3^{x-a} \xrightarrow{\text{verm. } y \sim a, b} y = 3^{\frac{1}{5}x-a}$$

$$g(x) = 81 \cdot 27^x = 3^4 \cdot (3^3)^x = 3^4 \cdot 3^{3x} = 3^{3x+4}$$

$$\text{Dus } b = \frac{1}{5} \text{ en } a = -4.$$

$$y = {}^3\log(x) \xrightarrow{\text{translatie } (-4, 0)} y = {}^3\log(x+4) \xrightarrow{\text{verm. } y \sim a, \frac{1}{3}} y = {}^3\log(3x+4)$$

$$\text{Er geldt } {}^3\log(3x+4) = {}^3\log(3(x+\frac{4}{3})) = {}^3\log(3) + {}^3\log(x+\frac{4}{3}) = 1 + {}^3\log(x+\frac{4}{3}).$$

$$\text{Dus } p = 1 \text{ en } q = \frac{4}{3}.$$

$$13 \quad \mathbf{a} \quad 3 + {}^2\log(3) = {}^2\log(2^3) + {}^2\log(3) = {}^2\log(8) + {}^2\log(3) = {}^2\log(8 \cdot 3) = {}^2\log(24)$$

$$\mathbf{b} \quad {}^2\log(x+1) = 3 + {}^2\log(3)$$

$${}^2\log(x+1) = {}^2\log(24)$$

$$x+1 = 24$$

$$x = 23$$

Bladzijde 14

$$14 \quad \mathbf{a} \quad {}^5\log(x) = 3 \cdot {}^5\log(2) - 2 \cdot {}^5\log(3)$$

$${}^5\log(x) = {}^5\log(2^3) - {}^5\log(3^2)$$

$${}^5\log(x) = {}^5\log(8) - {}^5\log(9)$$

$${}^5\log(x) = {}^5\log(\frac{8}{9})$$

$$x = \frac{8}{9}$$

$$\mathbf{b} \quad {}^2\log(x) = 4 - {}^2\log(3)$$

$${}^2\log(x) = {}^2\log(2^4) - {}^2\log(3)$$

$${}^2\log(x) = {}^2\log(16) - {}^2\log(3)$$

$${}^2\log(x) = {}^2\log(\frac{16}{3})$$

$$x = 5\frac{1}{3}$$

$$\mathbf{c} \quad {}^2\log(x+3) = 3 + {}^2\log(x)$$

$${}^2\log(x+3) = {}^2\log(2^3) + {}^2\log(x)$$

$${}^2\log(x+3) = {}^2\log(8) + {}^2\log(x)$$

$${}^2\log(x+3) = {}^2\log(8x)$$

$$x+3 = 8x$$

$$-7x = -3$$

$$x = \frac{3}{7}$$

$$\mathbf{d} \quad {}^3\log(2x) = 1 + {}^3\log(x+1)$$

$${}^3\log(2x) = {}^3\log(3) + {}^3\log(x+1)$$

$${}^3\log(2x) = {}^3\log(3(x+1))$$

$${}^3\log(2x) = {}^3\log(3x+3)$$

$$2x = 3x+3$$

$$-x = 3$$

$$x = -3$$

$$\text{vold. niet}$$

$$15 \quad \mathbf{a} \quad 5 \cdot \log(x) = 5 - \log(3125)$$

$$5 \cdot \log(x) = 5 - \log(5^5)$$

$$5 \cdot \log(x) = 5 - 5 \cdot \log(5)$$

$$\log(x) = 1 - \log(5)$$

$$\log(x) = \log(10) - \log(5)$$

$$\log(x) = \log(\frac{10}{5})$$

$$x = 2$$

$$\mathbf{b} \quad {}^{\frac{1}{2}}\log(2x-1) = 2 + {}^{\frac{1}{2}}\log(x+2)$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(x+2)$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}) + {}^{\frac{1}{2}}\log(x+2)$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}(x+2))$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}x + \frac{1}{2})$$

$$2x-1 = \frac{1}{4}x + \frac{1}{2}$$

$$\frac{3}{4}x = \frac{3}{2}$$

$$x = \frac{6}{3}$$

$$\mathbf{c} \quad {}^3\log(x+2) = 1 - {}^3\log(x)$$

$${}^3\log(x+2) = {}^3\log(3) - {}^3\log(x)$$

$${}^3\log(x+2) = {}^3\log(\frac{3}{x})$$

$$x+2 = \frac{3}{x}$$

$$x^2 + 2x = 3$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1 \vee x = -3$$

$$\text{vold. niet}$$

$$\mathbf{d} \quad 2 \cdot {}^3\log(x) + 1 = {}^3\log(5x-2)$$

$${}^3\log(x^2) + {}^3\log(3) = {}^3\log(5x-2)$$

$${}^3\log(3x^2) = {}^3\log(5x-2)$$

$$3x^2 = 5x-2$$

$$3x^2 - 5x + 2 = 0$$

$$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1$$

$$x = \frac{5+1}{6} = 1 \vee x = \frac{5-1}{6} = \frac{2}{3}$$

16 a ${}^5\log(x) = 2 + \frac{1}{2} \cdot {}^5\log(3)$
 ${}^5\log(x) = {}^5\log(5^2) + {}^5\log(3^{\frac{1}{2}})$
 ${}^5\log(x) = {}^5\log(25) + {}^5\log(\sqrt{3})$
 ${}^5\log(x) = {}^5\log(25\sqrt{3})$
 $x = 25\sqrt{3}$

b ${}^3\log(x+4) + 1 = 2 \cdot {}^3\log(x-2)$
 ${}^3\log(x+4) + {}^3\log(3) = {}^3\log((x-2)^2)$
 ${}^3\log(3(x+4)) = {}^3\log(x^2 - 4x + 4)$
 ${}^3\log(3x + 12) = {}^3\log(x^2 - 4x + 4)$
 $3x + 12 = x^2 - 4x + 4$
 $x^2 - 7x - 8 = 0$
 $(x+1)(x-8) = 0$
 $x = -1 \vee x = 8$
vold. niet

c ${}^2\log(2x) - {}^2\log(x+3) = {}^2\log(x) - 2$
 ${}^2\log\left(\frac{2x}{x+3}\right) = {}^2\log(x) - {}^2\log(2^2)$
 ${}^2\log\left(\frac{2x}{x+3}\right) = {}^2\log\left(\frac{x}{4}\right)$
 $\frac{2x}{x+3} = \frac{x}{4}$
 $x^2 + 3x = 8x$
 $x^2 - 5x = 0$
 $x(x-5) = 0$
 $x = 0 \vee x = 5$
vold. niet

d ${}^3\log(x) = 2 - {}^3\log(x-1)$
 ${}^3\log(x) = {}^3\log(3^2) - {}^3\log(x-1)$
 ${}^3\log(x) = {}^3\log\left(\frac{9}{x-1}\right)$
 $x = \frac{9}{x-1}$
 $x^2 - x = 9$
 $x^2 - x - 9 = 0$
 $D = (-1)^2 - 4 \cdot 1 \cdot -9 = 37$
 $x = \frac{1 + \sqrt{37}}{2} \vee x = \frac{1 - \sqrt{37}}{2}$
 $x = \frac{1}{2} + \frac{1}{2}\sqrt{37} \vee x = \frac{1}{2} - \frac{1}{2}\sqrt{37}$
vold. niet

Bladzijde 15

17 $y = {}^2\log(x) \xrightarrow{\text{verm. } x\text{-as, } a} y = a \cdot {}^2\log(x) \xrightarrow{\text{verm. } y\text{-as, } b} y = a \cdot {}^2\log\left(\frac{1}{b}x\right)$

Er geldt $a \cdot {}^2\log\left(\frac{1}{b}x\right) = a \cdot ({}^2\log(x) - {}^2\log(b)) = -a \cdot {}^2\log(b) + a \cdot {}^2\log(x)$.

Dus $a = 10$ en $-10 \cdot {}^2\log(b) = -5$

$${}^2\log(b) = \frac{1}{2}$$

$$b = 2^{\frac{1}{2}} = \sqrt{2}$$

18 a $p^{\log(g)} = g$
 $(p^{\log(g)})^{\log(a)} = g^{\log(a)}$
 $p^{\log(g) \cdot \log(a)} = g^{\log(a)}$
 $p^{\log(g) \cdot \log(a)} = a$

b $p^{\log(g) \cdot \log(a)} = p^{\log(a)}$
 $p^{\log(g) \cdot \log(a)} = p^{\log(a)}$
 $\log(a) = \frac{\log(a)}{\log(g)}$

Bladzijde 16

19 a Omdat is gegeven dat ${}^4\log(x) = {}^2\log(x-2)$ zal gelden dat $a = b$ als ${}^4\log(x) = a$ en ${}^2\log(x-2) = b$.

Substitutie van ${}^4\log(x) = a$ en ${}^2\log(x-2) = b$ in ${}^4\log(x) = {}^2\log(x-2)$ geeft $a = b$.

Uit ${}^8\log(x) = y$ geeft $x = g^y$ volgt ${}^4\log(x) = a$ geeft $x = 4^a$ en ${}^2\log(x-2) = b$ geeft $x-2 = 2^b$ oftewel $x = 2^b + 2$.

b Uit $x = 4^a$, $x = 2^b + 2$ en $a = b$ volgt $4^a = 2^a + 2$

$$(2^2)^a = 2^a + 2$$

$$(2^a)^2 - 2^a - 2 = 0$$

$$\text{Stel } 2^a = u.$$

$$u^2 - u - 2 = 0$$

$$(u + 1)(u - 2) = 0$$

$$u = -1 \vee u = 2$$

$$2^a = -1 \vee 2^a = 2$$

$$a = 1$$

$a = 1$ geeft $x = 4$

Bladzijde 17

20 a ${}^3\log(3x - 5) + {}^{\frac{1}{3}}\log(x - 1) = 0$

$${}^3\log(3x - 5) - {}^3\log(x - 1) = 0$$

$${}^3\log(3x - 5) = {}^3\log(x - 1)$$

$$3x - 5 = x - 1$$

$$2x = 4$$

$$x = 2$$

b ${}^5\log(3x) + 2 \cdot {}^{\frac{1}{5}}\log(x) = 0$

$${}^5\log(3x) + {}^{\frac{1}{5}}\log(x^2) = 0$$

$${}^5\log(3x) - {}^5\log(x^2) = 0$$

$${}^5\log(3x) = {}^5\log(x^2)$$

$$3x = x^2$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0 \vee x = 3$$

vold. niet

c $2x \cdot {}^{\frac{1}{2}}\log(3x + 5) = {}^{\frac{1}{2}}\log(3x + 5)$

$${}^{\frac{1}{2}}\log(3x + 5) = 0 \vee 2x = 1$$

$$3x + 5 = 1 \vee x = \frac{1}{2}$$

$$3x = -4 \vee x = \frac{1}{2}$$

$$x = -\frac{1}{3} \vee x = \frac{1}{2}$$

d ${}^2\log(x) = {}^4\log(x + 20)$

$${}^2\log(x) = \frac{{}^2\log(x + 20)}{{}^2\log(4)}$$

$${}^2\log(x) = \frac{{}^2\log(x + 20)}{2}$$

$$2 \cdot {}^2\log(x) = {}^2\log(x + 20)$$

$${}^2\log(x^2) = {}^2\log(x + 20)$$

$$x^2 = x + 20$$

$$x^2 - x - 20 = 0$$

$$(x + 4)(x - 5) = 0$$

$$x = -4 \vee x = 5$$

vold. niet

21 a $-2 \cdot {}^{\frac{1}{2}}\log(x) = 2 + {}^2\log(3 - x)$

$$2 \cdot {}^2\log(x) = {}^2\log(2^2) + {}^2\log(3 - x)$$

$${}^2\log(x^2) = {}^2\log(4(3 - x))$$

$$x^2 = 12 - 4x$$

$$x^2 + 4x - 12 = 0$$

$$(x - 2)(x + 6) = 0$$

$$x = 2 \vee x = -6$$

vold. niet

b ${}^9\log(2x) = {}^3\log(x - 4)$

$$\frac{{}^3\log(2x)}{{}^3\log(9)} = {}^3\log(x - 4)$$

$$\frac{{}^3\log(2x)}{2} = {}^3\log(x - 4)$$

$${}^3\log(2x) = 2 \cdot {}^3\log(x - 4)$$

$${}^3\log(2x) = {}^3\log((x - 4)^2)$$

$$2x = (x - 4)^2$$

$$2x = x^2 - 8x + 16$$

$$x^2 - 10x + 16 = 0$$

$$(x - 2)(x - 8) = 0$$

$$x = 2 \vee x = 8$$

vold. niet

c $4x \cdot {}^4\log(2x - 1) + 3 \cdot {}^4\log(2x - 1) = 0$

$$(4x + 3) \cdot {}^4\log(2x - 1) = 0$$

$$4x + 3 = 0 \vee {}^4\log(2x - 1) = 0$$

$$4x = -3 \vee 2x - 1 = 1$$

$$x = -\frac{3}{4} \vee 2x = 2$$

$$x = -\frac{3}{4} \vee x = 1$$

vold. niet

d $x^2 \cdot {}^5\log(2x + 1) + 9 \cdot {}^{\frac{1}{5}}\log(2x + 1) = 0$

$$x^2 \cdot {}^5\log(2x + 1) - 9 \cdot {}^5\log(2x + 1) = 0$$

$$(x^2 - 9) \cdot {}^5\log(2x + 1) = 0$$

$$x^2 - 9 = 0 \vee {}^5\log(2x + 1) = 0$$

$$x^2 = 9 \vee 2x + 1 = 1$$

$$x = 3 \vee x = -3 \vee 2x = 0$$

$$x = 3 \vee x = -3 \vee x = 0$$

vold. niet

22 ${}^3\log^2(x) = 2 \cdot {}^3\log(x) + 15$
 Stel ${}^3\log(x) = u$.
 $u^2 = 2u + 15$
 $u^2 - 2u - 15 = 0$
 $(u + 3)(u - 5) = 0$
 $u = -3 \vee u = 5$
 ${}^3\log(x) = -3 \vee {}^3\log(x) = 5$
 $x = 3^{-3} = \frac{1}{27} \vee x = 3^5 = 243$

23 a $2\log^2(x) = 2 \cdot 2\log(x) + 3$
 Stel $2\log(x) = u$.
 $u^2 = 2u + 3$
 $u^2 - 2u - 3 = 0$
 $(u + 1)(u - 3) = 0$
 $u = -1 \vee u = 3$
 $2\log(x) = -1 \vee 2\log(x) = 3$
 $x = \frac{1}{2} \vee x = 8$

b $\frac{1}{2}\log^2(x + 2) + 3 \cdot \frac{1}{2}\log(x + 2) = 0$
 Stel $\frac{1}{2}\log(x + 2) = u$.
 $u^2 + 3u = 0$
 $u(u + 3) = 0$
 $u = 0 \vee u = -3$
 $\frac{1}{2}\log(x + 2) = 0 \vee \frac{1}{2}\log(x + 2) = -3$
 $x + 2 = 1 \vee x + 2 = 8$
 $x = -1 \vee x = 6$

c $2 \cdot {}^3\log^2(x) + 2 = 5 \cdot {}^3\log(x)$
 Stel ${}^3\log(x) = u$.
 $2u^2 + 2 = 5u$
 $2u^2 - 5u + 2 = 0$
 $D = (-5)^2 - 4 \cdot 2 \cdot 2 = 9$
 $u = \frac{5+3}{4} = 2 \vee u = \frac{5-3}{4} = \frac{1}{2}$
 ${}^3\log(x) = 2 \vee {}^3\log(x) = \frac{1}{2}$
 $x = 9 \vee x = \sqrt[3]{3}$

d ${}^5\log^2(x) + 3 \cdot \frac{1}{5}\log(x) + 2 = 0$
 ${}^5\log^2(x) - 3 \cdot {}^5\log(x) + 2 = 0$
 Stel ${}^5\log(x) = u$.
 $u^2 - 3u + 2 = 0$
 $(u - 1)(u - 2) = 0$
 $u = 1 \vee u = 2$
 ${}^5\log(x) = 1 \vee {}^5\log(x) = 2$
 $x = 5 \vee x = 25$

9.2 Exponentiële en logaritmische formules

Bladzijde 19

- 24 a** Jaarlijkse toename van 5%, dus de vermenigvuldigingsfactor is 1,05.
 Het bedrag op 1 januari 2021 is $1000 \cdot 1,05 = 1050$ euro.
 Het bedrag op 1 januari 2025 is $1000 \cdot 1,05^5 \approx 1276,28$ euro.
- b** $g = 1,05$
- c** procentuele toename $\approx \frac{1276,28 - 1000}{1000} \cdot 100\% \approx 27,6\%$

Bladzijde 21

- 25 a** $g_{\text{jaar}} = 1,127$
b $g_{\text{maand}} = 0,932$
c Het groeipercentage per maand is 73,5%.
d De afname per dag is 15,5%.
e Het groeipercentage per jaar is 142%.
f $g_{\text{dag}} = 0,993$
- 26 a** $g_{\text{kwartier}} = 1,12$
 $g_{\text{uur}} = 1,12^4 \approx 1,574$
 De toename is 57,4% per uur.
- b** $g_{\text{kwartier}} = 1,12$
 $g_{5 \text{ minuten}} = 1,12^{\frac{1}{3}} \approx 1,038$
 De procentuele toename per vijf minuten is 3,8%.
- c** $g_{\text{kwartier}} = 1,12$
 $g_{5 \text{ uur}} = 1,12^{20} \approx 9,65$
 De toename is 865% per vijf uur.

- 27 a** $g_{\text{dag}} = 1,3$
 $g_{\text{week}} = 1,3^7 \approx 6,27$
 Het groeipercentage per week is 527%.
- b** $g_{\text{dag}} = 1,3$
 $g_{4 \text{ uur}} = 1,3^{\frac{1}{4}} \approx 1,045$
 Het groeipercentage per vier uur is 4,5%.

- 28 a** $g_{\text{dag}} = 0,84$
 $g_{\text{week}} = 0,84^7 \approx 0,295$
- b** $g_{\text{dag}} = 0,84$
 $g_{\text{uur}} = 0,84^{\frac{1}{24}} \approx 0,993$
 De hoeveelheid neemt met 0,7% per uur af.
- c** $g_{\text{dag}} = 0,84$
 $g_{\text{kwartier}} = 0,84^{\frac{1}{24 \cdot 4}} \approx 0,9982$
 Het groeipercentage per kwartier is -0,18%.

Bladzijde 22

- 29 a** $g_{\text{uur}} = 0,805$
 $g_{\text{kwartier}} = 0,805^{\frac{1}{4}} \approx 0,947$
 De afname is 5,3% per kwartier.
- b** $g_{\text{jaar}} = 1,086$
 $g_{25 \text{ jaar}} = 1,086^{25} \approx 7,87$
 De toename is 687% per 25 jaar.
- c** $g_{\text{week}} = 2,80$
 $g_{\text{dag}} = 2,80^{\frac{1}{7}} \approx 1,158$
- 30 a** De hoeveelheid N neemt in $10 - 3 = 7$ uur toe van 1600 tot 4100, dus $g_{7 \text{ uur}} = \frac{4100}{1600}$.
- b** $g_{\text{uur}} = \left(\frac{4100}{1600}\right)^{\frac{1}{7}} = 1,1438\dots$
- c**
$$\left. \begin{aligned} N &= b \cdot 1,1438\dots^t \\ t &= 3 \text{ en } N = 1600 \end{aligned} \right\} \begin{aligned} b \cdot 1,1438\dots^3 &= 1600 \\ b &= \frac{1600}{1,1438\dots^3} \approx 1070 \end{aligned}$$

 Dus op $t = 0$ is $N \approx 1070$.
 De formule van N is $N = 1070 \cdot 1,144^t$.
- 31 a** Stel $N = b \cdot g^t$.
 $g_{6 \text{ uur}} = \frac{1250}{150} = 8,333\dots$
 $g_{\text{uur}} = 8,333\dots^{\frac{1}{6}} = 1,4238\dots$

$$\left. \begin{aligned} N &= b \cdot 1,4238\dots^t \\ t &= 2 \text{ en } N = 150 \end{aligned} \right\} \begin{aligned} b \cdot 1,4238\dots^2 &= 150 \\ b &= \frac{150}{1,4238\dots^2} \approx 74 \end{aligned}$$

 Dus $N = 74 \cdot 1,424^t$.
- b** Stel $H = b \cdot g^t$.
 $g_{3 \text{ dagen}} = \frac{0,47}{0,60} = 0,7833\dots$
 $g_{\text{dag}} = 0,7833\dots^{\frac{1}{3}} = 0,9218\dots$

$$\left. \begin{aligned} H &= b \cdot 0,9218\dots^t \\ t &= 5 \text{ en } H = 0,60 \end{aligned} \right\} \begin{aligned} b \cdot 0,9218\dots^5 &= 0,60 \\ b &= \frac{0,60}{0,9218\dots^5} = 0,901\dots \end{aligned}$$

 Dus $H = 0,90 \cdot 0,922^t$.

32 Stel $A = b \cdot g^t$.

$$g_{4 \text{ dagen}} = \frac{11}{31} = 0,3548\dots$$

$$g_{\text{dag}} = 0,3548\dots^{\frac{1}{4}} = 0,7718\dots$$

$$\left. \begin{aligned} A &= b \cdot 0,7718\dots^t \\ t = 3 \text{ en } A &= 31 \end{aligned} \right\} b \cdot 0,7718\dots^3 = 31$$

$$b = \frac{31}{0,7718\dots^3} = 67,4\dots$$

$$60 \text{ uur} = \frac{60}{24} \text{ dag} = 2,5 \text{ dag}$$

$$t = 2,5 \text{ geeft } A = 67,4\dots \cdot 0,7718\dots^{2,5} \approx 35$$

Na 60 uur was de oppervlakte van de wond ongeveer 35 mm².

33 a Voer in $y_1 = 17,0 \cdot 1,031^x$ en $y_2 = 34,0$.

De optie snijpunt geeft $x = 22,70\dots$

Na 22,7 jaar is het aantal verdubbeld.

b Voer in $y_1 = 12,9 \cdot 1,031^x$ en $y_2 = 25,8$.

De optie snijpunt geeft $x = 22,70\dots$

Na 22,7 jaar is het aantal verdubbeld.

Bladzijde 23

34 a $g_{\text{jaar}} = 1,131$

$$1,131^T = 2$$

$$T = {}^{1,131}\log(2) = 5,630\dots$$

De verdubbelingstijd is 5 jaar en $0,630\dots \cdot 12 \approx 8$ maanden.

b $g_{\text{week}} = 0,915$

$$0,915^T = \frac{1}{2}$$

$$T = {}^{0,915}\log\left(\frac{1}{2}\right) = 7,802\dots$$

De halveringstijd is 7 weken en $0,802\dots \cdot 7 \approx 6$ dagen.

Bladzijde 24

35 a $g_{\text{jaar}} = 1,0325$

$$1,0325^T = 2$$

$$T = {}^{1,0325}\log(2) = 21,6\dots$$

De verdubbelingstijd is 22 jaar.

b $g_{10 \text{ jaar}} = 1,081$

$$1,081^T = 2$$

$$T = {}^{1,081}\log(2) = 8,89\dots$$

De verdubbelingstijd is $8,89\dots \cdot 10 \text{ jaar} \approx 89 \text{ jaar}$.

36 a $g_{25 \text{ jaar}} = 2$

$$g_{\text{jaar}} = 2^{\frac{1}{25}} \approx 1,028$$

Het groeipercentage per jaar is 2,8%.

b $g_{28 \text{ jaar}} = \frac{1}{2}$

$$g_{\text{jaar}} = \left(\frac{1}{2}\right)^{\frac{1}{28}} \approx 0,976$$

De hoeveelheid radioactieve stof neemt per jaar met 2,4% af.

37 a $g_{\text{dag}} = 0,917$

$$0,917^T = \frac{1}{2}$$

$$T = {}^{0,917}\log\left(\frac{1}{2}\right) = 7,9\dots$$

De halveringstijd is 8 dagen.

b $g_{\text{dag}} = 0,917$

$$0,917^T = 0,1$$

$$T = {}^{0,917}\log(0,1) = 26,5\dots$$

Na 27 dagen is nog 10% van de beginhoeveelheid over.

- 38 a** $g_{\text{dag}} = 0,81$
 $g_{\text{week}} = 0,81^7 = 0,2287...$
 De afname per week is 77,1%.
- b** $g_{\text{week}} = 0,38$
 $g_{\text{dag}} = 0,38^{\frac{1}{7}} = 0,8709...$
 De afname per dag is 12,9%.
- c** De groeifactor per dag is 0,845.
 Dus $BZV = 300 \cdot 0,845^t$.
- d** $0,845^T = \frac{1}{2}$
 $T = {}^{0,845}\log(\frac{1}{2}) = 4,115...$
 De halveringstijd is 4 dagen en $0,115... \cdot 24 \approx 3$ uur.
- e** $300 \cdot 0,845^t = 10$
 $0,845^t = \frac{10}{300} = \frac{1}{30}$
 $t = {}^{0,845}\log(\frac{1}{30}) = 20,1...$
 Dus na 20 dagen zuiveren is het BZV afgenomen tot 10 mg/liter.
- f** $T = 10$ geeft $h = 7,6 \cdot 0,96^{10} = 5,0527...$
 Bij 10 °C is de groeifactor per dag 0,8709... (zie vraag b).
 $0,8709...^{5,0527...} \approx 0,5$, dus de formule klopt voor $T = 10$.
 $T = 20$ geeft $h = 7,6 \cdot 0,96^{20} = 3,3592...$
 Bij 20 °C is de groeifactor per dag 0,81 (zie vraag a).
 $0,81^{3,3592...} \approx 0,5$, dus de formule klopt voor $T = 20$.

Bladzijde 25

- 39** $y = 2^{x-1}$
 $2^{x-1} = y$
 $x - 1 = {}^2\log(y)$
 $x = 1 + {}^2\log(y)$

Bladzijde 26

- 40 a** $t = 1 + {}^2\log(0,2N)$
 $t = {}^2\log(2) + {}^2\log(0,2N)$
 $t = {}^2\log(0,4N)$
 $t = \frac{\log(0,4N)}{\log(2)}$
 $t \approx 3,32 \cdot \log(0,4N)$
 Dus $t = 3,32 \cdot \log(0,4N)$.
- b** $t = 1 + {}^2\log(0,2N)$
 $t = 1 + {}^2\log(0,2) + {}^2\log(N)$
 $t = 1 + {}^2\log(0,2) + \frac{{}^3\log(N)}{{}^3\log(2)}$
 $t \approx -1,32 + 1,58 \cdot {}^3\log(N)$
 Dus $t = -1,32 + 1,58 \cdot {}^3\log(N)$.
- c** $x = 64 \cdot 8^y$
 $x = 64 \cdot (3^{\log(8)})^y$
 $x \approx 64 \cdot (3^{1,893})^y$
 $x \approx 64 \cdot 3^{1,893y}$
 Dus $x = 64 \cdot 3^{1,893y}$.

41 a $N = 34 \cdot 3^{2t+3}$

$$34 \cdot 3^{2t+3} = N$$

$$3^{2t+3} = \frac{1}{34}N$$

$$2t+3 = {}^3\log\left(\frac{1}{34}N\right)$$

$$2t = -3 + {}^3\log\left(\frac{1}{34}\right) + {}^3\log(N)$$

$$t = -1\frac{1}{2} + \frac{1}{2} \cdot {}^3\log\left(\frac{1}{34}\right) + \frac{1}{2} \cdot {}^3\log(N)$$

$$t \approx -3,10 + 0,5 \cdot {}^3\log(N)$$

$$\text{Dus } t = -3,10 + 0,5 \cdot {}^3\log(N).$$

b $y = 13 \cdot 2^{3x-1}$

$$13 \cdot 2^{3x-1} = y$$

$$2^{3x-1} = \frac{1}{13}y$$

$$3x-1 = {}^2\log\left(\frac{1}{13}y\right)$$

$$3x = 1 + {}^2\log\left(\frac{1}{13}y\right)$$

$$3x = {}^2\log(2) + {}^2\log\left(\frac{1}{13}y\right)$$

$$3x = {}^2\log\left(\frac{2}{13}y\right)$$

$$x = \frac{1}{3} \cdot {}^2\log\left(\frac{2}{13}y\right)$$

$$x \approx 0,33 \cdot {}^2\log(0,15y)$$

$$\text{Dus } x = 0,33 \cdot {}^2\log(0,15y).$$

c $y = 3 \cdot \log(x) - 5$

$$3 \cdot \log(x) = y + 5$$

$$\log(x) = \frac{1}{3}y + \frac{5}{3}$$

$$x = 10^{\frac{1}{3}y + \frac{5}{3}} = 10^{\frac{1}{3}y} \cdot 10^{\frac{5}{3}} \approx 46,42 \cdot 10^{0,33y}$$

$$\text{Dus } x = 46,42 \cdot 10^{0,33y}.$$

d $K = 1 - \frac{1}{2} \cdot {}^3\log(2v)$

$$\frac{1}{2} \cdot {}^3\log(2v) = 1 - K$$

$${}^3\log(2v) = 2 - 2K$$

$$2v = 3^{2-2K}$$

$$v = \frac{1}{2} \cdot 3^2 \cdot 3^{-2K}$$

$$v = \frac{1}{2} \cdot 9 \cdot (3^{-2})^K$$

$$v = 4\frac{1}{2} \cdot \left(\frac{1}{9}\right)^K$$

$$\text{Dus } v = 4\frac{1}{2} \cdot \left(\frac{1}{9}\right)^K.$$

42 a ${}^2\log(4t-1) = 5N+3$

$$4t-1 = 2^{5N+3}$$

$$4t = 1 + (2^5)^N \cdot 2^3$$

$$4t = 1 + 8 \cdot 32^N$$

$$4t = 1 + 8 \cdot (10^{\log(32)})^N$$

$$4t \approx 1 + 8 \cdot (10^{1,51})^N$$

$$t \approx \frac{1}{4} + 2 \cdot 10^{1,51N}$$

$$\text{Dus } t = \frac{1}{4} + 2 \cdot 10^{1,51N}.$$

b $M = 14 \cdot 1,3^{4N-3}$

$$14 \cdot 1,3^{4N-3} = M$$

$$1,3^{4N-3} = \frac{1}{14}M$$

$$4N-3 = {}^{1,3}\log\left(\frac{1}{14}M\right)$$

$$4N = 3 + {}^{1,3}\log\left(\frac{1}{14}\right) + {}^{1,3}\log(M)$$

$$4N = 3 + {}^{1,3}\log\left(\frac{1}{14}\right) + \frac{\log(M)}{\log(1,3)}$$

$$N = \frac{3 + {}^{1,3}\log\left(\frac{1}{14}\right)}{4} + \frac{\log(M)}{4 \cdot \log(1,3)}$$

$$N \approx -1,76 + 2,19 \cdot \log(M)$$

$$\text{Dus } N = -1,76 + 2,19 \cdot \log(M).$$

c $w = 125 \cdot \log(3T+15) - 8$

$$125 \cdot \log(3T+15) = w + 8$$

$$\log(3T+15) = 0,008w + 0,064$$

$$3T+15 = 10^{0,008w + 0,064}$$

$$3T = -15 + (10^{0,008})^w \cdot 10^{0,064}$$

$$T \approx -5 + 0,386 \cdot 1,019^w$$

$$\text{Dus } T = -5 + 0,386 \cdot 1,019^w.$$

d $p = 10 \cdot 5^{0,1q-3}$

$$10 \cdot 5^{0,1q-3} = p$$

$$5^{0,1q-3} = 0,1p$$

$$0,1q-3 = {}^5\log(0,1p)$$

$$0,1q = 3 + {}^5\log(0,1p)$$

$$0,1q = {}^5\log(125) + {}^5\log(0,1p)$$

$$0,1q = {}^5\log(12,5p)$$

$$0,1q = \frac{\log(12,5p)}{\log(5)}$$

$$q = \frac{10 \cdot \log(12,5p)}{\log(5)}$$

$$q \approx 14,31 \cdot \log(12,5p)$$

$$\text{Dus } q = 14,31 \cdot \log(12,5p).$$

43 a $f(x) = g_a(x)$ geeft ${}^2\log(x) = {}^2\log(x\sqrt{x}) + a$

$${}^2\log(x) = {}^2\log(x^{1\frac{1}{2}}) + a$$

$${}^2\log(x) = 1\frac{1}{2} \cdot {}^2\log(x) + a$$

$$\frac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -2a$$

$$x = 2^{-2a} = (2^{-2})^a = \left(\frac{1}{4}\right)^a$$

$$\left. \begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -2a \end{array} \right\} y = -2a$$

$$\text{Dus } S_a\left(\left(\frac{1}{4}\right)^a, -2a\right).$$

- b** Uit $A_a B_a = 4$ en $y_{A_a} = 0$ volgt $y_{B_a} = 4 \vee y_{B_a} = -4$.

$$g_a(x) = 0 \text{ geeft } {}^2\log(x\sqrt{x}) + a = 0$$

$${}^2\log(x^{1\frac{1}{2}}) = -a$$

$$1\frac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -\frac{2}{3}a$$

$$\left. \begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -\frac{2}{3}a \end{array} \right\} y_{B_a} = -\frac{2}{3}a$$

Dus $-\frac{2}{3}a = 4 \vee -\frac{2}{3}a = -4$ en dit geeft $a = -6 \vee a = 6$.

- c** De grafiek van g_a^{inv} snijdt de y -as in $(0, 64)$, dus de grafiek van g_a snijdt de x -as in $(64, 0)$.

$$g_a(64) = 0 \text{ geeft } {}^2\log(64\sqrt{64}) + a = 0$$

$$a = -{}^2\log(64 \cdot 8)$$

$$a = -{}^2\log(512)$$

$$a = -{}^2\log(2^9)$$

$$a = -9$$

$$\text{Dus } g_{-9}(x) = {}^2\log(x\sqrt{x}) - 9.$$

Voor g_{-9} geldt $y = {}^2\log(x\sqrt{x}) - 9$, dus voor g_{-9}^{inv} geldt $x = {}^2\log(y\sqrt{y}) - 9$.

$$x = {}^2\log(y\sqrt{y}) - 9 \text{ geeft } {}^2\log(y\sqrt{y}) = x + 9$$

$${}^2\log(y^{1\frac{1}{2}}) = x + 9$$

$$1\frac{1}{2} \cdot {}^2\log(y) = x + 9$$

$${}^2\log(y) = \frac{2}{3}x + 6$$

$$y = 2^{\frac{2}{3}x + 6} = 2^{\frac{2}{3}x} \cdot 2^6 = 64 \cdot 2^{\frac{2}{3}x}$$

$$\text{Dus } g_{-9}^{\text{inv}}(x) = 64 \cdot 2^{\frac{2}{3}x}.$$

44 ${}^2\log(N) = -1,9 + 0,3 \cdot {}^3\log(t)$

$$0,3 \cdot {}^3\log(t) = 1,9 + {}^2\log(N)$$

$${}^3\log(t) = \frac{1,9}{0,3} + \frac{1}{0,3} \cdot {}^2\log(N)$$

$$t = 3^{\frac{1,9}{0,3} + \frac{1}{0,3} \cdot {}^2\log(N)}$$

$$t = 3^{\frac{1,9}{0,3}} \cdot 3^{\frac{1}{0,3} \cdot \frac{{}^3\log(N)}{{}^3\log(2)}}$$

$$t = 3^{\frac{1,9}{0,3}} \cdot (3^{{}^3\log(N)})^{\frac{1}{0,3 \cdot {}^3\log(2)}}$$

$$t = 3^{\frac{1,9}{0,3}} \cdot N^{\frac{1}{0,3 \cdot {}^3\log(2)}}$$

$$t \approx 1051N^{5,28}$$

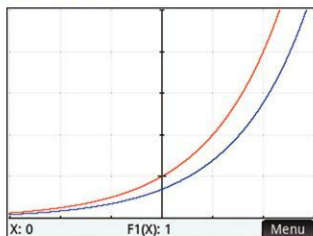
$$\text{Dus } t = 1051N^{5,28}.$$

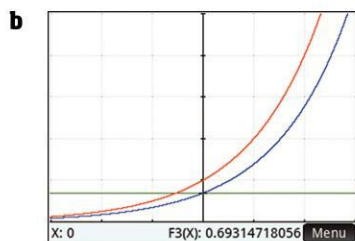
9.3 Het grondtal e

Bladzijde 28

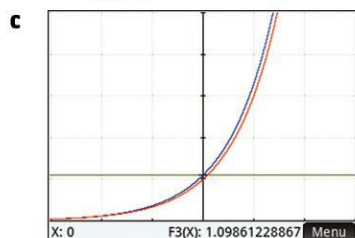
45

a





$$c \approx 0,6931$$



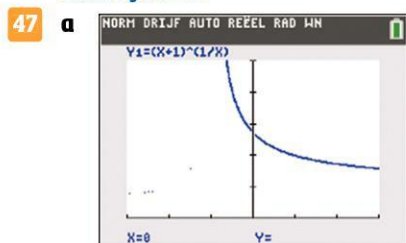
Dus ook hier is $\frac{y_2}{y_1}$ constant en wel ongeveer 1,0986.

46 a
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h \rightarrow 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim_{h \rightarrow 0} \frac{2^x \cdot (2^h - 1)}{h} = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^x$$

b
$$f'(0) = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^0 = \lim_{h \rightarrow 0} \frac{2^h - 1}{h}$$

c
$$f'(x) = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^x = f'(0) \cdot 2^x$$

Bladzijde 29



b Omdat voor $x = 0$ de exponent $\frac{1}{x}$ niet bestaat.

c

x	y_1
0,01	2,7048
0,001	2,7169
0,0001	2,7181
0,00001	2,7183

d $g \approx 2,718$

Bladzijde 30

48 a Dit mag omdat $e^x \neq 0$ voor elke x , dus $e^x = 0$ geeft geen oplossing.

b $e^x = 1$ geeft $x = 0$ en $e^x = -3$ heeft geen oplossing.

49 **a** $2e^2 - e^2 = e^2$
b $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$
c $5e^2 \cdot 3e^3 = 15e^5$
d $\frac{12e^6}{4e^2} = 3e^4$
e $e^{5x} \cdot e^x = e^{6x}$
f $e^x \cdot e^2 = e^{x+2}$
g $5e^x - 3e^x = 2e^x$
h $e^x(e^2 + 1) = e^{x+2} + e^x$
i $e^x(e^x + 1) = e^{2x} + e^x$
j $(e^x + 1)^2 = (e^x)^2 + 2 \cdot e^x \cdot 1 + 1 = e^{2x} + 2e^x + 1$
k $(e^{3x} + 3)^2 = (e^{3x})^2 + 2 \cdot e^{3x} \cdot 3 + 9 = e^{6x} + 6e^{3x} + 9$
l $\frac{6e^{2x} - e^x}{e^x} = 6e^x - 1$

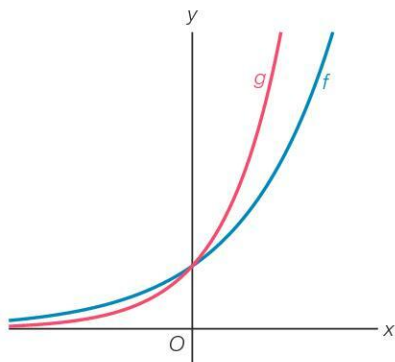
50 **a** $(2 + 3e^{\frac{1}{2}x})^2 = 4 + 2 \cdot 2 \cdot 3e^{\frac{1}{2}x} + (3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$
b $(e^x + e^{-x})^2 = (e^x)^2 + 2 \cdot e^x \cdot e^{-x} + (e^{-x})^2 = e^{2x} + 2 + e^{-2x}$
c $\frac{e^{2x} - 4}{e^x + 2} = \frac{(e^x + 2)(e^x - 2)}{e^x + 2} = e^x - 2$

51 **a** $(2x + 4)e^x = 0$
 $2x + 4 = 0$
 $2x = -4$
 $x = -2$
b $x^2 e^x = 3x e^x$
 $x^2 = 3x$
 $x = 0 \vee x = 3$
c $x^2 e^x = e^x$
 $x^2 = 1$
 $x = 1 \vee x = -1$
d $e^{3x} - e^x = 0$
 $e^{3x} = e^x$
 $3x = x$
 $x = 0$
e $e^{4x} - 1 = 0$
 $e^{4x} = 1$
 $4x = 0$
 $x = 0$
f $e^x \cdot e^x = e^6$
 $e^{2x} = e^6$
 $2x = 6$
 $x = 3$

52 **a** $e^x + e^x = 2e^6$
 $2e^x = 2e^6$
 $e^x = e^6$
 $x = 6$
b $\frac{e^{5x}}{e^x} = e$
 $e^{4x} = e^1$
 $4x = 1$
 $x = \frac{1}{4}$
c $2x e^x + e^x = 0$
 $e^x(2x + 1) = 0$
 $2x + 1 = 0$
 $2x = -1$
 $x = -\frac{1}{2}$
d $e^{x+2} - \sqrt{e} = 0$
 $e^{x+2} = \sqrt{e}$
 $e^{x+2} = e^{\frac{1}{2}}$
 $x + 2 = \frac{1}{2}$
 $x = -1\frac{1}{2}$
e $e^{2x} + e^x = 2$
 $(e^x)^2 + e^x = 2$
 Stel $e^x = u$.
 $u^2 + u = 2$
 $u^2 + u - 2 = 0$
 $(u - 1)(u + 2) = 0$
 $u = 1 \vee u = -2$
 $e^x = 1 \vee e^x = -2$
 $x = 0$
f $e^{6x} + 1 = 2e^{3x}$
 $(e^{3x})^2 + 1 = 2e^{3x}$
 Stel $e^{3x} = u$.
 $u^2 + 1 = 2u$
 $u^2 - 2u + 1 = 0$
 $(u - 1)^2 = 0$
 $u = 1$
 $e^{3x} = 1$
 $3x = 0$
 $x = 0$

53 $e^{2x} + e^3 = (e + 1)e^{x+1}$
 $e^{2x} + e^3 = (e + 1)e^x \cdot e$
 $e^{2x} + e^3 = (e^2 + e)e^x$
 $(e^x)^2 - (e^2 + e)e^x + e^3 = 0$
 Stel $e^x = u$.
 $u^2 - (e^2 + e)u + e^3 = 0$
 $(u - e^2)(u - e) = 0$
 $u = e^2 \vee u = e$
 $e^x = e^2 \vee e^x = e$
 $x = 2 \vee x = 1$

54 a



- b $2 < e < 3$, dus voor $x > 0$ is $2^x < e^x < 3^x$ en voor $x < 0$ is $3^x < e^x < 2^x$.
 Voor $x = 0$ is $2^x = e^x = 3^x = 1$.
 Dus de grafiek van $h(x) = e^x$ ligt voor $x \neq 0$ tussen de grafieken van $f(x) = 2^x$ en $g(x) = 3^x$.

Bladzijde 32

- 55 De productregel is gebruikt in de onderdelen b en c.
 De quotiëntregel is gebruikt in het onderdeel d.
 De kettingregel is gebruikt in de onderdelen c en d.

56 a $f(x) = e^x + 2$ geeft $f'(x) = e^x$

b $f(x) = 2e^x + \frac{1}{x} = 2e^x + x^{-1}$ geeft $f'(x) = 2e^x - x^{-2} = 2e^x - \frac{1}{x^2}$

c $f(x) = xe^x + 4$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (x + 1)e^x$

d $f(x) = \frac{x}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \frac{(1 - x)e^x}{(e^x)^2} = \frac{1 - x}{e^x}$

e $f(x) = \frac{2e^x}{x - 1}$ geeft $f'(x) = \frac{(x - 1) \cdot 2e^x - 2e^x \cdot 1}{(x - 1)^2} = \frac{2xe^x - 2e^x - 2e^x}{(x - 1)^2} = \frac{(2x - 4)e^x}{(x - 1)^2}$

f $f(x) = (2x - 4)e^x$ geeft $f'(x) = 2 \cdot e^x + (2x - 4) \cdot e^x = (2x - 2)e^x$

Bladzijde 33

57 a $f(x) = e^{x^2+x}$ geeft $f'(x) = e^{x^2+x} \cdot (2x + 1) = (2x + 1)e^{x^2+x}$

b $f(x) = x^2 + 2e^{3x}$ geeft $f'(x) = 2x + 2 \cdot e^{3x} \cdot 3 = 2x + 6e^{3x}$

c $f(x) = xe^{x^2}$ geeft $f'(x) = 1 \cdot e^{x^2} + x \cdot e^{x^2} \cdot 2x = (2x^2 + 1)e^{x^2}$

d $f(x) = \frac{2e^{-x-1}}{x^2}$ geeft

$$f'(x) = \frac{x^2 \cdot 2e^{-x-1} \cdot -1 - 2e^{-x-1} \cdot 2x}{x^4} = \frac{-2x^2e^{-x-1} - 4xe^{-x-1}}{x^4} = \frac{-2x(x+2)e^{-x-1}}{x^4} = \frac{-2(x+2)e^{-x-1}}{x^3}$$

e $f(x) = 3xe^{2x-1}$ geeft $f'(x) = 3 \cdot e^{2x-1} + 3x \cdot e^{2x-1} \cdot 2 = (6x + 3)e^{2x-1}$

f $f(x) = \frac{e^{2x}}{e^{2x} + 1}$ geeft $f'(x) = \frac{(e^{2x} + 1) \cdot e^{2x} \cdot 2 - e^{2x} \cdot e^{2x} \cdot 2}{(e^{2x} + 1)^2} = \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x} + 1)^2} = \frac{2e^{2x}}{(e^{2x} + 1)^2}$

58 a $-\frac{1}{e^2} \approx -0,135$ **c** $1\frac{1}{3}e^3 \approx 26,781$

b $\frac{3e}{(e+2)^2} \approx 0,366$ **d** $\frac{e^2}{2e-3} \approx 3,033$

59 a $f(x) = xe^x + 2$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$

b $f'(x) = 0$ geeft $(1+x)e^x = 0$

$$1+x=0$$

$$x=-1$$

min. is $f(-1) = -e^{-1} + 2 = 2 - \frac{1}{e}$

c $f'(x) = (1+x)e^x$ geeft $f''(x) = 1 \cdot e^x + (1+x) \cdot e^x = (2+x)e^x$

d $f''(x) = 0$ geeft $(2+x)e^x = 0$

$$2+x=0$$

$$x=-2$$

$$f(-2) = -2e^{-2} + 2 = 2 - \frac{2}{e^2}$$

Dus de coördinaten van het buigpunt zijn $\left(-2, 2 - \frac{2}{e^2}\right)$.

e Stel $k: y = ax + b$ met $a = f'(-2) = (1-2)e^{-2} = -\frac{1}{e^2}$.

$$\left. \begin{array}{l} y = -\frac{1}{e^2}x + b \\ \text{door } \left(-2, 2 - \frac{2}{e^2}\right) \end{array} \right\} \begin{array}{l} -\frac{1}{e^2} \cdot -2 + b = 2 - \frac{2}{e^2} \\ \frac{2}{e^2} + b = 2 - \frac{2}{e^2} \\ b = 2 - \frac{4}{e^2} \end{array}$$

Dus $k: y = -\frac{1}{e^2}x + 2 - \frac{4}{e^2}$.

60 a $f(x) = -xe^x$ geeft $f'(x) = -1 \cdot e^x + -x \cdot e^x = (-x-1)e^x$

$f'(x) = 0$ geeft $(-x-1)e^x = 0$

$$-x-1=0$$

$$-x=1$$

$$x=-1$$

max. is $f(-1) = e^{-1} = \frac{1}{e}$

b $f'(x) = (-x-1)e^x$ geeft $f''(x) = -1 \cdot e^x + (-x-1) \cdot e^x = (-x-2)e^x$

$f''(x) = 0$ geeft $(-x-2)e^x = 0$

$$-x-2=0$$

$$-x=2$$

$$x=-2$$

Stel $k: y = ax + b$ met $a = f'(-2) = (2-1)e^{-2} = \frac{1}{e^2}$.

$$\left. \begin{array}{l} y = \frac{1}{e^2}x + b \\ f(-2) = 2e^{-2} = \frac{2}{e^2} \end{array} \right\} \begin{array}{l} \frac{1}{e^2} \cdot -2 + b = \frac{2}{e^2} \\ \frac{-2}{e^2} + b = \frac{2}{e^2} \\ b = \frac{4}{e^2} \end{array}$$

Dus $k: y = \frac{1}{e^2}x + \frac{4}{e^2}$.

61 a $f(x) = 0$ geeft $(x^2 - 3)e^x = 0$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \sqrt{3} \vee x = -\sqrt{3}$$

De nulpunten zijn $\sqrt{3}$ en $-\sqrt{3}$.

b $f(x) = (x^2 - 3)e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2 - 3) \cdot e^x = (x^2 + 2x - 3)e^x$

$$f'(x) = 0 \text{ geeft } (x^2 + 2x - 3)e^x = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x - 1)(x + 3) = 0$$

$$x = 1 \vee x = -3$$

max. is $f(-3) = (9 - 3)e^{-3} = \frac{6}{e^3}$ en min. is $f(1) = (1 - 3)e^1 = -2e$.

c $f'(x) = (x^2 + 2x - 3)e^x$ geeft $f''(x) = (2x + 2) \cdot e^x + (x^2 + 2x - 3) \cdot e^x = (x^2 + 4x - 1)e^x$

$$f''(x) = 0 \text{ geeft } (x^2 + 4x - 1)e^x = 0$$

$$x^2 + 4x - 1 = 0$$

$$D = 4^2 - 4 \cdot 1 \cdot -1 = 20, \text{ dus } \sqrt{D} = \sqrt{20} = 2\sqrt{5}$$

$$x = \frac{-4 + 2\sqrt{5}}{2} \vee x = \frac{-4 - 2\sqrt{5}}{2}$$

$$x = -2 + \sqrt{5} \vee x = -2 - \sqrt{5}$$

De x-coördinaten van de buigpunten zijn $-2 + \sqrt{5}$ en $-2 - \sqrt{5}$.

Bladzijde 34

62 a $f(x) = \frac{1}{2}e^{2x}$ geeft $f'(x) = \frac{1}{2} \cdot e^{2x} \cdot 2 = e^{2x}$

Stel $k: y = ax + b$ met $a = f'(-1) = e^{-2} = \frac{1}{e^2}$.

$$y = \frac{1}{e^2}x + b$$

$$f(-1) = \frac{1}{2e^2}, \text{ dus } A\left(-1, \frac{1}{2e^2}\right) \left\{ \begin{array}{l} \frac{1}{e^2} \cdot -1 + b = \frac{1}{2e^2} \\ -\frac{2}{2e^2} + b = \frac{1}{2e^2} \end{array} \right.$$

$$b = \frac{3}{2e^2}$$

Dus $k: y = \frac{1}{e^2}x + \frac{3}{2e^2}$.

$$g(x) = \frac{1}{e^{x+3}} = e^{-x-3} \text{ geeft } g'(x) = e^{-x-3} \cdot -1 = -e^{-x-3}$$

Stel $l: y = ax + b$ met $a = g'(-1) = -e^{1-3} = -e^{-2} = -\frac{1}{e^2}$.

$$y = -\frac{1}{e^2}x + b$$

$$g(-1) = e^{1-3} = \frac{1}{e^2}, \text{ dus } B\left(-1, \frac{1}{e^2}\right) \left\{ \begin{array}{l} -\frac{1}{e^2} \cdot -1 + b = \frac{1}{e^2} \\ \frac{1}{e^2} + b = \frac{1}{e^2} \end{array} \right.$$

$$b = 0$$

Dus $l: y = -\frac{1}{e^2}x$.

k en l snijden geeft $\frac{1}{e^2}x + \frac{3}{2e^2} = -\frac{1}{e^2}x$

$$\frac{2}{e^2}x = -\frac{3}{2e^2}$$

$$x = -\frac{3}{2e^2} \cdot \frac{e^2}{2}$$

$$x = -\frac{3}{4}$$

b $h(x) = f(x) + g(x) = \frac{1}{2}e^{2x} + \frac{1}{e^{x+3}}$ geeft $h'(x) = e^{2x} - e^{-x-3}$

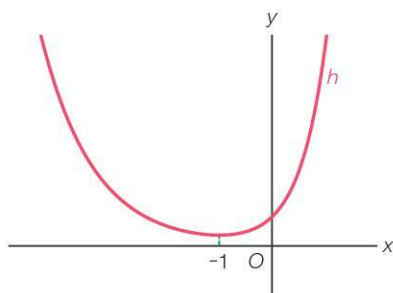
$h'(x) = 0$ geeft $e^{2x} - e^{-x-3} = 0$

$$e^{2x} = e^{-x-3}$$

$$2x = -x - 3$$

$$3x = -3$$

$$x = -1$$



min. is $h(-1) = \frac{1}{2}e^{-2} + \frac{1}{e^2} = \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}$

Het bereik is dus $B_h = \left[\frac{3}{2e^2}, \rightarrow \right)$.

63 a $h(4) = 5$ geeft $2(e^{\frac{1}{4} \cdot 4} + e^{\frac{1}{4} \cdot 4}) + c = 5$

$$2e^1 + 2e^{-1} + c = 5$$

$$c = 5 - 2e - \frac{2}{e}$$

b $h(x) = 5(e^{0,1x} + e^{-0,1x}) - 5,81$ geeft $h'(x) = 5(e^{0,1x} \cdot 0,1 + e^{-0,1x} \cdot -0,1) = 0,5(e^{0,1x} - e^{-0,1x})$

De helling in B is $h'(4) = 0,5(e^{0,1 \cdot 4} - e^{-0,1 \cdot 4}) = 0,5(e^{0,4} - e^{-0,4}) \approx 0,41$.

c $h_k(0) = 0$ geeft $\frac{1}{2k}(e^0 + e^0 - e^{4k} - e^{-4k}) + 5 = 0$ oftewel $\frac{1}{2k}(2 - e^{4k} - e^{-4k}) + 5 = 0$.

Voer in $y_1 = \frac{1}{2x}(2 - e^{4x} - e^{-4x}) + 5$.

De optie nulpunt geeft $x = 0,469...$

Dus $k \approx 0,47$.

Bladzijde 35

64 a $f(x) = 2xe^{-x^2}$ geeft $f'(x) = 2 \cdot e^{-x^2} + 2x \cdot e^{-x^2} \cdot -2x = (2 - 4x^2)e^{-x^2}$

$f'(x) = 0$ geeft $(2 - 4x^2)e^{-x^2} = 0$

$$2 - 4x^2 = 0$$

$$4x^2 = 2$$

$$x^2 = \frac{1}{2}$$

$$x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \vee x = -\frac{1}{2}\sqrt{2}$$

$$y_A = 2 \cdot -\frac{1}{2}\sqrt{2} \cdot e^{-\frac{1}{2}} = -\frac{\sqrt{2}}{e^{\frac{1}{2}}} = -\frac{\sqrt{2}}{\sqrt{e}}$$

$$y_B = 2 \cdot \frac{1}{2}\sqrt{2} \cdot e^{\frac{1}{2}} = \sqrt{\frac{2}{e}}$$

$$y_B - y_A = \sqrt{\frac{2}{e}} - -\sqrt{\frac{2}{e}} = 2\sqrt{\frac{2}{e}}$$

Dus $a = \frac{2}{e}$.

b $f'(x) = (2 - 4x^2)e^{-x^2}$ geeft
 $f''(x) = -8x \cdot e^{-x^2} + (2 - 4x^2) \cdot e^{-x^2} \cdot -2x = -8x \cdot e^{-x^2} + (8x^3 - 4x) \cdot e^{-x^2} = (8x^3 - 12x)e^{-x^2}$
 $f''(x) = 0$ geeft $(8x^3 - 12x)e^{-x^2} = 0$
 $8x^3 - 12x = 0$
 $4x(2x^2 - 3) = 0$
 $4x = 0 \vee 2x^2 = 3$
 $x = 0 \vee x^2 = 1\frac{1}{2}$
 $x = 0 \vee x = \sqrt{1\frac{1}{2}} = \frac{1}{2}\sqrt{6} \vee x = -\sqrt{1\frac{1}{2}} = -\frac{1}{2}\sqrt{6}$

$$f(0) = 0$$

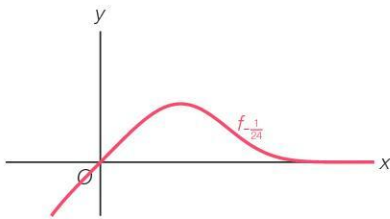
$$f\left(\frac{1}{2}\sqrt{6}\right) = \sqrt{6} \cdot e^{-1\frac{1}{2}} = \frac{\sqrt{6}}{e\sqrt{e}}$$

$$f\left(-\frac{1}{2}\sqrt{6}\right) = -\sqrt{6} \cdot e^{-1\frac{1}{2}} = -\frac{\sqrt{6}}{e\sqrt{e}}$$

De coördinaten van de buigpunten van de grafiek van f zijn $(0, 0)$,

$$\left(\frac{1}{2}\sqrt{6}, \frac{\sqrt{6}}{e\sqrt{e}}\right) \text{ en } \left(-\frac{1}{2}\sqrt{6}, -\frac{\sqrt{6}}{e\sqrt{e}}\right).$$

65 $f_a(x) = xe^{ax^3}$ geeft $f'_a(x) = 1 \cdot e^{ax^3} + x \cdot e^{ax^3} \cdot 3ax^2 = (1 + 3ax^3)e^{ax^3}$
 $f'_a(2) = 0$ geeft $(1 + 3a \cdot 2^3)e^{a \cdot 2^3} = 0$
 $(1 + 3a \cdot 8)e^{8a} = 0$
 $(1 + 24a)e^{8a} = 0$
 $1 + 24a = 0$
 $24a = -1$
 $a = -\frac{1}{24}$
 $a = -\frac{1}{24}$ geeft $f_{-\frac{1}{24}}(x) = xe^{-\frac{1}{24}x^3}$



De extreme waarde voor $x = 2$ is een maximum.

9.4 De natuurlijke logaritme

Bladzijde 37

66 **a** $f(x) = 2^x$ geeft $f^{\text{inv}}(x) = {}^2\log(x)$
b $g(x) = e^x$ geeft $g^{\text{inv}}(x) = {}^e\log(x)$

Bladzijde 38

67 **a** $\ln(e) = 1$
b $\ln(e\sqrt{e}) = \ln(e \cdot e^{\frac{1}{2}}) = \ln(e^{1\frac{1}{2}}) = 1\frac{1}{2}$
c $\ln\left(\frac{1}{e}\right) = \ln(e^{-1}) = -1$
d $\ln(1) = 0$
e $3\ln(e \cdot \sqrt[3]{e}) = 3\ln(e \cdot e^{\frac{1}{3}}) = 3\ln(e^{1\frac{1}{3}}) = 3 \cdot 1\frac{1}{3} = 4$
f $\ln^2(e^3) = 3^2 = 9$
g $\ln^3(e^2) = 2^3 = 8$
h $e^{\ln(7)} + e^{2\ln(7)} = 7 + e^{\ln(7^2)} = 7 + 7^2 = 56$
i $e^{\frac{1}{2}\ln(5)} = e^{\ln(5^{\frac{1}{2}})} = 5^{\frac{1}{2}} = \sqrt{5}$
j $e^{\ln(10)} \cdot e^{\ln(3)} = 10 \cdot 3 = 30$

68 a $e^{3x} = 12$
 $3x = \ln(12)$
 $x = \frac{1}{3} \ln(12) \approx 0,828$

b $5e^{2x} = 60$
 $e^{2x} = 12$
 $2x = \ln(12)$
 $x = \frac{1}{2} \ln(12) \approx 1,242$

c $6 + e^{0,5x+2} = 10$
 $e^{0,5x+2} = 4$
 $0,5x + 2 = \ln(4)$
 $x + 4 = 2 \ln(4)$
 $x = 2 \ln(4) - 4 \approx -1,227$

d $\frac{3}{5e^{2x-1}} = 10$
 $50e^{2x-1} = 3$
 $e^{2x-1} = \frac{3}{50}$
 $2x - 1 = \ln\left(\frac{3}{50}\right)$
 $2x = 1 + \ln\left(\frac{3}{50}\right)$
 $x = \frac{1}{2} + \frac{1}{2} \ln\left(\frac{3}{50}\right) \approx -0,907$

69 a $2 \ln(3) + \ln(4) = \ln(3^2) + \ln(4) = \ln(9) + \ln(4) = \ln(9 \cdot 4) = \ln(36)$

b $\ln(20) - 3 \ln(2) = \ln(20) - \ln(2^3) = \ln(20) - \ln(8) = \ln\left(\frac{20}{8}\right) = \ln\left(2\frac{1}{2}\right)$

c $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$

d $1 + \ln(10) = \ln(e) + \ln(10) = \ln(10e)$

e $\frac{1}{2} + 2 \ln(6) = \ln(e^{\frac{1}{2}}) + \ln(6^2) = \ln(\sqrt{e}) + \ln(36) = \ln(36\sqrt{e})$

f $e + \ln(2) = \ln(e^e) + \ln(2) = \ln(2e^e)$

70 a $\ln(x) = -1$
 $x = e^{-1} = \frac{1}{e}$

b $4 \ln(x) = 2$
 $\ln(x) = \frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e}$

c $\ln(3x) = 3$
 $3x = e^3$
 $x = \frac{1}{3}e^3$

d $\ln(-x + 2) = -2$
 $-x + 2 = e^{-2}$
 $-x = -2 + \frac{1}{e^2}$
 $x = 2 - \frac{1}{e^2}$

e $\ln^2(x) = \frac{1}{4}$
 $\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e} \vee x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

f $\ln(x) = 1 + \ln(5)$
 $\ln(x) = \ln(e) + \ln(5)$
 $\ln(x) = \ln(5e)$
 $x = 5e$

Bladzijde 39

71 a $3x \ln(x) = 2 \ln(x)$
 $\ln(x) = 0 \vee 3x = 2$
 $x = 1 \vee x = \frac{2}{3}$

b $\ln^2(x) - \ln(x) = 0$
 $\ln(x)(\ln(x) - 1) = 0$
 $\ln(x) = 0 \vee \ln(x) = 1$
 $x = 1 \vee x = e$

c $x^2 \ln(x + 1) = 4 \ln(x + 1)$
 $\ln(x + 1) = 0 \vee x^2 = 4$
 $x + 1 = 1 \vee x = 2 \vee x = -2$
 $x = 0 \vee x = 2 \vee x = -2$
 vold. niet

d $4e^{1-3x} = 20$
 $e^{1-3x} = 5$
 $1 - 3x = \ln(5)$
 $-3x = -1 + \ln(5)$
 $x = \frac{1}{3} - \frac{1}{3} \ln(5)$

e $\ln^2(x) - 2 \ln(x) - 3 = 0$
 Stel $\ln(x) = u$.
 $u^2 - 2u - 3 = 0$
 $(u + 1)(u - 3) = 0$
 $u = -1 \vee u = 3$
 $\ln(x) = -1 \vee \ln(x) = 3$
 $x = e^{-1} = \frac{1}{e} \vee x = e^3$

f $\ln(x + 3) - \ln(x - 1) = \ln(2)$
 $\ln\left(\frac{x+3}{x-1}\right) = \ln(2)$
 $\frac{x+3}{x-1} = 2$
 $2x - 2 = x + 3$
 $x = 5$

$$\begin{array}{ll} \text{g} & 2 \ln(x) = \ln(2) + \ln(x+4) \\ & \ln(x^2) = \ln(2(x+4)) \\ & x^2 = 2x + 8 \\ & x^2 - 2x - 8 = 0 \\ & (x+2)(x-4) = 0 \\ & x = -2 \vee x = 4 \\ & \text{vold. niet} \end{array} \quad \begin{array}{l} \text{h} \quad e^{x^2} = 100 \\ x^2 = \ln(100) \\ x = \sqrt{\ln(100)} \vee x = -\sqrt{\ln(100)} \end{array}$$

$$\begin{array}{ll} 72 \text{ a} & G = 100 \text{ geeft } F = 16(0,6 + \ln(100)) \approx 83 \\ & \text{De hartslagfrequentie is 83 slagen per minuut.} \\ \text{b} & F = 78 \text{ geeft } 16(0,6 + \ln(G)) = 78 \\ & 0,6 + \ln(G) = 4,875 \\ & \ln(G) = 4,275 \\ & G = e^{4,275} \approx 72 \end{array}$$

Dus het hiermee corresponderende gewicht is 72 kg.

$$\begin{array}{l} \text{c} \quad F = 16(0,6 + \ln(G)) \\ 16(0,6 + \ln(G)) = F \\ 0,6 + \ln(G) = \frac{1}{16}F \\ \ln(G) = \frac{1}{16}F - 0,6 \\ G = e^{\frac{1}{16}F - 0,6} \\ G = e^{\frac{1}{16}F} \cdot e^{-0,6} \\ G = e^{-0,6} \cdot (e^{\frac{1}{16}})^F \\ G \approx 0,549 \cdot 1,064^F \\ \text{Dus } G = 0,549 \cdot 1,064^F. \end{array}$$

$$\begin{array}{ll} 73 \text{ a} & 2^x = (e^{\ln(2)})^x = e^{\ln(2) \cdot x} \\ \text{b} & f(x) = 2^x = e^{\ln(2) \cdot x} \text{ geeft } f'(x) = e^{\ln(2) \cdot x} \cdot \ln(2) = 2^x \cdot \ln(2) \end{array}$$

$$\begin{array}{ll} 74 \text{ a} & e^{\ln(x)} = x \text{ geeft } [e^{\ln(x)}]' = x' \\ & e^{\ln(x)} \cdot [\ln(x)]' = 1 \\ & e^{\ln(x)} \cdot f'(x) = 1 \end{array}$$

$$\begin{array}{l} \text{b} \quad e^{\ln(x)} \cdot f'(x) = 1 \\ x \cdot f'(x) = 1 \\ f'(x) = \frac{1}{x} \end{array}$$

$$\text{c} \quad h(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)} \cdot \ln(x) \text{ geeft } h'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{x} = \frac{1}{x \ln(2)}$$

Bladzijde 40

$$75 \text{ a} \quad f(x) = \ln(ax) \text{ geeft } f'(x) = \frac{1}{ax} \cdot a = \frac{1}{x}$$

$$\begin{array}{l} \text{b} \quad f(x) = \ln(2x) \text{ geeft } f'(x) = \frac{1}{x} \\ g(x) = \ln(x\sqrt{2}) \text{ geeft } g'(x) = \frac{1}{x} \end{array}$$

$$\text{c} \quad f(x) = \ln(x^n) = n \cdot \ln(x) \text{ geeft } f'(x) = n \cdot \frac{1}{x} = \frac{n}{x}$$

$$\begin{array}{l} \text{d} \quad f(x) = \ln(x^2) \text{ geeft } f'(x) = \frac{2}{x} \\ g(x) = \ln\left(\frac{1}{x}\right) \text{ geeft } g'(x) = -\frac{1}{x} \\ h(x) = \ln(\sqrt{x}) \text{ geeft } h'(x) = \frac{1}{2x} \end{array}$$

Bladzijde 41

76 a $f(x) = 3^{4x-2}$ geeft $f'(x) = 3^{4x-2} \cdot \ln(3) \cdot 4 = 4 \cdot 3^{4x-2} \cdot \ln(3)$

b $f(x) = \frac{1 - \ln(x)}{x}$ geeft $f'(x) = \frac{x \cdot -\frac{1}{x} - (1 - \ln(x)) \cdot 1}{x^2} = \frac{-1 - 1 + \ln(x)}{x^2} = \frac{\ln(x) - 2}{x^2}$

c $f(x) = {}^2\log(4x - 1)$ geeft $f'(x) = \frac{1}{(4x - 1)\ln(2)} \cdot 4 = \frac{4}{(4x - 1)\ln(2)}$

d $f(x) = \frac{\ln(3x)}{x}$ geeft $f'(x) = \frac{x \cdot \frac{1}{x} - \ln(3x) \cdot 1}{x^2} = \frac{1 - \ln(3x)}{x^2}$

e $f(x) = x \ln(x^3)$ geeft $f'(x) = 1 \cdot \ln(x^3) + x \cdot \frac{3}{x} = \ln(x^3) + 3$

f $f(x) = \ln(x^2 + x)$ geeft $f'(x) = \frac{1}{x^2 + x} \cdot (2x + 1) = \frac{2x + 1}{x^2 + x}$

77 a $f(x) = \ln(2^x) = x \ln(2)$ geeft $f'(x) = \ln(2)$

b $f(x) = {}^2\log(x^2 + 1)$ geeft $f'(x) = \frac{1}{(x^2 + 1)\ln(2)} \cdot 2x = \frac{2x}{(x^2 + 1)\ln(2)}$

c $f(x) = x \ln^2(x)$ geeft $f'(x) = 1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} = \ln^2(x) + 2 \ln(x)$

d $f(x) = x^2 \cdot {}^3\log(4x)$ geeft $f'(x) = 2x \cdot {}^3\log(4x) + x^2 \cdot \frac{1}{x \ln(3)} = 2x \cdot {}^3\log(4x) + \frac{x}{\ln(3)}$

e $f(x) = (2^x - 1) \cdot 2^x = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = 2^x \cdot \ln(2) \cdot (2 \cdot 2^x - 1)$

f $f(x) = \ln^2(4x)$ geeft $f'(x) = 2 \ln(4x) \cdot \frac{1}{x} = \frac{2 \ln(4x)}{x}$

78 a $x^n = (e^{\ln(x)})^n = e^{n \ln(x)}$

b $f(x) = e^{n \ln(x)}$ geeft $f'(x) = e^{n \ln(x)} \cdot \frac{n}{x}$

c $e^{n \ln(x)} \cdot \frac{n}{x} = x^n \cdot \frac{n}{x} = n \cdot \frac{x^n}{x} = nx^{n-1}$

Er is geen gebruikgemaakt van enige beperking van n , dus de regel geldt ook voor elke niet-gehele n van $\mathbb{R}$.

79 a $x - 2 > 0$ oftewel $x > 2$, dus $D_f = \langle 2, \rightarrow \rangle$.

$7 - x > 0$ oftewel $x < 7$, dus $D_g = \langle \leftarrow, 7 \rangle$.

$f(x) = g(x)$ geeft $\ln(x - 2) = \ln(7 - x)$

$$x - 2 = 7 - x$$

$$2x = 9$$

$$x = 4\frac{1}{2}$$

$f(x) \leq g(x)$ geeft $2 < x \leq 4\frac{1}{2}$

b $s(x) = f(x) + g(x) = \ln(x-2) + \ln(7-x)$ geeft $s'(x) = \frac{1}{x-2} + \frac{1}{7-x} \cdot -1 = \frac{1}{x-2} - \frac{1}{7-x}$

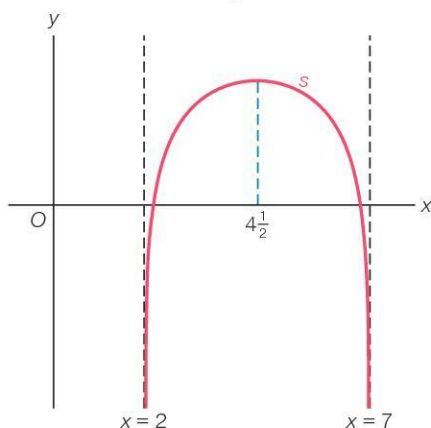
$$s'(x) = 0 \text{ geeft } \frac{1}{x-2} - \frac{1}{7-x} = 0$$

$$\frac{1}{x-2} = \frac{1}{7-x}$$

$$x-2 = 7-x$$

$$2x = 9$$

$$x = 4\frac{1}{2}$$



max. is $s(4\frac{1}{2}) = \ln(2\frac{1}{2}) + \ln(2\frac{1}{2}) = 2\ln(2\frac{1}{2})$

80 a $f(x) = \frac{x}{\ln(x)}$ geeft $f'(x) = \frac{\ln(x) \cdot 1 - x \cdot \frac{1}{x}}{\ln^2(x)} = \frac{\ln(x) - 1}{\ln^2(x)}$

Stel $k: y = ax + b$ met $a = f'\left(\frac{1}{e}\right) = \frac{\ln\left(\frac{1}{e}\right) - 1}{\ln^2\left(\frac{1}{e}\right)} = \frac{-1 - 1}{(-1)^2} = -2$.

$$\left. \begin{array}{l} y = -2x + b \\ f\left(\frac{1}{e}\right) = \frac{\frac{1}{e}}{\ln\left(\frac{1}{e}\right)} = \frac{\frac{1}{e}}{-1} = -\frac{1}{e}, \text{ dus } A\left(\frac{1}{e}, -\frac{1}{e}\right) \end{array} \right\} \begin{array}{l} -2 \cdot \frac{1}{e} + b = -\frac{1}{e} \\ b = \frac{1}{e} \end{array}$$

Dus $k: y = -2x + \frac{1}{e}$.

b $f'(x) = -6$ geeft $\frac{\ln(x) - 1}{\ln^2(x)} = -6$

$$\ln(x) - 1 = -6\ln^2(x)$$

Stel $\ln(x) = u$.

$$u - 1 = -6u^2$$

$$6u^2 + u - 1 = 0$$

$$D = 1^2 - 4 \cdot 6 \cdot -1 = 25$$

$$u = \frac{-1 \pm 5}{12} = \frac{1}{3} \vee u = \frac{-1 - 5}{12} = -\frac{1}{2}$$

$$\ln(x) = \frac{1}{3} \vee \ln(x) = -\frac{1}{2}$$

$$x = e^{\frac{1}{3}} \vee x = e^{-\frac{1}{2}}$$

$$f(e^{\frac{1}{3}}) = \frac{e^{\frac{1}{3}}}{\ln(e^{\frac{1}{3}})} = \frac{e^{\frac{1}{3}}}{\frac{1}{3}} = 3e^{\frac{1}{3}}, \text{ dus raakpunt } (e^{\frac{1}{3}}, 3e^{\frac{1}{3}}) = (\sqrt[3]{e}, 3 \cdot \sqrt[3]{e}).$$

$$f(e^{-\frac{1}{2}}) = \frac{e^{-\frac{1}{2}}}{\ln(e^{-\frac{1}{2}})} = \frac{e^{-\frac{1}{2}}}{-\frac{1}{2}} = -2e^{-\frac{1}{2}}, \text{ dus raakpunt } (e^{-\frac{1}{2}}, -2e^{-\frac{1}{2}}) = \left(\frac{1}{\sqrt{e}}, -\frac{2}{\sqrt{e}}\right).$$

81 a $f(x) = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = (2^{2x+1} - 2^x) \ln(2) = (2^{2x+1} - 2^x) \ln(2)$
 $f'(x) = 0$ geeft $(2^{2x+1} - 2^x) \ln(2) = 0$
 $2^{2x+1} - 2^x = 0$
 $2^{2x+1} = 2^x$
 $2x + 1 = x$
 $x = -1$

min. is $f(-1) = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, dus $B_f = [-\frac{1}{4}, \rightarrow)$.

b $f'(x) = (2^{2x+1} - 2^x) \ln(2)$ geeft $f''(x) = (2^{2x+1} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2)) \ln(2) = (2^{2x+2} - 2^x) \ln^2(2)$
 $f''(x) = 0$ geeft $(2^{2x+2} - 2^x) \ln^2(2) = 0$
 $2^{2x+2} - 2^x = 0$
 $2^{2x+2} = 2^x$
 $2x + 2 = x$
 $x = -2$
 $f(-2) = 2^{-4} - 2^{-2} = \frac{1}{16} - \frac{1}{4} = -\frac{3}{16}$, dus buigpunt $(-2, -\frac{3}{16})$.

82 a $f(x) = \sqrt{x} \ln(\sqrt{x})$ geeft $f'(x) = \frac{1}{2\sqrt{x}} \cdot \ln(\sqrt{x}) + \sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{\ln(\sqrt{x}) + 1}{2\sqrt{x}}$

$f'(x) = 0$ geeft $\frac{\ln(\sqrt{x}) + 1}{2\sqrt{x}} = 0$

$\ln(\sqrt{x}) + 1 = 0$

$\ln(\sqrt{x}) = -1$

$\sqrt{x} = e^{-1}$

$x = e^{-2} = \frac{1}{e^2}$

$f\left(\frac{1}{e^2}\right) = e^{-1} \cdot \ln(e^{-1}) = \frac{1}{e} \cdot -1 = -\frac{1}{e}$, dus $A\left(\frac{1}{e^2}, -\frac{1}{e}\right)$.

b $f'(x) = \frac{\ln(\sqrt{x}) + 1}{2\sqrt{x}}$ geeft

$f''(x) = \frac{2\sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - (\ln(\sqrt{x}) + 1) \cdot \frac{2}{2\sqrt{x}}}{(2\sqrt{x})^2} = \frac{\frac{1}{\sqrt{x}} - \frac{(\ln(\sqrt{x}) + 1)}{\sqrt{x}}}{4x} = \frac{\frac{\sqrt{x} - (\ln(\sqrt{x}) + 1)}{\sqrt{x}}}{4x} = \frac{1 - (\ln(\sqrt{x}) + 1)}{4x\sqrt{x}} = \frac{-\ln(\sqrt{x})}{4x\sqrt{x}}$

$f''(x) = 0$ geeft $-\ln(\sqrt{x}) = 0$

$\ln(\sqrt{x}) = 0$

$\sqrt{x} = 1$

$x = 1$

Stel $k: y = ax + b$ met $a = f'(1) = \frac{\ln(\sqrt{1}) + 1}{2\sqrt{1}} = \frac{0 + 1}{2} = \frac{1}{2}$.

$y = \frac{1}{2}x + b$
 $f(1) = \sqrt{1} \ln(\sqrt{1}) = 0 \left\{ \begin{array}{l} \frac{1}{2} \cdot 1 + b = 0 \\ b = -\frac{1}{2} \end{array} \right.$

Dus $k: y = \frac{1}{2}x - \frac{1}{2}$.

83 a Op $t = 0$ is $T = 80$, dus $20 + p e^0 = 80$
 $20 + p \cdot 1 = 80$
 $p = 60$

b $T = 45$ geeft $20 + 60 e^{-\frac{1}{6}t} = 45$
 $60 e^{-\frac{1}{6}t} = 25$
 $e^{-\frac{1}{6}t} = \frac{25}{60}$
 $-\frac{1}{6}t = \ln\left(\frac{25}{60}\right)$
 $t = -6 \ln\left(\frac{25}{60}\right) = 5,252...$

Dus na 5 minuten en 0,252... $\cdot 60 \approx 15$ seconden is de temperatuur van de koffie 45 °C.

c $T = 20 + 60e^{-\frac{1}{6}t}$ geeft $\frac{dT}{dt} = 60e^{-\frac{1}{6}t} \cdot -\frac{1}{6} = -10e^{-\frac{1}{6}t}$

$t = 2$ geeft $\frac{dT}{dt} = -10e^{-\frac{1}{3}} \approx -7,2$

Twee minuten nadat de koffie is ingeschonken neemt de temperatuur van de koffie af met een snelheid van $7,2^\circ\text{C}$ per minuut.

d $T = 20 + 60e^{-\frac{1}{6}t}$

$60e^{-\frac{1}{6}t} = T - 20$

$e^{-\frac{1}{6}t} = \frac{1}{60}T - \frac{1}{3}$

$-\frac{1}{6}t = \ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

$t = -6\ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

Dus $a = -6$, $b = \frac{1}{60}$ en $c = -\frac{1}{3}$.

84 a $f_2(x) = \ln\left(\frac{2x}{2x+1}\right)$ geeft

$$f_2'(x) = \frac{1}{\left(\frac{2x}{2x+1}\right)} \cdot \frac{(2x+1) \cdot 2 - 2x \cdot 2}{(2x+1)^2} = \frac{2x+1}{2x} \cdot \frac{4x+2-4x}{(2x+1)^2} = \frac{2(2x+1)}{2x(2x+1)^2} = \frac{1}{x(2x+1)}$$

$rc_k = \frac{1}{3}$, dus $f_2'(x) = \frac{1}{3}$

$$\frac{1}{x(2x+1)} = \frac{1}{3}$$

$$x(2x+1) = 3$$

$$2x^2 + x - 3 = 0$$

$$D = 1^2 - 4 \cdot 2 \cdot -3 = 25$$

$$x = \frac{-1 \pm 5}{4} = 1 \vee x = \frac{-1 - 5}{4} = -1\frac{1}{2}$$

vold. niet

$$f_2(1) = \ln\left(\frac{2}{2+1}\right) = \ln\left(\frac{2}{3}\right), \text{ dus } A(1, \ln(\frac{2}{3})).$$

b Voor f_a geldt $y = \ln\left(\frac{ax}{2x+1}\right)$, dus voor f_a^{inv} geldt $x = \ln\left(\frac{ay}{2y+1}\right)$.

$$x = \ln\left(\frac{ay}{2y+1}\right) \text{ geeft } \ln\left(\frac{ay}{2y+1}\right) = x$$

$$\frac{ay}{2y+1} = e^x$$

$$ay = (2y+1)e^x$$

$$ay = 2ye^x + e^x$$

$$ay - 2ye^x = e^x$$

$$y(a - 2e^x) = e^x$$

$$y = \frac{e^x}{a - 2e^x}$$

Dus $f_a^{\text{inv}}(x) = \frac{e^x}{a - 2e^x}$.

Dus voor $a = 3$ is $g(x) = \frac{e^x}{3 - 2e^x}$ de inverse van f_a .

Bladzijde 43

85 a $f(x) = 1$ geeft $e^{\ln^2(x) + 2\ln(x) - 3} = 1$

$$\ln^2(x) + 2\ln(x) - 3 = 0$$

Stel $\ln(x) = u$.

$$u^2 + 2u - 3 = 0$$

$$(u-1)(u+3) = 0$$

$$u = 1 \vee u = -3$$

$$\ln(x) = 1 \vee \ln(x) = -3$$

$$x = e \vee x = e^{-3} = \frac{1}{e^3}$$

$f(x) \leq 1$ geeft $\frac{1}{e^3} \leq x \leq e$

b $f(x) = e^{\ln^2(x) + 2 \ln(x) - 3}$ geeft

$$f'(x) = e^{\ln^2(x) + 2 \ln(x) - 3} \cdot \left(2 \ln(x) \cdot \frac{1}{x} + \frac{2}{x} \right) = e^{\ln^2(x) + 2 \ln(x) - 3} \cdot \left(\frac{2 \ln(x) + 2}{x} \right)$$

$$f'(x) = 0 \text{ geeft } e^{\ln^2(x) + 2 \ln(x) - 3} \cdot \left(\frac{2 \ln(x) + 2}{x} \right) = 0$$

$$\frac{2 \ln(x) + 2}{x} = 0$$

$$2 \ln(x) + 2 = 0$$

$$\ln(x) = -1$$

$$x = e^{-1}$$

$$\text{min. is } f(e^{-1}) = e^{(-1)^2 + 2 \cdot (-1) - 3} = e^{-4} = \frac{1}{e^4}, \text{ dus } B_f = \left[\frac{1}{e^4}, \rightarrow \right).$$

86 a $f(x) = x^x = (e^{\ln(x)})^x = e^{x \ln(x)}$ geeft $f'(x) = e^{x \ln(x)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x} \right) = (\ln(x) + 1) \cdot x^x$

b $g_a(x) = x^{\frac{a}{x}} = (e^{\ln(x)})^{\frac{a}{x}} = e^{\frac{a}{x} \ln(x)} = e^{ax^{-1} \cdot \ln(x)}$ geeft

$$g_a'(x) = e^{ax^{-1} \cdot \ln(x)} \cdot \left(-ax^{-2} \cdot \ln(x) + ax^{-1} \cdot \frac{1}{x} \right) = x^{\frac{a}{x}} \cdot \left(\frac{-a \ln(x)}{x^2} + \frac{a}{x^2} \right) = \left(\frac{a - a \ln(x)}{x^2} \right) \cdot x^{\frac{a}{x}}$$

Voor raken geldt $f(x) = g_a(x) \wedge f'(x) = g_a'(x)$.

$$f(x) = g_a(x) \text{ geeft } x^x = x^{\frac{a}{x}}$$

$$x = \frac{a}{x}$$

$$a = x^2$$

$$f'(x) = g_a'(x) \text{ met } a = x^2 \text{ geeft } (\ln(x) + 1) \cdot x^x = \left(\frac{x^2 - x^2 \ln(x)}{x^2} \right) \cdot x^{\frac{x^2}{x}}$$

$$(\ln(x) + 1) \cdot x^x = \left(\frac{x^2(1 - \ln(x))}{x^2} \right) \cdot x^x$$

$$\ln(x) + 1 = 1 - \ln(x)$$

$$2 \ln(x) = 0$$

$$x = 1$$

Dus $a = 1^2 = 1$.

Stel $k: y = mx + n$ met $m = f'(1) = (\ln(1) + 1) \cdot 1^1 = 1$.

$$\left. \begin{array}{l} y = x + n \\ f(1) = 1, \text{ dus } A(1, 1) \end{array} \right\} \begin{array}{l} 1 + n = 1 \\ n = 0 \end{array}$$

Dus $k: y = x$.

Diagnostische toets

Bladzijde 46

1 a ${}^2\log(18) - {}^2\log(36) + 3 = {}^2\log\left(\frac{18}{36}\right) + 3 = {}^2\log\left(\frac{1}{2}\right) + 3 = -1 + 3 = 2$

b $2 \cdot {}^3\log(5) - {}^3\log(75) + {}^2\log(8\sqrt{2}) = {}^3\log(5^2) - {}^3\log(75) + {}^2\log(2^3 \cdot 2^{\frac{1}{2}}) = {}^3\log(25) - {}^3\log(75) + {}^2\log(2^{3\frac{1}{2}}) = {}^3\log\left(\frac{25}{75}\right) + 3\frac{1}{2} = {}^3\log\left(\frac{1}{3}\right) + 3\frac{1}{2} = -1 + 3\frac{1}{2} = 2\frac{1}{2}$

2 a $4 + {}^2\log(a) + \log(100) = 4 + {}^2\log(a) + 2 = 6 + {}^2\log(a) = {}^2\log(2^6) + {}^2\log(a) = {}^2\log(64) + {}^2\log(a) = {}^2\log(64a)$

b $2 - {}^3\log(6a) = {}^3\log(3^2) - {}^3\log(6a) = {}^3\log(9) - {}^3\log(6a) = {}^3\log\left(\frac{9}{6a}\right) = {}^3\log\left(\frac{3}{2a}\right)$

3 a $3 \cdot {}^3\log(36) = 3 \cdot {}^3\log(9 \cdot 4) = 3 \cdot ({}^3\log(9) + {}^3\log(4)) = 3 \cdot (2 + {}^3\log(4)) = 6 + 3 \cdot {}^3\log(4) = 6 + {}^3\log(4^3) = 6 + {}^3\log(64)$

b $3 + \log(800) = 3 + \log(100 \cdot 8) = 3 + \log(100) + \log(8) = 3 + 2 + \log(8) = 5 + \log(8)$

- 4 a** $y = {}^5\log(x) \xrightarrow{\text{translatie } (0, 2)} y = {}^5\log(x) + 2$
 Er geldt ${}^5\log(x) + 2 = {}^5\log(x) + {}^5\log(5^2) = {}^5\log(x) + {}^5\log(25) = {}^5\log(25x)$.
 Dus de vermenigvuldiging ten opzichte van de y -as met $\frac{1}{25}$ levert dezelfde beeldgrafiek op.

- b** $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = {}^3\log(3x)$
 Er geldt ${}^3\log(3x) = {}^3\log(3) + {}^3\log(x) = 1 + {}^3\log(x)$.
 Dus de translatie $(0, 1)$ levert dezelfde beeldgrafiek op.

- 5 a** $2 \cdot {}^2\log(x-1) = 1 + {}^2\log(18)$
 ${}^2\log((x-1)^2) = {}^2\log(2) + {}^2\log(18)$
 ${}^2\log((x-1)^2) = {}^2\log(2 \cdot 18)$
 ${}^2\log((x-1)^2) = {}^2\log(36)$
 $(x-1)^2 = 36$
 $x-1 = 6 \vee x-1 = -6$
 $x = 7 \vee x = -5$

vold. niet

- b** ${}^3\log(2x-1) + \frac{1}{3}\log(x+2) = 0$
 ${}^3\log(2x-1) - {}^3\log(x+2) = 0$
 ${}^3\log(2x-1) = {}^3\log(x+2)$
 $2x-1 = x+2$
 $x = 3$

- c** ${}^2\log(2x-1) = {}^4\log(x)$
 ${}^2\log(2x-1) = \frac{{}^2\log(x)}{{}^2\log(4)}$
 ${}^2\log(2x-1) = \frac{{}^2\log(x)}{2}$
 $2 \cdot {}^2\log(2x-1) = {}^2\log(x)$
 ${}^2\log((2x-1)^2) = {}^2\log(x)$
 $(2x-1)^2 = x$
 $4x^2 - 4x + 1 = x$
 $4x^2 - 5x + 1 = 0$
 $D = (-5)^2 - 4 \cdot 4 \cdot 1 = 9$
 $x = \frac{5+3}{8} = 1 \vee x = \frac{5-3}{8} = \frac{1}{4}$

vold. niet

- d** $\log^2(x) = \log(x) + 2$
 Stel $\log(x) = u$.
 $u^2 = u + 2$
 $u^2 - u - 2 = 0$
 $(u+1)(u-2) = 0$
 $u = -1 \vee u = 2$
 $\log(x) = -1 \vee \log(x) = 2$
 $x = \frac{1}{10} \vee x = 100$

- e** ${}^2\log(x) = 3 - {}^2\log(x+2)$
 ${}^2\log(x) = {}^2\log(8) - {}^2\log(x+2)$
 ${}^2\log(x) + {}^2\log(x+2) = {}^2\log(8)$
 ${}^2\log(x(x+2)) = {}^2\log(8)$
 $x(x+2) = 8$
 $x^2 + 2x - 8 = 0$
 $(x-2)(x+4) = 0$
 $x = 2 \vee x = -4$

vold. niet

- f** ${}^2\log^2(x) + 12 = 7 \cdot {}^2\log(x)$
 Stel ${}^2\log(x) = u$.
 $u^2 + 12 = 7u$
 $u^2 - 7u + 12 = 0$
 $(u-3)(u-4) = 0$
 $u = 3 \vee u = 4$
 ${}^2\log(x) = 3 \vee {}^2\log(x) = 4$
 $x = 8 \vee x = 16$

- 6 a** $g_{\text{dag}} = 1,1$
 $g_{\text{week}} = 1,1^7 \approx 1,949$
 Het groeipercentage per week is 94,9%.

- b** $g_{\text{dag}} = 1,1$
 $g_{8 \text{ uur}} = 1,1^{\frac{1}{3}} \approx 1,032$
 Het groeipercentage per 8 uur is 3,2%.

- 7 a** $g_{\text{jaar}} = 0,64$
 $g_{\text{maand}} = 0,64^{\frac{1}{12}} \approx 0,963$
 De afname per maand is 3,7%.

- b** $g_{\text{jaar}} = 0,64$
 $g_{5 \text{ jaar}} = 0,64^5 \approx 0,107$
 De afname per 5 jaar is 89,3%.

- 8 a** $g_{\text{maand}} = 1,002$
 $1,002^T = 2$
 $T = {}^{1,002}\log(2) = 346,920\dots$
 De verdubbelingstijd is $346,920\dots : 12 \approx 29$ jaar.
- b** $g_{\text{week}} = 0,8$
 $0,8^T = \frac{1}{2}$
 $T = {}^{0,8}\log(\frac{1}{2}) = 3,106\dots$
 De halveringstijd is $3,106\dots \cdot 7 \approx 22$ dagen.

- 9 a** $y = 4 \cdot \log(3x + 12) - 1$
 $4 \cdot \log(3x + 12) = y + 1$
 $\log(3x + 12) = 0,25y + 0,25$
 $3x + 12 = 10^{0,25y + 0,25}$
 $3x + 12 = 10^{0,25y} \cdot 10^{0,25}$
 $3x = -12 + (10^{0,25})^y \cdot 10^{0,25}$
 $x \approx -4 + 0,59 \cdot 1,78^y$
 Dus $x = -4 + 0,59 \cdot 1,78^y$.
- b** $p = 34 \cdot 2,7^{5q+2}$
 $34 \cdot 2,7^{5q+2} = p$
 $2,7^{5q+2} = \frac{1}{34}p$
 $5q + 2 = {}^{2,7}\log(\frac{1}{34}p)$
 $5q + 2 = {}^{2,7}\log(\frac{1}{34}) + {}^{2,7}\log(p)$
 $5q = -2 + \frac{\log(\frac{1}{34})}{\log(2,7)} + \frac{\log(p)}{\log(2,7)}$
 $q = -\frac{2}{5} + \frac{\log(\frac{1}{34})}{5\log(2,7)} + \frac{\log(p)}{5\log(2,7)}$
 $q \approx -1,11 + 0,46 \cdot \log(p)$
 Dus $q = -1,11 + 0,46 \cdot \log(p)$.

Bladzijde 47

- 10 a** $\frac{3e^3 - e^3}{e^2} = \frac{2e^3}{e^2} = 2e$
- b** $\frac{e^{3x} - e^x}{e^x} = e^{2x} - 1$
- c** $(e^{3x} - 5)^2 = (e^{3x})^2 - 2 \cdot e^{3x} \cdot 5 + 25 = e^{6x} - 10e^{3x} + 25$
- 11 a** $3xe^x - e^x = 0$
 $e^x(3x - 1) = 0$
 $3x - 1 = 0$
 $3x = 1$
 $x = \frac{1}{3}$
- b** $e^{2x-1} - \sqrt[3]{e^2} = 0$
 $e^{2x-1} = \sqrt[3]{e^2}$
 $e^{2x-1} = e^{\frac{2}{3}}$
 $2x - 1 = \frac{2}{3}$
 $2x = 1\frac{2}{3}$
 $x = \frac{5}{6}$
- c** $e^{2x} + 2e^x = 3$
 $(e^x)^2 + 2e^x - 3 = 0$
 Stel $e^x = u$.
 $u^2 + 2u - 3 = 0$
 $(u - 1)(u + 3) = 0$
 $u = 1 \vee u = -3$
 $e^x = 1 \vee e^x = -3$
 $x = 0$

- 12 a** $f(x) = 2e^x - 3x^2$ geeft $f'(x) = 2e^x - 6x$
- b** $f(x) = \frac{x^2 + 1}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 2x - (x^2 + 1) \cdot e^x}{(e^x)^2} = \frac{-x^2 + 2x - 1}{e^x}$
- c** $f(x) = (x^2 + 1)e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2 + 1) \cdot e^x = (x^2 + 2x + 1)e^x$
- d** $f(x) = \frac{e^x}{x^2 + 1}$ geeft $f'(x) = \frac{(x^2 + 1) \cdot e^x - e^x \cdot 2x}{(x^2 + 1)^2} = \frac{(x^2 - 2x + 1)e^x}{(x^2 + 1)^2}$
- e** $f(x) = x^2 \cdot e^{2x-1}$ geeft $f'(x) = 2x \cdot e^{2x-1} + x^2 \cdot e^{2x-1} \cdot 2 = (2x^2 + 2x)e^{2x-1}$
- f** $f(x) = e^{x^2+9}$ geeft $f'(x) = e^{x^2+9} \cdot 2x = 2xe^{x^2+9}$

- 13 a** $f(x) = (x^2 - x + 1)e^x$ geeft $f'(x) = (2x - 1) \cdot e^x + (x^2 - x + 1) \cdot e^x = (x^2 + x)e^x$
 $f'(x) = 0$ geeft $(x^2 + x)e^x = 0$
 $x^2 + x = 0$
 $x(x + 1) = 0$
 $x = 0 \vee x = -1$

max. is $f(-1) = \frac{3}{e}$ en min. is $f(0) = 1$.

- b** $f'(x) = (x^2 + x)e^x$ geeft $f''(x) = (2x + 1) \cdot e^x + (x^2 + x) \cdot e^x = (x^2 + 3x + 1)e^x$
 $f''(x) = 0$ geeft $(x^2 + 3x + 1)e^x = 0$
 $x^2 + 3x + 1 = 0$
 $D = 3^2 - 4 \cdot 1 \cdot 1 = 5$
 $x = \frac{-3 + \sqrt{5}}{2} \vee x = \frac{-3 - \sqrt{5}}{2}$
 $x = -1\frac{1}{2} + \frac{1}{2}\sqrt{5} \vee x = -1\frac{1}{2} - \frac{1}{2}\sqrt{5}$

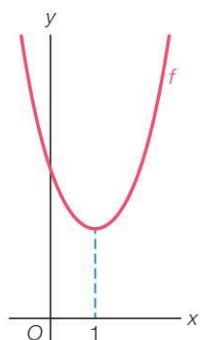
- c** Stel $k: y = ax + b$ met $a = f'(1) = (1 + 1)e = 2e$.
 $\left. \begin{array}{l} k: y = 2ex + b \\ f(1) = e, \text{ dus } A(1, e) \end{array} \right\} \begin{array}{l} 2e + b = e \\ b = -e \end{array}$
Dus $k: y = 2ex - e$.

- 14 a** $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$
- b** $\ln(10) - 4\ln(2) = \ln(10) - \ln(2^4) = \ln(10) - \ln(16) = \ln(\frac{10}{16}) = \ln(\frac{5}{8})$

- 15 a** $2e^{5x} = 16$
 $e^{5x} = 8$
 $5x = \ln(8)$
 $x = \frac{1}{5}\ln(8)$
- b** $\ln^2(5x) = 16$
 $\ln(5x) = 4 \vee \ln(5x) = -4$
 $5x = e^4 \vee 5x = e^{-4}$
 $x = \frac{1}{5}e^4 \vee x = \frac{1}{5e^4}$
- c** $2\ln^2(x) - \ln(x) = 0$
 $\ln(x)(2\ln(x) - 1) = 0$
 $\ln(x) = 0 \vee 2\ln(x) - 1 = 0$
 $x = 1 \vee 2\ln(x) = 1$
 $x = 1 \vee \ln(x) = \frac{1}{2}$
 $x = 1 \vee x = e^{\frac{1}{2}} = \sqrt{e}$
- d** $\ln(9x + 1) - \ln(x + 2) = \ln(4)$
 $\ln(9x + 1) = \ln(x + 2) + \ln(4)$
 $\ln(9x + 1) = \ln(4(x + 2))$
 $9x + 1 = 4(x + 2)$
 $9x + 1 = 4x + 8$
 $5x = 7$
 $x = 1\frac{2}{5}$

- 16 a** $f(x) = 2^{3x-4}$ geeft $f'(x) = 2^{3x-4} \cdot \ln(2) \cdot 3 = 3 \cdot 2^{3x-4} \cdot \ln(2)$
- b** $f(x) = x \cdot 3^x$ geeft $f'(x) = 1 \cdot 3^x + x \cdot 3^x \cdot \ln(3) = (1 + x\ln(3)) \cdot 3^x$
- c** $f(x) = \ln(x \cdot \sqrt[3]{x}) = \ln(x^{1\frac{1}{3}}) = \frac{1}{3}\ln(x)$ geeft $f'(x) = \frac{1}{3} \cdot \frac{1}{x} = \frac{1}{3x}$
- d** $f(x) = {}^2\log(4x) = {}^2\log(4) + {}^2\log(x)$ geeft $f'(x) = \frac{1}{x\ln(2)}$
- e** $f(x) = {}^3\log(5x - 6)$ geeft $f'(x) = \frac{1}{(5x - 6)\ln(3)} \cdot 5 = \frac{5}{(5x - 6)\ln(3)}$
- f** $f(x) = \ln(3x^2 + 3)$ geeft $f'(x) = \frac{1}{3x^2 + 3} \cdot 6x = \frac{2x}{x^2 + 1}$

17 $f(x) = 3^{x-1} + 3^{-x+1}$ geeft $f'(x) = 3^{x-1} \cdot \ln(3) + 3^{-x+1} \cdot \ln(3) \cdot -1 = (3^{x-1} - 3^{-x+1}) \ln(3)$
 $f'(x) = 0$ geeft $(3^{x-1} - 3^{-x+1}) \ln(3) = 0$
 $3^{x-1} - 3^{-x+1} = 0$
 $3^{x-1} = 3^{-x+1}$
 $x-1 = -x+1$
 $2x = 2$
 $x = 1$



min. is $f(1) = 3^0 + 3^0 = 1 + 1 = 2$, dus $B_f = [2, \rightarrow)$.

18 a $f(x) = x^2 \ln(x) - \frac{1}{2}x^2$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} - x = 2x \ln(x)$
 $f'(x) = 0$ geeft $2x \ln(x) = 0$
 $x = 0 \vee \ln(x) = 0$
 $x = 0 \vee x = 1$
vold. niet
 $f(1) = 1 \cdot 0 - \frac{1}{2} \cdot 1 = -\frac{1}{2}$, dus $A(1, -\frac{1}{2})$.

b $f'(x) = 2x \ln(x)$ geeft $f''(x) = 2 \cdot \ln(x) + 2x \cdot \frac{1}{x} = 2 \ln(x) + 2$
 $f''(x) = 0$ geeft $2 \ln(x) + 2 = 0$
 $2 \ln(x) = -2$
 $\ln(x) = -1$
 $x = e^{-1} = \frac{1}{e}$

Stel $k: y = ax + b$ met $a = f'\left(\frac{1}{e}\right) = 2 \cdot \frac{1}{e} \cdot -1 = -\frac{2}{e}$.

$$k: y = -\frac{2}{e}x + b \left\{ \begin{array}{l} -\frac{2}{e} \cdot \frac{1}{e} + b = -\frac{3}{2e^2} \\ f\left(\frac{1}{e}\right) = -\frac{3}{2e^2} \end{array} \right. \left\{ \begin{array}{l} -\frac{4}{2e^2} + b = -\frac{3}{2e^2} \\ b = \frac{1}{2e^2} \end{array} \right.$$

Dus $k: y = -\frac{2}{e}x + \frac{1}{2e^2}$.

10 Meetkunde met vectoren

Voorkennis Afstanden en middens

Bladzijde 52

1 $P(\frac{1}{2}(5+11), \frac{1}{2}(-1+5)) = P(8, 2)$

$$Q(\frac{1}{2}(11+(-3)), \frac{1}{2}(5+7)) = Q(4, 6)$$

$$d(A, P) = \sqrt{(8-5)^2 + (2-(-1))^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$d(A, Q) = \sqrt{(4-5)^2 + (6-(-1))^2} = \sqrt{(-1)^2 + 7^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$d(P, Q) = \sqrt{(4-8)^2 + (6-2)^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

De omtrek van driehoek APQ is $3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$.

2 a $p = 3$ geeft $A(-2, 3)$, $B(4, 5)$ en

$$d(A, B) = \sqrt{(4-(-2))^2 + (5-3)^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}.$$

b $\frac{1}{2}(-2+p+1) = -6$

$$-2+p+1 = -12$$

$$p = -11$$

$p = -11$ geeft $A(-2, -11)$, $B(-10, 5)$ en

$$d(A, B) = \sqrt{(-10-(-2))^2 + (5-(-11))^2} = \sqrt{(-8)^2 + 16^2} = \sqrt{64+256} = \sqrt{320} = 8\sqrt{5}.$$

c $M(\frac{1}{2}(-2+p+1), \frac{1}{2}(p+5)) = M(\frac{1}{2}p - \frac{1}{2}, \frac{1}{2}p + \frac{5}{2})$

$$M \text{ op } y = -x \text{ geeft } \frac{1}{2}p + \frac{5}{2} = -(\frac{1}{2}p - \frac{1}{2})$$

$$\frac{1}{2}p + \frac{5}{2} = -\frac{1}{2}p + \frac{1}{2}$$

$$p = -2$$

$p = -2$ geeft $A(-2, -2)$, $M(-1\frac{1}{2}, 1\frac{1}{2})$ en

$$d(A, M) = \sqrt{(-1\frac{1}{2}-(-2))^2 + (1\frac{1}{2}-(-2))^2} = \sqrt{(\frac{1}{2})^2 + (3\frac{1}{2})^2} = \sqrt{\frac{1}{4} + 12\frac{1}{4}} = \sqrt{12\frac{1}{2}} = 2\frac{1}{2}\sqrt{2}.$$

d $d(A, B) = \sqrt{(p+1-(-2))^2 + (5-p)^2} = \sqrt{(p+3)^2 + (5-p)^2} = \sqrt{p^2 + 6p + 9 + 25 - 10p + p^2}$
 $= \sqrt{2p^2 - 4p + 34}$

e $d(A, B) = 8$ geeft $\sqrt{2p^2 - 4p + 34} = 8$

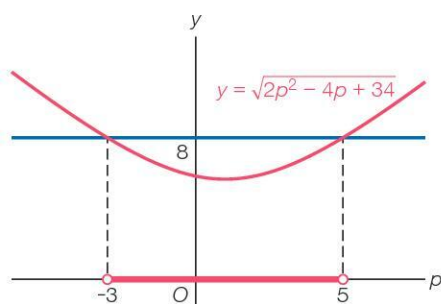
$$2p^2 - 4p + 34 = 64$$

$$2p^2 - 4p - 30 = 0$$

$$p^2 - 2p - 15 = 0$$

$$(p+3)(p-5) = 0$$

$$p = -3 \vee p = 5$$



Dus $d(A, B) < 8$ voor $-3 < p < 5$.

10.1 Vectoren

Bladzijde 53

1 $d(O, A) = \sqrt{(5-0)^2 + (2-0)^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$

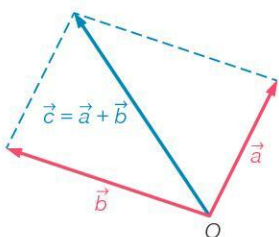
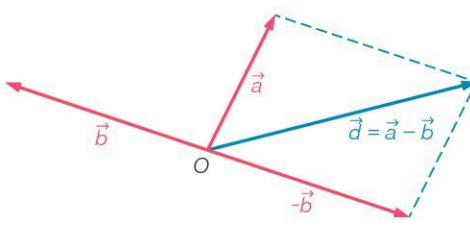
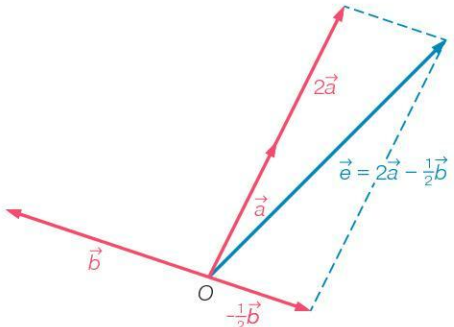
$$d(A, B) = \sqrt{(2-5)^2 + (6-2)^2} = \sqrt{(-3)^2 + 4^2} = 5$$

Dus de lengte van de wandeling is $5 + \sqrt{29}$.

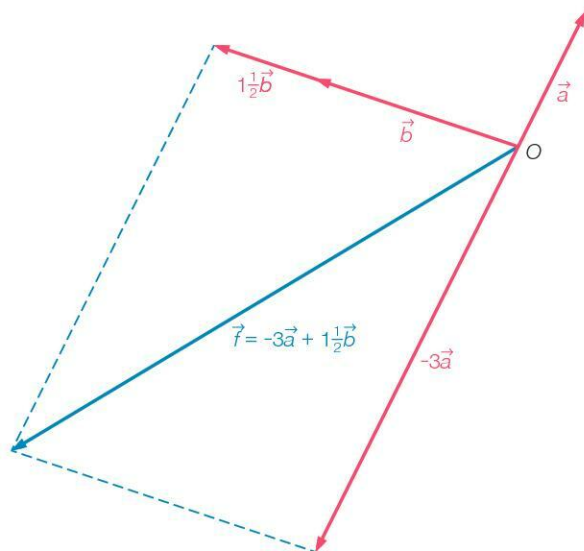
Bladzijde 56

- 2 a** $|\vec{a}| = 8 \sin(17^\circ) \approx 2,34$ en $|\vec{b}| = 8 \cos(17^\circ) \approx 7,65$
- b** $1,5 = |\vec{v}| \cdot \sin(26^\circ)$ geeft $|\vec{v}| = \frac{1,5}{\sin(26^\circ)} \approx 3,42$ en $|\vec{b}| = 3,42 \dots \cos(26^\circ) \approx 3,08$.
- 3 a** $|\vec{a}| = \left| \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right| = \sqrt{4^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17}$
- b** $|\vec{b}| = \left| \begin{pmatrix} \sqrt{3} \\ \sqrt{6} \end{pmatrix} \right| = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} = \sqrt{3 + 6} = \sqrt{9} = 3$
- c** $|\vec{c}| = \left| \begin{pmatrix} 0 \\ 5 \end{pmatrix} \right| = \sqrt{0^2 + 5^2} = \sqrt{0 + 25} = \sqrt{25} = 5$
- d** $|\vec{d}| = \left| \begin{pmatrix} -0,3 \\ 0,4 \end{pmatrix} \right| = \sqrt{(-0,3)^2 + 0,4^2} = \sqrt{0,09 + 0,16} = \sqrt{0,25} = 0,5$
- e** $|\vec{e}| = 2 \cdot \left| \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right| = 2 \cdot \sqrt{1^2 + 3^2} = 2 \cdot \sqrt{1 + 9} = 2\sqrt{10}$
- f** $|\vec{f}| = 0,6 \cdot \left| \begin{pmatrix} 6 \\ 8 \end{pmatrix} \right| = 0,6 \cdot \sqrt{6^2 + 8^2} = 0,6 \cdot \sqrt{36 + 64} = 0,6 \cdot \sqrt{100} = 0,6 \cdot 10 = 6$
- 4 a** $\vec{c} = 2\vec{a} + 3\vec{b} = 2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \end{pmatrix}$
- b** $\vec{d} = \vec{a} - 2\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$
- c** $\vec{e} = 4\vec{a} - 3\vec{b} = 4 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$
- d** $\vec{f} = 2 \cdot (\vec{a} + \vec{b}) = 2 \cdot \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} \right) = 2 \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix}$

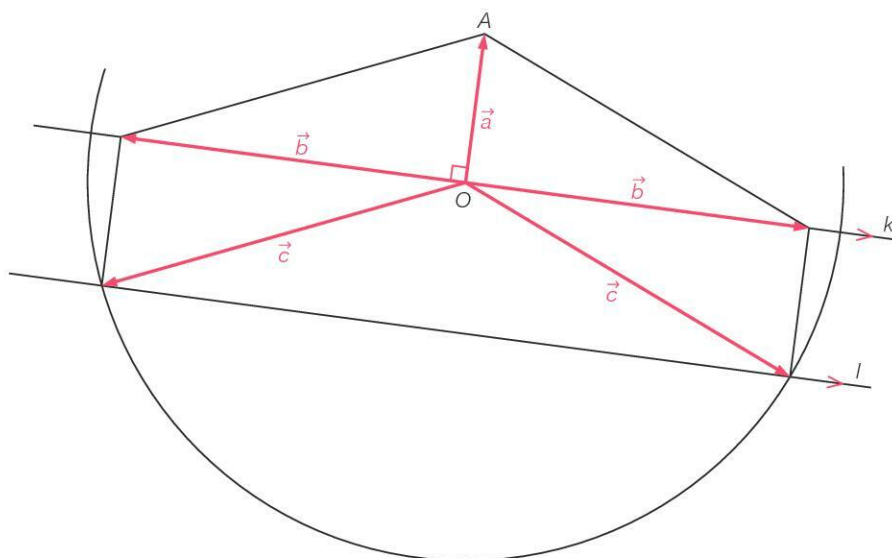
Bladzijde 57

- 5 a**
- 
- b**
- 
- c**
- 

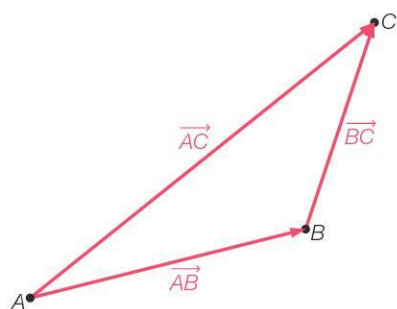
d



- 6 Teken de cirkel met middelpunt O en straal 5.
Teken de lijn l evenwijdig met k , op afstand 2 van k .
De snijpunten van l met de cirkel zijn de eindpunten van de vectoren c .
Met de parallellogrammen krijg je de vectoren b .



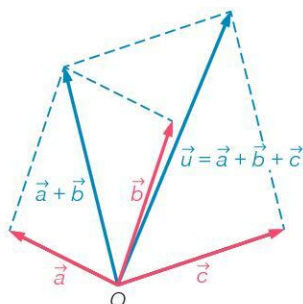
7 a



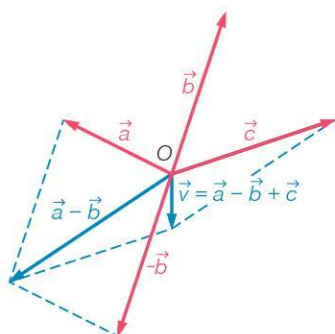
- b $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$
c $\overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{DF}$
d $\overrightarrow{AK} + \overrightarrow{KL} = \overrightarrow{AL}$
e $\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$

- 8 a $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$, dus $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$.
 b $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \vec{a} + \frac{1}{2}(\vec{b} - \vec{a}) = \vec{a} + \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} = \frac{1}{2}(\vec{a} + \vec{b})$
 c $\overrightarrow{PQ} = \vec{q} - \vec{p} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$
 $\overrightarrow{QP} = \vec{p} - \vec{q} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$
 $\overrightarrow{OR} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 7 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 10 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 3\frac{1}{2} \end{pmatrix}$
 $\overrightarrow{QR} = \vec{r} - \vec{q} = \begin{pmatrix} 5 \\ 3\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1\frac{1}{2} \end{pmatrix}$

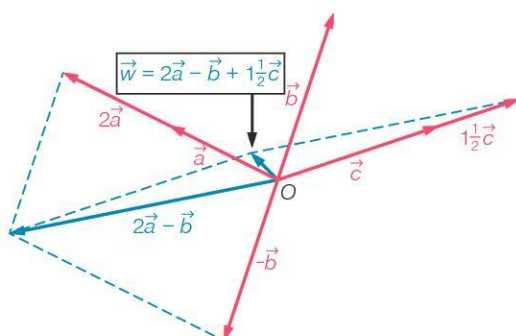
9 a



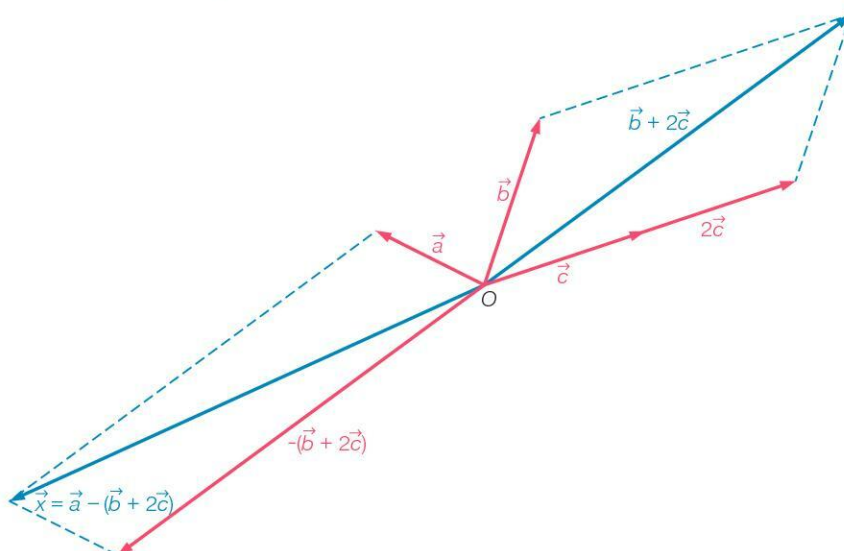
b



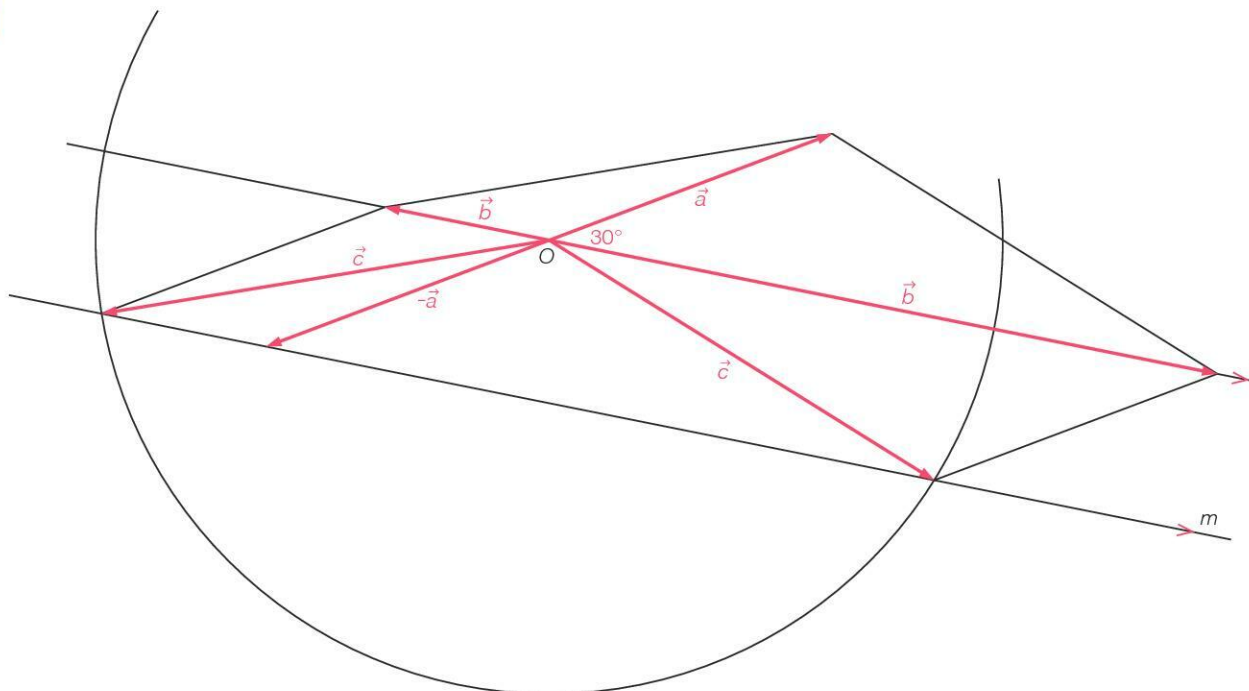
c



d

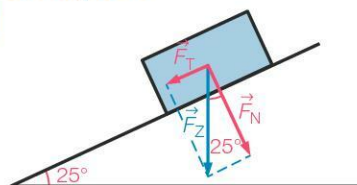


10



Bladzijde 58

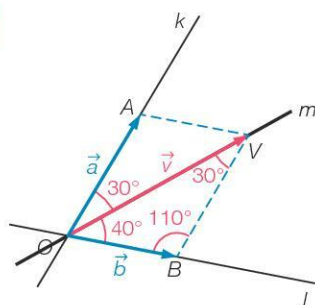
11



De component langs de helling is $F_T = 750 \sin(25^\circ) \approx 317 \text{ N}$.

De component loodrecht op de helling is $F_N = 750 \cos(25^\circ) \approx 680 \text{ N}$.

12



De sinusregel in $\triangle OBV$ geeft $\frac{12}{\sin(110^\circ)} = \frac{|\vec{b}|}{\sin(30^\circ)} = \frac{|\vec{a}|}{\sin(40^\circ)}$.

Dus $|\vec{a}| = \frac{12 \sin(40^\circ)}{\sin(110^\circ)} \approx 8,2$ en $|\vec{b}| = \frac{12 \sin(30^\circ)}{\sin(110^\circ)} \approx 6,4$.

10.2 Vectoren en rotaties

Bladzijde 60

13

a $\vec{a}_R = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ en $\vec{a}_L = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

b $\vec{b}_R = \begin{pmatrix} q \\ -p \end{pmatrix}$

Bladzijde 61

14 $\vec{d} = \vec{m} + \overrightarrow{MD} = \vec{m} + \overrightarrow{AM_L} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

Dus $D(-1, 5)$.

Bladzijde 62

15 a $\vec{b} = \vec{a} + \overrightarrow{AB} = \vec{a} + \overrightarrow{AD_R}$
 $\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, dus $\overrightarrow{AD_R} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

Dit geeft $\vec{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$.

Dus $B(9, 5)$.

b $\vec{n} = \vec{b} + \overrightarrow{BN} = \vec{b} + \overrightarrow{BM_R}$
 $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}(\vec{d} - \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ 5 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} -7 \\ 1 \end{pmatrix} = \begin{pmatrix} -3\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$, dus $\overrightarrow{BM_R} = \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix}$.

Dit geeft $\vec{n} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 9\frac{1}{2} \\ 8\frac{1}{2} \end{pmatrix}$.

Dus $N(9\frac{1}{2}, 8\frac{1}{2})$.

16 $\vec{n} = \frac{1}{2}(\vec{b} + \vec{d})$ met $\vec{d} = \vec{m} + \overrightarrow{MD} = \vec{m} + \overrightarrow{AM_L}$

$\vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\overrightarrow{AM_L} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

Dit geeft $\vec{d} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ en $\vec{n} = \frac{1}{2}\left(\begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}\right) = \begin{pmatrix} 5\frac{1}{2} \\ 5\frac{1}{2} \end{pmatrix}$.

Dus $N(5\frac{1}{2}, 5\frac{1}{2})$.

17 $\vec{q} = \vec{a} + \overrightarrow{AB} + \overrightarrow{BP} + \overrightarrow{PQ} = \vec{a} + \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{AB} = \vec{a} + 1\frac{1}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC}$

$\overrightarrow{BC} = \overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ en $\overrightarrow{AB} = \overrightarrow{AD_R} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\vec{q} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 1\frac{1}{3}\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 + 5\frac{1}{3} - 1 \\ 0 + 4 + 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix}$

$\vec{r} = \vec{q} + \frac{1}{3}\overrightarrow{BC} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \begin{pmatrix} -1 \\ 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 6\frac{1}{3} \\ 6\frac{2}{3} \end{pmatrix}$

Dus $Q(7\frac{1}{3}, 5\frac{1}{3})$ en $R(6\frac{1}{3}, 6\frac{2}{3})$.

18 a $\vec{p} = \vec{a} + \overrightarrow{AP} = \vec{a} + \overrightarrow{AC_R}$

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{AD_R}$

$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ d \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ d \end{pmatrix}$ en $\overrightarrow{AD_R} = \begin{pmatrix} d \\ a \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -a \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} -a + d \\ a + d \end{pmatrix}$ en $\overrightarrow{AC_R} = \begin{pmatrix} a + d \\ a - d \end{pmatrix}$

$\vec{p} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} a + d \\ a - d \end{pmatrix} = \begin{pmatrix} 2a + d \\ a - d \end{pmatrix}$

Dus $P(2a + d, a - d)$.

b $\vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{AC} = \begin{pmatrix} 2a + d \\ a - d \end{pmatrix} + \begin{pmatrix} -a + d \\ a + d \end{pmatrix} = \begin{pmatrix} a + 2d \\ 2a \end{pmatrix}$

Dus $Q(a + 2d, 2a)$.

Bladzijde 63

19 a $\vec{m} = \frac{1}{2}(\vec{a} + \vec{c})$ met $\vec{c} = \vec{d} + \vec{DC} = \vec{d} + \vec{AD}_R$
 $\vec{AD} = \begin{pmatrix} -a \\ d \end{pmatrix}$ en $\vec{DC} = \vec{AD}_R = \begin{pmatrix} d \\ a \end{pmatrix}$
 $\vec{c} = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} d \\ a+d \end{pmatrix}$
 $\vec{m} = \frac{1}{2} \left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} d \\ a+d \end{pmatrix} \right) = \frac{1}{2} \begin{pmatrix} a+d \\ a+d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}d \\ \frac{1}{2}a + \frac{1}{2}d \end{pmatrix}$
Dus $M(\frac{1}{2}a + \frac{1}{2}d, \frac{1}{2}a + \frac{1}{2}d)$.

Voor alle waarden van a en d is $x_M = y_M$, dus M ligt op de lijn $y = x$.

b $OM = \sqrt{(\frac{1}{2}a + \frac{1}{2}d)^2 + (\frac{1}{2}a + \frac{1}{2}d)^2} = \sqrt{2(\frac{1}{2}a + \frac{1}{2}d)^2} = \sqrt{2(\frac{1}{4}a^2 + \frac{1}{2}ad + \frac{1}{4}d^2)}$
 $= \sqrt{\frac{1}{2}(a^2 + 2ad + d^2)} = \sqrt{\frac{1}{2}(a+d)^2} = \sqrt{\frac{1}{2}} \cdot (a+d) = \frac{1}{2}\sqrt{2} \cdot (a+d)$

20 $\vec{t} = \vec{b} + \vec{BS} + \vec{ST} = \vec{b} + \vec{BA}_L + \vec{SP}_R$ en $\vec{u} = \vec{t} + \vec{TU} = \vec{t} + \vec{SP}$
 $\vec{BA} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} a-b \\ -c \end{pmatrix}$, dus $\vec{BA}_L = \begin{pmatrix} c \\ a-b \end{pmatrix}$.
 $\vec{SP} = \vec{BP} - \vec{BS} = \vec{b}_L - \vec{BA}_L = \begin{pmatrix} -c \\ b \end{pmatrix} - \begin{pmatrix} c \\ a-b \end{pmatrix} = \begin{pmatrix} -2c \\ -a+2b \end{pmatrix}$, dus $\vec{SP}_R = \begin{pmatrix} -a+2b \\ 2c \end{pmatrix}$.
 $\vec{t} = \begin{pmatrix} b \\ c \end{pmatrix} + \begin{pmatrix} c \\ a-b \end{pmatrix} + \begin{pmatrix} -a+2b \\ 2c \end{pmatrix} = \begin{pmatrix} -a+3b+c \\ a-b+3c \end{pmatrix}$
 $\vec{u} = \begin{pmatrix} -a+3b+c \\ a-b+3c \end{pmatrix} + \begin{pmatrix} -2c \\ -a+2b \end{pmatrix} = \begin{pmatrix} -a+3b-c \\ b+3c \end{pmatrix}$
Dus $T(-a+3b+c, a-b+3c)$ en $U(-a+3b-c, b+3c)$.

21 $\vec{n} = \vec{m} + \vec{MN} = \vec{m} + \vec{AB}$
 $\vec{AD} = \begin{pmatrix} 0 \\ 2a \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ 2a \end{pmatrix}$ en $\vec{AB} = \vec{AD}_R = \begin{pmatrix} 2a \\ a \end{pmatrix}$
 $\vec{m} = \frac{1}{2}(\vec{d} + \vec{b}) = \frac{1}{2}(\vec{d} + \vec{a} + \vec{AB}) = \frac{1}{2} \left(\begin{pmatrix} 0 \\ 2a \end{pmatrix} + \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} \right) = \frac{1}{2} \begin{pmatrix} 3a \\ 3a \end{pmatrix} = \begin{pmatrix} \frac{3}{2}a \\ \frac{3}{2}a \end{pmatrix}$
 $\vec{n} = \begin{pmatrix} \frac{3}{2}a \\ \frac{3}{2}a \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} = \begin{pmatrix} \frac{7}{2}a \\ \frac{5}{2}a \end{pmatrix}$
 $y = mx$
door $N(3\frac{1}{2}a, 2\frac{1}{2}a)$ $\left. \vphantom{\begin{matrix} y = mx \\ \text{door } N(3\frac{1}{2}a, 2\frac{1}{2}a) \end{matrix}} \right\} 3\frac{1}{2}am = 2\frac{1}{2}a$
 $m = \frac{5}{7}$

Bladzijde 64

22 a $\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} b \\ c \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} b-a \\ c \end{pmatrix}$
 $\vec{d} = \vec{c}_L = \begin{pmatrix} -c \\ b \end{pmatrix}$ en $\vec{b} = \vec{a}_L = \begin{pmatrix} 0 \\ a \end{pmatrix}$ geeft $\vec{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -c \\ b \end{pmatrix} - \begin{pmatrix} 0 \\ a \end{pmatrix} = \begin{pmatrix} -c \\ b-a \end{pmatrix}$.
b Uit vraag a volgt $\vec{BD} = \vec{AC}_L$, dus $AC = BD$ en $AC \perp BD$.

23 $M(\frac{1}{2}a, 0)$
 $\vec{p} = \vec{a} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$
 $\vec{q} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{b}_L = \frac{1}{2}\begin{pmatrix} b \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix}$
 $\vec{MP} = \vec{p} - \vec{m} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b+c \\ a-b+c \end{pmatrix}$
 $\vec{MQ} = \vec{q} - \vec{m} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a+b-c \\ b+c \end{pmatrix}$
 $\vec{MQ} = \vec{MP}_L$, dus $MP = MQ$ en $MP \perp MQ$.
Dus $\triangle MPQ$ is een gelijkbenige rechthoekige driehoek.

24 $\vec{p} = \vec{a} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$
 $\vec{q} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{b}_L = \frac{1}{2}\begin{pmatrix} b \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix}$
 $\vec{r} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{a}_R = \frac{1}{2}\begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ -a \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}$
 $\vec{QR} = \vec{r} - \vec{q} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a-b+c \\ -a-b-c \end{pmatrix}$
 $\vec{QR} = \vec{OP}_L$, dus $OP \perp QR$.
 $\vec{AQ} = \vec{q} - \vec{a} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a+\frac{1}{2}b-\frac{1}{2}c \\ \frac{1}{2}b+\frac{1}{2}c \end{pmatrix}$
 $\vec{PR} = \vec{r} - \vec{p} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}b-\frac{1}{2}c \\ -a+\frac{1}{2}b-\frac{1}{2}c \end{pmatrix}$
 $\vec{PR} = \vec{AQ}_L$, dus $AQ \perp PR$.
 $\vec{BR} = \vec{r} - \vec{b} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a-b \\ -\frac{1}{2}a-c \end{pmatrix}$
 $\vec{PQ} = \vec{q} - \vec{p} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}a-c \\ -\frac{1}{2}a+b \end{pmatrix}$
 $\vec{PQ} = \vec{BR}_R$, dus $BR \perp PQ$.

Bladzijde 65

25 a $\vec{p} = \vec{a} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$
 $\vec{q} = \vec{c} + \frac{1}{2}\vec{CB} + \frac{1}{2}\vec{CB}_L = \begin{pmatrix} d \\ e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-d \\ c-e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c+e \\ b-d \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix}$
 $\vec{r} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{c}_L = \frac{1}{2}\begin{pmatrix} d \\ e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -e \\ d \end{pmatrix} = \frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix}$
 $\vec{s} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{a}_R = \frac{1}{2}\begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ -a \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}$
b $\vec{PR} = \vec{r} - \vec{p} = \frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a-b-c+d-e \\ -a+b-c+d+e \end{pmatrix}$
 $\vec{QS} = \vec{s} - \vec{q} = \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a-b+c-d-e \\ -a-b-c+d+e \end{pmatrix}$
 $\vec{QS} = \vec{PR}_L$, dus $PR = QS$ en $PR \perp QS$.

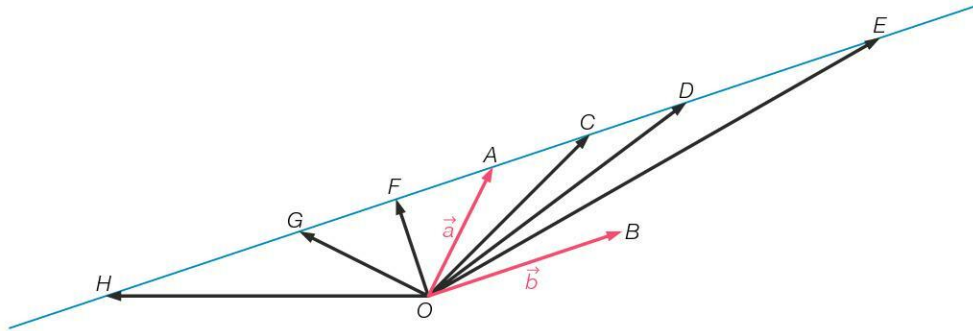
26 $\vec{k} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} a+2b+d+e \\ a+2c-d+e \end{pmatrix}$
 $\vec{l} = \frac{1}{2}(\vec{q} + \vec{r}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} b-c+2d \\ b+c+2e \end{pmatrix}$
 $\vec{m} = \frac{1}{2}(\vec{r} + \vec{s}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} d-e \\ d+e \end{pmatrix} + \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} a+d-e \\ -a+d+e \end{pmatrix}$
 $\vec{n} = \frac{1}{2}(\vec{p} + \vec{s}) = \frac{1}{2}\left(\frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} a \\ -a \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} 2a+b+c \\ -b+c \end{pmatrix}$
 $\vec{KM} = \vec{m} - \vec{k} = \frac{1}{4}\begin{pmatrix} a+d-e \\ -a+d+e \end{pmatrix} - \frac{1}{4}\begin{pmatrix} a+2b+d+e \\ a+2c-d+e \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -2b-2e \\ -2a-2c+2d \end{pmatrix}$
 $\vec{LN} = \vec{n} - \vec{l} = \frac{1}{4}\begin{pmatrix} 2a+b+c \\ -b+c \end{pmatrix} - \frac{1}{4}\begin{pmatrix} b-c+2d \\ b+c+2e \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2a+2c-2d \\ -2b-2e \end{pmatrix}$
 $\vec{LN} = \vec{KM}_L$, dus $KM = LN$ en $KM \perp LN$.

10.3 Vectoren en lijnen

Bladzijde 67

27

a



- b De eindpunten van de in a getekende vectoren liggen op de lijn door het eindpunt van de vector $\vec{a}$ en die evenwijdig is met de vector $\vec{b}$.

Bladzijde 69

28

- a Je kunt ook $\vec{q}$ als steunvector nemen.

$$\left. \begin{aligned} l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{q} + t \cdot (\vec{q} - \vec{p}) \\ \vec{q} &= \begin{pmatrix} 5 \\ 6 \end{pmatrix} \text{ en } \vec{q} - \vec{p} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{aligned} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

- b $\begin{pmatrix} 2 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ is ook een richtingsvector van l .

$t = -\frac{1}{2}$ geeft $\begin{pmatrix} 5 \\ 6 \end{pmatrix} + -\frac{1}{2} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, dus $(4, 5)$ is een punt van l en dus is ook $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ een steunvector van l .

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

- c $(3, 4)$ op l .

$x = 3$ geeft $6 + t = 3$, dus $t = -3$.

$t = -3$ en $y = 4$ geeft $p - 3 \cdot 1 = 4$, dus $p = 7$.

29

- a $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{a})$
 $\vec{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ en $\vec{b} - \vec{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix} \triangleq \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ } $k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

- b $l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t \cdot (\vec{d} - \vec{c})$
 $\vec{c} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ en $\vec{d} - \vec{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ } $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

- c $m: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{e} + t \cdot (\vec{f} - \vec{e})$
 $\vec{e} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ en $\vec{f} - \vec{e} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ } $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

30 $x = 7$ geeft $-2 + 3t = 7$

$$3t = 9$$

$$t = 3$$

$t = 3$ geeft $y = 5 - 4 \cdot 3 = 5 - 12 = -7$

Dus A ligt op k .

$x = -13$ geeft $-2 + 3t = -13$

$$3t = -11$$

$$t = -3\frac{2}{3}$$

$t = -3\frac{2}{3}$ geeft $y = 5 - 4 \cdot -3\frac{2}{3} = 5 + 14\frac{2}{3} = 19\frac{2}{3}$

Dus B ligt niet op k .

$x = -3\frac{1}{2}$ geeft $-2 + 3t = -3\frac{1}{2}$

$$3t = -1\frac{1}{2}$$

$$t = -\frac{1}{2}$$

$t = -\frac{1}{2}$ geeft $y = 5 - 4 \cdot -\frac{1}{2} = 5 + 2 = 7$

Dus C ligt op k .

31 lijn k

t	0	1
punt	(1, 2)	(4, 3)

lijn l

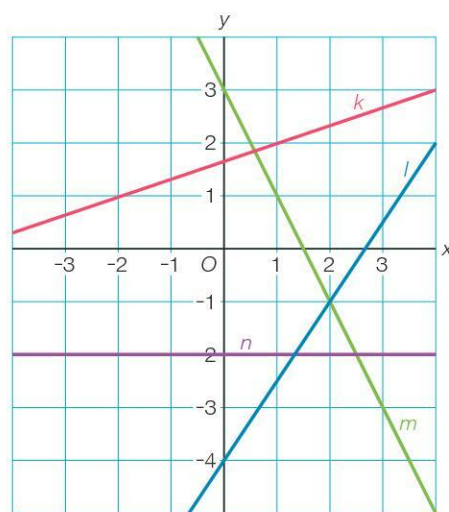
u	0	1
punt	(2, -1)	(4, 2)

lijn m

v	0	1
punt	(0, 3)	(1, 1)

lijn n

w	0	1
punt	(1, -2)	(2, -2)

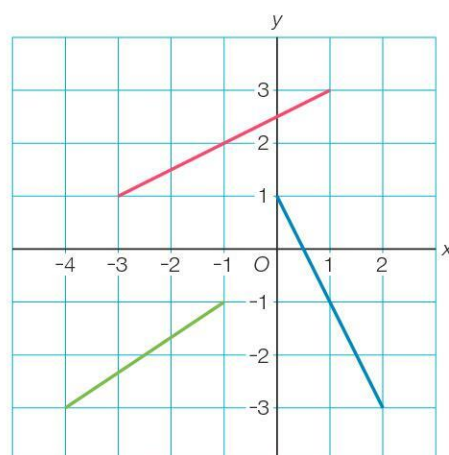


32

t	-1	1
punt	(-3, 1)	(1, 3)

u	1	3
punt	(0, 1)	(2, -3)

v	-1	0
punt	(-4, -3)	(-1, -1)



33 a $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{c})$

$$\left. \begin{matrix} \vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \end{matrix} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$$\begin{aligned} \mathbf{b} \quad \vec{m} &= \frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ -2 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \\ l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + t \cdot (\vec{b} - \vec{m}) \\ \vec{b} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{m} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \end{aligned} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$$\begin{aligned} \mathbf{c} \quad \vec{n} &= \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \\ m: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{n} + t \cdot (\vec{a} - \vec{c}) \\ \vec{n} &= \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \end{aligned} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

d Stel eerst een vectorvoorstelling op van de lijn door BC .

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + t \cdot (\vec{b} - \vec{c}) \\ \vec{b} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \end{aligned} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

Dus een vectorvoorstelling van het lijnstuk BC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix} \wedge -1 \leq t \leq 0$.

Bladzijde 70

34 a De lijn door de middens van AD en BC is evenwijdig met AB en CD en heeft dus dezelfde richting als AB en CD . Dus een richtingsvector van deze lijn is $\vec{DC} = \vec{c} - \vec{d}$. De vector vanuit O naar het midden van BC is $\frac{1}{2}(\vec{b} + \vec{c})$. Dus $\frac{1}{2}(\vec{b} + \vec{c})$ is een steunvector van de lijn door de middens van AD en BC . Dus $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}(\vec{b} + \vec{c}) + t \cdot (\vec{c} - \vec{d})$ is een vectorvoorstelling van de lijn door de middens van AD en BC .

$$\begin{aligned} \mathbf{b} \quad & \bullet k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{d}) \\ & \bullet l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + u \cdot \left(\frac{1}{2}(\vec{c} + \vec{d}) - \vec{b} \right) \\ & \bullet m: \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}(\vec{a} + \vec{c}) + v \cdot (\vec{c} - \vec{b}) \end{aligned}$$

c I gaat door B en is evenwijdig met AC .
II gaat door het midden van AB en is evenwijdig met BD .
III gaat door A en het midden van BC .

$$\begin{aligned} \mathbf{35 a} \quad \text{Substitutie van } x = 1 - 3t \text{ en } y = 2 + t \text{ in } y = 4x - 28 \text{ geeft } 2 + t &= 4(1 - 3t) - 28 \\ 2 + t &= 4 - 12t - 28 \\ 13t &= -26 \\ t &= -2 \end{aligned}$$

$$t = -2 \text{ geeft } x = 1 - 2 \cdot 3 = 7 \text{ en } y = 2 - 2 \cdot 1 = 0$$

Dus $S(7, 0)$.

$$\begin{aligned} \mathbf{b} \quad \text{Substitutie van } x = 7 + 3u \text{ en } y = -3 + u \text{ in } 2x - 5y = 11 \text{ geeft } 2(7 + 3u) - 5(-3 + u) &= 11 \\ 14 + 6u + 15 - 5u &= 11 \\ u &= -18 \end{aligned}$$

$$u = -18 \text{ geeft } x = 7 - 18 \cdot 3 = -47 \text{ en } y = -3 - 18 \cdot 1 = -21$$

Dus $T(-47, -21)$.

$$\begin{aligned} \mathbf{36 a} \quad \text{Substitutie van } x = 3t \text{ en } y = 2 + 2t \text{ in } 2x - 5y = 6 \text{ geeft } 2 \cdot 3t - 5(2 + 2t) &= 6 \\ 6t - 10 - 10t &= 6 \\ -4t &= 16 \\ t &= -4 \end{aligned}$$

$$t = -4 \text{ geeft } x = -4 \cdot 3 = -12 \text{ en } y = 2 - 4 \cdot 2 = -6$$

Dus het snijpunt is $(-12, -6)$.

b Substitutie van $x = 2 - u$ en $y = -5 + 3u$ in $3x + 4y = 10$ geeft $3(2 - u) + 4(-5 + 3u) = 10$
 $6 - 3u - 20 + 12u = 10$
 $9u = 24$
 $u = 2\frac{2}{3}$

$u = 2\frac{2}{3}$ geeft $x = 2 + 2\frac{2}{3} \cdot -1 = -\frac{2}{3}$ en $y = -5 + 2\frac{2}{3} \cdot 3 = 3$

Dus het snijpunt is $(-\frac{2}{3}, 3)$.

37 a $\begin{cases} x = 4 + 2t \\ y = 1 - 5t \end{cases} \begin{vmatrix} 5 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 5x = 20 + 10t \\ 2y = 2 - 10t \end{cases} +$
 $5x + 2y = 22$

Dus k : $5x + 2y = 22$.

b $\begin{cases} x = 3 - t \\ y = 2 + 3t \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 3x = 9 - 3t \\ y = 2 + 3t \end{cases} +$
 $3x + y = 11$

Dus l : $3x + y = 11$.

c $\begin{cases} x = 3t - 1 \\ y = 5t + 1 \end{cases} \begin{vmatrix} 5 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 5x = 15t - 5 \\ 3y = 15t + 3 \end{cases} -$
 $5x - 3y = -8$

Dus m : $5x - 3y = -8$.

Bladzijde 71

38 a Neem je $x = 4t$ in $y = 2x + 3$, dan krijg je $y = 2 \cdot 4t + 3$ oftewel $y = 8t + 3$.
Dus k : $x = 4t \wedge y = 8t + 3$ is een parametervoorstelling van k .

b $x = t + 5$ geeft $y = 2 \cdot (t + 5) + 3$
 $y = 2t + 10 + 3$
 $y = 2t + 13$

Dus k : $x = t + 5 \wedge y = 2t + 13$.

c $x = -3t + 2$ geeft $y = 2 \cdot (-3t + 2) + 3$
 $y = -6t + 4 + 3$
 $y = -6t + 7$

Dus k : $x = -3t + 2 \wedge y = -6t + 7$.

d $y = 4t$ geeft $4t = 2x + 3$
 $-2x = -4t + 3$
 $x = 2t - 1\frac{1}{2}$

Dus k : $x = 2t - 1\frac{1}{2} \wedge y = 4t$.

39 a $\begin{cases} x = 5 + 4t \\ y = 3 - 2t \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix}$ geeft $\begin{cases} x = 5 + 4t \\ 2y = 6 - 4t \end{cases} +$
 $x + 2y = 11$

Dus k : $x + 2y = 11$.

b Neem $x = 2t$ in $3x - 2y = 6$.

Dit geeft $3 \cdot 2t - 2y = 6$

$$6t - 2y = 6$$

$$-2y = -6t + 6$$

$$y = 3t - 3$$

Dus l : $x = 2t \wedge y = 3t - 3$.

c $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -9 \\ 2 \end{pmatrix}$

d $n: x = 4 + 3t \wedge y = -1 + 2t$

40 a $\begin{cases} x = 5t + p \\ y = 4t + 3 \end{cases} \begin{vmatrix} 4 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 4x = 20t + 4p \\ 5y = 20t + 15 \end{cases} -$
 $4x - 5y = 4p - 15$

Er moet gelden $4p - 15 = -9$ oftewel $4p = 6$, dus $p = 1\frac{1}{2}$.

$$\begin{aligned} \mathbf{b} \quad \begin{cases} x = 2t - 1 \\ y = -3t + 5 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 3x = 6t - 3 \\ 2y = -6t + 10 \end{cases} + \\ 3x + 2y = 7 \\ 2y = -3x + 7 \\ y = -1\frac{1}{2}x + 3\frac{1}{2} \end{aligned}$$

41 a Substitutie van $x = 4 + 2t$ en $y = 5 + 3t$ in $x - 2y = 10$ geeft $4 + 2t - 2(5 + 3t) = 10$
 $4 + 2t - 10 - 6t = 10$
 $-4t = 16$
 $t = -4$

$t = -4$ geeft $x = 4 - 4 \cdot 2 = -4$ en $y = 5 - 4 \cdot 3 = -7$
Dus $S(-4, -7)$.

$$\mathbf{b} \quad \begin{cases} -2 + 3u = -3a + 1 \\ 5 - 4u = 2a \end{cases} \begin{vmatrix} 4 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} -8 + 12u = -12a + 4 \\ 15 - 12u = 6a \end{cases} + \\ 7 = -6a + 4 \\ 6a = -3 \\ a = -\frac{1}{2}$$

Dus $A(-3 \cdot -\frac{1}{2} + 1, 2 \cdot -\frac{1}{2}) = A(2\frac{1}{2}, -1)$.

$$\mathbf{c} \quad \begin{cases} 2v + q = 5 \\ -v - 2q = 2 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 2v + q = 5 \\ -2v - 4q = 4 \end{cases} + \\ -3q = 9 \\ q = -3$$

42 a $(3, 4)$ op $l_c: 2x + 5y = c$ geeft $c = 2 \cdot 3 + 5 \cdot 4 = 26$, dus $l: 2x + 5y = 26$.
 $(3, 4)$ op $k_{a,b}: x = at - 3 \wedge y = bt + 1$ geeft $at - 3 = 3$ oftewel $at = 6$ en $bt + 1 = 4$ oftewel $bt = 3$.

Dus $at = 2bt$ oftewel $a = 2b$.

Kies bijvoorbeeld $b = 1$, dan is $a = 2$.

Dit geeft $k: x = 2t - 3 \wedge y = t + 1$.

$$\begin{cases} x = 2t - 3 \\ y = t + 1 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} x = 2t - 3 \\ 2y = 2t + 2 \end{cases} - \\ x - 2y = -5$$

Omdat $\frac{2}{1} \neq \frac{5}{-2}$ snijden $k: x - 2y = -5$ en $l: 2x + 5y = 26$ elkaar en vallen ze niet samen.

Dus mogelijke waarden zijn $a = 2$, $b = 1$ en $c = 26$.

$$\mathbf{b} \quad \begin{cases} x = at - 3 \\ y = bt + 1 \end{cases} \begin{vmatrix} b \\ a \end{vmatrix} \text{ geeft } \begin{cases} bx = abt - 3b \\ ay = abt + a \end{cases} - \\ bx - ay = -a - 3b$$

$k_{a,b}: bx - ay = -a - 3b$ en $l_c: 2x + 5y = c$ vallen samen als geldt $\frac{b}{2} = \frac{-a}{5} = \frac{-a - 3b}{c}$.

Uit $\frac{b}{2} = \frac{-a}{5}$ volgt $5b = -2a$.

Kies bijvoorbeeld $a = 5$, dan is $b = -2$.

$$\begin{aligned} \frac{-a}{5} = \frac{-a - 3b}{c} \quad \left. \begin{aligned} \frac{-5}{5} &= \frac{-5 - 3 \cdot -2}{c} \\ a = 5 \text{ en } b = -2 \end{aligned} \right\} \begin{aligned} \frac{-5}{5} &= \frac{1}{c} \\ c &= -1 \end{aligned} \end{aligned}$$

Dus mogelijke waarden zijn $a = 5$, $b = -2$ en $c = -1$.

43 $x = -t - 3 \wedge y = pt - 8$ geeft $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ p \end{pmatrix}$
 $\begin{pmatrix} -1 \\ p \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ geeft $p = -2\frac{1}{2}$
 $(1, q)$ op $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -2\frac{1}{2} \end{pmatrix}$ geeft $-3 - t = 1$
 $-t = 4$
 $t = -4$
 $t = -4$ geeft $q = -8 - 4 \cdot -2\frac{1}{2} = -8 + 10 = 2$
Dus $p = -2\frac{1}{2}$ en $q = 2$.

10.4 Vectoren en hoeken

Bladzijde 73

44 a $A(a_x, a_y)$, dus $\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$ en $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$.
Kwadrateren geeft $|\vec{a}|^2 = a_x^2 + a_y^2$.
 $B(b_x, b_y)$, dus $\vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$ en $|\vec{b}| = \sqrt{b_x^2 + b_y^2}$.
Kwadrateren geeft $|\vec{b}|^2 = b_x^2 + b_y^2$.
b Voor de punten $A(x_A, y_A)$ en $B(x_B, y_B)$ geldt $d(A, B) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$.
Dus voor de punten $A(a_x, a_y)$ en $B(b_x, b_y)$ geldt $d(A, B) = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$.
De lengte van het lijnstuk AB wordt behalve als $d(A, B)$ ook genoteerd als AB , dus
 $AB = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$.
Kwadrateren geeft $AB^2 = (b_x - a_x)^2 + (b_y - a_y)^2$
 $= b_x^2 - 2a_x b_x + a_x^2 + b_y^2 - 2a_y b_y + a_y^2$
 $= a_x^2 + a_y^2 + b_x^2 + b_y^2 - 2a_x b_x - 2a_y b_y$

Bladzijde 75

45 a $\angle(k, l) \approx 180^\circ - 108,4^\circ = 71,6^\circ$
b $\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right|} = \frac{2 \cdot -3 + 3 \cdot 1}{\sqrt{13} \cdot \sqrt{10}} = \frac{-3}{\sqrt{130}}$
Dus $\angle(\vec{r}_m, \vec{r}_n) \approx 105,3^\circ$.
46 a $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -6 \end{pmatrix} = 2 \cdot -1 + 3 \cdot -6 = -2 - 18 = -20$
b $\vec{c} \cdot \vec{d} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = -1 \cdot 2 + 0 \cdot 4 = -2 + 0 = -2$
c $\vec{e} \cdot \vec{f} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 2 \cdot 3 + -3 \cdot 2 = 6 - 6 = 0$
d $\vec{g} \cdot \vec{h} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2\sqrt{2} \\ -\sqrt{3} \end{pmatrix} = \sqrt{2} \cdot 2\sqrt{2} + \sqrt{3} \cdot -\sqrt{3} = 4 - 3 = 1$
47 a $\cos(\angle(\vec{a}, \vec{b})) = \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{3 \cdot 2 + 4 \cdot 1}{\sqrt{25} \cdot \sqrt{5}} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$
Dus $\angle(\vec{a}, \vec{b}) \approx 26,6^\circ$.
b $\cos(\angle(\vec{c}, \vec{d})) = \frac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right|} = \frac{4 \cdot -1 + -1 \cdot 3}{\sqrt{17} \cdot \sqrt{10}} = \frac{-7}{\sqrt{170}}$
Dus $\angle(\vec{c}, \vec{d}) \approx 125,5^\circ$.

$$\text{c} \quad \cos(\angle(\vec{e}, \vec{f})) = \frac{\begin{pmatrix} 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} 0 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right|} = \frac{0 \cdot 2 + 3 \cdot 5}{3 \cdot \sqrt{29}} = \frac{15}{3\sqrt{29}} = \frac{5}{\sqrt{29}}$$

Dus $\angle(\vec{e}, \vec{f}) \approx 21,8^\circ$.

$$\text{d} \quad \cos(\angle(\vec{g}, \vec{h})) = \frac{\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|} = \frac{2 \cdot 5 + (-5) \cdot 2}{\sqrt{29} \cdot \sqrt{29}} = \frac{0}{29} = 0$$

Dus $\angle(\vec{g}, \vec{h}) = 90^\circ$.

$$48 \quad \text{a} \quad \cos(\angle(k, l)) = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{1 \cdot 2 + 2 \cdot 1}{\sqrt{5} \cdot \sqrt{5}} = \frac{4}{5}$$

Dus $\angle(k, l) \approx 36,9^\circ$.

$$\text{b} \quad \cos(\angle(m, n)) = \frac{\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right|} = \frac{3 \cdot (-1) + 1 \cdot 2}{\sqrt{10} \cdot \sqrt{5}} = \frac{1}{\sqrt{50}}$$

Dus $\angle(m, n) \approx 81,9^\circ$.

$$\text{c} \quad \cos(\angle(p, q)) = \frac{\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{-1 \cdot 2 + 1 \cdot 1}{\sqrt{2} \cdot \sqrt{5}} = \frac{1}{\sqrt{10}}$$

Dus $\angle(p, q) \approx 71,6^\circ$.

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$$49 \quad \text{a} \quad \cos(\angle(k, l)) = \frac{\begin{pmatrix} 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right|} = \frac{3 \cdot 3 + (-1) \cdot 5}{\sqrt{10} \cdot \sqrt{34}} = \frac{4}{\sqrt{340}}$$

Dus $\angle(k, l) \approx 77,5^\circ$.

$$\text{b} \quad n: x = 3u - 2 \wedge y = 10u + 3 \text{ oftewel } n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + u \cdot \begin{pmatrix} 3 \\ 10 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 10 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 10 \end{pmatrix} \right|} = \frac{2 \cdot 3 + (-1) \cdot 10}{\sqrt{5} \cdot \sqrt{109}} = \frac{4}{\sqrt{545}}$$

Dus $\angle(m, n) \approx 80,1^\circ$.

$$50 \quad \text{a} \quad \overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{BA}, \overrightarrow{BC})) = \frac{\begin{pmatrix} -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right|} = \frac{-2 \cdot (-1) + (-2) \cdot 3}{\sqrt{8} \cdot \sqrt{10}} = \frac{-4}{\sqrt{80}}$$

$\angle(\overrightarrow{BA}, \overrightarrow{BC}) \approx 116,6^\circ$

Dus $\angle B \approx 116,6^\circ$.

Alternatieve uitwerking

$\tan(\alpha) = \text{rc}_{AB} = 1$ geeft $\alpha = 45^\circ$

$\tan(\beta) = \text{rc}_{BC} = -3$ geeft $\beta = -71,56\dots^\circ$

$\alpha - \beta = 45^\circ - (-71,56\dots^\circ) = 116,56\dots^\circ$

Dus $\angle B \approx 116,6^\circ$.

$$\begin{aligned}\mathbf{b} \quad \vec{r}_{AD} &= \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \\ \vec{r}_{BE} &= \vec{e} - \vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix} \\ \cos(\angle(AD, BE)) &= \frac{\left| \begin{pmatrix} -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -5 \\ 1 \end{pmatrix} \right|} = \frac{|-1 \cdot -5 + 6 \cdot 1|}{\sqrt{37} \cdot \sqrt{26}} = \frac{11}{\sqrt{962}}\end{aligned}$$

$$\angle(AD, BE) \approx 69,2^\circ$$

Dus de hoek tussen de diagonalen AD en BE is $69,2^\circ$.

$$\begin{aligned}\mathbf{51} \quad \mathbf{a} \quad \vec{r}_{AC} &= \vec{c} - \vec{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \\ \vec{r}_{BC} &= \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 4 \end{pmatrix} \\ \cos(\angle(AC, BC)) &= \frac{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right|} = \frac{|2 \cdot 1 + -5 \cdot 4|}{\sqrt{29} \cdot \sqrt{17}} = \frac{18}{\sqrt{493}}\end{aligned}$$

$$\text{Dus } \angle(AC, BC) \approx 35,8^\circ.$$

$$\begin{aligned}\mathbf{b} \quad \vec{BA} &= \vec{a} - \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \\ \vec{BC} &= \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \\ \cos(\angle(\vec{BA}, \vec{BC})) &= \frac{\left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|} = \frac{-3 \cdot -1 + 1 \cdot -4}{\sqrt{10} \cdot \sqrt{17}} = \frac{-1}{\sqrt{170}}\end{aligned}$$

$$\angle(\vec{BA}, \vec{BC}) \approx 94,4^\circ$$

Dus $\angle ABC \approx 94,4^\circ$.

$$\mathbf{52} \quad \mathbf{a} \quad S(3, -1) \text{ en } (2, 1) \text{ op } k_{a,b} \text{ geeft } \vec{r}_{k_{a,b}} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

$$\cos(\angle(k, l)) = \frac{\left| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|} = \frac{|1 \cdot 3 + -2 \cdot -4|}{\sqrt{5} \cdot \sqrt{25}} = \frac{11}{5\sqrt{5}}$$

$$\text{Dus } \angle(k, l) \approx 10,3^\circ.$$

$$\mathbf{b} \quad \cos(\angle(k_{a,1}, l)) = \frac{\left| \begin{pmatrix} a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} a \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|} = \frac{|a \cdot 3 + 1 \cdot -4|}{\sqrt{a^2 + 1} \cdot \sqrt{25}} = \frac{|3a - 4|}{5\sqrt{a^2 + 1}}$$

$$\text{Voer in } y_1 = \frac{|3x - 4|}{5\sqrt{x^2 + 1}} \text{ en } y_2 = \cos(80).$$

De optie snijpunt geeft $x = 0,936...$ en $x = 1,973...$

Dus $a \approx 0,94 \vee a \approx 1,97$.

$$\begin{aligned}\mathbf{53} \quad \vec{a} \cdot \vec{b} &= \begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} q \\ -p \end{pmatrix} = pq - pq = 0 \\ \vec{a} \cdot \vec{b} &= 0, \text{ dus } \cos(\angle(\vec{a}, \vec{b})) = 0 \text{ en hieruit volgt } \angle(\vec{a}, \vec{b}) = 90^\circ.\end{aligned}$$

$$\mathbf{54} \quad \mathbf{a} \quad \text{Een richtingsvector gaat van het punt } (0, 2) \text{ naar het punt } (3, 0).$$

$$\text{Dit is de vector } \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

$$\text{Dus } \vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

- b** De vector $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op $\vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.
 Dus $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op de lijn k .
- c** De vector $\vec{n}_l = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ staat loodrecht op de lijn l .

Bladzijde 78

- 55 a** $\vec{r}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.
 $k: 4x - 3y = c \left\{ \begin{array}{l} c = 4 \cdot 2 - 3 \cdot -1 = 11 \\ (2, -1) \text{ op } k \end{array} \right.$
 Dus $k: 4x - 3y = 11$.
 $\vec{r}_l = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.
 $l: 7x + 4y = c \left\{ \begin{array}{l} c = 0 \\ (0, 0) \text{ op } l \end{array} \right.$
 Dus $l: 7x + 4y = 0$.
- b** $\vec{n}_m = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, dus $\vec{r}_m = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
 $(-3, 0) \text{ op } m$, dus $\vec{s}_m = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.
 Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
 $\vec{n}_n = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\vec{r}_n = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.
 $(0, 0) \text{ op } n$, dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
 Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = u \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

Bladzijde 79

- 56 a** $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
 $l: 5x + 2y = c \left\{ \begin{array}{l} c = 5 \cdot 4 + 2 \cdot 1 = 22 \\ \text{door } A(4, 1) \end{array} \right.$
 Dus $l: 5x + 2y = 22$.
- b** $n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 $n: 3x + 2y = c \left\{ \begin{array}{l} c = 3 \cdot 5 + 2 \cdot -1 = 13 \\ \text{door } B(5, -1) \end{array} \right.$
 Dus $n: 3x + 2y = 13$.
- 57 a** $\vec{n}_k = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 $A(5, 2) \text{ op } l$, dus $\vec{s}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
 Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
- b** $n \perp m$, dus $\vec{r}_n = \vec{n}_m = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.
 $B(1, -3) \text{ op } n$, dus $\vec{s}_n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.
 Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + u \cdot \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

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$$\mathbf{a} \quad \begin{cases} 3+t=-1+3u \\ 3+2t=5-4u \end{cases} \text{ oftewel } \begin{cases} t-3u=-4 \\ 2t+4u=2 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 2t-6u=-8 \\ 2t+4u=2 \end{cases} -$$

$$\frac{-10u=-10}{u=1}$$

$$u=1 \text{ geeft } x=-1+3=2 \wedge y=5-4=1$$

Dus $S(2, 1)$.

$$\mathbf{b} \quad m \text{ door } A(-3, 5) \text{ en } B(3, -1), \text{ dus } \vec{r}_m = \begin{pmatrix} 3-3 \\ -1-5 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

$$m \text{ door } A(-3, 5), \text{ dus } \vec{s}_m = \begin{pmatrix} -3 \\ 5 \end{pmatrix}.$$

$$\text{Dus } m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

n staat loodrecht op $p: -x+5y=4$, dus $n: 5x+y=c$.

$$\left. \begin{array}{l} n: 5x+y=c \\ \text{door } C(-4, -2) \end{array} \right\} c = 5 \cdot -4 + -2 = -22$$

Dus $n: 5x+y=-22$.

Substitutie van $x=-3+t$ en $y=5-t$ in $5x+y=-22$ geeft $5(-3+t)+5-t=-22$

$$-15+5t+5-t=-22$$

$$4t=-12$$

$$t=-3$$

$$t=-3 \text{ geeft } x=-3+-3=-6 \text{ en } y=5--3=8$$

Dus $T(-6, 8)$.

$$\mathbf{c} \quad \vec{r}_q = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, \text{ dus } \vec{n}_q = \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

$$\left. \begin{array}{l} q: x+4y=c \\ \text{door } (5, 3) \end{array} \right\} c = 5+4 \cdot 3 = 17$$

Dus $q: x+4y=17$.

Substitutie van $x=2w-4$ en $y=w-6$ in $x+4y=17$ geeft $2w-4+4(w-6)=17$

$$2w-4+4w-24=17$$

$$6w=45$$

$$w=7\frac{1}{2}$$

$$w=7\frac{1}{2} \text{ geeft } x=2 \cdot 7\frac{1}{2}-4=11 \text{ en } y=7\frac{1}{2}-6=1\frac{1}{2}$$

Dus $U(11, 1\frac{1}{2})$.

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$\mathbf{a}$ Snijden van k met de x -as geeft $y=0$

$$3+t=0$$

$$t=-3$$

$t=-3$ geeft $x=-2+-3 \cdot 5=-17$, dus $A(-17, 0)$.

$$l \perp k, \text{ dus } \vec{n}_l = \vec{r}_k = \begin{pmatrix} 5 \\ 1 \end{pmatrix}.$$

$$\left. \begin{array}{l} l: 5x+y=c \\ \text{door } A(-17, 0) \end{array} \right\} c = 5 \cdot -17 + 0 = -85$$

Dus $l: 5x+y=-85$.

$$\mathbf{b} \quad \vec{r}_m = \vec{b} - \vec{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$n // m, \text{ dus } \vec{r}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ en } (0, 0) \text{ op } n, \text{ dus } \vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$\text{Dus } n: \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

$$\mathbf{c} \quad \vec{r}_q = \vec{e} - \vec{o} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$p \perp q, \text{ dus } \vec{n}_q = \vec{r}_p = \begin{pmatrix} 1 \\ -4 \end{pmatrix}.$$

$$\left. \begin{array}{l} q: x-4y=c \\ \text{door } D(-4, 7) \end{array} \right\} c = -4-4 \cdot 7 = -32$$

Dus $q: x-4y=-32$.

60 a $\vec{n}_l = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left| \begin{pmatrix} -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} -4 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|} = \frac{|-4 \cdot 5 + 1 \cdot 2|}{\sqrt{17} \cdot \sqrt{29}} = \frac{18}{\sqrt{493}}$$

Dus $\angle(k, l) \approx 35,8^\circ$.

b $\vec{n}_m = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$ en $\vec{n}_n = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 3 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 3 \\ -7 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right|} = \frac{|3 \cdot 1 + -7 \cdot 2|}{\sqrt{58} \cdot \sqrt{5}} = \frac{11}{\sqrt{290}}$$

Dus $\angle(m, n) = \angle(\vec{n}_m, \vec{n}_n) \approx 49,8^\circ$.

c $\tan(\alpha) = \text{rc}_p = 2$ geeft $\alpha = 63,43\dots^\circ$

$\tan(\beta) = \text{rc}_q = -1$ geeft $\beta = -45^\circ$

$\alpha - \beta = 63,43\dots^\circ - -45^\circ = 108,43\dots^\circ$

Dus $\angle(p, q) = 180 - 108,43\dots^\circ \approx 71,6^\circ$.

61 a $k_p // l_p$, dus $\vec{r}_{k_p} = \vec{r}_{l_p} = \begin{pmatrix} 2p \\ p+5 \end{pmatrix}$ en $\vec{n}_{l_p} = \begin{pmatrix} p+5 \\ -2p \end{pmatrix}$.

$l_p: (p+5)x - 2py = c$
door $B(-1, -2)$ $\left\} c = -(p+5) + 4p$

$l_p: (p+5)x - 2py = c$
door $C(p, 4)$ $\left\} c = (p+5)p - 8p$

$-(p+5) + 4p = (p+5)p - 8p$

$-p - 5 + 4p = p^2 + 5p - 8p$

$p^2 - 6p + 5 = 0$

$(p-1)(p-5) = 0$

$p = 1 \vee p = 5$

$p = 1$ geeft $l_1: (1+5)x - 2y = -(1+5) + 4$ oftewel $l_1: 6x - 2y = -2$, dus $l_1: 3x - y = -1$.

$p = 5$ geeft $l_5: (5+5)x - 2 \cdot 5y = -(5+5) + 4 \cdot 5$ oftewel $l_5: 10x - 10y = 10$, dus

$l_5: x - y = 1$.

b $D(-2, p-4)$ op $m_p: (p+3)x + (p-1)y = -12$ geeft $(p+3) \cdot -2 + (p-1)(p-4) = -12$

$-2p - 6 + p^2 - 4p - p + 4 = -12$

$p^2 - 7p + 10 = 0$

$(p-2)(p-5) = 0$

$p = 2 \vee p = 5$

$p = 2$ geeft $D(-2, -2)$, $m_2: 5x + y = -12$ en $n_{2,q}: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$.

$m_2 \perp n_{2,q}$, dus $\vec{n}_{m_2} = \vec{r}_{n_{2,q}}$, dus $\begin{pmatrix} 5 \\ 1 \end{pmatrix} = u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$. Dit geeft $5 = qu$ en $1 = u(3-q)$

$1 = 3u - qu$

$1 = 3u - 5$

$3u = 6$

$u = 2$

$5 = qu$ en $u = 2$ geeft $q = 2\frac{1}{2}$.

$p = 5$ geeft $D(-2, 1)$, $m_5: 8x + 4y = -12$ oftewel $m_5: 2x + y = -3$ en $n_{5,q}: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$.

$m_5 \perp n_{5,q}$, dus $\vec{n}_{m_5} = \vec{r}_{n_{5,q}}$, dus $\begin{pmatrix} 2 \\ 1 \end{pmatrix} = u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$. Dit geeft $2 = qu$ en $1 = u(3-q)$

$1 = 3u - qu$

$1 = 3u - 2$

$3u = 3$

$u = 1$

$2 = qu$ en $u = 1$ geeft $q = 2$.

Dus $(p = 2 \wedge q = 2\frac{1}{2}) \vee (p = 5 \wedge q = 2)$.

62 a P op BD , dus $\vec{p} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix}$.

$$\overrightarrow{CP} = \vec{p} - \vec{c} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix}$$

b $CP \perp BD$, dus $\overrightarrow{CP} \cdot \vec{r}_{BD} = 0$

$$\begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0$$

$$t-1+9t+12=0$$

$$10t = -11$$

$$t = -1\frac{1}{10}$$

$$t = -1\frac{1}{10} \text{ geeft } P(2 - 1\frac{1}{10}, 5 + 3 \cdot -1\frac{1}{10}) = P(\frac{9}{10}, 1\frac{7}{10})$$

c $CP = OP$, dus $|\overrightarrow{CP}| = |\overrightarrow{OP}|$

$$\left| \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \right| = \left| \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} \right|$$

$$\sqrt{(t-1)^2 + (3t+4)^2} = \sqrt{(2+t)^2 + (5+3t)^2}$$

kwadrateren geeft

$$(t-1)^2 + (3t+4)^2 = (2+t)^2 + (5+3t)^2$$

$$t^2 - 2t + 1 + 9t^2 + 24t + 16 = 4 + 4t + t^2 + 25 + 30t + 9t^2$$

$$-12t = 12$$

$$t = -1$$

$$t = -1 \text{ geeft } P(2 + -1, 5 + 3 \cdot -1) = P(1, 2)$$

63 a $y = 0$ geeft $4x = 12$, dus $x = 3$ en $A(3, 0)$.

$x = 0$ geeft $3y = 12$, dus $y = 4$ en $B(0, 4)$.

P op l , dus $\vec{p} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix}$.

$$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix}$$

$$\overrightarrow{BP} = \vec{p} - \vec{b} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix}$$

$AP \perp BP$, dus $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$

$$\begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix} \cdot \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix} = 0$$

$$4t^2 + 12t + 6t + 18 + 9t^2 + 15t + 3t + 5 = 0$$

$$13t^2 + 36t + 23 = 0$$

$$D = 36^2 - 4 \cdot 13 \cdot 23 = 100$$

$$t = \frac{-36 \pm 10}{26} = -1 \vee t = \frac{-36 - 10}{26} = -1\frac{10}{13}$$

$$t = -1 \text{ geeft } P(6 + 2 \cdot -1, -1 - 3 \cdot -1) = P(4, 2)$$

$$t = -1\frac{10}{13} \text{ geeft } P(6 + 2 \cdot -1\frac{10}{13}, -1 - 3 \cdot -1\frac{10}{13}) = P(2\frac{6}{13}, 4\frac{4}{13})$$

b $AP = BP$, dus $|\overrightarrow{AP}| = |\overrightarrow{BP}|$

$$\left| \begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix} \right| = \left| \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix} \right|$$

$$\sqrt{(2t+3)^2 + (-3t-1)^2} = \sqrt{(2t+6)^2 + (-3t-5)^2}$$

kwadrateren geeft

$$(2t+3)^2 + (-3t-1)^2 = (2t+6)^2 + (-3t-5)^2$$

$$4t^2 + 12t + 9 + 9t^2 + 6t + 1 = 4t^2 + 24t + 36 + 9t^2 + 30t + 25$$

$$-36t = 51$$

$$t = -1\frac{5}{12}$$

$$t = -1\frac{5}{12} \text{ geeft } P(6 + 2 \cdot -1\frac{5}{12}, -1 - 3 \cdot -1\frac{5}{12}) = P(3\frac{1}{6}, 3\frac{1}{4})$$

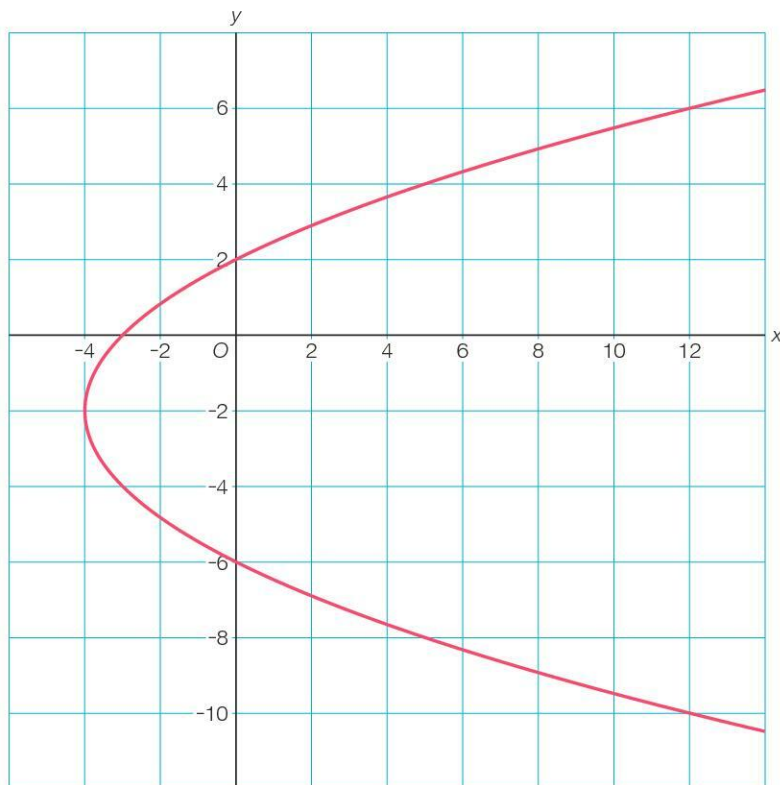
10.5 Vectoren bij snelheid en versnelling

Bladzijde 82

64 a $x(0) = 0 - 0 = 0$ en $y(0) = 0 - 6 = -6$, dus op $t = 0$ is P in het punt $(0, -6)$.

b

t	-2	-1	0	1	2	3	4	5	6
x	12	5	0	-3	-4	-3	0	5	12
y	-10	-8	-6	-4	-2	0	2	4	6



c De verplaatsing van P op $[2, 3]$ is $\begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Bladzijde 85

65 a Evenwijdig met de y -as, dus $x'(t) = 0 \wedge y'(t) \neq 0$
 $t^2 - 4 = 0 \wedge 2t - 2 \neq 0$
 $t^2 = 4 \wedge 2t \neq 2$
 $(t = 2 \vee t = -2) \wedge t \neq 1$
 $t = 2 \vee t = -2$

$$x(2) = \frac{1}{3} \cdot 2^3 - 4 \cdot 2 = -5\frac{1}{3} \text{ en } y(2) = 2^2 - 2 \cdot 2 = 0$$

$$x(-2) = \frac{1}{3} \cdot (-2)^3 - 4 \cdot (-2) = 5\frac{1}{3} \text{ en } y(-2) = (-2)^2 - 2 \cdot (-2) = 8$$

Dus $(-5\frac{1}{3}, 0)$ en $(5\frac{1}{3}, 8)$.

b Door de oorsprong, dus $x(t) = 0 \wedge y(t) = 0$
 $\frac{1}{3}t^3 - 4t = 0 \wedge t^2 - 2t = 0$
 $\frac{1}{3}t(t^2 - 12) = 0 \wedge t(t - 2) = 0$
 $(t = 0 \vee t^2 = 12) \wedge (t = 0 \vee t = 2)$
 $t = 0$

Stel $l: ax + by = 0$.

$$\vec{r}_l = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 0 - 4 \\ 0 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \text{ dus } \vec{n}_l = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Dus $l: x - 2y = 0$.

c Naar links bewegen betekent $x'(t) < 0$ en omhoog bewegen betekent $y'(t) > 0$, dus $x'(t) < 0 \wedge y'(t) > 0$ en dit geeft $t^2 - 4 < 0 \wedge 2t - 2 > 0$.

$$\begin{aligned} \text{d } t^2 - 4 &< 0 \wedge 2t - 2 > 0 \\ t^2 &< 4 \wedge 2t > 2 \\ -2 &< t < 2 \wedge t > 1 \\ 1 &< t < 2 \end{aligned}$$

66

a $x(t) = t^2 - 2$ geeft $x'(t) = 2t$

$y(t) = t^3 - 4t$ geeft $y'(t) = 3t^2 - 4$

Evenwijdig met de x -as, dus $y'(t) = 0 \wedge x'(t) \neq 0$

$$3t^2 - 4 = 0 \wedge 2t \neq 0$$

$$3t^2 = 4 \wedge t \neq 0$$

$$t^2 = \frac{4}{3} \wedge t \neq 0$$

$$t = \sqrt{\frac{4}{3}} = \frac{2}{3}\sqrt{3} \vee t = -\frac{2}{3}\sqrt{3}$$

$$x(\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3} \text{ en } y(\frac{2}{3}\sqrt{3}) = (\frac{2}{3}\sqrt{3})^3 - 4 \cdot \frac{2}{3}\sqrt{3} = \frac{8}{9}\sqrt{3} - \frac{8}{3}\sqrt{3} = -1\frac{7}{9}\sqrt{3}$$

$$x(-\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3} \text{ en } y(-\frac{2}{3}\sqrt{3}) = (-\frac{2}{3}\sqrt{3})^3 - 4 \cdot -\frac{2}{3}\sqrt{3} = -\frac{8}{9}\sqrt{3} + \frac{8}{3}\sqrt{3} = 1\frac{7}{9}\sqrt{3}$$

Dus de punten zijn $(-\frac{2}{3}, -1\frac{7}{9}\sqrt{3})$ en $(-\frac{2}{3}, 1\frac{7}{9}\sqrt{3})$.

b $v(-1) = \sqrt{(x'(-1))^2 + (y'(-1))^2} = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$

c $x'(t) > 0 \wedge y'(t) < 0$ geeft $2t > 0 \wedge 3t^2 - 4 < 0$

$$t > 0 \wedge 3t^2 < 4$$

$$t > 0 \wedge t^2 < \frac{4}{3}$$

$$t > 0 \wedge -\sqrt{\frac{4}{3}} < t < \sqrt{\frac{4}{3}}$$

$$t > 0 \wedge -\frac{2}{3}\sqrt{3} < t < \frac{2}{3}\sqrt{3}$$

$$0 < t < \frac{2}{3}\sqrt{3}$$

d $x(t) = 2$ geeft $t^2 - 2 = 2$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$$y(2) = 2^3 - 4 \cdot 2 = 0 \text{ en } y(-2) = (-2)^3 - 4 \cdot -2 = 0.$$

$t = 2$ geeft het punt $(2, 0)$ en $t = -2$ geeft het punt $(-2, 0)$, dus de baan snijdt zichzelf in het punt $(2, 0)$.

$$\vec{v}(2) = \begin{pmatrix} x'(2) \\ y'(2) \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\vec{v}(-2) = \begin{pmatrix} x'(-2) \\ y'(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right|} = \frac{|1 \cdot 1 + 2 \cdot -2|}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5}$$

Dus $\varphi \approx 53,1^\circ$.

67

a $x(t) = 0$ geeft $2t - \frac{1}{6}t^3 = 0$

$$\frac{1}{6}t(12 - t^2) = 0$$

$$t = 0 \vee t^2 = 12$$

$$t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$$

$t = 0$ geeft $(0, 0)$

$$y(2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot 2\sqrt{3} = 3 - 4\sqrt{3}, \text{ dus } B(0, 3 - 4\sqrt{3}) \text{ en}$$

$$y(-2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot -2\sqrt{3} = 3 + 4\sqrt{3}, \text{ dus } A(0, 3 + 4\sqrt{3}).$$

$$d(A, B) = 3 + 4\sqrt{3} - (3 - 4\sqrt{3}) = 8\sqrt{3}$$

b $x'(t) = 2 - \frac{1}{2}t^2$ en $y'(t) = \frac{1}{2}t - 2$

Evenwijdig met de x -as, dus $y'(t) = 0 \wedge x'(t) \neq 0$

$$\frac{1}{2}t - 2 = 0 \wedge 2 - \frac{1}{2}t^2 \neq 0$$

$$\frac{1}{2}t = 2 \wedge \frac{1}{2}t^2 \neq 2$$

$$t = 4 \wedge t^2 \neq 4$$

$$t = 4 \wedge t \neq 2 \wedge t \neq -2$$

$$t = 4$$

$$x(4) = 2 \cdot 4 - \frac{1}{6} \cdot 4^3 = -\frac{2}{3} \text{ en } y(4) = \frac{1}{4} \cdot 4^2 - 2 \cdot 4 = -4, \text{ dus } (-\frac{2}{3}, -4).$$

Evenwijdig met de y -as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$2 - \frac{1}{2}t^2 = 0 \wedge \frac{1}{2}t - 2 \neq 0$$

$$(t = 2 \vee t = -2) \wedge t \neq 4$$

$$t = 2 \vee t = -2$$

$$x(2) = 2 \cdot 2 - \frac{1}{6} \cdot 2^3 = \frac{2}{3} \text{ en } y(2) = \frac{1}{4} \cdot 2^2 - 2 \cdot 2 = -3$$

$$x(-2) = 2 \cdot -2 - \frac{1}{6} \cdot (-2)^3 = -\frac{2}{3} \text{ en } y(-2) = \frac{1}{4} \cdot (-2)^2 - 2 \cdot -2 = 5$$

Dus $(\frac{2}{3}, -3)$ en $(-\frac{2}{3}, 5)$.

c Bij de oorsprong hoort $t = 0$.

$$v(0) = \sqrt{(x'(0))^2 + (y'(0))^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

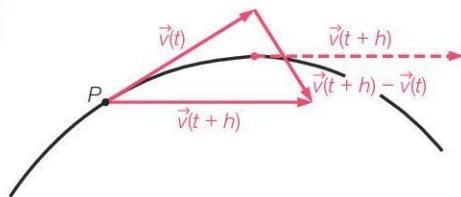
d $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(2 - \frac{1}{2}t^2)^2 + (\frac{1}{2}t - 2)^2}$

$$\text{Voer in } y_1 = \sqrt{(2 - \frac{1}{2}x^2)^2 + (\frac{1}{2}x - 2)^2}.$$

De optie minimum geeft $x = 2,109...$ en $y = 0,971...$

Dus de minimale baansnelheid is ongeveer 0,97 voor $t \approx 2,11$.

68



Bladzijde 86

69

$$\vec{v}(t) = \begin{pmatrix} 2t \\ t^3 - 2 \end{pmatrix} \text{ geeft } \vec{a}(t) = \begin{pmatrix} 2 \\ 3t^2 \end{pmatrix}$$

$$\vec{v}(t) \cdot \vec{a}(t) = \begin{pmatrix} 2t \\ t^3 - 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3t^2 \end{pmatrix} = 4t + 3t^5 - 6t^2 = 3t^5 - 6t^2 + 4t$$

$$|\vec{v}(t)| = \sqrt{t^6 - 4t^3 + 4t^2 + 4}$$

$$\text{Dit geeft } a(t) = \frac{3t^5 - 6t^2 + 4t}{\sqrt{t^6 - 4t^3 + 4t^2 + 4}}.$$

Bladzijde 87

70

a Snijden met de x -as, dus $y(t) = 0$ geeft $t^3 - 3t = 0$

$$t(t^2 - 3) = 0$$

$$t = 0 \vee t^2 = 3$$

$$t = 0 \vee t = \sqrt{3} \vee t = -\sqrt{3}$$

De baan passeert D twee keer, dat is dus op $t = -\sqrt{3}$ en $t = \sqrt{3}$. Dus op $t = 0$ is P in C .

Snijden met de y -as, dus $x(t) = 0$ geeft $t^2 - 4 = 0$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$y(2) = 2^3 - 3 \cdot 2 = 2$, dus de baan passeert A op $t = 2$ en B op $t = -2$.

Dus de punten worden in de volgorde B, D, C, D, A doorlopen.

b $\vec{r}(t) = \begin{pmatrix} t^2 - 4 \\ t^3 - 3t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 2t \\ 3t^2 - 3 \end{pmatrix}$ en

$$v(t) = \sqrt{(2t)^2 + (3t^2 - 3)^2} = \sqrt{4t^2 + 9t^4 - 18t^2 + 9} = \sqrt{9t^4 - 14t^2 + 9}.$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{9t^4 - 14t^2 + 9}} \cdot (36t^3 - 28t) = \frac{18t^3 - 14t}{\sqrt{9t^4 - 14t^2 + 9}}$$

c $v(2) = \sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9} = \sqrt{97}$

$$a(2) = \frac{18 \cdot 2^3 - 14 \cdot 2}{\sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9}} = \frac{116}{\sqrt{97}} = 1\frac{19}{97}\sqrt{97}$$

d $a(t) = 0$ geeft $18t^3 - 14t = 0$

$$2t(9t^2 - 7) = 0$$

$$t = 0 \vee t^2 = \frac{7}{9}$$

$$t = 0 \vee t = \sqrt{\frac{7}{9}} \vee t = -\sqrt{\frac{7}{9}}$$

$$t = 0 \vee t = \frac{1}{3}\sqrt{7} \vee t = -\frac{1}{3}\sqrt{7}$$

$$v(0) = \sqrt{0 + 9} = 3$$

$$v\left(\frac{1}{3}\sqrt{7}\right) = \sqrt{9 \cdot \left(\frac{7}{9}\right)^2 - 14 \cdot \frac{7}{9} + 9} = \sqrt{\frac{32}{9}} = \frac{1}{3}\sqrt{32}$$

$$v\left(-\frac{1}{3}\sqrt{7}\right) = \sqrt{9 \cdot \left(\frac{7}{9}\right)^2 - 14 \cdot \frac{7}{9} + 9} = \frac{1}{3}\sqrt{32}$$

e Voer in $y_1 = \sqrt{9x^4 - 14x^2 + 9}$.

De optie minimum geeft $y = 1,8856...$ voor $x = -0,8819...$ en $x \approx 0,8819...$

Dus de minimale baansnelheid is ongeveer 1,886 voor $t \approx -0,882$ en voor $t \approx 0,882$.

f $v(t) = 2$ geeft $\sqrt{9t^4 - 14t^2 + 9} = 2$

kwadrateren geeft

$$9t^4 - 14t^2 + 9 = 4$$

$$9t^4 - 14t^2 + 5 = 0$$

Stel $t^2 = u$.

$$9u^2 - 14u + 5 = 0$$

$$D = (-14)^2 - 4 \cdot 9 \cdot 5 = 16$$

$$u = \frac{14 \pm 4}{18} = 1 \vee \frac{14 - 4}{18} = \frac{5}{9}$$

$$t^2 = 1 \vee t^2 = \frac{5}{9}$$

$$t = 1 \vee t = -1 \vee t = \sqrt{\frac{5}{9}} \vee t = -\sqrt{\frac{5}{9}}$$

$$t = 1 \vee t = -1 \vee t = \frac{1}{3}\sqrt{5} \vee t = -\frac{1}{3}\sqrt{5}$$

g $t = \sqrt{3}$ geeft $x = (\sqrt{3})^2 - 4 = -1$ en $y = 0$

$$t = -\sqrt{3} \text{ geeft } x = (-\sqrt{3})^2 - 4 = -1 \text{ en } y = 0$$

Dus de baan snijdt zichzelf in $D(-1, 0)$.

$$\vec{v}(\sqrt{3}) = \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \text{ en } \vec{v}(-\sqrt{3}) = \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix}.$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix} \right|} = \frac{|2\sqrt{3} \cdot -2\sqrt{3} + 6 \cdot 6|}{(\sqrt{12 + 36})^2} = \frac{24}{48} = \frac{1}{2}$$

Dus $\varphi = 60^\circ$.

Bladzijde 88

71 a Snijden met de x -as, dus $y(t) = 0$ geeft $\frac{1}{2}t^2 - 2t = 0$

$$\frac{1}{2}t(t - 4) = 0$$

$$t = 0 \vee t = 4$$

$t = 0$ geeft $(-2, 0)$, dus $t = 0$ hoort bij A en $t = 4$ hoort bij B .

Dus de punten worden in de volgorde C, A, D, B doorlopen.

b $\vec{r}(t) = \begin{pmatrix} \frac{1}{2}t^2 - 2 \\ \frac{1}{2}t^2 - 2t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} t \\ t - 2 \end{pmatrix}$ en $v(t) = \sqrt{t^2 + (t - 2)^2} = \sqrt{t^2 + t^2 - 4t + 4} = \sqrt{2t^2 - 4t + 4}$.

$$a(t) = v'(t) = \frac{1}{2\sqrt{2t^2 - 4t + 4}} \cdot (4t - 4) = \frac{2t - 2}{\sqrt{2t^2 - 4t + 4}}$$

c $\vec{v}(t) = \begin{pmatrix} t \\ t - 2 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\vec{v}(t) \perp \vec{a}(t) \text{ geeft } \vec{v}(t) \cdot \vec{a}(t) = 0$$

$$\begin{pmatrix} t \\ t - 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

$$t + t - 2 = 0$$

$$2t = 2$$

$$t = 1$$

$$x(1) = \frac{1}{2} - 2 = -\frac{1}{2} \text{ en } y(1) = \frac{1}{2} - 2 = -\frac{1}{2}, \text{ dus in } \left(-\frac{1}{2}, -\frac{1}{2}\right).$$

d $a(t) = 0$ geeft $2t - 2 = 0$

$$2t = 2$$

$$t = 1$$

$$v(1) = \sqrt{2 - 4 + 4} = \sqrt{2}$$

Dus de minimale baansnelheid is $\sqrt{2}$ voor $t = 1$.

e Bij $t = 4$ hoort B , dus de baansnelheid in B is $v(4) = \sqrt{4^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5}$.

Snijden met de y -as, dus $x(t) = 0$ geeft $\frac{1}{2}t^2 - 2 = 0$

$$\frac{1}{2}t^2 = 2$$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

Bij $t = -2$ hoort dus C en dit geeft $v(-2) = \sqrt{(-2)^2 + (-2 - 2)^2} = \sqrt{20} = 2\sqrt{5}$.

Dus de baansnelheden in B en C zijn gelijk.

f $E(-1\frac{1}{2}, 2\frac{1}{2})$ invullen in $\begin{cases} x(t) = \frac{1}{2}t^2 - 2 \\ y(t) = \frac{1}{2}t^2 - 2t \end{cases}$ geeft $\begin{cases} \frac{1}{2}t^2 - 2 = -1\frac{1}{2} \\ \frac{1}{2}t^2 - 2t = 2\frac{1}{2} \end{cases}$

$$2t - 2 = -4$$

$$2t = -2$$

$$t = -1$$

$$\vec{r}_k = \vec{v}(-1) = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \text{ geeft } \vec{n}_k = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\left. \begin{array}{l} k: 3x - y = c \\ \text{door } (-1\frac{1}{2}, 2\frac{1}{2}) \end{array} \right\} c = 3 \cdot -1\frac{1}{2} - 2\frac{1}{2} = -7$$

Dus $k: 3x - y = -7$.

72 Q en R op een verticale lijn, dus $x_Q = x_R$.

Dit geeft $\frac{1}{2}t_Q^2 - 2 = \frac{1}{2}t_R^2 - 2$, dus $t_Q = -t_R$.

$$\left. \begin{array}{l} y_R - y_Q = 4 \text{ geeft } \frac{1}{2}t_R^2 - 2t_R - (\frac{1}{2}t_Q^2 - 2t_Q) = 4 \\ t_Q = -t_R \end{array} \right\} \begin{cases} \frac{1}{2}t_R^2 - 2t_R - \frac{1}{2}t_R^2 + 2t_R = 4 \\ -4t_R = 4 \end{cases}$$

$$t_R = -1$$

Dus $t_Q = 1$.

$$y(1) = \frac{1}{2} - 2 = -1\frac{1}{2} \text{ en } y(-1) = \frac{1}{2} - 2 = 2\frac{1}{2}.$$

Dus $Q(-1\frac{1}{2}, -1\frac{1}{2})$ en $R(-1\frac{1}{2}, 2\frac{1}{2})$.

73 a $\vec{r}(t) = \begin{pmatrix} 25t \\ -5t^2 + 15t + 3 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$

$$v(0) = \sqrt{25^2 + 15^2} \approx 29,2 \text{ m/s}$$

b Een richtingsvector van een horizontale lijn is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\vec{v}(0) = \begin{pmatrix} 25 \\ 15 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 25 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 25 \\ 15 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{|25 \cdot 1 + 15 \cdot 0|}{\sqrt{850}} = \frac{25}{\sqrt{850}}$$

Dus $\varphi \approx 31^\circ$.

c $y(t) = -5t^2 + 15t + 3$ geeft $y'(t) = -10t + 15$

$$y'(t) = 0 \text{ geeft } -10t + 15 = 0$$

$$-10t = -15$$

$$t = 1,5$$

$$y(1,5) = -5 \cdot 1,5^2 + 15 \cdot 1,5 + 3 = 14,25$$

Dus de maximale hoogte is 14,25 meter.

- d** $y(t) = 0$ geeft $-5t^2 + 15t + 3 = 0$
 $D = 15^2 - 4 \cdot (-5) \cdot 3 = 285$
 $t = \frac{-15 + \sqrt{285}}{-10} \vee t = \frac{-15 - \sqrt{285}}{-10}$
 $t = 1,5 - 0,1\sqrt{285} \vee t = 1,5 + 0,1\sqrt{285}$
 vold. niet
 $x(1,5 + 0,1\sqrt{285}) = 25(1,5 + 0,1\sqrt{285}) = 79,70\dots$
 Dus de speer komt 77,7 meter achter de afworplijn neer.
- e** $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$
 De versnellingsvector is de valversnelling van -10 m/s^2 .

Bladzijde 89

- 74 a** Snijden met de x -as, dus $y(t) = 0$ geeft $t^2 - 4 = 0$
 $t^2 = 4$
 $t = 2 \vee t = -2$
- $\vec{r}(t) = \begin{pmatrix} \frac{1}{3}t^3 - 4t \\ t^2 - 4 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix}$
 $\vec{v}(2) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ en $\vec{v}(-2) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$.
- De snelheidsvectoren in A en B zijn verticaal, dus de raaklijnen aan de baan in A en B zijn verticaal.
- b** $y(t) = 5$ geeft $t^2 - 4 = 5$
 $t^2 = 9$
 $t = 3 \vee t = -3$
 $x(3) = \frac{1}{3} \cdot 3^3 - 4 \cdot 3 = -3$, dus $t = 3$ hoort bij C .
 $\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix}$ geeft $v(t) = \sqrt{(t^2 - 4)^2 + (2t)^2} = \sqrt{t^4 - 8t^2 + 16 + 4t^2} = \sqrt{t^4 - 4t^2 + 16}$
 $a(t) = v'(t) = \frac{1}{2\sqrt{t^4 - 4t^2 + 16}} \cdot (4t^3 - 8t) = \frac{2t^3 - 4t}{\sqrt{t^4 - 4t^2 + 16}}$
 De baansnelheid in C is $v(3) = \sqrt{3^4 - 4 \cdot 3^2 + 16} = \sqrt{61}$.
 De baanversnelling in C is $a(3) = \frac{2 \cdot 3^3 - 4 \cdot 3}{\sqrt{3^4 - 4 \cdot 3^2 + 16}} = \frac{42}{\sqrt{61}} = \frac{42}{61}\sqrt{61}$.
- c** $v(t) = 6\frac{1}{2}$ geeft $\sqrt{t^4 - 4t^2 + 16} = 6\frac{1}{2}$
 kwadrateren geeft
 $t^4 - 4t^2 + 16 = 42\frac{1}{4}$
 $t^4 - 4t^2 - 26\frac{1}{4} = 0$
 Stel $t^2 = u$.
 $u^2 - 4u - 26\frac{1}{4} = 0$
 $D = (-4)^2 - 4 \cdot 1 \cdot (-26\frac{1}{4}) = 121$
 $u = \frac{4 + 11}{2} = 7\frac{1}{2} \vee u = \frac{4 - 11}{2} = -3\frac{1}{2}$
 $t^2 = 7\frac{1}{2} \vee t^2 = -3\frac{1}{2}$
 $t = \sqrt{7\frac{1}{2}} = \frac{1}{2}\sqrt{30} \vee t = -\sqrt{7\frac{1}{2}} = -\frac{1}{2}\sqrt{30}$
 $x(\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot \frac{1}{2}\sqrt{30} - 4 \cdot \frac{1}{2}\sqrt{30} = 1\frac{1}{4}\sqrt{30} - 2\sqrt{30} = -\frac{3}{4}\sqrt{30}$ en $y(\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}$.
 $x(-\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot -\frac{1}{2}\sqrt{30} - 4 \cdot -\frac{1}{2}\sqrt{30} = -1\frac{1}{4}\sqrt{30} + 2\sqrt{30} = \frac{3}{4}\sqrt{30}$ en $y(-\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}$.
 Dus de punten zijn $(-\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$ en $(\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$.
- d** Voer in $y_1 = \frac{2x^3 - 4x}{\sqrt{x^4 - 4x^2 + 16}}$.
 De optie minimum geeft $x = 0,8510\dots$ en $y = -0,5882\dots$.
 Dus de baanversnelling heeft voor $t \approx 0,851$ een minimum van ongeveer $-0,588$.

e Snijden met de y-as, dus $x(t) = 0$ geeft $\frac{1}{3}t^3 - 4t = 0$
 $\frac{1}{3}t(t^2 - 12) = 0$
 $t = 0 \vee t^2 = 12$
 $t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$

$y(2\sqrt{3}) = 12 - 4 = 8$ en $y(-2\sqrt{3}) = 12 - 4 = 8$.

Dus de baan snijdt zichzelf in $S(0, 8)$.

$\vec{v}(2\sqrt{3}) = \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix}$ en $\vec{v}(-2\sqrt{3}) = \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}$.

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix} \right|}{\left| \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix} \right|} = \frac{|8 \cdot 8 + 4\sqrt{3} \cdot -4\sqrt{3}|}{(\sqrt{64 + 48})^2} = \frac{16}{112} = \frac{1}{7}$$

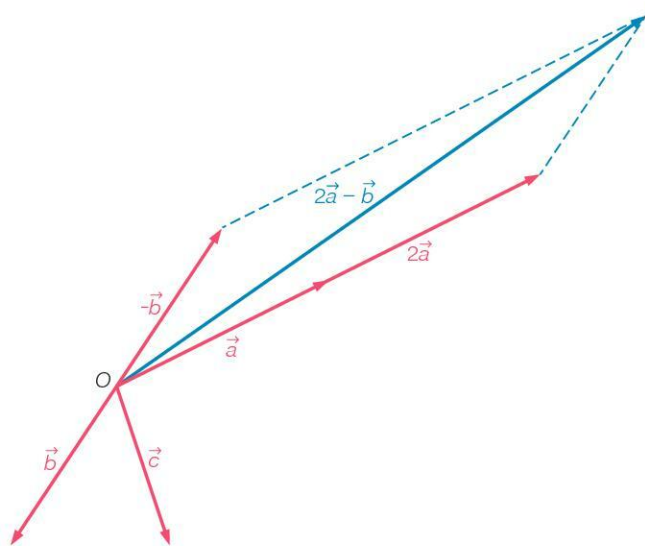
Dus $\varphi \approx 82^\circ$.

Diagnostische toets

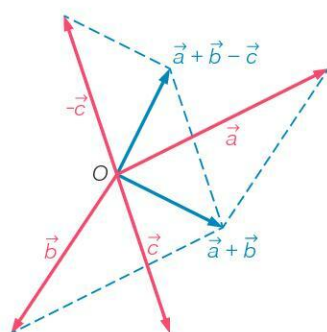
Bladzijde 92

- 1 a $\vec{c} = \vec{a} + 2\vec{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$
 $|\vec{c}| = 10$
b $\vec{d} = 2\vec{a} - 3\vec{b} = 2 \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} - 3 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$
 $|\vec{d}| = \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$
c $\vec{e} = \vec{b} - \frac{1}{2}\vec{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
 $|\vec{e}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

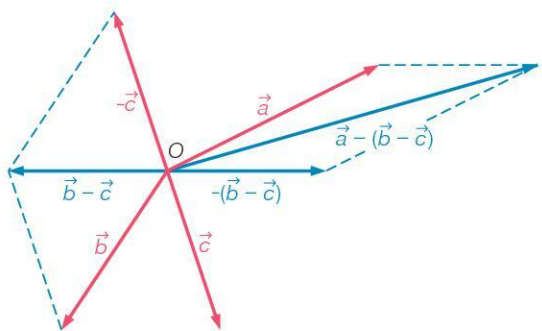
2 a



b

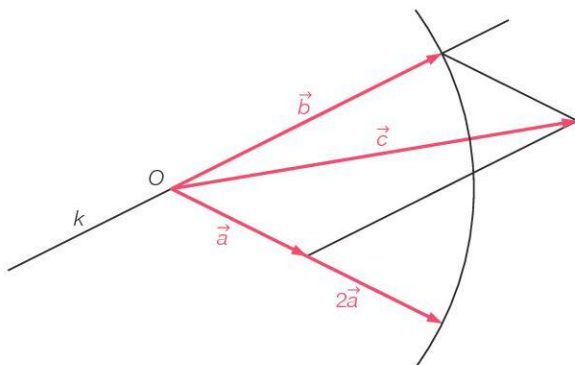
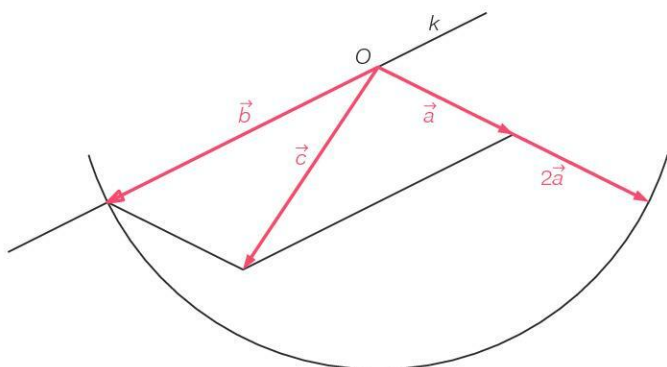


c

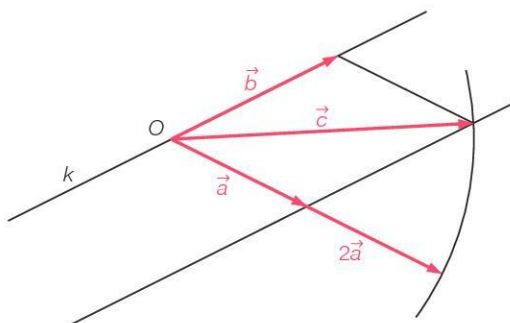
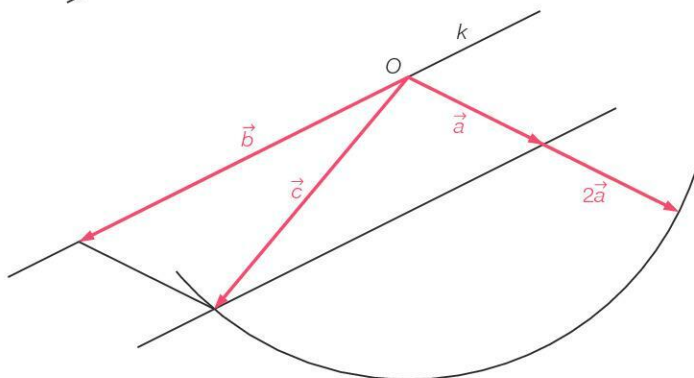


3

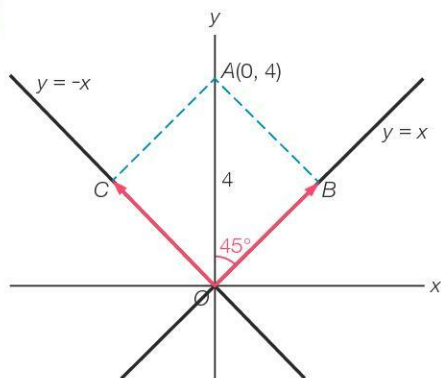
a



b



4



In $\triangle OAB$ is $OA = 4$ en $\angle O = 45^\circ$.

$$\text{Dus } OB = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

Ook $OC = 2\sqrt{2}$, dus de lengte van beide componenten is $2\sqrt{2}$.

5

M is het midden van zijde AB .

$$\vec{c} = \vec{b} + \vec{BC} = \vec{b} + \vec{MA}_R \text{ en } \vec{d} = \vec{b} + \vec{AD} = \vec{b} + \vec{BC}$$

$$M\left(\frac{1}{2}(4+8), \frac{1}{2}(0+6)\right) = M(6, 3)$$

$$\vec{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, \text{ dus } \vec{MA}_R = \begin{pmatrix} -3 \\ 2 \end{pmatrix}.$$

$$\vec{c} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$\vec{d} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Dus $C(5, 8)$ en $D(1, 2)$.

6

$$\vec{CM} = \vec{m} - \vec{c} = \frac{1}{2}(\vec{a} + \vec{b}) - \vec{c} = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 0 \end{pmatrix}\right) - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b - c \\ -d \end{pmatrix}$$

$$\vec{d} = \vec{c} + \vec{CD} = \vec{c} + \vec{CA}_R \text{ en } \vec{h} = \vec{c} + \vec{CH} = \vec{c} + \vec{CB}_L$$

$$\vec{CA} = \vec{a} - \vec{c} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a - c \\ -d \end{pmatrix} \text{ en } \vec{CB} = \vec{b} - \vec{c} = \begin{pmatrix} b \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} b - c \\ -d \end{pmatrix}$$

$$\vec{d} = \vec{c} + \vec{CD} = \vec{c} + \vec{CA}_R = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} -d \\ c - a \end{pmatrix} = \begin{pmatrix} c - d \\ -a + c + d \end{pmatrix}$$

$$\vec{h} = \vec{c} + \vec{CH} = \vec{c} + \vec{CB}_L = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} d \\ b - c \end{pmatrix} = \begin{pmatrix} c + d \\ b - c + d \end{pmatrix}$$

$$\vec{DH} = \vec{h} - \vec{d} = \begin{pmatrix} c + d \\ b - c + d \end{pmatrix} - \begin{pmatrix} c - d \\ -a + c + d \end{pmatrix} = \begin{pmatrix} 2d \\ a + b - 2c \end{pmatrix} = 2 \cdot \begin{pmatrix} d \\ \frac{1}{2}a + \frac{1}{2}b - c \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b - c \\ -d \end{pmatrix}_L = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b - c \\ d \end{pmatrix}, \text{ dus } CM \perp DH \text{ en } DH = 2 \cdot CM.$$

7

$$\text{a } k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + t \cdot (\vec{a} - \vec{c}) \quad \left. \begin{array}{l} \vec{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \end{array} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

$$\text{b } \vec{m} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2}\left(\begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{m} - \vec{a}) \quad \left. \begin{array}{l} \vec{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ en } \vec{m} - \vec{a} = \begin{pmatrix} -1\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} -7\frac{1}{2} \\ -7\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{array} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

c Stel eerst een vectorvoorstelling op van de lijn n door A en C .

$$n: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{a} - \vec{c}) \quad \left. \begin{array}{l} \vec{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \end{array} \right\} n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

Dus een vectorvoorstelling van het lijnstuk AC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix} \wedge -1 \leq t \leq 0$.

8 Substitutie van $x = -2 + 2t$ en $y = 3 - t$ in $2x - 3y = 22$ geeft $2(-2 + 2t) - 3(3 - t) = 22$
 $-4 + 4t - 9 + 3t = 22$
 $7t = 35$
 $t = 5$

$t = 5$ geeft $x = -2 + 5 \cdot 2 = 8$ en $y = 3 - 5 = -2$, dus $S(8, -2)$.

9 a $k: x = 1 + 3t \wedge y = 2 + 4t$
 b $x = t$ geeft $3t - y = 6$ oftewel $y = 3t - 6$.

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

c $\begin{cases} x = 4 - 2t \\ y = 6 + 3t \end{cases} \Big| \begin{matrix} 3 \\ 2 \end{matrix}$ geeft $\begin{cases} 3x = 12 - 6t \\ 2y = 12 + 6t \end{cases} +$
 $3x + 2y = 24$

Dus $m: 3x + 2y = 24$.

10 a $2 + t = 2p$ geeft $t = 2p - 2$
 $\begin{cases} 3 - 2t = 3 - p \\ t = 2p - 2 \end{cases} \Rightarrow \begin{cases} 3 - 2(2p - 2) = 3 - p \\ 3 - 4p + 4 = 3 - p \end{cases}$
 $-3p = -4$
 $p = 1\frac{1}{3}$

Dus $A(2 \cdot 1\frac{1}{3}, 3 - 1\frac{1}{3}) = A(2\frac{2}{3}, 1\frac{2}{3})$.

b $3 - 2t = 6$
 $-2t = 3$
 $t = -1\frac{1}{2}$
 $\begin{cases} q + t = 1 \\ t = -1\frac{1}{2} \end{cases} \Rightarrow q = 2\frac{1}{2}$

11 a $\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$
 $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$
 $\cos(\angle(AB, BC)) = \frac{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ -5 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -7 \\ -5 \end{pmatrix} \right|} = \frac{|6 \cdot -7 + 1 \cdot -5|}{\sqrt{37} \cdot \sqrt{74}} = \frac{47}{\sqrt{2738}}$

Dus $\angle(AB, BC) \approx 26,1^\circ$.

b $\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ en $\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$
 $\cos(\angle(\vec{AB}, \vec{AC})) = \frac{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|} = \frac{-6 - 4}{\sqrt{37} \cdot \sqrt{17}} = \frac{-10}{\sqrt{629}}$
 Dus $\angle BAC = \angle(\vec{AB}, \vec{AC}) \approx 113,5^\circ$.

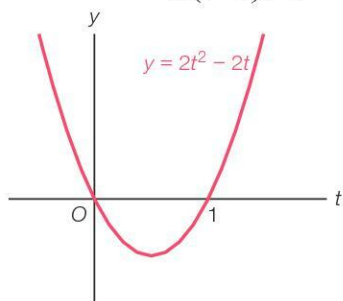
12 a $\vec{r}_k = \vec{r}_l = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.
 $k: 3x + 5y = c$
 door $(3, 4)$ $\Rightarrow c = 3 \cdot 3 + 5 \cdot 4 = 29$
 Dus $k: 3x + 5y = 29$.

b $(1, 1)$ op m , dus $\vec{s}_m = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
 $m \perp n$, dus $\vec{r}_m = \vec{n}_n = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.
 Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

c $p \perp r$, dus $\vec{n}_p = \vec{r}_r = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.
 $p: 4x - y = c$
 door $(-2, 3)$ } $c = 4 \cdot -2 - 3 = -11$
 Dus $p: 4x - y = -11$.

- 13 Neem $x = 3t$ bij k , dan is $y = \frac{1}{3} \cdot 3t + 3 = t + 3$.
 Dus een parametervoorstelling van k is $k: x = 3t \wedge y = t + 3$.
 $\vec{r}_k = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ en $\vec{r}_l = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ geeft $\vec{r}_k \cdot \vec{r}_l = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} = 3 - 3 = 0$
 Dus $\angle(k, l) = 90^\circ$.

- 14 a $x(t) = 4t - t^2$ geeft $x'(t) = 4 - 2t$
 $y(t) = \frac{2}{3}t^3 - t^2$ geeft $y'(t) = 2t^2 - 2t$
 $x'(t) > 0$ geeft $4 - 2t > 0$
 $-2t > -4$
 $t < 2$
 $y'(t) > 0$ geeft $2t^2 - 2t > 0$
 $2t(t - 1) > 0$



- $2t^2 - 2t > 0$ geeft $t < 0 \vee t > 1$
 $t < 2 \wedge (t < 0 \vee t > 1)$ geeft $t < 0 \vee 1 < t < 2$
 Dus voor $t < 0$ en voor $1 < t < 2$ beweegt P zowel naar rechts als omhoog.
 b $y'(t) = 0$ geeft $t = 0 \vee t = 1$ (zie a) met de bijbehorende punten $(0, 0)$ en $(3, -\frac{1}{3})$.
 $x'(t) = 0$ geeft $t = 2$ (zie a) met het bijbehorende punt $(4, 1\frac{1}{3})$.
 Dus evenwijdig met de x -as in $(0, 0)$ en $(3, -\frac{1}{3})$ en evenwijdig met de y -as in $(4, 1\frac{1}{3})$.
 c $y(t) = 0$ geeft $\frac{2}{3}t^3 - t^2 = 0$
 $t^2(\frac{2}{3}t - 1) = 0$
 $t = 0 \vee \frac{2}{3}t = 1$
 $t = 0 \vee t = 1\frac{1}{2}$
 vold. niet
 $\vec{v}(1\frac{1}{2}) = \begin{pmatrix} x'(1\frac{1}{2}) \\ y'(1\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} 4 - 2 \cdot 1\frac{1}{2} \\ 2 \cdot (1\frac{1}{2})^2 - 2 \cdot 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix}$
 Bij de x -as hoort richtingsvector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
 $\cos(\varphi) = \frac{\left| \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{1}{\sqrt{3\frac{1}{4}}}$
 Dus $\varphi \approx 56,3^\circ$.
 d $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(4 - 2t)^2 + (2t^2 - 2t)^2}$
 $v(1\frac{1}{2}) = \sqrt{(4 - 3)^2 + (4\frac{1}{2} - 3)^2} = \sqrt{1 + 2\frac{1}{4}} = \sqrt{3\frac{1}{4}} = \frac{1}{2}\sqrt{13}$
 e Voer in $y_1 = \sqrt{(4 - 2x)^2 + (2x^2 - 2x)^2}$.
 De optie minimum geeft $x = 1,317\dots$ en $y = 1,600\dots$
 Dus de minimale baansnelheid van P is ongeveer 1,60 voor $t \approx 1,32$.

f $\vec{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix} = \begin{pmatrix} -2 \\ 4t-2 \end{pmatrix}$

$\vec{a}(t)$ is evenwijdig met de x -as geeft $y''(t) = 0$

$$4t - 2 = 0$$

$$4t = 2$$

$$t = \frac{1}{2}$$

$$x\left(\frac{1}{2}\right) = 4 \cdot \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 1\frac{3}{4} \text{ en } y\left(\frac{1}{2}\right) = \frac{2}{3} \cdot \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 = -\frac{1}{6}$$

Dus in het punt $(1\frac{3}{4}, -\frac{1}{6})$.

g $v(t) = \sqrt{(4-2t)^2 + (2t^2-2t)^2} = \sqrt{16-16t+4t^2+4t^4-8t^3+4t^2} = \sqrt{4t^4-8t^3+8t^2-16t+16}$

$$a(t) = v'(t) = \frac{1}{2\sqrt{4t^4-8t^3+8t^2-16t+16}} \cdot (16t^3-24t^2+16t-16) = \frac{8t^3-12t^2+8t-8}{\sqrt{4t^4-8t^3+8t^2-16t+16}}$$

$$a(3) = \frac{216-108+24-8}{\sqrt{324-216+72-48+16}} = \frac{124}{\sqrt{148}} = \frac{124}{2\sqrt{37}} = \frac{1}{37}^{25}\sqrt{37}$$

11 Integraalrekening

Voorkennis Differentiaalrekening

Bladzijde 98

- 1 a** $f(x) = 4 \ln^2(x) - 3 \ln(x)$ geeft $f'(x) = 8 \ln(x) \cdot \frac{1}{x} - 3 \cdot \frac{1}{x} = \frac{8 \ln(x) - 3}{x}$
- b** $f(x) = \frac{10 \cdot 3^x}{\ln(3)} = \frac{10}{\ln(3)} \cdot 3^x$ geeft $f'(x) = \frac{10}{\ln(3)} \cdot 3^x \cdot \ln(3) = 10 \cdot 3^x$
- c** $f(x) = x^2 \cdot \sqrt{2x-1}$ geeft
 $f'(x) = 2x \cdot \sqrt{2x-1} + x^2 \cdot \frac{1}{2\sqrt{2x-1}} \cdot 2 = 2x\sqrt{2x-1} + \frac{x^2}{\sqrt{2x-1}} = \frac{2x(2x-1) + x^2}{\sqrt{2x-1}} = \frac{5x^2 - 2x}{\sqrt{2x-1}}$
- d** $f(x) = \frac{e^{2x} + 3}{4e^x} = \frac{1}{4}e^x + \frac{3}{4}e^{-x}$ geeft $f'(x) = \frac{1}{4}e^x + \frac{3}{4}e^{-x} \cdot -1 = \frac{e^x}{4} - \frac{3}{4e^x} = \frac{e^{2x} - 3}{4e^x}$
- 2 a** $f(x) = x \ln(x) - x$ geeft $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$
- b** $f(x) = x\sqrt{4x-3}$ geeft
 $f'(x) = 1 \cdot \sqrt{4x-3} + x \cdot \frac{1}{2\sqrt{4x-3}} \cdot 4 = \sqrt{4x-3} + \frac{2x}{\sqrt{4x-3}} = \frac{4x-3+2x}{\sqrt{4x-3}} = \frac{6x-3}{\sqrt{4x-3}}$
- c** $f(x) = \frac{e^{4x} - 5}{2e^{3x}} = \frac{1}{2}e^x - \frac{5}{2}e^{-3x}$ geeft $f'(x) = \frac{1}{2}e^x - \frac{5}{2}e^{-3x} \cdot -3 = \frac{1}{2}e^x + \frac{15}{2e^{3x}} = \frac{e^{4x} + 15}{2e^{3x}}$
- d** $f(x) = x^2 \ln^2(x) - x^2 \ln(x)$ geeft
 $f'(x) = 2x \cdot \ln^2(x) + x^2 \cdot 2 \ln(x) \cdot \frac{1}{x} - 2x \cdot \ln(x) - x^2 \cdot \frac{1}{x} = 2x \ln^2(x) + 2x \ln(x) - 2x \ln(x) - x = 2x \ln^2(x) - x$
- e** $f(x) = 6x^2 \sqrt{x^2+1}$ geeft
 $f'(x) = 12x \cdot \sqrt{x^2+1} + 6x^2 \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x = 12x\sqrt{x^2+1} + \frac{6x^3}{\sqrt{x^2+1}} = \frac{12x(x^2+1) + 6x^3}{\sqrt{x^2+1}} = \frac{18x^3 + 12x}{\sqrt{x^2+1}}$
- f** $f(x) = \frac{5 \cdot 2^x}{\ln(2)} - \frac{e^{2x} - 1}{e^x} = \frac{5}{\ln(2)} \cdot 2^x - (e^x - e^{-x}) = \frac{5}{\ln(2)} \cdot 2^x - e^x + e^{-x}$ geeft
 $f'(x) = \frac{5}{\ln(2)} \cdot 2^x \cdot \ln(2) - e^x + e^{-x} \cdot -1 = 5 \cdot 2^x - \left(e^x + \frac{1}{e^x}\right) = 5 \cdot 2^x - \frac{e^{2x} + 1}{e^x}$
- 3 a** $f(x) = x + \frac{1}{1+e^x} - \ln(1+e^x) = x + (1+e^x)^{-1} - \ln(1+e^x)$ geeft
 $f'(x) = 1 - (1+e^x)^{-2} \cdot e^x - \frac{1}{1+e^x} \cdot e^x = 1 - \frac{e^x}{(1+e^x)^2} - \frac{e^x}{1+e^x}$
 $= \frac{(1+e^x)^2 - e^x - e^x(1+e^x)}{(1+e^x)^2} = \frac{1 + 2e^x + e^{2x} - e^x - e^x - e^{2x}}{(1+e^x)^2} = \frac{1}{(1+e^x)^2}$
- b** $f(x) = \sin(x) - \frac{1}{3} \sin^3(x)$ geeft
 $f'(x) = \cos(x) - \sin^2(x) \cdot \cos(x) = \cos(x) - (1 - \cos^2(x)) \cdot \cos(x) = \cos(x) - \cos(x) + \cos^3(x) = \cos^3(x)$
- c** $f(x) = x + 2 - \frac{4}{x+2} - 4 \ln(x+2) = x + 2 - 4(x+2)^{-1} - 4 \ln(x+2)$ geeft
 $f'(x) = 1 + 4(x+2)^{-2} - 4 \cdot \frac{1}{x+2} = 1 + \frac{4}{(x+2)^2} - \frac{4}{x+2}$
 $= \frac{(x+2)^2 + 4 - 4(x+2)}{(x+2)^2} = \frac{x^2 + 4x + 4 + 4 - 4x - 8}{(x+2)^2} = \frac{x^2}{(x+2)^2}$

d $f(x) = (0,2x^2 - 0,4x + 1,2)\sqrt{2x+3}$ geeft

$$f'(x) = (0,4x - 0,4) \cdot \sqrt{2x+3} + (0,2x^2 - 0,4x + 1,2) \cdot \frac{1}{2\sqrt{2x+3}} \cdot 2$$

$$= (0,4x - 0,4)\sqrt{2x+3} + \frac{0,2x^2 - 0,4x + 1,2}{\sqrt{2x+3}} = \frac{(0,4x - 0,4)(2x+3)}{\sqrt{2x+3}} + \frac{0,2x^2 - 0,4x + 1,2}{\sqrt{2x+3}}$$

$$= \frac{0,8x^2 + 1,2x - 0,8x - 1,2 + 0,2x^2 - 0,4x + 1,2}{\sqrt{2x+3}} = \frac{x^2}{\sqrt{2x+3}}$$

11.1 Primitieven en integralen

Bladzijde 99

- 1** $g(x) = x^3 + c$ geeft $g'(x) = 3x^2$ en dit is $f(x)$.
 $g(2) = 15$ geeft $2^3 + c = 15$ oftewel $8 + c = 15$, dus $c = 7$.

Bladzijde 100

- 2 a** $F(x) = \frac{1}{9}(3x+4)^3$ geeft $F'(x) = \frac{1}{3}(3x+4)^2 \cdot 3 = (3x+4)^2$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- b** $F(x) = x - \ln(x+2)$ geeft $F'(x) = 1 - \frac{1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+1}{x+2}$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- c** $F(x) = \frac{-1}{2(x^2+1)} = -\frac{1}{2}(x^2+1)^{-1}$ geeft $F'(x) = \frac{1}{2}(x^2+1)^{-2} \cdot 2x = \frac{x}{(x^2+1)^2}$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- 3 a** $F(x) = (x^2+1)^6 + 1$ geeft $F'(x) = 6(x^2+1)^5 \cdot 2x = 12x(x^2+1)^5$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- b** $G(x) = (\frac{1}{2}x - \frac{1}{4})e^{2x} - 2$ geeft $G'(x) = \frac{1}{2} \cdot e^{2x} + (\frac{1}{2}x - \frac{1}{4}) \cdot e^{2x} \cdot 2 = (\frac{1}{2} + x - \frac{1}{2}) \cdot e^{2x} = x e^{2x}$
Dus $G'(x) = g(x)$ oftewel G is een primitieve van g .
- c** $H(x) = \ln^2(x) + 2 \ln(x) + 3$ geeft $H'(x) = 2 \ln(x) \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2 \ln(x) + 2}{x}$
Dus $H'(x) = h(x)$ oftewel H is een primitieve van h .
- d** $J(x) = \frac{e^{3x} - 10}{2e^x} - 4 = \frac{1}{2}e^{2x} - 5e^{-x} - 4$ geeft $J'(x) = \frac{1}{2}e^{2x} \cdot 2 - 5e^{-x} \cdot -1 = e^{2x} + \frac{5}{e^x} = \frac{e^{3x} + 5}{e^x}$
Dus $J'(x) = j(x)$ oftewel J is een primitieve van j .
- e** $K(x) = \sin^3(x)$ geeft $K'(x) = 3 \sin^2(x) \cdot \cos(x) = 3(1 - \cos^2(x)) \cdot \cos(x) = 3 \cos(x) - 3 \cos^3(x)$
Dus $K'(x) = k(x)$ oftewel K is een primitieve van k .

- 4** $F(x) = \ln(x + \sqrt{x^2-1})$ geeft
- $$F'(x) = \frac{1}{x + \sqrt{x^2-1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right) = \frac{1}{x + \sqrt{x^2-1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{1}{x + \sqrt{x^2-1}} \cdot \frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}}$$
- $$= \frac{\sqrt{x^2-1} + x}{x + \sqrt{x^2-1}} \cdot \frac{1}{\sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$$
- Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .

- 5 a** $F(x) = \frac{1}{5}x^5$ geeft $F'(x) = x^4$
Dus $F(x) = \frac{1}{5}x^5$ is een primitieve van $f(x) = x^4$.
- b** $G(x) = \frac{1}{4}e^{4x+1}$ geeft $G'(x) = \frac{1}{4}e^{4x+1} \cdot 4 = e^{4x+1}$
Dus $G(x) = \frac{1}{4}e^{4x+1}$ is een primitieve van $g(x) = e^{4x+1}$.
- c** $H(x) = \frac{3^x}{\ln(3)} = \frac{1}{\ln(3)} \cdot 3^x$ geeft $H'(x) = \frac{1}{\ln(3)} \cdot 3^x \cdot \ln(3) = 3^x$
Dus $H(x) = \frac{3^x}{\ln(3)}$ is een primitieve van $h(x) = 3^x$.
- d** $K(x) = \frac{1}{2} \sin(2x)$ geeft $K'(x) = \frac{1}{2} \cdot \cos(2x) \cdot 2 = \cos(2x)$
Dus $K(x) = \frac{1}{2} \sin(2x)$ is een primitieve van $k(x) = \cos(2x)$.

Bladzijde 101

6 $F(x) = a \cdot \frac{1}{n+1} x^{n+1} + c$ geeft $F'(x) = a \cdot \frac{1}{n+1} \cdot (n+1)x^n = ax^n$

Dus $F(x) = a \cdot \frac{1}{n+1} x^{n+1} + c$ zijn de primitieven van $f(x) = ax^n$.

$$F(x) = \frac{g^x}{\ln(g)} + c = \frac{1}{\ln(g)} \cdot g^x + c \text{ geeft } F'(x) = \frac{1}{\ln(g)} \cdot g^x \cdot \ln(g) = g^x$$

Dus $F(x) = \frac{g^x}{\ln(g)} + c$ zijn de primitieven van $f(x) = g^x$.

$$F(x) = e^x + c \text{ geeft } F'(x) = e^x$$

Dus $F(x) = e^x + c$ zijn de primitieven van $f(x) = e^x$.

$$F(x) = \ln|x| + c = \begin{cases} \ln(x) + c & \text{voor } x > 0 \\ \ln(-x) + c & \text{voor } x < 0 \end{cases} \text{ geeft } F'(x) = \begin{cases} \frac{1}{x} & \text{voor } x > 0 \\ -\frac{1}{x} \cdot -1 = \frac{1}{x} & \text{voor } x < 0 \end{cases}$$

Dus $F(x) = \ln|x| + c$ zijn de primitieven van $f(x) = \frac{1}{x}$.

$$F(x) = x \ln(x) - x + c \text{ geeft } F'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x)$$

Dus $F(x) = x \ln(x) - x + c$ zijn de primitieven van $f(x) = \ln(x)$.

$$F(x) = \frac{1}{\ln(g)} (x \ln(x) - x) + c \text{ geeft}$$

$$F'(x) = \frac{1}{\ln(g)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 \right) = \frac{1}{\ln(g)} \cdot \ln(x) = \frac{\ln(x)}{\ln(g)} = {}^g\log(x)$$

Dus $F(x) = \frac{1}{\ln(g)} (x \ln(x) - x) + c$ zijn de primitieven van $f(x) = {}^g\log(x)$.

$$F(x) = -\cos(x) + c \text{ geeft } F'(x) = \sin(x)$$

Dus $F(x) = -\cos(x) + c$ zijn de primitieven van $f(x) = \sin(x)$.

$$F(x) = \sin(x) + c \text{ geeft } F'(x) = \cos(x)$$

Dus $F(x) = \sin(x) + c$ zijn de primitieven van $f(x) = \cos(x)$.

Bladzijde 102

7 a $f(x) = ax^{-1}$ geeft $F(x) = a \cdot \frac{1}{-1+1} x^{-1+1} + c$ kan niet kloppen, want $\frac{1}{-1+1} = \frac{1}{0}$ bestaat niet en $x^0 = 1$ voor $x \neq 0$.

b $f(x) = x^{-1} = \frac{1}{x}$ geeft $F(x) = \ln|x| + c$

c $g(x) = a \cdot F(x)$ geeft $g'(x) = a \cdot F'(x) = a \cdot f(x)$

Dus als F een primitieve is van f , dan is $a \cdot F$ een primitieve van $a \cdot f$.

8 a $f(x) = 6x^2$ geeft $F(x) = 6 \cdot \frac{1}{3} x^3 + c = 2x^3 + c$

b $f(x) = 2x^3 + 5x^4$ geeft $F(x) = 2 \cdot \frac{1}{4} x^4 + 5 \cdot \frac{1}{5} x^5 + c = \frac{1}{2} x^4 + x^5 + c$

c $f(x) = \frac{x^4 - 2x}{2x^3} = \frac{1}{2} x - x^{-2}$ geeft $F(x) = \frac{1}{4} x^2 + x^{-1} + c = \frac{1}{4} x^2 + \frac{1}{x} + c$

d $f(x) = 10^x$ geeft $F(x) = \frac{10^x}{\ln(10)} + c$

e $f(x) = 5 \cdot 2^x$ geeft $F(x) = 5 \cdot \frac{2^x}{\ln(2)} + c = \frac{5 \cdot 2^x}{\ln(2)} + c$

f $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3} x^3 - \cos(x) + c$

g $f(x) = \frac{x^3 + 2}{x^4} = \frac{1}{x} + 2x^{-4}$ geeft $F(x) = \ln|x| + 2 \cdot -\frac{1}{3} \cdot x^{-3} + c = \ln|x| - \frac{2}{3x^3} + c$

h $f(x) = x\sqrt{x} - 2\cos(x) = x^{1\frac{1}{2}} - 2\cos(x)$ geeft

$$F(x) = \frac{1}{2\frac{1}{2}} x^{2\frac{1}{2}} - 2\sin(x) + c = \frac{2}{5} x^2 \cdot \sqrt{x} - 2\sin(x) + c$$

- 9** a $f(x) = x^3 - 3x$ geeft $F(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + c$
 b $f(x) = 5e^x$ geeft $F(x) = 5e^x + c$
 c $f(x) = \frac{x^4 - 6}{2x^3} = \frac{1}{2}x - 3x^{-3}$ geeft $F(x) = \frac{1}{4}x^2 + \frac{3}{2}x^{-2} + c = \frac{1}{4}x^2 + \frac{3}{2x^2} + c$
 d $f(x) = 3^x + x^3$ geeft $F(x) = \frac{3^x}{\ln(3)} + \frac{1}{4}x^4 + c$
 e $f(x) = 2\ln(x)$ geeft $F(x) = 2(x\ln(x) - x) + c = 2x\ln(x) - 2x + c$
 f $f(x) = \ln(2x) = \ln(2) + \ln(x)$ geeft $F(x) = \ln(2) \cdot x + x\ln(x) - x + c = x\ln(2) + x\ln(x) - x + c$

- 10** a $f(x) = e^{x+1} = e^x \cdot e = e \cdot e^x$ geeft $F(x) = e \cdot e^x + c = e^{x+1} + c$
 b $f(x) = \frac{8}{x^3} = 8x^{-3}$ geeft $F(x) = 8 \cdot -\frac{1}{2}x^{-2} + c = -\frac{4}{x^2} + c$
 c $f(x) = \frac{-x^2 + 2x + 3}{x^4} = -x^{-2} + 2x^{-3} + 3x^{-4}$ geeft
 $F(x) = x^{-1} - x^{-2} - x^{-3} + c = \frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^3} + c = \frac{x^2 - x - 1}{x^3} + c$
 d $f(x) = \ln(x\sqrt{x}) = \ln(x^{1/2}) = \frac{1}{2}\ln(x)$ geeft $F(x) = \frac{1}{2}(x\ln(x) - x) + c = \frac{1}{2}x\ln(x) - \frac{1}{2}x + c$
 e $f(x) = 2\log\left(\frac{1}{x}\right) = 2\log(x^{-1}) = -2\log(x)$ geeft
 $F(x) = -\frac{1}{\ln(2)}(x\ln(x) - x) + c = -\frac{x\ln(x) - x}{\ln(2)} + c$
 f $f(x) = 5 \cdot \log(2x) = 5 \cdot \log(2) + 5 \cdot \log(x)$ geeft
 $F(x) = 5 \cdot \log(2) \cdot x + 5 \cdot \frac{1}{\ln(10)} \cdot (x\ln(x) - x) + c = 5x\log(2) + \frac{5x\ln(x) - 5x}{\ln(10)} + c$

- 11** $F_q(x) = (3x + q)x\sqrt{x} = 3x^{2\frac{1}{2}} + qx^{1\frac{1}{2}}$ geeft $F_q'(x) = 7\frac{1}{2}x^{\frac{1}{2}} + 1\frac{1}{2}qx^{\frac{1}{2}} = 7\frac{1}{2}x\sqrt{x} + 1\frac{1}{2}q\sqrt{x}$
 Dit moet gelijk zijn aan $f_p(x) = p\sqrt{x} + 2px\sqrt{x}$, dus $2p = 7\frac{1}{2}$ en $p = 1\frac{1}{2}q$
 $p = 3\frac{3}{4}$ en $q = \frac{2}{3}p = \frac{2}{3} \cdot \frac{15}{4} = 2\frac{1}{2}$

Dus voor $p = 3\frac{3}{4}$ en $q = 2\frac{1}{2}$.

- 12** a $f(x) = 2x - 3$ geeft $F(x) = x^2 - 3x + c$
 b $F(1) = 2$ geeft $1 - 3 + c = 2$, dus $c = 4$.
 Dus $F(x) = x^2 - 3x + 4$.
 c De grafiek van $F(x) = x^2 - 3x + c$ raakt de x -as, dus

$$\left. \begin{array}{l} D = 0 \\ D = (-3)^2 - 4 \cdot 1 \cdot c \end{array} \right\} \begin{array}{l} 9 - 4c = 0 \\ -4c = -9 \\ c = 2\frac{1}{4} \end{array}$$

Dus $F(x) = x^2 - 3x + 2\frac{1}{4}$.

- 13** $f(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1$ geeft $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$
 $F(1) = 7$ geeft $\frac{1}{5} - \frac{2}{3} + 1 + c = 7$, dus $c = 6\frac{7}{15}$.
 Dus $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 6\frac{7}{15}$.

- 14** $F(x) = (ax^2 + bx + c)e^{2x}$ geeft
 $F'(x) = (2ax + b) \cdot e^{2x} + (ax^2 + bx + c) \cdot e^{2x} \cdot 2 = (2ax^2 + (2a + 2b)x + b + 2c)e^{2x}$
 Dit moet gelijk zijn aan $f(x) = x^2 e^{2x}$, dus $2a = 1 \wedge 2a + 2b = 0 \wedge b + 2c = 0$.
 $2a = 1$ geeft $a = \frac{1}{2}$
 $2a + 2b = 0$ en $a = \frac{1}{2}$ geeft $1 + 2b = 0$, dus $b = -\frac{1}{2}$.
 $b + 2c = 0$ en $b = -\frac{1}{2}$ geeft $-\frac{1}{2} + 2c = 0$, dus $c = \frac{1}{4}$.
 Dus $a = \frac{1}{2}$, $b = -\frac{1}{2}$ en $c = \frac{1}{4}$.

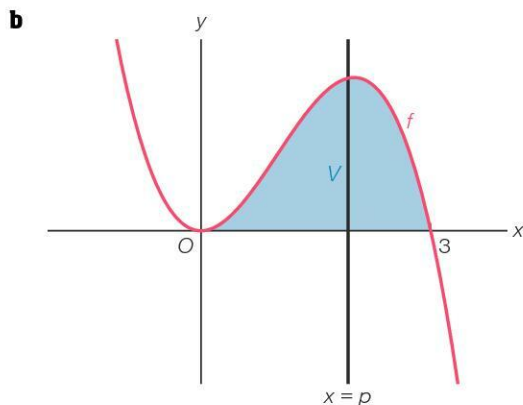
Bladzijde 103

- 15 a** $A(p) = O(\text{rechthoek}) + O(\text{driehoek}) = p \cdot b + \frac{1}{2}p \cdot (ap + b - b) = bp + \frac{1}{2}p \cdot ap = \frac{1}{2}ap^2 + bp$
b $A'(p) = ap + b$
 Dus $A'(p) = f(p)$.

Bladzijde 105

- 16 a** $f(x) = 0$ geeft $3x^2 - x^3 = 0$
 $x^2(3 - x) = 0$
 $x = 0 \vee x = 3$

$$O(V) = \int_0^3 (3x^2 - x^3) dx = \left[x^3 - \frac{1}{4}x^4 \right]_0^3 = 3^3 - \frac{1}{4} \cdot 3^4 - 0 = 27 - 20\frac{1}{4} = 6\frac{3}{4}$$



$$\int_0^p (3x^2 - x^3) dx = \frac{1}{2} \cdot 6\frac{3}{4}$$

$$\left[x^3 - \frac{1}{4}x^4 \right]_0^p = 3\frac{3}{8}$$

$$p^3 - \frac{1}{4}p^4 - 0 = 3\frac{3}{8}$$

$$p^3 - \frac{1}{4}p^4 = 3\frac{3}{8}$$

Voer in $y_1 = x^3 - \frac{1}{4}x^4$ en $y_2 = 3\frac{3}{8}$.

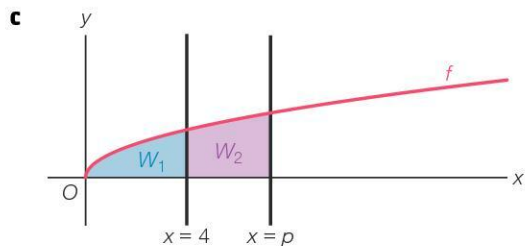
De optie snijpunt geeft $x \approx 1,84$.

Dus $p \approx 1,84$.

- 17** $O(W) = \frac{1}{2} O(V)$
 $\int_{\frac{1}{3}\pi}^p \sin(x) dx = \frac{1}{2} \cdot \int_0^\pi \sin(x) dx$
 $\left[-\cos(x) \right]_{\frac{1}{3}\pi}^p = \frac{1}{2} \left[-\cos(x) \right]_0^\pi$
 $-\cos(p) + \cos\left(\frac{1}{3}\pi\right) = \frac{1}{2}(-\cos(\pi) + \cos(0))$
 $-\cos(p) + \frac{1}{2} = \frac{1}{2}(1 + 1)$
 $-\cos(p) + \frac{1}{2} = 1$
 $-\cos(p) = \frac{1}{2}$
 $\cos(p) = -\frac{1}{2}$
 $p = \frac{2}{3}\pi$

18 a $O(V) = \int_0^{12} \sqrt{x} \, dx = \int_0^{12} x^{\frac{1}{2}} \, dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^{12} = \left[\frac{2}{3} x \sqrt{x} \right]_0^{12} = \frac{2}{3} \cdot 12 \sqrt{12} - 0 = 8 \sqrt{12} = 16 \sqrt{3}$

b $O(V) = 18$ geeft $\int_0^p \sqrt{x} \, dx = 18$
 $\left[\frac{2}{3} x \sqrt{x} \right]_0^p = 18$
 $\frac{2}{3} p \sqrt{p} = 18$
 $p \sqrt{p} = 27$
 $p^{\frac{3}{2}} = 3^3$
 $p = 3^2$
 $p = 9$



$$O(W_1) = O(W_2)$$

$$\int_0^4 \sqrt{x} \, dx = \int_4^p \sqrt{x} \, dx$$

$$\left[\frac{2}{3} x \sqrt{x} \right]_0^4 = \left[\frac{2}{3} x \sqrt{x} \right]_4^p$$

$$\frac{2}{3} \cdot 4 \sqrt{4} - 0 = \frac{2}{3} p \sqrt{p} - \frac{2}{3} \cdot 4 \sqrt{4}$$

$$\frac{2}{3} p \sqrt{p} = 2 \cdot \frac{2}{3} \cdot 4 \sqrt{4}$$

$$p^{\frac{3}{2}} = 16$$

$$p = 16^{\frac{2}{3}}$$

$$p = (2^4)^{\frac{2}{3}}$$

$$p = 2^{2\frac{2}{3}}$$

$$p = 4 \cdot \sqrt[3]{4}$$

Bladzijde 106

19 a $O(V) = 5$

$$\int_0^p e^x \, dx = 5$$

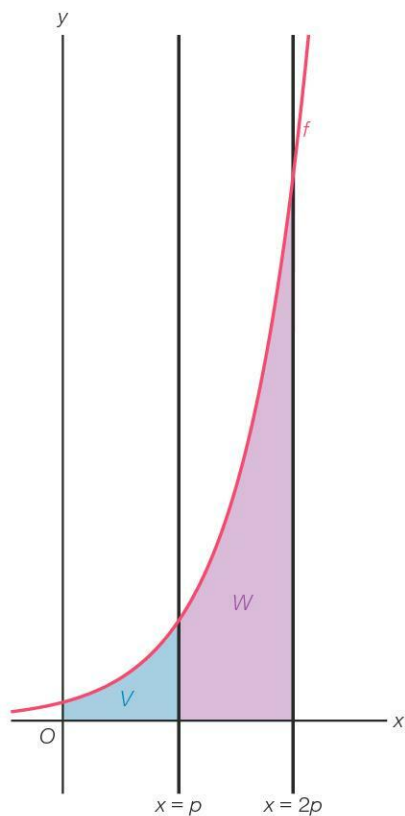
$$\left[e^x \right]_0^p = 5$$

$$e^p - e^0 = 5$$

$$e^p - 1 = 5$$

$$e^p = 6$$

$$p = \ln(6)$$

b

$$O(W) = 2 \cdot O(V)$$

$$\int_p^{2p} e^x dx = 2 \int_0^p e^x dx$$

$$[e^x]_p^{2p} = 2[e^x]_0^p$$

$$e^{2p} - e^p = 2(e^p - e^0)$$

$$e^{2p} - e^p = 2e^p - 2$$

$$(e^p)^2 - 3e^p + 2 = 0$$

Stel $e^p = u$.

$$u^2 - 3u + 2 = 0$$

$$(u - 1)(u - 2) = 0$$

$$u = 1 \vee u = 2$$

$$e^p = 1 \vee e^p = 2$$

$$p = 0 \vee p = \ln(2)$$

vold. niet

Dus $p = \ln(2)$.

20 a $O(V) = \int_1^{e^2} \ln(x) dx = [x \ln(x) - x]_1^{e^2} = e^2 \ln(e^2) - e^2 - (\ln(1) - 1) = 2e^2 - e^2 + 1 = e^2 + 1$

b $O(V) = 10$

$$\int_1^p \ln(x) dx = 10$$

$$[x \ln(x) - x]_1^p = 10$$

$$p \ln(p) - p - (\ln(1) - 1) = 10$$

$$p \ln(p) - p + 1 = 10$$

$$p \ln(p) - p = 9$$

Voer in $y_1 = x \ln(x) - x$ en $y_2 = 9$.

De optie snijpunt geeft $x \approx 8,174$.

Dus $p \approx 8,174$.

21 a $F_p(x) = \frac{1}{3}p \cos^3(x) - p \cos(x)$ geeft

$$F_p'(x) = p \cos^2(x) \cdot -\sin(x) - p \cdot -\sin(x) = -p \cos^2(x) \sin(x) + p \sin(x) \\ = p \sin(x)(-\cos^2(x) + 1) = p \sin(x) \cdot \sin^2(x) = p \sin^3(x)$$

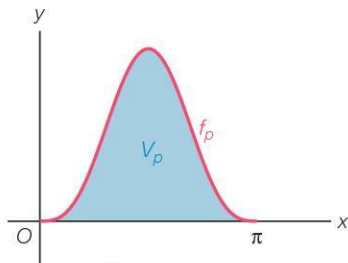
Dus $F_p(x) = \frac{1}{3}p \cos^3(x) - p \cos(x)$ is een primitieve van $f_p(x) = p \sin^3(x)$.

b $f_p(x) = 0$ geeft $p \sin^3(x) = 0$

$$\sin(x) = 0$$

$$x = 0 + k \cdot \pi$$

x in $[0, \pi]$ geeft $x = 0 \vee x = \pi$



$$O(V_p) = \int_0^\pi f_p(x) dx = \left[\frac{1}{3}p \cos^3(x) - p \cos(x) \right]_0^\pi \\ = \frac{1}{3}p \cdot (-1)^3 - p \cdot -1 - \left(\frac{1}{3}p \cdot 1^3 - p \cdot 1 \right) = -\frac{1}{3}p + p - \frac{1}{3}p + p = \frac{4}{3}p$$

$$O(V_p) = 4 \text{ geeft } \frac{4}{3}p = 4 \\ p = 3$$

11.2 Oppervlakten

Bladzijde 108

22 a $F(x) = a(3x+1)^6 + c$ geeft $F'(x) = a \cdot 6(3x+1)^5 \cdot 3 = 18a(3x+1)^5$

b $F'(x) = 18a(3x+1)^5$ moet gelijk zijn aan $f(x) = (3x+1)^5$, dus $18a = 1$ en dit geeft $a = \frac{1}{18}$.

Bladzijde 109

23 $\left[\frac{1}{a} F(ax+b) + c \right]' = \frac{1}{a} \cdot f(ax+b) \cdot a = f(ax+b)$

Dus $\frac{1}{a} F(ax+b) + c$ zijn de primitieven van $f(ax+b)$.

24 a $f(x) = (ax+b)^n$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{n+1} (ax+b)^{n+1} + c$

b $f(x) = g^{ax+b}$ geeft $F(x) = \frac{1}{a} \cdot \frac{g^{ax+b}}{\ln(g)} + c$

c $f(x) = \frac{1}{ax+b}$ geeft $F(x) = \frac{1}{a} \ln|ax+b| + c$

d $f(x) = \ln(ax+b)$ geeft $F(x) = \frac{1}{a} ((ax+b) \ln(ax+b) - (ax+b)) + c$

e $f(x) = {}^s\log(ax+b)$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{\ln(g)} ((ax+b) \ln(ax+b) - (ax+b)) + c$

f $f(x) = \cos(ax+b)$ geeft $F(x) = \frac{1}{a} \sin(ax+b) + c$

25 a $f(x) = (2x-1)^6$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{7} (2x-1)^7 + c = \frac{1}{14} (2x-1)^7 + c$

b $f(x) = \frac{1}{(3x+4)^3} = (3x+4)^{-3}$ geeft $F(x) = \frac{1}{3} \cdot -\frac{1}{2} (3x+4)^{-2} + c = -\frac{1}{6(3x+4)^2} + c$

c $f(x) = 4\sqrt{3-2x} = 4(3-2x)^{\frac{1}{2}}$ geeft $F(x) = 4 \cdot -\frac{1}{2} \cdot \frac{2}{3} (3-2x)^{\frac{3}{2}} + c = -\frac{4}{3} (3-2x)\sqrt{3-2x} + c$

d $f(x) = \frac{2}{\sqrt{1-x}} = 2(1-x)^{-\frac{1}{2}}$ geeft $F(x) = 2 \cdot -2(1-x)^{\frac{1}{2}} + c = -4\sqrt{1-x} + c$

e $f(x) = \sin(2x + \frac{1}{3}\pi)$ geeft $F(x) = -\frac{1}{2} \cos(2x + \frac{1}{3}\pi) + c$

f $f(x) = 3 \cos(\frac{1}{2}x - \frac{1}{6}\pi)$ geeft $F(x) = 3 \cdot 2 \sin(\frac{1}{2}x - \frac{1}{6}\pi) + c = 6 \sin(\frac{1}{2}x - \frac{1}{6}\pi) + c$

- 26** **a** $f(x) = \frac{1}{x-1}$ geeft $F(x) = \ln|x-1| + c$
b $f(x) = (2x+1)\sqrt{2x+1} = (2x+1)^{\frac{1}{2}}$ geeft
 $F(x) = \frac{1}{2} \cdot \frac{2}{5} (2x+1)^{\frac{5}{2}} + c = \frac{1}{5} (2x+1)^2 \cdot \sqrt{2x+1} + c$
c $f(x) = e^{4x-1}$ geeft $F(x) = \frac{1}{4} e^{4x-1} + c$
d $f(x) = \ln(4x-1)$ geeft
 $F(x) = \frac{1}{4} ((4x-1) \ln(4x-1) - (4x-1)) + c = (x - \frac{1}{4}) \ln(4x-1) - (x - \frac{1}{4}) + c$
e $f(x) = \frac{3}{2x-5}$ geeft $F(x) = 3 \cdot \frac{1}{2} \ln|2x-5| + c = \frac{3}{2} \ln|2x-5| + c$
f $f(x) = 2^{3x}$ geeft $F(x) = \frac{1}{3} \cdot \frac{2^{3x}}{\ln(2)} + c = \frac{2^{3x}}{3 \ln(2)} + c$
g $f(x) = 3^{2-5x}$ geeft $F(x) = -\frac{1}{5} \cdot \frac{3^{2-5x}}{\ln(3)} + c = -\frac{3^{2-5x}}{5 \ln(3)} + c$
h $f(x) = {}^2\log(5x+3)$ geeft
 $F(x) = \frac{1}{5} \cdot \frac{1}{\ln(2)} \cdot ((5x+3) \ln(5x+3) - (5x+3)) + c = \frac{(5x+3) \ln(5x+3) - (5x+3)}{5 \ln(2)} + c$

- 27** **a** $f(x) = \frac{3}{(2x-1)^4} = 3(2x-1)^{-4}$ geeft $F(x) = 3 \cdot \frac{1}{2} \cdot -\frac{1}{3} (2x-1)^{-3} + c = -\frac{1}{2(2x-1)^3} + c$
b $f(x) = (4x+3)\sqrt{4x+3} = (4x+3)^{\frac{1}{2}}$ geeft $F(x) = \frac{1}{4} \cdot \frac{2}{5} (4x+3)^{\frac{5}{2}} + c = \frac{1}{10} (4x+3)^2 \cdot \sqrt{4x+3} + c$
c $f(x) = 2 \sin(3x + \frac{1}{2}\pi)$ geeft $F(x) = 2 \cdot -\frac{1}{3} \cos(3x + \frac{1}{2}\pi) + c = -\frac{2}{3} \cos(3x + \frac{1}{2}\pi) + c$
d $f(x) = x^2 - \ln(\frac{1}{2}x + 1)$ geeft
 $F(x) = \frac{1}{3} x^3 - 2((\frac{1}{2}x + 1) \ln(\frac{1}{2}x + 1) - (\frac{1}{2}x + 1)) + c = \frac{1}{3} x^3 - (x + 2) \ln(\frac{1}{2}x + 1) + x + 2 + c$
e $f(x) = \frac{6}{3x+1}$ geeft $F(x) = 6 \cdot \frac{1}{3} \cdot \ln|3x+1| + c = 2 \ln|3x+1| + c$
f $f(x) = e^{4x+1} - 4e^x$ geeft $F(x) = \frac{1}{4} e^{4x+1} - \frac{1}{e} \cdot \frac{4e^x}{\ln(4)} + c = \frac{1}{4} e^{4x+1} - \frac{4e^x}{e \ln(4)} + c$
g $f(x) = 5 \cdot \log(3x+2)$ geeft
 $F(x) = 5 \cdot \frac{1}{3} \cdot \frac{1}{\ln(10)} \cdot ((3x+2) \ln(3x+2) - (3x+2)) + c = \frac{5((3x+2) \ln(3x+2) - (3x+2))}{3 \ln(10)} + c$
h $f(x) = 3x - \cos(3x+1)$ geeft $F(x) = \frac{1}{2} x^2 - \frac{1}{3} \sin(3x+1) + c$

- 28** **a** $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} (2 \sin(x - \frac{1}{6}\pi)) dx = [-2 \cos(x - \frac{1}{6}\pi)]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = -2 \cos(\frac{1}{6}\pi) + 2 \cos(0) = -\sqrt{3} + 2$
b $\int_0^{\frac{1}{3}\pi} (2x + \cos(\frac{1}{2}x)) dx = [x^2 + 2 \sin(\frac{1}{2}x)]_0^{\frac{1}{3}\pi} = (\frac{1}{3}\pi)^2 + 2 \sin(\frac{1}{6}\pi) - (0 + 0) = \frac{1}{9}\pi^2 + 2 \cdot \frac{1}{2} = \frac{1}{9}\pi^2 + 1$

29 $f(x) = 0$ geeft $1 + 2 \cos(\frac{1}{2}x - \frac{5}{6}\pi) = 0$

$$2 \cos(\frac{1}{2}x - \frac{5}{6}\pi) = -1$$

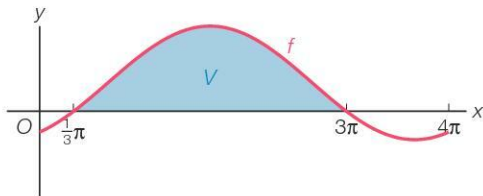
$$\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -\frac{1}{2}$$

$$\frac{1}{2}x - \frac{5}{6}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}x - \frac{5}{6}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$$

$$\frac{1}{2}x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = 3\pi + k \cdot 4\pi \vee x = \frac{1}{3}\pi + k \cdot 4\pi$$

x in $[0, 4\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 3\pi$



$$\begin{aligned} O(V) &= \int_{\frac{1}{3}\pi}^{3\pi} (1 + 2 \cos(\frac{1}{2}x - \frac{5}{6}\pi)) dx = \left[x + 4 \sin(\frac{1}{2}x - \frac{5}{6}\pi) \right]_{\frac{1}{3}\pi}^{3\pi} = 3\pi + 4 \sin(\frac{2}{3}\pi) - (\frac{1}{3}\pi + 4 \sin(-\frac{2}{3}\pi)) \\ &= 3\pi + 4 \cdot \frac{1}{2}\sqrt{3} - (\frac{1}{3}\pi + 4 \cdot -\frac{1}{2}\sqrt{3}) = 3\pi + 2\sqrt{3} - \frac{1}{3}\pi + 2\sqrt{3} = 2\frac{2}{3}\pi + 4\sqrt{3} \end{aligned}$$

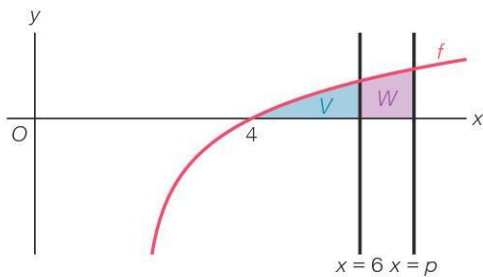
Bladzijde 110

30 $f(x) = 0$ geeft $\ln(\frac{1}{2}x - 1) = 0$

$$\frac{1}{2}x - 1 = 1$$

$$\frac{1}{2}x = 2$$

$$x = 4$$



$$f(x) = \ln(\frac{1}{2}x - 1) \text{ geeft } F(x) = 2((\frac{1}{2}x - 1) \ln(\frac{1}{2}x - 1) - (\frac{1}{2}x - 1)) = (x - 2) \ln(\frac{1}{2}x - 1) - x + 2$$

$$O(V) = \int_4^6 f(x) dx = \left[(x - 2) \ln(\frac{1}{2}x - 1) - x + 2 \right]_4^6 = 4 \ln(2) - 6 + 2 - (2 \ln(1) - 4 + 2) = 4 \ln(2) - 2$$

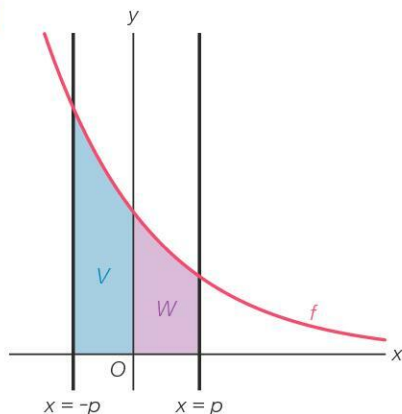
$$O(W) = \int_6^p f(x) dx = \left[(x - 2) \ln(\frac{1}{2}x - 1) - x + 2 \right]_6^p = (p - 2) \ln(\frac{1}{2}p - 1) - p + 2 - (4 \ln(2) - 4)$$

$$= (p - 2) \ln(\frac{1}{2}p - 1) - p + 6 - 4 \ln(2)$$

Voer in $y_1 = 4 \ln(2) - 2$ en $y_2 = (x - 2) \ln(\frac{1}{2}x - 1) - x + 6 - 4 \ln(2)$.

De optie snijpunt geeft $x = 2,119...$ en $x = 6,960...$

Dus $p \approx 6,96$.



$$O(V) = 2O(W)$$

$$\int_{-p}^0 2e^{1-\frac{1}{3}x} dx = 2 \int_0^p 2e^{1-\frac{1}{3}x} dx$$

$$\left[2 \cdot -3e^{1-\frac{1}{3}x} \right]_{-p}^0 = 2 \left[2 \cdot -3e^{1-\frac{1}{3}x} \right]_0^p$$

$$\left[-6e^{1-\frac{1}{3}x} \right]_{-p}^0 = 2 \left[-6e^{1-\frac{1}{3}x} \right]_0^p$$

$$-6e^1 + 6e^{1+\frac{1}{3}p} = 2(-6e^{1-\frac{1}{3}p} + 6e^1)$$

$$-6e + 6e \cdot e^{\frac{1}{3}p} = -12e \cdot e^{-\frac{1}{3}p} + 12e$$

$$-1 + e^{\frac{1}{3}p} = -2e^{-\frac{1}{3}p} + 2$$

$$\text{Stel } e^{\frac{1}{3}p} = u.$$

$$-1 + u = -\frac{2}{u} + 2$$

$$-u + u^2 = -2 + 2u$$

$$u^2 - 3u + 2 = 0$$

$$(u-1)(u-2) = 0$$

$$u = 1 \vee u = 2$$

$$e^{\frac{1}{3}p} = 1 \vee e^{\frac{1}{3}p} = 2$$

$$\frac{1}{3}p = 0 \vee \frac{1}{3}p = \ln(2)$$

$$p = 0 \vee p = 3 \ln(2)$$

vold. niet

$$\text{Dus } p = 3 \ln(2).$$

32 a De regel gaat over functies van de vorm $f(ax+b)$ en g is van de vorm $f(ax^2+b)$.

b $G(x) = a(4x^2 - 1)^{\frac{1}{2}}$ geeft $G'(x) = \frac{1}{2}a(4x^2 - 1)^{-\frac{1}{2}} \cdot 8x = 4ax\sqrt{4x^2 - 1}$

Er bestaat geen waarde van a waarvoor $G'(x) = \sqrt{4x^2 - 1}$.

33 a Stel A is het gebied begrensd door de x -as, de lijnen $x = a$ en $x = b$ en de grafiek van f .

Dit geeft $O(A) = \int_a^b f(x) dx$.

Stel B is het gebied begrensd door de x -as, de lijnen $x = a$ en $x = b$ en de grafiek van g .

Dit geeft $O(B) = \int_a^b g(x) dx$.

Dus $O(V) = O(A) - O(B) = \int_a^b f(x) dx - \int_a^b g(x) dx$.

b $O(V) = \int_a^b f(x) dx - \int_a^b g(x) dx = [F(x)]_a^b - [G(x)]_a^b = F(b) - F(a) - (G(b) - G(a))$
 $= F(b) - F(a) - G(b) + G(a) = F(b) - G(b) - F(a) + G(a)$
 $= F(b) - G(b) - (F(a) - G(a))$

c $O(V) = F(b) - G(b) - (F(a) - G(a)) = [F(x) - G(x)]_a^b = \int_a^b (f(x) - g(x)) dx$

Bladzijde 112

- 34** Verschuif de grafieken van f en g c omhoog waarbij c voldoende groot is om V geheel boven de x -as te krijgen. De beeldgrafieken zijn dan $y = f(x) + c$ en $y = g(x) + c$.

$$\text{Dus } O(V) = \int_a^b (f(x) + c - (g(x) + c)) dx = \int_a^b (f(x) + c - g(x) - c) dx = \int_a^b (f(x) - g(x)) dx.$$

35 a $O(V) = \int_0^\pi \sin(x) dx = [-\cos(x)]_0^\pi = -\cos(\pi) + \cos(0) = 1 + 1 = 2$

$$O(W) = -\int_\pi^{2\pi} \sin(x) dx = -[-\cos(x)]_\pi^{2\pi} = [\cos(x)]_\pi^{2\pi} = \cos(2\pi) - \cos(\pi) = 1 - (-1) = 2$$

b $\int_0^{2\pi} \sin(x) dx = [-\cos(x)]_0^{2\pi} = -\cos(2\pi) + \cos(0) = -1 + 1 = 0$

$$\int_0^{2\pi} \sin(x) dx = \int_0^\pi \sin(x) dx + \int_\pi^{2\pi} \sin(x) dx = O(V) + -O(W) = 2 - 2 = 0$$

Bladzijde 113

- 36** $f(x) = g(x)$ geeft $\sin(x) = \cos(2x)$

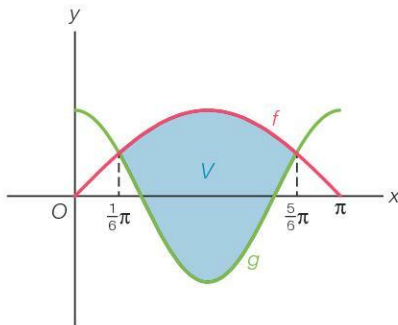
$$\cos(x - \frac{1}{2}\pi) = \cos(2x)$$

$$x - \frac{1}{2}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x + k \cdot 2\pi$$

$$-x = \frac{1}{2}\pi + k \cdot 2\pi \vee 3x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{2}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

x in $[0, \pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$



$$\begin{aligned} O(V) &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (f(x) - g(x)) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin(x) - \cos(2x)) dx = [-\cos(x) - \frac{1}{2}\sin(2x)]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \\ &= -\cos(\frac{5}{6}\pi) - \frac{1}{2}\sin(1\frac{2}{3}\pi) - (-\cos(\frac{1}{6}\pi) - \frac{1}{2}\sin(\frac{1}{3}\pi)) = \frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{3}) \\ &= \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} = 1\frac{1}{2}\sqrt{3} \end{aligned}$$

- 37 a** $x^3 = 2x$

$$x = 0 \vee x^2 = 2$$

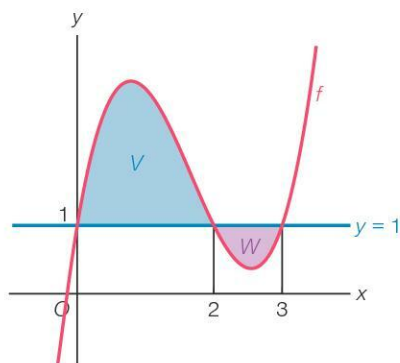
$$x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2}$$

$$O(V) = \int_0^{\sqrt{2}} (2x - x^3) dx = [x^2 - \frac{1}{4}x^4]_0^{\sqrt{2}} = 2 - \frac{1}{4} \cdot 4 - 0 = 1$$

$$\begin{aligned}
 \text{b } \int_0^p (2x - x^3) dx &= \frac{1}{2} \text{ met } 0 < p < \sqrt{2} \\
 \left[x^2 - \frac{1}{4}x^4 \right]_0^p &= \frac{1}{2} \\
 p^2 - \frac{1}{4}p^4 - 0 &= \frac{1}{2} \\
 4p^2 - p^4 &= 2 \\
 p^4 - 4p^2 + 2 &= 0 \\
 \text{Stel } p^2 &= u. \\
 u^2 - 4u + 2 &= 0 \\
 (u - 2)^2 - 4 + 2 &= 0 \\
 (u - 2)^2 &= 2 \\
 u - 2 &= \sqrt{2} \vee u - 2 = -\sqrt{2} \\
 u &= 2 + \sqrt{2} \vee u = 2 - \sqrt{2} \\
 p^2 &= 2 + \sqrt{2} \vee p^2 = 2 - \sqrt{2} \\
 \text{vold. niet} \quad p &= \sqrt{2 - \sqrt{2}} \vee p = -\sqrt{2 - \sqrt{2}} \\
 &\text{vold. niet} \\
 \text{Dus } p &= \sqrt{2 - \sqrt{2}}.
 \end{aligned}$$

38 $f(x) = 1$ geeft $x^3 - 5x^2 + 6x + 1 = 1$

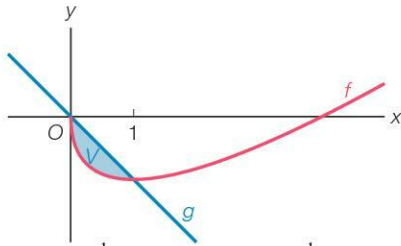
$$\begin{aligned}
 x^3 - 5x^2 + 6x &= 0 \\
 x(x^2 - 5x + 6) &= 0 \\
 x(x - 2)(x - 3) &= 0 \\
 x = 0 \vee x = 2 \vee x = 3
 \end{aligned}$$



$$\begin{aligned}
 O(V) &= \int_0^2 (f(x) - 1) dx = \int_0^2 (x^3 - 5x^2 + 6x) dx = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_0^2 \\
 &= \frac{1}{4} \cdot 2^4 - \frac{5}{3} \cdot 2^3 + 3 \cdot 2^2 - 0 = 2\frac{2}{3} \\
 O(W) &= \int_2^3 (1 - f(x)) dx = \int_2^3 (1 - (x^3 - 5x^2 + 6x + 1)) dx = \int_2^3 (1 - x^3 + 5x^2 - 6x - 1) dx \\
 &= \int_2^3 (-x^3 + 5x^2 - 6x) dx = \left[-\frac{1}{4}x^4 + \frac{5}{3}x^3 - 3x^2 \right]_2^3 \\
 &= -\frac{1}{4} \cdot 3^4 + \frac{5}{3} \cdot 3^3 - 3 \cdot 3^2 - \left(-\frac{1}{4} \cdot 2^4 + \frac{5}{3} \cdot 2^3 - 3 \cdot 2^2 \right) = \frac{5}{12} \\
 \frac{O(V)}{O(W)} &= \frac{2\frac{2}{3}}{\frac{5}{12}} = \frac{8}{3} \cdot \frac{12}{5} = \frac{32}{5}
 \end{aligned}$$

Dus $O(V)$ is $6\frac{2}{3}$ keer zo groot als $O(W)$.

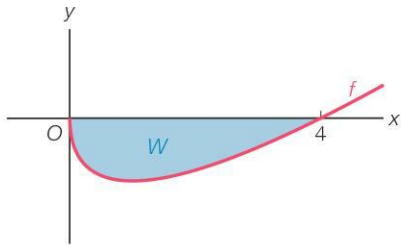
39 $f(x) = g(x)$ geeft $x - 2\sqrt{x} = -x$
 $2x = 2\sqrt{x}$
 $x = \sqrt{x}$
kwadrateren geeft
 $x^2 = x$
 $x = 0 \vee x = 1$



$$O(V) = \int_0^1 (g(x) - f(x)) dx = \int_0^1 (-x - (x - 2\sqrt{x})) dx = \int_0^1 (-2x + 2\sqrt{x}) dx = \int_0^1 (-2x + 2x^{\frac{1}{2}}) dx$$

$$= \left[-x^2 + 2 \cdot \frac{2}{3} x^{\frac{3}{2}} \right]_0^1 = \left[-x^2 + \frac{4}{3} x^{\frac{3}{2}} \right]_0^1 = -1^2 + \frac{4}{3} \cdot 1 \cdot \sqrt{1} - 0 = -1 + \frac{4}{3} = \frac{1}{3}$$

$f(x) = 0$ geeft $x - 2\sqrt{x} = 0$
 $x = 2\sqrt{x}$
kwadrateren geeft
 $x^2 = 4x$
 $x = 0 \vee x = 4$



$$O(W) = - \int_0^4 f(x) dx = - \int_0^4 (x - 2\sqrt{x}) dx = \int_0^4 (-x + 2x^{\frac{1}{2}}) dx = \left[-\frac{1}{2}x^2 + 2 \cdot \frac{2}{3} x^{\frac{3}{2}} \right]_0^4 = \left[-\frac{1}{2}x^2 + \frac{4}{3} x^{\frac{3}{2}} \right]_0^4$$

$$= -\frac{1}{2} \cdot 4^2 + \frac{4}{3} \cdot 4 \cdot \sqrt{4} - 0 = -8 + \frac{16}{3} \cdot 2 = -8 + \frac{32}{3} = \frac{8}{3}$$

$$8 \cdot O(V) = 8 \cdot \frac{1}{3} = \frac{8}{3} = O(W)$$

40 a $f(x) = -1\frac{1}{2}$ geeft $\frac{x^2 + x + 1}{x} = -1\frac{1}{2}$
 $x^2 + x + 1 = -1\frac{1}{2}x$
 $x^2 + 2\frac{1}{2}x + 1 = 0$
 $(x + \frac{1}{2})(x + 2) = 0$
 $x = -\frac{1}{2} \vee x = -2$

$$O(V) = \int_{-2}^{-\frac{1}{2}} \left(\frac{x^2 + x + 1}{x} - (-1\frac{1}{2}) \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x + 1 + \frac{1}{x} + 1\frac{1}{2} \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x + \frac{1}{x} + 2\frac{1}{2} \right) dx$$

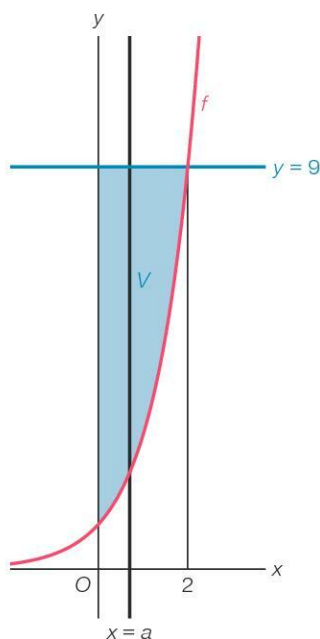
$$= \left[\frac{1}{2}x^2 + \ln|x| + 2\frac{1}{2}x \right]_{-2}^{-\frac{1}{2}} = \frac{1}{8} + \ln(\frac{1}{2}) - 1\frac{1}{4} - (2 + \ln(2) - 5) = \frac{1}{8} + \ln(\frac{1}{2}) - 1\frac{1}{4} - 2 - \ln(2) + 5$$

$$= 1\frac{7}{8} + \ln(\frac{1}{2}) - \ln(2) = 1\frac{7}{8} + \ln(\frac{1}{4})$$

$$\begin{aligned}
 \text{b } O(W) = 2 \text{ geeft } \int_1^p \left(\frac{x^2 + x + 1}{x} - (x + 1) \right) dx &= 2 \\
 \int_1^p \left(x + 1 + \frac{1}{x} - (x + 1) \right) dx &= 2 \\
 \int_1^p \frac{1}{x} dx &= 2 \\
 [\ln|x|]_1^p &= 2 \\
 \ln|p| - \ln(1) &= 2 \\
 \ln(p) &= 2 \\
 p &= e^2
 \end{aligned}$$

Bladzijde 114

41 $f(x) = 9$ geeft $3^x = 9$
 $x = 2$



$$O(V) = \int_0^2 (9 - 3^x) dx = \left[9x - \frac{3^x}{\ln(3)} \right]_0^2 = 18 - \frac{9}{\ln(3)} - \left(0 - \frac{1}{\ln(3)} \right) = 18 - \frac{8}{\ln(3)}$$

$$O(W) = \int_0^a (9 - 3^x) dx = \left[9x - \frac{3^x}{\ln(3)} \right]_0^a = 9a - \frac{3^a}{\ln(3)} - \left(0 - \frac{1}{\ln(3)} \right) = 9a + \frac{1 - 3^a}{\ln(3)}$$

Los op $9a + \frac{1 - 3^a}{\ln(3)} = 9 - \frac{4}{\ln(3)}$.

Voer in $y_1 = 9x + \frac{1 - 3^x}{\ln(3)}$ en $y_2 = 9 - \frac{4}{\ln(3)}$.

De optie snijpunt geeft $x = 0,716...$ en $x = 2,875...$

Dus $a \approx 0,72$.

42 $f(x) = 8$ geeft $\frac{8}{x^2} = 8$

$$x^2 = 1$$

$$x = 1 \vee x = -1$$

vold. niet

$$O(V) = 1 \cdot 8 + \int_1^8 \frac{8}{x^2} dx = 8 + \int_1^8 8x^{-2} dx = 8 + [-8x^{-1}]_1^8 = 8 + \left[-\frac{8}{x}\right]_1^8 = 8 - \frac{8}{8} + \frac{8}{1} = 15$$

$$O(V_1) = \frac{2}{3} O(V) \text{ geeft } 8 + \int_1^a \frac{8}{x^2} dx = \frac{2}{3} \cdot 15$$

$$8 + \int_1^a 8x^{-2} dx = 10$$

$$\int_1^a 8x^{-2} dx = 2$$

$$\left[-\frac{8}{x}\right]_1^a = 2$$

$$-\frac{8}{a} + \frac{8}{1} = 2$$

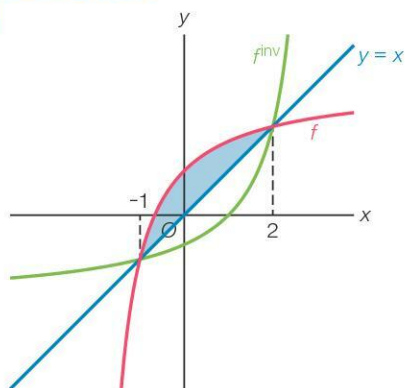
$$-\frac{8}{a} = -6$$

$$-6a = -8$$

$$a = 1\frac{1}{3}$$

Bladzijde 115

43



De snijpunten van de grafiek f en de grafiek van f^{inv} liggen op de lijn $y = x$.

$$f(x) = x \text{ geeft } 3 - \frac{4}{x+2} = x$$

$$3(x+2) - 4 = x(x+2)$$

$$3x + 6 - 4 = x^2 + 2x$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \vee x = 2$$

$$\begin{aligned} O(V) &= 2 \cdot \int_{-1}^2 (f(x) - x) dx = 2 \cdot \int_{-1}^2 \left(3 - \frac{4}{x+2} - x\right) dx = 2 \cdot \left[3x - 4 \ln|x+2| - \frac{1}{2}x^2\right]_{-1}^2 \\ &= 2(6 - 4 \ln(4) - 2 - 2(-3 - 4 \ln(1) - \frac{1}{2})) = 8 - 8 \ln(4) + 7 = 15 - 8 \ln(4) \end{aligned}$$

44 $P(p, 2e^{0,2p})$
 $O_{\text{rechthoek}} = p \cdot 2e^{0,2p}$
 $O(V) = O(W)$
 $O(W) = \frac{1}{2} O_{\text{rechthoek}}$

$$\int_0^p (2e^{0,2x}) dx = p \cdot e^{0,2p}$$

$$[10e^{0,2x}]_0^p = p \cdot e^{0,2p}$$

$$10e^{0,2p} - 10 = p \cdot e^{0,2p}$$

Voer in $y_1 = 10e^{0,2x} - 10$ en $y_2 = x \cdot e^{0,2x}$.

De optie snijpunt geeft $x = 7,968...$

Dus $p \approx 7,97$.

45 a $F_p(x) = -2p\sqrt{1-x}$ geeft $F_p'(x) = -2p \cdot \frac{1}{2\sqrt{1-x}} \cdot -1 = \frac{p}{\sqrt{1-x}}$

Dus $F_p(x) = -2p\sqrt{1-x}$ is een primitieve van $f_p(x) = \frac{p}{\sqrt{1-x}}$.

b $O(V_q) = \int_q^0 (f_6(x) - f_2(x)) dx = \int_q^0 \left(\frac{6}{\sqrt{1-x}} - \frac{2}{\sqrt{1-x}} \right) dx = \int_q^0 \left(\frac{4}{\sqrt{1-x}} \right) dx$
 $= [-8\sqrt{1-x}]_q^0 = -8\sqrt{1} + 8\sqrt{1-q} = -8 + 8\sqrt{1-q}$

$$O(V_q) = 12 \text{ geeft } -8 + 8\sqrt{1-q} = 12$$

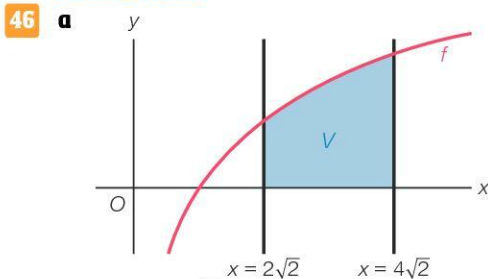
$$8\sqrt{1-q} = 20$$

$$\sqrt{1-q} = 2\frac{1}{2}$$

$$1-q = 6\frac{1}{4}$$

$$q = -5\frac{1}{4}$$

Bladzijde 116



$$O(V) = \int_{2\sqrt{2}}^{4\sqrt{2}} \ln(x) dx = [x \ln(x) - x]_{2\sqrt{2}}^{4\sqrt{2}} = 4\sqrt{2} \cdot \ln(4\sqrt{2}) - 4\sqrt{2} - (2\sqrt{2} \cdot \ln(2\sqrt{2}) - 2\sqrt{2})$$

$$= 4\sqrt{2} \cdot 2\frac{1}{2} \ln(2) - 4\sqrt{2} - 2\sqrt{2} \cdot 1\frac{1}{2} \ln(2) + 2\sqrt{2} = 10\sqrt{2} \cdot \ln(2) - 2\sqrt{2} - 3\sqrt{2} \cdot \ln(2)$$

$$= 7\sqrt{2} \cdot \ln(2) - 2\sqrt{2} = \sqrt{2} \cdot (7 \ln(2) - 2)$$

b $L = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{1 + (f'(x))^2} dx = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{1 + \frac{1}{x^2}} dx = \int_{2\sqrt{2}}^{4\sqrt{5}} \sqrt{\frac{x^2 + 1}{x^2}} dx = \int_{2\sqrt{2}}^{4\sqrt{5}} \frac{\sqrt{x^2 + 1}}{x} dx$

$$= \left[\sqrt{x^2 + 1} - \ln\left(\frac{1 + \sqrt{x^2 + 1}}{x}\right) \right]_{2\sqrt{2}}^{4\sqrt{5}} = \sqrt{80 + 1} - \ln\left(\frac{1 + \sqrt{80 + 1}}{4\sqrt{5}}\right) - \left(\sqrt{8 + 1} - \ln\left(\frac{1 + \sqrt{8 + 1}}{2\sqrt{2}}\right) \right)$$

$$= 9 - \ln\left(\frac{10}{4\sqrt{5}}\right) - \left(3 - \ln\left(\frac{4}{2\sqrt{2}}\right) \right) = 6 - \ln\left(\frac{1}{2}\sqrt{5}\right) + \ln(\sqrt{2}) = 6 + \ln\left(\frac{\sqrt{2}}{\frac{1}{2}\sqrt{5}}\right) = 6 + \ln\left(\frac{2}{\sqrt{5}}\sqrt{10}\right)$$

c $4\sqrt{5} - 2\sqrt{2} + \ln(4\sqrt{5}) + 6 + \ln\left(\frac{2}{\sqrt{5}}\sqrt{10}\right) + \ln(2\sqrt{2}) = 15,581...$

Dus de omtrek van W is ongeveer 15,58.

47 $G(x) = \sqrt{x^2 + 1} - \ln\left(\frac{1 + \sqrt{x^2 + 1}}{x}\right)$ geeft

$$\begin{aligned} G'(x) &= \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x - \frac{1}{\frac{1 + \sqrt{x^2 + 1}}{x}} \cdot \frac{x \cdot \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x - (1 + \sqrt{x^2 + 1}) \cdot 1}{x^2} \\ &= \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{1 + \sqrt{x^2 + 1}} \cdot \left(\frac{1}{\sqrt{x^2 + 1}} - \frac{1 + \sqrt{x^2 + 1}}{x^2} \right) = \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{1 + \sqrt{x^2 + 1}} \cdot \frac{x^2 - \sqrt{x^2 + 1} - x^2 - 1}{x^2 \sqrt{x^2 + 1}} \\ &= \frac{x}{\sqrt{x^2 + 1}} - \frac{x}{1 + \sqrt{x^2 + 1}} \cdot \frac{-1 - \sqrt{x^2 + 1}}{x^2 \sqrt{x^2 + 1}} = \frac{x}{\sqrt{x^2 + 1}} + \frac{1}{x\sqrt{x^2 + 1}} = \frac{x^2 + 1}{x\sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1}}{x} \end{aligned}$$

Dus $G'(x) = g(x)$ oftewel G is een primitieve van g .

11.3 Inhouden

Bladzijde 118

- 48 a** Het groene lichaam is te benaderen door de cilinder met straal $f(x)$ en hoogte h .

Dus de inhoud is ongeveer $\pi \cdot (f(x))^2 \cdot h$.

b $B'(x) = \lim_{h \rightarrow 0} \frac{B(x+h) - B(x)}{h} = \lim_{h \rightarrow 0} \frac{\pi \cdot (f(x))^2 \cdot h}{h} = \lim_{h \rightarrow 0} \pi \cdot (f(x))^2 = \pi \cdot (f(x))^2$

Dus $B(x)$ is een primitieve van $\pi \cdot (f(x))^2$.

Bladzijde 119

49 $f(x) = x$ geeft $2 - \sqrt{x} = x$
 $2 - x = \sqrt{x}$
 kwadrateren geeft
 $4 - 4x + x^2 = x$
 $x^2 - 5x + 4 = 0$
 $(x-1)(x-4) = 0$
 $x = 1 \vee x = 4$

vold. niet

$f(x) = 0$ geeft $2 - \sqrt{x} = 0$, oftewel $x = 4$.

$$\begin{aligned} I(K) &= \pi \int_0^1 x^2 dx + \pi \int_1^4 (2 - \sqrt{x})^2 dx = \pi \left[\frac{1}{3} x^3 \right]_0^1 + \pi \left[4x - \frac{8}{3} x \sqrt{x} + \frac{1}{2} x^2 \right]_1^4 \\ &= \frac{1}{3} \pi - 0 + \pi \left(4 \cdot 4 - \frac{8}{3} \cdot 4 \cdot 2 + \frac{1}{2} \cdot 4^2 - \left(4 \cdot 1 - \frac{8}{3} \cdot 1 \cdot 1 + \frac{1}{2} \cdot 1^2 \right) \right) = \frac{1}{3} \pi + \frac{5}{6} \pi = 1\frac{1}{6} \pi \end{aligned}$$

50 $g(x) = 2x$ geeft $3 - \sqrt{x} = 2x$
 $-2x + 3 = \sqrt{x}$
 kwadrateren geeft
 $4x^2 - 12x + 9 = x$
 $4x^2 - 13x + 9 = 0$
 $D = (-13)^2 - 4 \cdot 4 \cdot 9 = 25$
 $x = \frac{13 \pm 5}{8} = 2\frac{1}{4} \vee x = \frac{13-5}{8} = 1$

vold. niet

$g(x) = 0$ geeft $3 - \sqrt{x} = 0$, oftewel $x = 9$.

$$\begin{aligned} I(L) &= \pi \int_0^1 (2x)^2 dx + \pi \int_1^9 (3 - \sqrt{x})^2 dx = \pi \int_0^1 4x^2 dx + \pi \int_1^9 (9 - 6\sqrt{x} + x) dx \\ &= \pi \left[\frac{4}{3} x^3 \right]_0^1 + \pi \left[9x - 4x\sqrt{x} + \frac{1}{2} x^2 \right]_1^9 = \pi \left(\frac{4}{3} \cdot 1^3 - 0 \right) + \pi \left(81 - 108 + 40\frac{1}{2} - (9 - 4 + \frac{1}{2}) \right) \\ &= \frac{4}{3} \pi + 8\pi = 9\frac{1}{3} \pi \end{aligned}$$

S1 a $I(K) = \pi \int_0^1 (e^x)^2 dx = \pi \int_0^1 e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_0^1 = \pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)$

$$I(L_2) = \pi \int_1^2 (e^x)^2 dx = \pi \int_1^2 e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_1^2 = \pi \left(\frac{1}{2} e^4 - \frac{1}{2} e^2 \right)$$

$$\frac{I(L_2)}{I(K)} = \frac{\pi \left(\frac{1}{2} e^4 - \frac{1}{2} e^2 \right)}{\pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)} = \frac{\frac{1}{2} \pi e^2 (e^2 - 1)}{\frac{1}{2} \pi (e^2 - 1)} = e^2$$

Dus $I(L_2)$ is e^2 keer zo groot als $I(K)$.

b $I(L_p) = \pi \int_1^p (e^x)^2 dx = \pi \int_1^p e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_1^p = \pi \left(\frac{1}{2} e^{2p} - \frac{1}{2} e^2 \right)$

$$\frac{I(L_p)}{I(K)} = \frac{\pi \left(\frac{1}{2} e^{2p} - \frac{1}{2} e^2 \right)}{\pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)} = \frac{\frac{1}{2} \pi (e^{2p} - e^2)}{\frac{1}{2} \pi (e^2 - 1)} = \frac{e^{2p} - e^2}{e^2 - 1}$$

$$\frac{I(L_p)}{I(K)} = 10 \text{ geeft } \frac{e^{2p} - e^2}{e^2 - 1} = 10$$

$$e^{2p} - e^2 = 10e^2 - 10$$

$$e^{2p} = 11e^2 - 10$$

$$2p = \ln(11e^2 - 10)$$

$$p = \frac{1}{2} \ln(11e^2 - 10) = 2,1333...$$

Dus $p \approx 2,133$.

S2 a $g(x) = \frac{3}{x}$

b $I(M) = \pi \int_1^3 \left(\frac{3}{x} \right)^2 dx = \pi \int_1^3 9x^{-2} dx = \pi \left[-9x^{-1} \right]_1^3 = \pi \left[-\frac{9}{x} \right]_1^3 = \pi \left(-\frac{9}{3} + \frac{9}{1} \right) = 6\pi$

Wentelen van V om de lijn $y = 2$ levert hetzelfde lichaam als wentelen van W om de x -as omdat V en de lijn $y = 2$ beide 2 omlaag zijn geschoven.

S3 a $O(V) = \int_1^5 f(x) dx = \int_1^5 \left(3 + \frac{4}{2x-1} \right) dx = \left[3x + 4 \cdot \frac{1}{2} \ln|2x-1| \right]_1^5 = \left[3x + 2 \ln|2x-1| \right]_1^5$
 $= 15 + 2 \ln(9) - (3 + 2 \ln(1)) = 12 + 2 \ln(9)$

b De grafiek van f wentelen om de lijn $y = 3$ komt neer op het wentelen van de grafiek van $g(x) = f(x) - 3 = \frac{4}{2x-1}$ wentelen om de x -as.

$$I(K) = \pi \int_1^5 (g(x))^2 dx = \pi \int_1^5 \left(\frac{4}{2x-1} \right)^2 dx = \pi \int_1^5 \frac{16}{(2x-1)^2} dx = \pi \int_1^5 16(2x-1)^{-2} dx$$

$$= \pi \left[16 \cdot \frac{1}{2} \cdot -(2x-1)^{-1} \right]_1^5 = \pi \left[\frac{-8}{2x-1} \right]_1^5 = \pi \left(-\frac{8}{9} + 8 \right) = 7\frac{1}{9} \pi$$

$$\text{54 } I(L_1 + L_2) = \pi \int_2^8 (\sqrt{x-2})^2 dx = \pi \int_2^8 (x-2) dx = \pi \left[\frac{1}{2}x^2 - 2x \right]_2^8 = \pi(32 - 16) - \pi(2 - 4) = 18\pi$$

$$I(L_1) = \frac{1}{2} \cdot 18\pi \text{ geeft } \pi \int_2^a (\sqrt{x-2})^2 dx = 9\pi$$

$$\pi \int_2^a (x-2) dx = 9\pi$$

$$\int_2^a (x-2) dx = 9$$

$$\left[\frac{1}{2}x^2 - 2x \right]_2^a = 9$$

$$\left(\frac{1}{2}a^2 - 2a \right) - (2 - 4) = 9$$

$$\frac{1}{2}a^2 - 2a + 2 = 9$$

$$a^2 - 4a - 14 = 0$$

$$(a-2)^2 - 4 - 14 = 0$$

$$(a-2)^2 = 18$$

$$a-2 = \sqrt{18} \vee a-2 = -\sqrt{18}$$

$$a = 2 + 3\sqrt{2} \vee a = 2 - 3\sqrt{2}$$

vold. niet

Dus $a = 2 + 3\sqrt{2}$.

$$\text{55 a } O(V_p) = 2e \text{ geeft } \int_0^p e^{\frac{1}{2}x+1} dx = 2e$$

$$\left[2e^{\frac{1}{2}x+1} \right]_0^p = 2e$$

$$2e^{\frac{1}{2}p+1} - 2e^1 = 2e$$

$$2e \cdot e^{\frac{1}{2}p} = 4e$$

$$e^{\frac{1}{2}p} = 2$$

$$\frac{1}{2}p = \ln(2)$$

$$p = 2 \ln(2)$$

$$\text{b } I(L_p) = 9\pi e^2 \text{ geeft } \pi \int_0^p (e^{\frac{1}{2}x+1})^2 dx = 9\pi e^2$$

$$\int_0^p e^{x+2} dx = 9e^2$$

$$\left[e^{x+2} \right]_0^p = 9e^2$$

$$e^{p+2} - e^2 = 9e^2$$

$$e^p \cdot e^2 = 10e^2$$

$$e^p = 10$$

$$p = \ln(10)$$

$$\text{56 a } I(L) = 8\frac{2}{3}\pi - 2\frac{1}{6}\pi = 6\frac{1}{2}\pi$$

$$\text{b } \pi \int_1^3 (f(x) - g(x))^2 dx = \pi \int_1^3 (x - \frac{1}{2}x)^2 dx = \pi \int_1^3 (\frac{1}{2}x)^2 dx = \pi \int_1^3 \frac{1}{4}x^2 dx =$$

$$\pi \left[\frac{1}{12}x^3 \right]_1^3 = \pi \left(\frac{1}{12} \cdot 3^3 - \frac{1}{12} \cdot 1^3 \right) = 2\frac{1}{6}\pi$$

$$\pi \int_1^3 ((f(x))^2 - (g(x))^2) dx = \pi \int_1^3 (x^2 - (\frac{1}{2}x)^2) dx = \pi \int_1^3 (x^2 - \frac{1}{4}x^2) dx = \pi \int_1^3 \frac{3}{4}x^2 dx =$$

$$\pi \left[\frac{1}{4}x^3 \right]_1^3 = \pi \left(\frac{1}{4} \cdot 3^3 - \frac{1}{4} \cdot 1^3 \right) = 6\frac{1}{2}\pi$$

Dus Irma heeft gelijk.

Bladzijde 122

57 $f(x) = g(x)$ geeft $\sqrt{x} = \frac{1}{2}x$
 kwadrateren geeft
 $x = \frac{1}{4}x^2$
 $4x = x^2$
 $x = 0 \vee x = 4$

$$I(L) = \pi \int_0^4 ((f(x))^2 - (g(x))^2) dx = \pi \int_0^4 (x - \frac{1}{4}x^2) dx = \pi \left[\frac{1}{2}x^2 - \frac{1}{12}x^3 \right]_0^4 = \pi(8 - 5\frac{1}{3}) = 2\frac{2}{3}\pi$$

Bladzijde 123

58 $f(x) = g(x)$ geeft $e^{\frac{1}{2}x} = 2e - e^{\frac{1}{2}x}$
 $2e^{\frac{1}{2}x} = 2e$
 $\frac{1}{2}x = 1$
 $x = 2$

$$I(L) = \pi \int_0^2 ((2e - e^{\frac{1}{2}x})^2 - (e^{\frac{1}{2}x})^2) dx = \pi \int_0^2 (4e^2 - 2 \cdot 2e \cdot e^{\frac{1}{2}x} + e^x - e^x) dx = \pi \int_0^2 (4e^2 - 4e^{\frac{1}{2}x+1}) dx$$

$$= \pi [4e^2x - 8e^{\frac{1}{2}x+1}]_0^2 = \pi(8e^2 - 8e^2 - (0 - 8e)) = 8\pi e$$

59 a $f(x) = 10$ geeft $\frac{10}{x\sqrt{x}} = 10$
 $x\sqrt{x} = 1$
 $x = 1$

$$O(V) = \int_1^4 \left(10 - \frac{10}{x\sqrt{x}} \right) dx = \int_1^4 (10 - 10x^{-\frac{1}{2}}) dx = [10x - 10 \cdot -2x^{\frac{1}{2}}]_1^4 = \left[10x + \frac{20}{\sqrt{x}} \right]_1^4$$

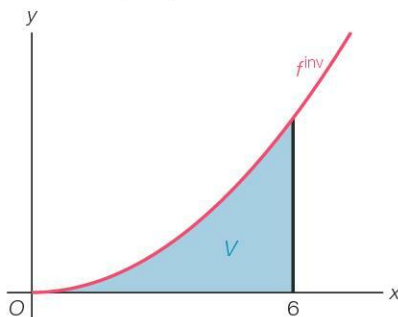
$$= 40 + 10 - (10 + 20) = 20$$

b $I(L) = \pi \int_1^4 (10^2 - (f(x))^2) dx = \pi \int_1^4 \left(100 - \frac{100}{x^3} \right) dx = \pi \int_1^4 (100 - 100x^{-3}) dx$

$$= \pi [100x + 50x^{-2}]_1^4 = \pi \left[100x + \frac{50}{x^2} \right]_1^4 = \pi \left(400 + \frac{50}{16} - (100 + 50) \right) = 253\frac{1}{8}\pi$$

60 a Voor f geldt $y = 3\sqrt{x}$, dus voor f^{inv} geldt $x = 3\sqrt{y}$
 $x^2 = 9y$ met $x \geq 0$
 $y = \frac{1}{9}x^2$ met $x \geq 0$

Dus $f^{\text{inv}}(x) = \frac{1}{9}x^2$ met $x \geq 0$.



$$I(L) = \pi \int_0^6 \frac{1}{81}x^4 dx = \pi \left[\frac{1}{405}x^5 \right]_0^6 = \pi \left(\frac{1}{405} \cdot 6^5 - 0 \right) = 19\frac{1}{5}\pi$$

De gebruikte grenzen zijn 0 en 6.

b $y = 3\sqrt{x}$
 $y^2 = 9x$
 $x = \frac{1}{9}y^2$
 $x^2 = \frac{1}{81}y^4$

c Jasmijn moet de grenzen 0 en 6 gebruiken, want y loopt van 0 tot 6.

d $I(L) = \pi \int_0^6 \frac{1}{81} x^4 dx$ is hetzelfde als bij onderdeel a.

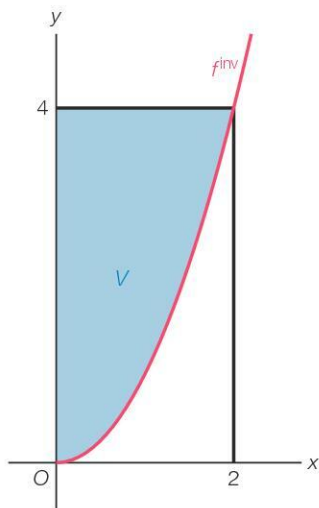
Bladzijde 125

61 a Gebruik 'buitenste' inhoud min 'binnenste' inhoud.

$$\text{Je krijgt dan } I(L) = \pi \int_0^2 (4^2 - x^2) dy = \pi \int_0^2 (16 - y^4) dy = \pi \left[16y - \frac{1}{5} y^5 \right]_0^2 = \pi \left(32 - \frac{32}{5} \right) = 25\frac{3}{5}\pi.$$

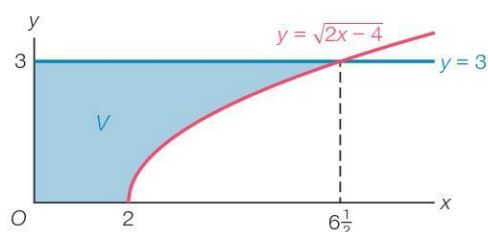
b Voor f geldt $y = \sqrt{x}$, dus voor f^{inv} geldt $x = \sqrt{y}$
 $y = x^2$ met $x \geq 0$

Dus $f^{\text{inv}}(x) = x^2$ met $x \geq 0$.



$$I(L) = \pi \int_0^2 (4^2 - (f^{\text{inv}}(x))^2) dx = \pi \int_0^2 (16 - x^4) dx = \pi \left[16x - \frac{1}{5} x^5 \right]_0^2 = \pi \left(32 - \frac{32}{5} \right) = 25\frac{3}{5}\pi$$

62 a



$$\begin{aligned} y = \sqrt{2x-4} \text{ geeft } y^2 &= 2x-4 \\ 2x-4 &= y^2 \\ 2x &= y^2+4 \\ x &= \frac{1}{2}y^2+2 \\ x^2 &= \left(\frac{1}{2}y^2+2\right)^2 \\ x^2 &= \frac{1}{4}y^4+2y^2+4 \end{aligned}$$

$$\begin{aligned} I(L) &= \pi \int_0^3 x^2 dy = \pi \int_0^3 \left(\frac{1}{4}y^4 + 2y^2 + 4\right) dy = \pi \left[\frac{1}{20}y^5 + \frac{2}{3}y^3 + 4y \right]_0^3 = \pi \left(\frac{1}{20} \cdot 3^5 + \frac{2}{3} \cdot 3^3 + 4 \cdot 3 - 0 \right) \\ &= \pi \left(12\frac{3}{20} + 18 + 12 \right) = 42\frac{3}{20}\pi \end{aligned}$$

b $y = 3$ geeft $\sqrt{2x-4} = 3$
 $2x-4 = 9$
 $2x = 13$
 $x = 6\frac{1}{2}$

$$\begin{aligned} I(M) &= I(\text{cilinder}) - \pi \int_2^{6\frac{1}{2}} (\sqrt{2x-4})^2 dx = \pi \cdot 3^2 \cdot 6\frac{1}{2} - \pi \int_2^{6\frac{1}{2}} (2x-4) dx = 58\frac{1}{2}\pi - \pi \left[x^2 - 4x \right]_2^{6\frac{1}{2}} \\ &= 58\frac{1}{2}\pi - \pi \left(\left(6\frac{1}{2}\right)^2 - 4 \cdot 6\frac{1}{2} - (2^2 - 4 \cdot 2) \right) = 58\frac{1}{2}\pi - 20\frac{1}{4}\pi = 38\frac{1}{4}\pi \end{aligned}$$

63 a $I(L_4) = \pi \int_0^4 x^2 dy = \pi \int_0^4 y dy = \pi \left[\frac{1}{2} y^2 \right]_0^4 = \pi(8 - 0) = 8\pi$

b $P(p, q)$ ligt op de parabool, dus $q = p^2$.

$$I(L_p) = I(M_q) \text{ geeft } \pi \int_0^p y^2 dx = \pi \int_0^q x^2 dy$$

$$\int_0^p x^4 dx = \int_0^{p^2} y dy$$

$$\left[\frac{1}{5} x^5 \right]_0^p = \left[\frac{1}{2} y^2 \right]_0^{p^2}$$

$$\frac{1}{5} p^5 - 0 = \frac{1}{2} p^4 - 0$$

$$2p^5 = 5p^4$$

$$p^4 = 0 \vee 2p = 5$$

$$p = 0 \vee p = 2\frac{1}{2}$$

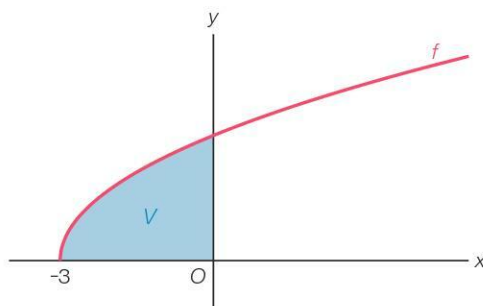
$p = 0$ voldoet niet en $p = 2\frac{1}{2}$ geeft $q = 6\frac{1}{4}$.

Dus voor $p = 2\frac{1}{2}$ en $q = 6\frac{1}{4}$.

64 a $f(x) = 0$ geeft $2x + 6 = 0$

$$2x = -6$$

$$x = -3$$



$$I(L) = \pi \int_{-3}^0 (\sqrt{2x+6})^2 dx = \pi \int_{-3}^0 (2x+6) dx = \pi [x^2 + 6x]_{-3}^0 = \pi(0 - (9 - 18)) = 9\pi$$

b $f(0) = \sqrt{6}$
 $y = \sqrt{2x+6}$ geeft $y^2 = 2x+6$

$$2x + 6 = y^2$$

$$2x = y^2 - 6$$

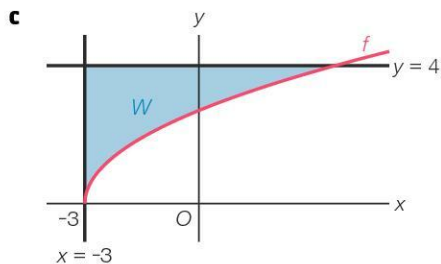
$$x = \frac{1}{2} y^2 - 3$$

$$x^2 = \left(\frac{1}{2} y^2 - 3 \right)^2$$

$$x^2 = \frac{1}{4} y^4 - 3y^2 + 9$$

$$I(M) = \pi \int_0^{\sqrt{6}} x^2 dy = \pi \int_0^{\sqrt{6}} \left(\frac{1}{4} y^4 - 3y^2 + 9 \right) dy = \pi \left[\frac{1}{20} y^5 - y^3 + 9y \right]_0^{\sqrt{6}}$$

$$= \pi \left(\frac{1}{20} \cdot (\sqrt{6})^5 - (\sqrt{6})^3 + 9 \cdot \sqrt{6} - 0 \right) = \pi \left(\frac{1}{20} \cdot 36\sqrt{6} - 6\sqrt{6} + 9\sqrt{6} \right) = 4\frac{4}{5} \pi \sqrt{6}$$



$$y = \sqrt{2x+6}$$

↓ translatie (3, 0)

$$y = \sqrt{2(x-3)+6}$$

oftewel $y = \sqrt{2x}$

$$y = \sqrt{2x} \text{ geeft } y^2 = 2x$$

$$2x = y^2$$

$$x = \frac{1}{2}y^2$$

$$x^2 = \frac{1}{4}y^4$$

$$I(N) = \pi \int_0^4 x^2 dy = \pi \int_0^4 \frac{1}{4} y^4 dy = \pi \left[\frac{1}{20} y^5 \right]_0^4 = \pi \left(\frac{1}{20} \cdot 4^5 - 0 \right) = 51 \frac{1}{5} \pi$$

Bladzijde 126

65 a $I(L) = \pi \int_{-4}^4 (x^2 + 4) dx = \pi \left[\frac{1}{3} x^3 + 4x \right]_{-4}^4 = \pi \left(\frac{64}{3} + 16 - \left(-\frac{64}{3} - 16 \right) \right) = 74 \frac{2}{3} \pi$

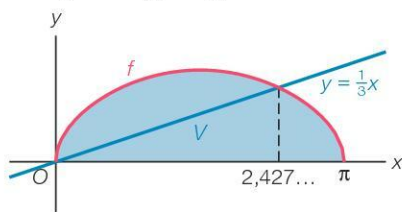
b $O(V) = \int_{-4}^4 \sqrt{x^2 + 4} dx$ en van $\sqrt{x^2 + 4}$ is geen primitieve te vinden met de regels die tot nu toe zijn geweest.

66 Voer in $y_1 = \sqrt{x^2 + 4}$.

De optie integraal geeft $O(V) = \int_{-4}^4 \sqrt{x^2 + 4} dx \approx 23,66$.

67 Voer in $y_1 = \sqrt{\sin(x)}$ en $y_2 = \frac{1}{3}x$.

De optie snijpunt geeft $x = 0$ en $x = 2,427...$



De optie integraal geeft $\int_0^{2,427...} (\sqrt{\sin(x)} - \frac{1}{3}x) dx = 1,019...$ en $\int_0^{\pi} \sqrt{\sin(x)} dx = 2,396...$

1,019... is niet gelijk aan de helft van 2,396..., dus de lijn $y = \frac{1}{3}x$ verdeelt het vlakdeel V niet in twee delen met gelijke oppervlakte.

Bladzijde 127

68 a $f(x) = 0$ geeft $2 \ln(x) - \ln^2(x) = 0$
 $\ln(x)(2 - \ln(x)) = 0$
 $\ln(x) = 0 \vee \ln(x) = 2$
 $x = 1 \vee x = e^2$

Dus de nulpunten zijn 1 en e^2 .

b $f(x) = 2 \ln(x) - \ln^2(x)$ geeft $f'(x) = \frac{2}{x} - 2 \ln(x) \cdot \frac{1}{x} = \frac{2 - 2 \ln(x)}{x}$

$f'(x) = 0$ geeft $2 - 2 \ln(x) = 0$

$\ln(x) = 1$

$x = e$

$f(e) = 2 \cdot 1 - 1^2 = 1$, dus de top is $(e, 1)$.

c $F(x) = 4x \ln(x) - x \ln^2(x) - 4x$ geeft

$$F'(x) = 4 \cdot \ln(x) + 4x \cdot \frac{1}{x} - \left(1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} \right) - 4 = 4 \ln(x) + 4 - \ln^2(x) - 2 \ln(x) - 4$$

$$= 2 \ln(x) - \ln^2(x)$$

Dus $F(x) = 4x \ln(x) - x \ln^2(x) - 4x$ is een primitieve van $f(x) = 2 \ln(x) - \ln^2(x)$.

d $O(V) = \int_1^{e^2} (2 \ln(x) - \ln^2(x)) dx = [4x \ln(x) - x \ln^2(x) - 4x]_1^{e^2}$

$$= 4e^2 \cdot \ln(e^2) - e^2 \cdot (\ln(e^2))^2 - 4e^2 - (4 \ln(1) - (\ln(1))^2 - 4) = 8e^2 - 4e^2 - 4e^2 - (0 - 0 - 4) = 4$$

e De optie integraal geeft $\pi \int_1^{e^2} (2 \ln(x) - \ln^2(x))^2 dx = 9,7780...$

Dus $I(L) \approx 9,778$.

11.4 Toepassingen van integralen

Bladzijde 129

69 $I_{\text{bol}} = 2 \cdot \pi \int_0^r y^2 dx = 2\pi \int_0^r (r^2 - x^2) dx = 2\pi \left[r^2 x - \frac{1}{3} x^3 \right]_0^r = 2\pi \left(r^3 - \frac{1}{3} r^3 - 0 \right) = 2\pi \cdot \frac{2}{3} r^3 = \frac{4}{3} \pi r^3$

Bladzijde 130

70 a $I(T) = \pi \int_{\frac{1}{4}r}^{\frac{1}{2}r} (r^2 - x^2) dx = \pi \left[r^2 x - \frac{1}{3} x^3 \right]_{\frac{1}{4}r}^{\frac{1}{2}r} = \pi \left(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot \left(\frac{1}{2}r \right)^3 - \left(r^2 \cdot \frac{1}{4}r - \frac{1}{3} \cdot \left(\frac{1}{4}r \right)^3 \right) \right)$

$$= \pi \left(\frac{1}{2} r^3 - \frac{1}{24} r^3 - \frac{1}{4} r^3 + \frac{1}{192} r^3 \right) = \frac{41}{192} \pi r^3$$

$$\frac{I(T)}{I(S)} = \frac{\frac{41}{192} \pi r^3}{\frac{29}{192} \pi r^3} = \frac{41}{29} = 1 \frac{12}{29}$$

Dus de inhoud van bolschijf T is $1 \frac{12}{29}$ keer zo groot als de inhoud van bolschijf S .

b $I_p = 24\pi$ geeft $\frac{1}{3} \cdot \pi \cdot \left(\frac{2}{3} \cdot 6 \right)^2 \cdot 6 - \frac{1}{3} \cdot \pi \cdot \left(\frac{2}{3} p \right)^2 \cdot p = 24\pi$

$$32 - \frac{4}{27} p^3 = 24$$

$$-\frac{4}{27} p^3 = -8$$

$$p^3 = 54$$

$$p = \sqrt[3]{54}$$

Bladzijde 131

71 $I = \pi \int_{\frac{1}{3}r}^r y^2 dx = \pi \int_{\frac{1}{3}r}^r (r^2 - x^2) dx = \pi \left[r^2 x - \frac{1}{3} x^3 \right]_{\frac{1}{3}r}^r = \pi \left(r^3 - \frac{1}{3} r^3 - \left(r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot \left(\frac{1}{3}r \right)^3 \right) \right)$

$$= \pi \left(r^3 - \frac{1}{3} r^3 - \frac{1}{3} r^3 + \frac{1}{81} r^3 \right) = \frac{28}{81} \pi r^3$$

$$72 \quad I(S_p) = \pi \int_{-3}^p y^2 dx = \pi \int_{-3}^p (36 - x^2) dx = \pi \left[36x - \frac{1}{3}x^3 \right]_{-3}^p = \pi \left(36p - \frac{1}{3}p^3 - \left(36 \cdot -3 - \frac{1}{3} \cdot (-3)^3 \right) \right) = \pi \left(36p - \frac{1}{3}p^3 + 99 \right)$$

$$I_{\text{bol}} = \frac{4}{3}\pi r^3 = \frac{4}{3} \cdot \pi \cdot 6^3 = 288\pi$$

$$I(S_p) = \frac{1}{2} \cdot 288\pi \text{ geeft } 36p - \frac{1}{3}p^3 + 99 = 144$$

$$36p - \frac{1}{3}p^3 = 45$$

Voer in $y_1 = 36x - \frac{1}{3}x^3$ en $y_2 = 45$.

De optie snijpunt geeft $x = 1,268\dots$ en $x = 9,699\dots$

Dus $p \approx 1,27$.

$$73 \quad \text{rc}_{OA} = \frac{4-0}{3-0} = \frac{4}{3}$$

$$k \perp OA, \text{ dus } \text{rc}_k \cdot \text{rc}_{OA} = -1 \quad \left. \begin{array}{l} \text{rc}_{OA} = \frac{4}{3} \\ \text{rc}_k = -\frac{3}{4} \end{array} \right\} \text{rc}_k \cdot \frac{4}{3} = -1$$

$$y = -\frac{3}{4}x + b \quad \left. \begin{array}{l} \text{door } A(3, 4) \\ -\frac{3}{4} \cdot 3 + b = 4 \end{array} \right\} -\frac{3}{4} \cdot 3 + b = 4$$

$$-2\frac{1}{4} + b = 4$$

$$b = 6\frac{1}{4}$$

Dus $k: y = -\frac{3}{4}x + 6\frac{1}{4}$.

k snijden met de x -as geeft $-\frac{3}{4}x + 6\frac{1}{4} = 0$

$$-\frac{3}{4}x = -6\frac{1}{4}$$

$$x = 8\frac{1}{3}$$

$$I(L) = \pi \int_3^{8\frac{1}{3}} \left(-\frac{3}{4}x + 6\frac{1}{4} \right)^2 dx - \pi \int_3^5 (25 - x^2) dx = \pi \left[-\frac{4}{3} \cdot \frac{1}{3} \cdot \left(-\frac{3}{4}x + 6\frac{1}{4} \right)^3 \right]_3^{8\frac{1}{3}} - \pi \left[25x - \frac{1}{3}x^3 \right]_3^5$$

$$= \pi \left[-\frac{4}{9} \left(-\frac{3}{4}x + 6\frac{1}{4} \right)^3 \right]_3^{8\frac{1}{3}} - \pi \left[25x - \frac{1}{3}x^3 \right]_3^5$$

$$= \pi \left(-\frac{4}{9} \left(-\frac{3}{4} \cdot \frac{25}{3} + 6\frac{1}{4} \right)^3 + \frac{4}{9} \left(-\frac{3}{4} \cdot 3 + 6\frac{1}{4} \right)^3 \right) - \pi \left(25 \cdot 5 - \frac{1}{3} \cdot 5^3 - \left(25 \cdot 3 - \frac{1}{3} \cdot 3^3 \right) \right)$$

$$= \pi \left(-\frac{4}{9} \cdot 0^3 + \frac{4}{9} \cdot 4^3 \right) - \pi \left(125 - \frac{1}{3} \cdot 5^3 - \left(75 - \frac{1}{3} \cdot 3^3 \right) \right) = \pi \left(0 + \frac{4}{9} \cdot 64 \right) - \pi \left(83\frac{1}{3} - 66 \right) = 11\frac{1}{9}\pi$$

$$74 \quad \text{a} \quad \text{De gemiddelde snelheid gedurende de eerste zes seconden is } \frac{0+10}{2} = 5 \text{ m/s.}$$

Er wordt $5 \cdot 6 = 30$ m afgelegd gedurende deze zes seconden.

b De afgelegde afstand is een primitieve van de snelheid. Dus de afstand gedurende de

$$\text{eerste zes seconden is } \int_0^6 v(t) dt. \text{ Ook is } O(\triangle OAB) = \int_0^6 v(t) dt.$$

Dus de oppervlakte van driehoek OAB geeft de afgelegde afstand gedurende de eerste zes seconden.

c De afgelegde afstand op het interval $[0, 15]$ is $O(\text{gehele driehoek}) = \frac{1}{2} \cdot 15 \cdot 10 = 75$ m.

Bladzijde 133

$$75 \quad \text{Stel } s(0) = c. \text{ Dan geldt } s(t) = -\frac{1}{12}t^3 + 2\frac{1}{2}t^2 + 4t + c \text{ en dit geeft } s(10) = 206\frac{2}{3} + c \text{ meter.}$$

In de eerste 10 seconden wordt dan afgelegd $s(10) - s(0) = 206\frac{2}{3} + c - c = 206\frac{2}{3}$ meter.

Je krijgt dus hetzelfde antwoord.

76 Stel $a(t) = mt + n$.

$$m = \frac{a(20) - a(0)}{20 - 0} = \frac{5 - 0}{20} = \frac{1}{4}$$

$$a(t) = \frac{1}{4}t + n \text{ en } a(0) = 0 \text{ geeft } a(t) = \frac{1}{4}t.$$

$$a(t) = \frac{1}{4}t \text{ en } v(0) = 1 \text{ geeft } v(t) = \frac{1}{8}t^2 + 1$$

$$v(t) = \frac{1}{8}t^2 + 1 \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{24}t^3 + t$$

De afgelegde afstand gedurende deze 20 seconden is $s(20) = \frac{1}{24} \cdot 20^3 + 20 = 353\frac{1}{3}$ meter.

77 a $a(t) = -t^2 + 6t$ en $v(0) = 0$ geeft $v(t) = -\frac{1}{3}t^3 + 3t^2$

$$v(6) = -\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2 = 36$$

Dus de snelheid op $t = 6$ is 36 m/s.

b $v(t) = -\frac{1}{3}t^3 + 3t^2$ en $s(0) = 0$ geeft $s(t) = -\frac{1}{12}t^4 + t^3$

De afgelegde afstand gedurende deze 6 seconden is $s(6) = -\frac{1}{12} \cdot 6^4 + 6^3 = 108$ meter.

c $s(6) = 108$ en $v(6) = 36$ geeft $s(10) = 108 + 4 \cdot 36 = 252$

Dus op $t = 10$ is in totaal 252 meter afgelegd.

d $500 - 108 = 392$

392 meter met een snelheid van 36 m/s duurt $\frac{392}{36} = 10\frac{8}{9}$ seconden.

Dus op $t = 6 + 10\frac{8}{9} = 16\frac{8}{9}$ is in totaal 500 meter afgelegd.

78 a 54 km/uur = 15 m/s

De constante versnelling is a .

$$v(t) = at + b \text{ en } v(0) = 15 \text{ geeft } v(t) = at + 15.$$

$$v(t) = at + 15 \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{2}at^2 + 15t.$$

Noem de remtijd t_r , dan geldt $v(t_r) = 0$ en $s(t_r) = 0,75$.

$$v(t_r) = 0 \text{ geeft } at_r + 15 = 0$$

$$at_r = -15$$

$$s(t_r) = 0,75 \text{ geeft } \frac{1}{2}at_r^2 + 15t_r = 0,75 \text{ oftewel } \frac{1}{2}at_r \cdot t_r + 15t_r = 0,75.$$

$$\text{Substitutie van } at_r = -15 \text{ in } \frac{1}{2}at_r \cdot t_r + 15t_r = 0,75 \text{ geeft } -7,5t_r + 15t_r = 0,75$$

$$7,5t_r = 0,75$$

$$t_r = 0,1$$

De botsing duurt 0,1 seconde.

Alternatieve uitwerking

Het rekenwerk wordt eenvoudiger als het probleem 'achterstevoren' wordt aangepakt.

Noem de remtijd t_r en neem $v(0) = 0$, $s(0) = 0$, $v(t_r) = 15$ en $s(t_r) = 0,75$.

Dit geeft $v(t) = at$ en $s(t) = \frac{1}{2}at^2$. Dan geldt

$$\left. \begin{array}{l} at_r = 15 \\ \frac{1}{2}at_r^2 = 0,75 \text{ oftewel } \frac{1}{2}at_r \cdot t_r = 0,75 \end{array} \right\} \begin{array}{l} 7,5t_r = 0,75 \\ t_r = 0,1 \end{array}$$

De botsing duurt 0,1 seconde.

b $at_r = -15$ $\left\{ \begin{array}{l} a \cdot 0,1 = -15 \\ a = -150 \end{array} \right.$

De versnelling is -150 m/s^2 .

Dit is 15 keer zo groot als de versnelling van de zwaartekracht.

Bladzijde 134

79

a $v(t) = -0,2t^2 + 4t$ en $s(0) = 0$ geeft $s(t) = -\frac{2}{30}t^3 + 2t^2$

$$s(10) = -\frac{2}{30} \cdot 10^3 + 2 \cdot 10^2 = 133\frac{1}{3}$$

$$v(t) = 0,8t^2 - 24t + 180 \text{ geeft } s(t) = \frac{4}{15}t^3 - 12t^2 + 180t + c$$

$$s(10) = 133\frac{1}{3} \text{ geeft } \frac{4}{15} \cdot 10^3 - 12 \cdot 10^2 + 180 \cdot 10 + c = 133\frac{1}{3}$$

$$866\frac{2}{3} + c = 133\frac{1}{3}$$

$$c = -733\frac{1}{3}$$

$$\text{Dus } s(t) = \frac{4}{15}t^3 - 12t^2 + 180t - 733\frac{1}{3}.$$

$$s(15) = \frac{4}{15} \cdot 15^3 - 12 \cdot 15^2 + 180 \cdot 15 - 733\frac{1}{3} = 166\frac{2}{3}$$

De totale afstand die de auto gedurende deze 15 seconden aflegt is $166\frac{2}{3}$ meter.

b gemiddelde snelheid $= \frac{166\frac{2}{3}}{15} = 11\frac{1}{9}$ m/s

$$\text{Voer in } y_1 = -0,2x^2 + 4x, y_2 = 0,8x^2 - 24x + 180 \text{ en } y_3 = 11\frac{1}{9}.$$

De optie snijpunt met y_1 en y_3 geeft $x = 3,333...$

De optie snijpunt met y_2 en y_3 geeft $x = 11,273...$

Dus op $t \approx 3,33$ en $t \approx 11,27$ is de snelheid gelijk aan de gemiddelde snelheid.

c Voer in $y_4 = -\frac{2}{30}x^3 + 2x^2$ en $y_5 = 100$.

De optie snijpunt met y_4 en y_5 geeft $x = 8,317...$

Dus na ongeveer 8,3 seconde heeft de auto 100 meter afgelegd.

80

a $a(t) = -4e^{-0,1t}$

$$v(t) = 40e^{-0,1t} + c \quad \left. \begin{array}{l} 40e^{-0,3} + c = 32 \\ 29,632... + c = 32 \end{array} \right\}$$

$$v(3) = 32 \quad \left. \begin{array}{l} 29,632... + c = 32 \\ c = 2,367... \end{array} \right\}$$

$$c = 2,367...$$

$$v(t) = 40e^{-0,1t} + 2,367...$$

$$v(0) = 40 + 2,367... = 42,367...$$

De snelheid is 42,4 m/s.

b $v(t) = 40e^{-0,1t} + 2,367...$

$$\left. \begin{array}{l} s(t) = -400e^{-0,1t} + 2,367...t + c \\ s(0) = 0 \end{array} \right\} \begin{array}{l} -400 + 0 + c = 0 \\ c = 400 \end{array}$$

$$\text{Dus } s(t) = -400e^{-0,1t} + 2,367...t + 400.$$

$$\text{Voer in } y_1 = -400e^{-0,1x} + 2,367...x + 400 \text{ en } y_2 = 800.$$

De optie snijpunt geeft $x = 168,970...$

$$v(168,970...) = 2,367...$$

De snelheid waarmee de parachutiste op de grond komt is ongeveer 2,4 m/s.

Bladzijde 135

81

a $a(t) = 5e^{0,012t}$ geeft

$$\left. \begin{array}{l} v(t) = 416\frac{2}{3}e^{0,012t} + c \\ v(0) = 0 \end{array} \right\} \begin{array}{l} 416\frac{2}{3} + c = 0 \\ c = -416\frac{2}{3} \end{array}$$

$$\text{Dus } v(t) = 416\frac{2}{3}e^{0,012t} - 416\frac{2}{3}.$$

$$v(t) = 416\frac{2}{3}e^{0,012t} - 416\frac{2}{3} \text{ geeft}$$

$$\left. \begin{array}{l} s(t) = 34722\frac{2}{9}e^{0,012t} - 416\frac{2}{3}t + c \\ s(0) = 0 \end{array} \right\} \begin{array}{l} 34722\frac{2}{9} + c = 0 \\ c = -34722\frac{2}{9} \end{array}$$

$$\text{Dus } s(t) = 34722\frac{2}{9}e^{0,012t} - 416\frac{2}{3}t - 34722\frac{2}{9}.$$

$$v(170) = 416\frac{2}{3}e^{0,012 \cdot 170} - 416\frac{2}{3} = 2787,7... \text{ m/s} \approx 10036 \text{ km/uur}$$

$$s(170) = 34722\frac{2}{9}e^{0,012 \cdot 170} - 416\frac{2}{3} \cdot 170 - 34722\frac{2}{9} = 161479,4... \text{ m} \approx 161 \text{ km}$$

Dus de snelheid na 170 seconden is 10036 km/uur en de Apollo heeft dan 161 km afgelegd.

b $a(t) = 4e^{0,0045(t-170)}$ geeft

$$\begin{aligned} v(t) &= 888\frac{8}{9}e^{0,0045(t-170)} + c \\ v(170) &= 2787,7... \end{aligned} \quad \left. \begin{aligned} &888\frac{8}{9} + c = 2787,7... \\ &c = 1898,8... \end{aligned} \right\}$$

Dus $v(t) = 888\frac{8}{9}e^{0,0045(t-170)} + 1898,8...$

$v(t) = 888\frac{8}{9}e^{0,0045(t-170)} + 1898,8...$ geeft

$$\begin{aligned} s(t) &= 197530,8...e^{0,0045(t-170)} + 1898,8...t + c \\ s(170) &= 161479,4... \end{aligned} \quad \left. \begin{aligned} &197530,8... + 1898,8... \cdot 170 + c = 161479,4... \\ &520337,9... + c = 161479,4... \\ &c = -358858,4... \end{aligned} \right\}$$

Dus $s(t) = 197530,8...e^{0,0045(t-170)} + 1898,8...t - 358858,4...$

$v(560) = 888\frac{8}{9}e^{0,0045(560-170)} + 1898,8... = 7039,7... \text{ m/s} \approx 25342 \text{ km/uur}$

$s(560) = 197530,8...e^{0,0045(560-170)} + 1898,8... \cdot 560 - 358858,4... = 1846915,3... \text{ m} \approx 1847 \text{ km}$

Dus op $t = 560$ is de snelheid 25342 km/uur en de afgelegde afstand 1847 km.

Diagnostische toets

Bladzijde 138

1 a $F(x) = x^3 e^x$ geeft $F'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = (x^3 + 3x^2)e^x$
Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .

b $G(x) = \frac{1}{2}x^2 - x + 2 \ln(x+1)$ geeft $G'(x) = x - 1 + 2 \cdot \frac{1}{x+1} = \frac{(x-1)(x+1) + 2}{x+1}$
Dus $G'(x) = g(x)$ oftewel G is een primitieve van g .

2 a $f(x) = \frac{2x+6}{x^2} = 2 \cdot \frac{1}{x} + 6x^{-2}$ geeft $F(x) = 2 \ln|x| - 6x^{-1} + c = 2 \ln|x| - \frac{6}{x} + c$

b $f(x) = 3 \cdot 2^x$ geeft $F(x) = \frac{3}{\ln(2)} \cdot 2^x + c$

c $f(x) = 6e^x + x^2$ geeft $F(x) = 6e^x + \frac{1}{3}x^3 + c$

d $f(x) = \ln(x^5) = 5 \ln(x)$ geeft $F(x) = 5(x \ln(x) - x) + c = 5x \ln(x) - 5x + c$

e $f(x) = {}^2\log(4x)$ geeft $F(x) = \frac{1}{4} \cdot \frac{1}{\ln(2)} (4x \ln(4x) - 4x) + c = \frac{1}{\ln(2)} \cdot (x \ln(4x) - x) + c$

f $f(x) = 3 \sin(x) + 2 \cos(x)$ geeft $F(x) = -3 \cos(x) + 2 \sin(x) + c$

3 $f(x) = 4x - \ln(x)$ geeft $F(x) = 2x^2 - (x \ln(x) - x) + c = 2x^2 + x - x \ln(x) + c$

$F(1) = 7$ geeft $2 \cdot 1^2 + 1 - 1 \cdot \ln(1) + c = 7$

$2 + 1 - 0 + c = 7$

$c = 4$

Dus $F(x) = 2x^2 + x - x \ln(x) + 4$.

4 a $O(V) = \int_0^2 \frac{2}{x+1} dx = [2 \ln|x+1|]_0^2 = 2 \ln(3) - 2 \ln(1) = 2 \ln(3)$

b $\int_0^p \frac{2}{x+1} dx = [2 \ln|x+1|]_0^p = 2 \ln(p+1) - 2 \ln(1) = 2 \ln(p+1)$

$\int_0^p \frac{2}{x+1} dx = \frac{1}{2} O(V)$ geeft $2 \ln(p+1) = \ln(3)$

$\ln(p+1) = \frac{1}{2} \ln(3)$

$\ln(p+1) = \ln(\sqrt{3})$

$p+1 = \sqrt{3}$

$p = -1 + \sqrt{3}$

5 a $f(x) = (2x+6)^5 + \frac{10}{(3x-1)^2} = (2x+6)^5 + 10(3x-1)^{-2}$ geeft
 $F(x) = \frac{1}{2} \cdot \frac{1}{6} (2x+6)^6 + 10 \cdot -\frac{1}{3} (3x-1)^{-1} + c = \frac{1}{12} (2x+6)^6 - \frac{10}{3(3x-1)} + c$

b $f(x) = (5x+2)^2 \cdot \sqrt{5x+2} = (5x+2)^{2\frac{1}{2}}$ geeft
 $F(x) = \frac{1}{5} \cdot \frac{2}{\frac{7}{2}} (5x+2)^{3\frac{1}{2}} + c = \frac{2}{35} (5x+2)^3 \cdot \sqrt{5x+2} + c$

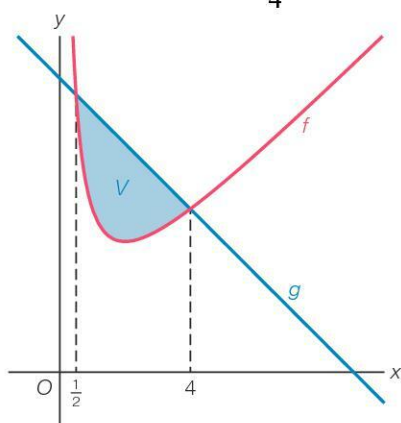
c $f(x) = 4e^{2x+3}$ geeft $F(x) = 4 \cdot \frac{1}{2} e^{2x+3} + c = 2e^{2x+3} + c$

d $f(x) = 8 \cdot 2^{2x-1}$ geeft $F(x) = 8 \cdot \frac{1}{2} \cdot \frac{1}{\ln(2)} \cdot 2^{2x-1} + c = \frac{4}{\ln(2)} \cdot 2^{2x-1} + c$

e $f(x) = \ln(2x+3)$ geeft
 $F(x) = \frac{1}{2} ((2x+3) \ln(2x+3) - (2x+3)) + c = (x + \frac{1}{2}) \ln(2x+3) - (x + \frac{1}{2}) + c$

f $f(x) = \frac{6}{2x+5}$ geeft $F(x) = 6 \cdot \frac{1}{2} \cdot \ln|2x+5| + c = 3 \ln|2x+5| + c$

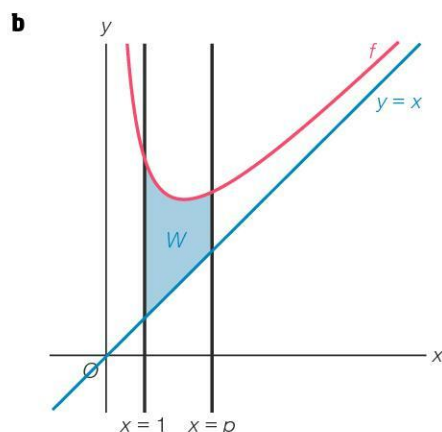
6 a $f(x) = g(x)$ geeft $\frac{x^2+4}{x} = -x+9$
 $x^2+4 = -x^2+9x$
 $2x^2-9x+4=0$
 $D = (-9)^2 - 4 \cdot 2 \cdot 4 = 49$
 $x = \frac{9+7}{4} = 4 \vee x = \frac{9-7}{4} = \frac{1}{2}$



$$O(V) = \int_{\frac{1}{2}}^4 \left(-x + 9 - \frac{x^2+4}{x} \right) dx = \int_{\frac{1}{2}}^4 \left(-x + 9 - x - \frac{4}{x} \right) dx = \int_{\frac{1}{2}}^4 \left(-2x + 9 - \frac{4}{x} \right) dx$$

$$= \left[-x^2 + 9x - 4 \ln|x| \right]_{\frac{1}{2}}^4 = -16 + 36 - 4 \ln(4) - \left(-\frac{1}{4} + 4\frac{1}{2} - 4 \ln\left(\frac{1}{2}\right) \right) = 20 - 4 \ln(4) - 4\frac{1}{4} + 4 \ln\left(\frac{1}{2}\right)$$

$$= 15\frac{3}{4} - 8 \ln(2) - 4 \ln(2) = 15\frac{3}{4} - 12 \ln(2)$$



$$O(W) = \int_1^p \left(\frac{x^2+4}{x} - x \right) dx = \int_1^p \frac{4}{x} dx = [4 \ln|x|]_1^p = 4 \ln(p) - 0 = 4 \ln(p)$$

$O(W) = 3$ geeft $4 \ln(p) = 3$
 $\ln(p) = \frac{3}{4}$
 $p = e^{\frac{3}{4}} = \sqrt[4]{e^3}$

7 $f(x) = 0$ geeft $(2x+1)\sqrt{2x+1} = 0$

$$(2x+1)^{\frac{1}{2}} = 0$$

$$2x+1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$I(K) = \pi \int_{-\frac{1}{2}}^0 (2x+1)^3 dx = \pi \left[\frac{1}{2} \cdot \frac{1}{4} (2x+1)^4 \right]_{-\frac{1}{2}}^0 = \pi \left(\frac{1}{8} - 0 \right) = \frac{1}{8} \pi$$

$$I(L_p) = \pi \int_0^p (2x+1)^3 dx = \pi \left[\frac{1}{2} \cdot \frac{1}{4} (2x+1)^4 \right]_0^p = \pi \left(\frac{1}{8} (2p+1)^4 - \frac{1}{8} \cdot 1^4 \right) = \frac{1}{8} \pi ((2p+1)^4 - 1)$$

$$I(L_p) = 624 \cdot I(K) = 624 \cdot \frac{1}{8} \pi = 78\pi \text{ geeft } \frac{1}{8} \pi ((2p+1)^4 - 1) = 78\pi$$

$$(2p+1)^4 - 1 = 624$$

$$(2p+1)^4 = 625$$

$$2p+1 = 5 \vee 2p+1 = -5$$

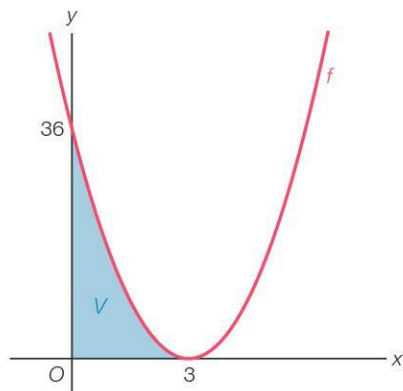
$$2p = 4 \vee 2p = -6$$

$$p = 2 \vee p = -3$$

vold. niet

Dus $p = 2$.

8 $f(0) = (-6)^2 = 36$



$$y = (2x-6)^2 \text{ en } 0 \leq x \leq 3 \text{ geeft } 2x-6 = -\sqrt{y}$$

$$2x = -\sqrt{y} + 6$$

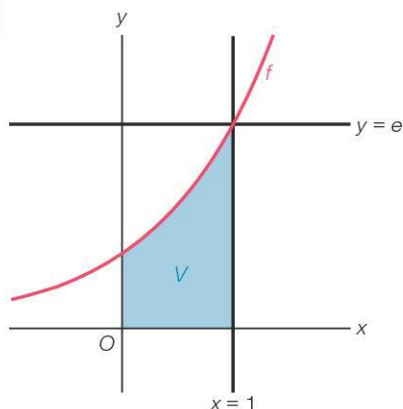
$$x = -\frac{1}{2}\sqrt{y} + 3$$

$$x^2 = \left(-\frac{1}{2}\sqrt{y} + 3\right)^2$$

$$x^2 = \frac{1}{4}y - 3\sqrt{y} + 9$$

$$\begin{aligned} I(L) &= \pi \int_0^{36} x^2 dy = \pi \int_0^{36} \left(\frac{1}{4}y - 3y^{\frac{1}{2}} + 9 \right) dy = \pi \left[\frac{1}{8}y^2 - 3 \cdot \frac{2}{3}y^{\frac{1}{2}} + 9y \right]_0^{36} = \pi \left[\frac{1}{8}y^2 - 2y\sqrt{y} + 9y \right]_0^{36} \\ &= \pi(162 - 432 + 324 - 0) = 54\pi \end{aligned}$$

9



$$f(x) = e^x$$

↓ translatie (0, -e)

$$y = e^x - e$$

Wentelen om de lijn $y = e$ geeft dezelfde inhoud als het vlakdeel ingesloten door de y -as, de lijn $x = 1$, de lijn $y = -e$ en de grafiek van $y = e^x - e$ wentelen om de x -as.

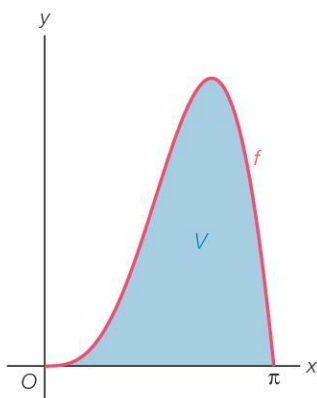
$$\begin{aligned} I(L) &= \pi \int_0^1 ((-e)^2 - (e^x - e)^2) dx = \pi \int_0^1 (e^2 - (e^{2x} - 2e \cdot e^x + e^2)) dx = \pi \int_0^1 (-e^{2x} + 2e \cdot e^x) dx \\ &= \pi \left[-\frac{1}{2} e^{2x} + 2e \cdot e^x \right]_0^1 = \pi \left(-\frac{1}{2} e^2 + 2e^2 - \left(-\frac{1}{2} + 2e \right) \right) = \left(1\frac{1}{2} e^2 - 2e + \frac{1}{2} \right) \pi \approx 19,31 \end{aligned}$$

10 $f(x) = 0$ geeft $x^2 \sin(x) = 0$

$$x^2 = 0 \vee \sin(x) = 0$$

$$x = 0 \vee x = k \cdot \pi$$

$$x = 0 \vee x = \pi$$



$$O(V) = \int_0^\pi x^2 \sin(x) dx$$

De optie integraal geeft $\int_0^\pi x^2 \sin(x) dx = 5,8696...$

Dus $O(V) \approx 5,870$.

11 OA: $y = 2x$

$$\begin{aligned} I(L) &= \pi \int_0^1 (2x)^2 dx + \pi \int_1^{\sqrt{5}} (5 - x^2) dx = \pi \int_0^1 4x^2 dx + \pi \int_1^{\sqrt{5}} (5 - x^2) dx = \pi \left[\frac{4}{3} x^3 \right]_0^1 + \pi \left[5x - \frac{1}{3} x^3 \right]_1^{\sqrt{5}} \\ &= \pi \left(\frac{4}{3} \cdot 1^3 - \frac{4}{3} \cdot 0^3 \right) + \pi \left(5\sqrt{5} - \frac{1}{3} (\sqrt{5})^3 - \left(5 \cdot 1 - \frac{1}{3} \cdot 1^3 \right) \right) = 1\frac{1}{3} \pi + \pi \left(5\sqrt{5} - \frac{1}{3} \cdot 5\sqrt{5} - 4\frac{2}{3} \right) \\ &= \pi \left(1\frac{1}{3} + 3\frac{1}{3} \sqrt{5} - 4\frac{2}{3} \right) = \pi \left(3\frac{1}{3} \sqrt{5} - 3\frac{1}{3} \right) = 3\frac{1}{3} \pi (\sqrt{5} - 1) \approx 12,94 \end{aligned}$$

12 a Stel $a(t) = mt + n$ met $m = \frac{a(60) - a(0)}{60} = \frac{68 - 8}{60} = 1$ $\left. \begin{array}{l} \\ a(0) = 8 \end{array} \right\} a(t) = t + 8$

$$a(t) = t + 8 \text{ en } v(0) = 0 \text{ geeft } v(t) = \frac{1}{2}t^2 + 8t$$

$$v(t) = \frac{1}{2}t^2 + 8t \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{6}t^3 + 4t^2$$

$$s(60) = \frac{1}{6} \cdot 60^3 + 4 \cdot 60^2 = 50\,400$$

Dus de raket bereikt in deze 60 seconden een hoogte van 50,4 km.

b Vanaf $t = 60$ is $a(t) = -10$, dus $v(t) = -10t + c$.

$$v(60) = 2280 \text{ geeft } -600 + c_1 = 2280, \text{ dus } c_1 = 2880 \text{ en } v(t) = -10t + 2880.$$

$$s(t) = -5t^2 + 2880t + c_2$$

$$s(60) = 50\,400 \text{ geeft } -5 \cdot 60^2 + 2880 \cdot 60 + c_2 = 50\,400$$

$$-18\,000 + 172\,800 + c_2 = 50\,400$$

$$c_2 = -104\,400$$

$$\text{Dus } s(t) = -5t^2 + 2880t - 104\,400.$$

$$s(288) = -5 \cdot 288^2 + 2880 \cdot 288 - 104\,400 = 310\,320$$

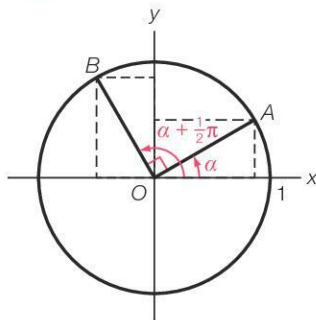
Dus de maximale hoogte is 310,32 km.

12 Goniometrische formules

Voorkennis Goniometrische formules herleiden

Bladzijde 143

1 a

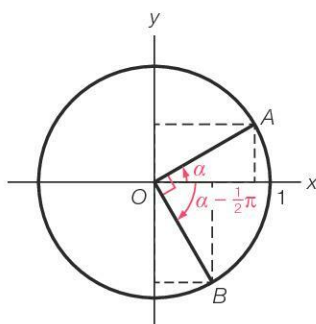


$$\sin(\alpha + \frac{1}{2}\pi) = y_B = x_A = \cos(\alpha) \text{ oftewel } \sin(\alpha + \frac{1}{2}\pi) = \cos(\alpha).$$

$$\cos(\alpha + \frac{1}{2}\pi) = x_B = -y_A = -\sin(\alpha) \text{ oftewel } \cos(\alpha + \frac{1}{2}\pi) = -\sin(\alpha).$$

b $\sin(\alpha) = -\cos(\alpha + \frac{1}{2}\pi) = \cos(\alpha + \frac{1}{2}\pi + \pi) = \cos(\alpha + 1\frac{1}{2}\pi) = \cos(\alpha + 1\frac{1}{2}\pi - 2\pi) = \cos(\alpha - \frac{1}{2}\pi)$

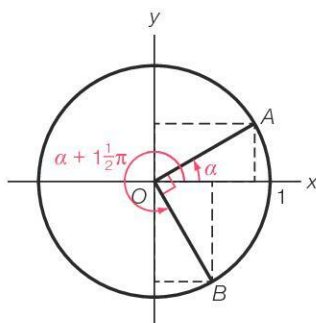
2 a



$$y = \sin(\alpha - \frac{1}{2}\pi) = y_B = -x_A = -\cos(\alpha), \text{ dus } y = -\cos(\alpha).$$

b $y = \cos(\alpha - \frac{1}{2}\pi) = x_B = y_A = \sin(\alpha), \text{ dus } y = \sin(\alpha).$

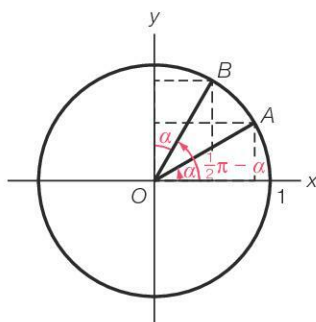
c



$$y = \sin(\alpha + 1\frac{1}{2}\pi) = y_B = -x_A = -\cos(\alpha), \text{ dus } y = -\cos(\alpha).$$

d $y = \cos(\alpha + 1\frac{1}{2}\pi) = x_B = y_A = \sin(\alpha), \text{ dus } y = \sin(\alpha).$

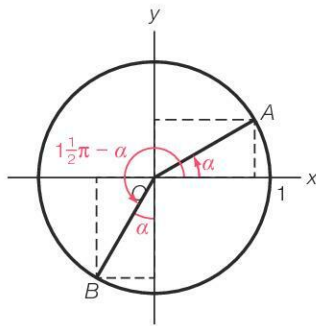
e



$$y = \sin(\frac{1}{2}\pi - \alpha) = y_B = x_A = \cos(\alpha), \text{ dus } y = \cos(\alpha).$$

f $y = \cos(\frac{1}{2}\pi - \alpha) = x_B = y_A = \sin(\alpha)$, dus $y = \sin(\alpha)$.

g



$y = \sin(\frac{1}{2}\pi - \alpha) = y_B = -x_A = -\cos(\alpha)$, dus $y = -\cos(\alpha)$.

h $y = \cos(\frac{1}{2}\pi - \alpha) = x_B = -y_A = -\sin(\alpha)$, dus $y = -\sin(\alpha)$.

Bladzijde 144

- 3 a** $y = \sin(2x - \frac{1}{3}\pi) = \cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(2x - \frac{5}{6}\pi)$
b $y = -\sin(3x - \frac{1}{4}\pi) = \sin(3x - \frac{1}{4}\pi + \pi) = \sin(3x + \frac{3}{4}\pi) = \cos(3x + \frac{3}{4}\pi - \frac{1}{2}\pi) = \cos(3x + \frac{1}{4}\pi)$
c $y = \cos(\frac{1}{2}x + \frac{1}{4}\pi) = \sin(\frac{1}{2}x + \frac{1}{4}\pi + \frac{1}{2}\pi) = \sin(\frac{1}{2}x + \frac{3}{4}\pi)$
d $y = -\cos(2(x - \frac{1}{6}\pi)) = \cos(2x - \frac{1}{3}\pi + \pi) = \cos(2x + \frac{2}{3}\pi) = \sin(2x + \frac{2}{3}\pi + \frac{1}{2}\pi) = \sin(2x + 1\frac{1}{6}\pi)$

12.1 Goniometrische formules bij vergelijkingen en herleidingen

Bladzijde 145

- 1 a** $\sin(2x) + \sin(x) = 0$
 $\sin(2x) = -\sin(x)$
 $\sin(2x) = \sin(x + \pi)$
b $2\sin^2(x) = 3\cos(x)$
 $2(1 - \cos^2(x)) = 3\cos(x)$
 $2 - 2\cos^2(x) = 3\cos(x)$
 $2\cos^2(x) + 3\cos(x) - 2 = 0$

Bladzijde 147

- 2** $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$
 $2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$

- 3 a** $\sin(x + \frac{1}{2}\pi) = \cos(2x)$
 $\cos(x) = \cos(2x)$
 $x = 2x + k \cdot 2\pi \vee x = -2x + k \cdot 2\pi$
 $-x = k \cdot 2\pi \vee 3x = k \cdot 2\pi$
 $x = k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = 2\pi$
b $\sin(3x) = -\cos(x)$
 $\cos(3x - \frac{1}{2}\pi) = \cos(x + \pi)$
 $3x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -(x + \pi) + k \cdot 2\pi$
 $2x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot \pi \vee 4x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot \pi \vee x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = 1\frac{3}{4}\pi \vee x = \frac{3}{8}\pi \vee x = \frac{7}{8}\pi \vee x = 1\frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi$

c $\sin^2(x) + \frac{1}{2}\cos(x) = 1$
 $1 - \cos^2(x) + \frac{1}{2}\cos(x) - 1 = 0$
 $-\cos^2(x) + \frac{1}{2}\cos(x) = 0$
 $-\cos(x)(\cos(x) - \frac{1}{2}) = 0$
 $\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$
 $x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = \frac{5}{3}\pi$

d $\sin(\frac{1}{2}x) + \sqrt{3} \cdot \cos(\frac{1}{2}x) = 0$
 $\sin(\frac{1}{2}x) = -\sqrt{3} \cdot \cos(\frac{1}{2}x)$
 $\frac{\sin(\frac{1}{2}x)}{\cos(\frac{1}{2}x)} = -\sqrt{3}$
 $\tan(\frac{1}{2}x) = -\sqrt{3}$
 $\frac{1}{2}x = -\frac{1}{3}\pi + k \cdot \pi$
 $x = -\frac{2}{3}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{4}{3}\pi$

4 a $\cos(2\pi t) = \sin(\frac{1}{2}\pi t)$
 $\cos(2\pi t) = \cos(\frac{1}{2}\pi t - \frac{1}{2}\pi)$
 $2\pi t = \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -(\frac{1}{2}\pi t - \frac{1}{2}\pi) + k \cdot 2\pi$
 $\frac{1}{2}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi$
 $\frac{1}{2}t = -\frac{1}{2} + k \cdot 2 \vee 2\frac{1}{2}t = \frac{1}{2} + k \cdot 2\pi$
 $t = -\frac{1}{3} + k \cdot \frac{1}{3} \vee 2\frac{1}{2}t = \frac{1}{2} + k \cdot 2$
 $t = -\frac{1}{3} + k \cdot \frac{1}{3} \vee t = \frac{1}{5} + k \cdot \frac{4}{5}$
 t in $[0, 3]$ geeft $t = 1 \vee t = \frac{1}{3} \vee t = \frac{1}{5} \vee t = \frac{4}{5} \vee t = \frac{9}{5}$

b $\sin(\frac{1}{6}\pi t) = -\cos(\pi t)$
 $\cos(\frac{1}{6}\pi t - \frac{1}{2}\pi) = \cos(\pi t + \pi)$
 $\frac{1}{6}\pi t - \frac{1}{2}\pi = \pi t + \pi + k \cdot 2\pi \vee \frac{1}{6}\pi t - \frac{1}{2}\pi = -\pi t - \pi + k \cdot 2\pi$
 $\frac{1}{6}t - \frac{1}{2} = t + 1 + k \cdot 2 \vee \frac{1}{6}t - \frac{1}{2} = -t - 1 + k \cdot 2$
 $-\frac{5}{6}t = 1\frac{1}{2} + k \cdot 2 \vee \frac{1}{6}t = -\frac{1}{2} + k \cdot 2$
 $t = -1\frac{4}{5} + k \cdot 2\frac{2}{5} \vee t = -\frac{3}{7} + k \cdot 1\frac{5}{7}$
 t in $[0, 3]$ geeft $t = \frac{3}{5} \vee t = 3 \vee t = 1\frac{2}{7}$

5 a $2\sin^2(x) + \cos^2(x) + \cos(x) = 0$
 $2(1 - \cos^2(x)) + \cos^2(x) + \cos(x) = 0$
 $2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = 0$
 $\cos^2(x) - \cos(x) - 2 = 0$
 $(\cos(x) + 1)(\cos(x) - 2) = 0$
 $\cos(x) = -1 \vee \cos(x) = 2$
 $x = \pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \pi$

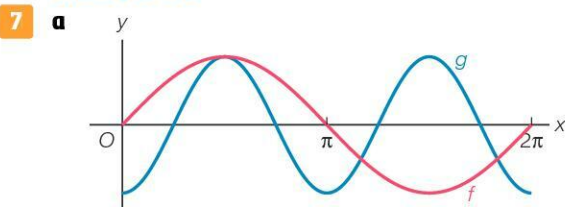
b $\sin(2x - \frac{1}{3}\pi) = -\cos(\frac{1}{2}x)$
 $\cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(\frac{1}{2}x + \pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(\frac{1}{2}x + \pi)$
 $2x - \frac{5}{6}\pi = \frac{1}{2}x + \pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -\frac{1}{2}x - \pi + k \cdot 2\pi$
 $\frac{1}{2}x = 1\frac{5}{6}\pi + k \cdot 2\pi \vee 2\frac{1}{2}x = -\frac{1}{6}\pi + k \cdot 2\pi$
 $x = 1\frac{2}{3}\pi + k \cdot \frac{4}{3} \vee x = -\frac{1}{15}\pi + k \cdot \frac{4}{5}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi \vee x = \frac{11}{15}\pi \vee x = 1\frac{8}{15}\pi$

c $\cos(\frac{1}{4}\pi t) = -\sin(\frac{2}{5}\pi t)$
 $\sin(\frac{1}{4}\pi t + \frac{1}{2}\pi) = \sin(\frac{2}{5}\pi t + \pi)$
 $\frac{1}{4}\pi t + \frac{1}{2}\pi = \frac{2}{5}\pi t + \pi + k \cdot 2\pi \vee \frac{1}{4}\pi t + \frac{1}{2}\pi = \pi - (\frac{2}{5}\pi t + \pi) + k \cdot 2\pi$
 $\frac{1}{4}t + \frac{1}{2} = \frac{2}{5}t + 1 + k \cdot 2 \vee \frac{1}{4}t + \frac{1}{2} = 1 - (\frac{2}{5}t + 1) + k \cdot 2$
 $-\frac{3}{20}t = \frac{1}{2} + k \cdot 2 \vee \frac{13}{20}t = -\frac{1}{2} + k \cdot 2$
 $t = -\frac{10}{3} + k \cdot \frac{40}{3} \vee t = -\frac{10}{13} + k \cdot \frac{40}{13}$
 $t \text{ in } [0, 10] \text{ geeft } t = 10 \vee t = 2\frac{4}{13} \vee t = 5\frac{5}{13} \vee t = 8\frac{6}{13}$

d $\cos(\frac{1}{3}\pi x) = \sqrt{3} \cdot \sin(\frac{1}{3}\pi x)$
 $\frac{\sin(\frac{1}{3}\pi x)}{\cos(\frac{1}{3}\pi x)} = \frac{1}{\sqrt{3}}$
 $\tan(\frac{1}{3}\pi x) = \frac{1}{\sqrt{3}}$
 $\frac{1}{3}\pi x = \frac{1}{6}\pi + k \cdot \pi$
 $\frac{1}{3}x = \frac{1}{6} + k \cdot 1$
 $x = \frac{1}{2} + k \cdot 3$
 $x \text{ in } [0, 10] \text{ geeft } x = \frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 6\frac{1}{2} \vee x = 9\frac{1}{2}$

- 6 a** $2\sin(x) = \sin(x)$
 $\sin(x) = 0$
b $\sin(2x) = \sin(x)$
 $2x = x + k \cdot 2\pi \vee 2x = \pi - x + k \cdot 2\pi$
c —
d —
e $\sin(2x) = \sin(x + \frac{1}{3}\pi)$
 $2x = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
f —

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- b** $f(x) = -\frac{1}{2}\sqrt{2}$ geeft $\sin(x) = -\frac{1}{2}\sqrt{2}$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \pi - \frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi$
c $g(x) = \frac{1}{2}$ geeft $-\cos(2x) = \frac{1}{2}$
 $\cos(2x) = -\frac{1}{2}$
 $2x = \frac{2}{3}\pi + k \cdot 2\pi \vee 2x = -\frac{2}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \pi \vee x = -\frac{1}{3}\pi + k \cdot \pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{2}{3}\pi$
d $f(x) = g(x)$ geeft $\sin(x) = -\cos(2x)$
 $\cos(x - \frac{1}{2}\pi) = \cos(2x + \pi)$
 $x - \frac{1}{2}\pi = 2x + \pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -(2x + \pi) + k \cdot 2\pi$
 $-x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x - \pi + k \cdot 2\pi$
 $x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = -1\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$
 $f(x) \leq g(x) \text{ geeft } x = \frac{1}{2}\pi \vee 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$

8 $f(x) = g(x)$ geeft $\sin(2x - \frac{1}{3}\pi) = -\cos(x - \frac{1}{6}\pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + \frac{5}{6}\pi)$
 $2x - \frac{5}{6}\pi = x + \frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{3}\pi + k \cdot 2\pi \vee 3x = k \cdot 2\pi$
 $x = 1\frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = 1\frac{2}{3}\pi \vee x = 0 \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi \vee x = \pi$

$$O(V) = \int_0^{\frac{2}{3}\pi} (f(x) - g(x)) dx = \int_0^{\frac{2}{3}\pi} (\sin(2x - \frac{1}{3}\pi) + \cos(x - \frac{1}{6}\pi)) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) + \sin(x - \frac{1}{6}\pi)\right]_0^{\frac{2}{3}\pi}$$

$$= -\frac{1}{2}\cos(\pi) + \sin(\frac{1}{2}\pi) - (-\frac{1}{2}\cos(-\frac{1}{3}\pi) + \sin(-\frac{1}{6}\pi)) = \frac{1}{2} + 1 - (-\frac{1}{4} - \frac{1}{2}) = 2\frac{1}{4}$$

$$O(W) = \int_{1\frac{1}{3}\pi}^{1\frac{2}{3}\pi} (f(x) - g(x)) dx = \int_{1\frac{1}{3}\pi}^{1\frac{2}{3}\pi} (\sin(2x - \frac{1}{3}\pi) + \cos(x - \frac{1}{6}\pi)) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) + \sin(x - \frac{1}{6}\pi)\right]_{1\frac{1}{3}\pi}^{1\frac{2}{3}\pi}$$

$$= -\frac{1}{2}\cos(3\pi) + \sin(1\frac{1}{2}\pi) - (-\frac{1}{2}\cos(2\frac{1}{3}\pi) + \sin(1\frac{1}{6}\pi)) = \frac{1}{2} - 1 - (-\frac{1}{4} - \frac{1}{2}) = \frac{1}{4}$$

De oppervlakte van V is $\frac{2\frac{1}{4}}{\frac{1}{4}} = 9$ keer zo groot als de oppervlakte van W .

9 $f(x) = \frac{1}{2}$ geeft $\sin(2x - \frac{1}{3}\pi) = \frac{1}{2}$
 $2x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi$
 $2x = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = 1\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{4}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$

Het is duidelijk dat het kleinste deel het deel boven de lijn $y = \frac{1}{2}$ is.

$$\text{Dus } O = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (f(x) - \frac{1}{2}) dx = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (\sin(2x - \frac{1}{3}\pi) - \frac{1}{2}) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) - \frac{1}{2}x\right]_{\frac{1}{4}\pi}^{\frac{7}{12}\pi}$$

$$= -\frac{1}{2}\cos(\frac{5}{6}\pi) - \frac{7}{24}\pi - (-\frac{1}{2}\cos(\frac{1}{6}\pi) - \frac{1}{8}\pi) = -\frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - \frac{7}{24}\pi + \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{8}\pi$$

$$= \frac{1}{4}\sqrt{3} + \frac{1}{4}\sqrt{3} - \frac{4}{24}\pi = \frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$$

10 Er geldt $\vec{a} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$, $\vec{b} = \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}$ en $\angle(\vec{a}, \vec{b}) = \alpha - \beta$.

$$\cos(\alpha - \beta) = \cos(\angle(\vec{a}, \vec{b})) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}}{1 \cdot 1} = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

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11 a $\cos(t - u) = \cos(t)\cos(u) + \sin(t)\sin(u)$

u vervangen door $-u$ geeft

$$\cos(t + u) = \cos(t)\cos(-u) + \sin(t)\sin(-u)$$

$$\cos(t + u) = \cos(t)\cos(u) + \sin(t) \cdot -\sin(u)$$

$$\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$$

b $\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$

u vervangen door $u - \frac{1}{2}\pi$ geeft

$$\cos(t + u - \frac{1}{2}\pi) = \cos(t)\cos(u - \frac{1}{2}\pi) - \sin(t)\sin(u - \frac{1}{2}\pi)$$

$$\sin(t + u) = \cos(t)\sin(u) - \sin(t) \cdot -\cos(u)$$

$$\sin(t + u) = \cos(t)\sin(u) + \sin(t)\cos(u)$$

$$\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$$

c $\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$

u vervangen door $-u$ geeft

$$\sin(t - u) = \sin(t)\cos(-u) + \cos(t)\sin(-u)$$

$$\sin(t - u) = \sin(t)\cos(u) + \cos(t) \cdot -\sin(u)$$

$$\sin(t - u) = \sin(t)\cos(u) - \cos(t)\sin(u)$$

12 a $\sin(t+u) = \sin(t)\cos(u) + \cos(t)\sin(u)$
 $t = A$ en $u = A$ geeft
 $\sin(A+A) = \sin(A)\cos(A) + \cos(A)\sin(A)$
 $\sin(2A) = 2\sin(A)\cos(A)$

$\cos(t+u) = \cos(t)\cos(u) - \sin(t)\sin(u)$
 $t = A$ en $u = A$ geeft
 $\cos(A+A) = \cos(A)\cos(A) - \sin(A)\sin(A)$
 $\cos(2A) = \cos^2(A) - \sin^2(A)$

b $\cos(2A) = \cos^2(A) - \sin^2(A)$
 $\cos(2A) = \cos^2(A) - (1 - \cos^2(A))$
 $\cos(2A) = \cos^2(A) - 1 + \cos^2(A)$
 $\cos(2A) = 2\cos^2(A) - 1$

$\cos(2A) = \cos^2(A) - \sin^2(A)$
 $\cos(2A) = 1 - \sin^2(A) - \sin^2(A)$
 $\cos(2A) = 1 - 2\sin^2(A)$

13 a $\cos(2A) = 2\cos^2(A) - 1$
 $2\cos^2(A) = 1 + \cos(2A)$
 $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$
b $\cos(2A) = 1 - 2\sin^2(A)$
 $2\sin^2(A) = 1 - \cos(2A)$
 $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

14 a $\sin(x - \frac{1}{4}\pi) = \sin(x)\cos(\frac{1}{4}\pi) - \cos(x)\sin(\frac{1}{4}\pi) = \sin(x) \cdot \frac{1}{2}\sqrt{2} - \cos(x) \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}(\sin(x) - \cos(x))$
b $\sin(\frac{1}{12}\pi) = \sin(\frac{1}{3}\pi - \frac{1}{4}\pi) = \sin(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi) - \cos(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$
c $\sin(x + \frac{1}{4}\pi) = \sin(x)\cos(\frac{1}{4}\pi) + \cos(x)\sin(\frac{1}{4}\pi) = \sin(x) \cdot \frac{1}{2}\sqrt{2} + \cos(x) \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}(\sin(x) + \cos(x))$
d $\sin(\frac{7}{12}\pi) = \sin(\frac{1}{3}\pi + \frac{1}{4}\pi) = \sin(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi) + \cos(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$

15 a $\cos(\frac{1}{12}\pi) = \cos(\frac{1}{3}\pi - \frac{1}{4}\pi) = \cos(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi) + \sin(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi) = \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2}$
b $\sin^2(\frac{1}{8}\pi) = \frac{1}{2} - \frac{1}{2}\cos(\frac{1}{4}\pi) = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2} - \frac{1}{4}\sqrt{2}$
 $\sin(\frac{1}{8}\pi) = \sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}} \vee \sin(\frac{1}{8}\pi) = -\sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}}$
vold. niet, want $\sin(\frac{1}{8}\pi)$ is positief

16 a $\cos(4x)\cos(x) + \sin(4x)\sin(x) = \cos(4x-x) = \cos(3x)$
b $\sin(\frac{1}{2}x)\cos(x) - \cos(\frac{1}{2}x)\sin(x) = \sin(\frac{1}{2}x-x) = \sin(\frac{1}{2}x)$
c $\cos(\frac{3}{4}x)\cos(\frac{1}{4}x) - \sin(\frac{3}{4}x)\sin(\frac{1}{4}x) = \cos(\frac{3}{4}x + \frac{1}{4}x) = \cos(2x)$
d $\sin(\frac{1}{3}x)\cos(\frac{2}{3}x) + \cos(\frac{1}{3}x)\sin(\frac{2}{3}x) = \sin(\frac{1}{3}x + \frac{2}{3}x) = \sin(x)$

17 $\sin(x)(\sin(4x) - \sin(2x)) + \cos(x)(\cos(4x) + \cos(2x)) =$
 $\sin(x)\sin(4x) - \sin(x)\sin(2x) + \cos(x)\cos(4x) + \cos(x)\cos(2x) =$
 $\cos(4x)\cos(x) + \sin(4x)\sin(x) + \cos(x)\cos(2x) - \sin(x)\sin(2x) =$
 $\cos(4x-x) + \cos(x+2x) = \cos(3x) + \cos(3x) = 2\cos(3x)$

18 a $2\sin(x)\cos(x) = \sin(2x)$
 $\sin(2x) = \sin(x-1)$
 $2x = x-1 + k \cdot 2\pi \vee 2x = \pi - (x-1) + k \cdot 2\pi$
 $x = -1 + k \cdot 2\pi \vee 2x = \pi - x + 1 + k \cdot 2\pi$
 $x = -1 + k \cdot 2\pi \vee 3x = \pi + 1 + k \cdot 2\pi$
 $x = -1 + k \cdot 2\pi \vee x = \frac{1}{3}\pi + \frac{1}{3} + k \cdot \frac{2}{3}\pi$

b Gebruik $\cos^2(A) = \frac{1}{2} + \frac{1}{2} \cos(2A)$.

$$\cos^2(2x) = \cos(4x) + \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{2} \cos(4x) = \cos(4x) + \frac{1}{2}$$

$$\frac{1}{2} \cos(4x) = \cos(4x)$$

$$\cos(4x) = 0$$

$$4x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi$$

c Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$.

$$\sin^2(\frac{1}{2}x) = \cos(x) + 1\frac{1}{4}$$

$$\frac{1}{2} - \frac{1}{2} \cos(x) = \cos(x) + 1\frac{1}{4}$$

$$-1\frac{1}{2} \cos(x) = \frac{3}{4}$$

$$\cos(x) = -\frac{1}{2}$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$$

d $(\sin(x) + \cos(x))^2 = 1\frac{1}{2}$

$$\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) = 1\frac{1}{2}$$

$$\sin^2(x) + \cos^2(x) + \sin(2x) = 1\frac{1}{2}$$

$$1 + \sin(2x) = 1\frac{1}{2}$$

$$\sin(2x) = \frac{1}{2}$$

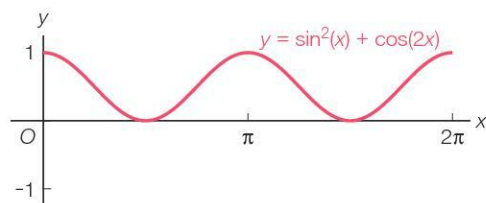
$$2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi$$

19

a



De evenwichtsstand is $\frac{1+0}{2} = \frac{1}{2}$, dus $a = \frac{1}{2}$.

De amplitude is $1 - \frac{1}{2} = \frac{1}{2}$, dus $b = \frac{1}{2}$.

De periode is π , dus $c = \frac{2\pi}{\pi} = 2$.

Een hoogste punt is $(0, 1)$, dus $y = \frac{1}{2} + \frac{1}{2} \cos(2x)$.

b Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$.

$$y = \sin^2(x) + \cos(2x)$$

$$= \frac{1}{2} - \frac{1}{2} \cos(2x) + \cos(2x)$$

$$= \frac{1}{2} + \frac{1}{2} \cos(2x)$$

20

$$\sin(3x) = \sin(2x + x)$$

$$= \sin(2x) \cos(x) + \cos(2x) \sin(x)$$

$$= 2 \sin(x) \cos(x) \cos(x) + (1 - 2 \sin^2(x)) \cdot \sin(x)$$

$$= 2 \sin(x) \cos^2(x) + \sin(x) - 2 \sin^3(x)$$

$$= 2 \sin(x)(1 - \sin^2(x)) + \sin(x) - 2 \sin^3(x)$$

$$= 2 \sin(x) - 2 \sin^3(x) + \sin(x) - 2 \sin^3(x)$$

$$= 3 \sin(x) - 4 \sin^3(x)$$

Bladzijde 151

21 a Gebruik $\sin^2(A) = \frac{1}{2} - \frac{1}{2} \cos(2A)$.

$$\begin{aligned} y &= 1 - \cos(x) - \sin^2\left(\frac{1}{2}x\right) \\ &= 1 - \cos(x) - \left(\frac{1}{2} - \frac{1}{2}\cos(x)\right) \\ &= 1 - \cos(x) - \frac{1}{2} + \frac{1}{2}\cos(x) \\ &= \frac{1}{2} - \frac{1}{2}\cos(x) \end{aligned}$$

b $\cos(3x) = \cos(2x + x)$

$$\begin{aligned} &= \cos(2x)\cos(x) - \sin(2x)\sin(x) \\ &= (2\cos^2(x) - 1) \cdot \cos(x) - 2\sin(x)\cos(x)\sin(x) \\ &= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x) \\ &= 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x)) \cdot \cos(x) \\ &= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x) \\ &= 4\cos^3(x) - 3\cos(x) \end{aligned}$$

22 a $-\cos(A) = \cos(A + \pi)$

b $\sin(A) = \cos(A - \frac{1}{2}\pi)$

c $\cos(2A) = 1 - 2\sin^2(A)$ oftewel $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

d $\cos(2A) = 2\cos^2(A) - 1$ oftewel $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$

e $\cos(A) = \sin(A + \frac{1}{2}\pi)$

12.2 Goniometrische formules bij symmetrie en primitiveren**Bladzijde 153**

23 $x_B = -x_A$ en $y_B = y_A$

$x_C = -x_A$ en $y_C = -y_A$

Bladzijde 155

24 a In voorbeeld a worden twee functiewaarden apart berekend die achteraf gelijk blijken te zijn.

In voorbeeld b wordt in één keer de som van twee functiewaarden berekend. Bij die berekening vallen verschillende termen tegen elkaar weg.

b $f(\frac{1}{4}\pi - p) + f(\frac{1}{4}\pi + p) =$

$$\begin{aligned} &\sin(\frac{1}{4}\pi - p) - \cos(\frac{1}{4}\pi - p) + 1\frac{1}{2} + \sin(\frac{1}{4}\pi + p) - \cos(\frac{1}{4}\pi + p) + 1\frac{1}{2} = \\ &\cos(\frac{1}{4}\pi + p) - \sin(\frac{1}{4}\pi + p) + 1\frac{1}{2} + \sin(\frac{1}{4}\pi + p) - \cos(\frac{1}{4}\pi + p) + 1\frac{1}{2} = 3 \end{aligned}$$

Er geldt $f(\frac{1}{4}\pi - p) + f(\frac{1}{4}\pi + p) = 2y_A$, dus de grafiek van f is symmetrisch in het punt $A(\frac{1}{4}\pi, 1\frac{1}{2})$.

25 a $f(-p) + f(p) = -p\cos(-p) + p\cos(p) = -p\cos(p) + p\cos(p) = 0$

Er geldt $f(-p) + f(p) = 2 \cdot 0$, dus de grafiek van f is symmetrisch in de oorsprong.

b $g(-p) = -p\sin(-p) = p\sin(p)$

$g(p) = p\sin(p)$

Er geldt $g(-p) = g(p)$, dus de grafiek van g is symmetrisch in de y -as.

Bladzijde 156

26 a $f(-p) + f(p) = \cos^2(-p)\sin(-p) + \cos^2(p)\sin(p) = \cos^2(p) \cdot -\sin(p) + \cos^2(p)\sin(p) = 0$

Er geldt $f(-p) + f(p) = 2 \cdot 0$, dus de grafiek van f is symmetrisch in O .

b $f(\frac{1}{2}\pi - p) = \cos^2(\frac{1}{2}\pi - p)\sin(\frac{1}{2}\pi - p)$

$$\begin{aligned} &= (\cos(\frac{1}{2}\pi)\cos(p) + \sin(\frac{1}{2}\pi)\sin(p))^2 (\sin(\frac{1}{2}\pi)\cos(p) - \cos(\frac{1}{2}\pi)\sin(p)) \\ &= (0 \cdot \cos(p) + 1 \cdot \sin(p))^2 (1 \cdot \cos(p) - 0 \cdot \sin(p)) = \sin^2(p)\cos(p) \end{aligned}$$

$$\begin{aligned} f(\frac{1}{2}\pi + p) &= \cos^2(\frac{1}{2}\pi + p)\sin(\frac{1}{2}\pi + p) \\ &= (\cos(\frac{1}{2}\pi)\cos(p) - \sin(\frac{1}{2}\pi)\sin(p))^2 (\sin(\frac{1}{2}\pi)\cos(p) + \cos(\frac{1}{2}\pi)\sin(p)) \\ &= (0 \cdot \cos(p) - 1 \cdot \sin(p))^2 (1 \cdot \cos(p) + 0 \cdot \sin(p)) = \sin^2(p)\cos(p) \end{aligned}$$

Er geldt $f(\frac{1}{2}\pi - p) = f(\frac{1}{2}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{2}\pi$.

27 a $f(\frac{1}{4}\pi - p) = \cos(\frac{1}{4}\pi - p) + \sin(\frac{1}{4}\pi - p) + 1$
 $= \cos(\frac{1}{4}\pi)\cos(p) + \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) - \cos(\frac{1}{4}\pi)\sin(p) + 1$
 $= \frac{1}{2}\sqrt{2}\cos(p) + \frac{1}{2}\sqrt{2}\sin(p) + \frac{1}{2}\sqrt{2}\cos(p) - \frac{1}{2}\sqrt{2}\sin(p) + 1 = \sqrt{2}\cos(p) + 1$
 $f(\frac{1}{4}\pi + p) = \cos(\frac{1}{4}\pi + p) + \sin(\frac{1}{4}\pi + p) + 1$
 $= \cos(\frac{1}{4}\pi)\cos(p) - \sin(\frac{1}{4}\pi)\sin(p) + \sin(\frac{1}{4}\pi)\cos(p) + \cos(\frac{1}{4}\pi)\sin(p) + 1$
 $= \frac{1}{2}\sqrt{2}\cos(p) - \frac{1}{2}\sqrt{2}\sin(p) + \frac{1}{2}\sqrt{2}\cos(p) + \frac{1}{2}\sqrt{2}\sin(p) + 1 = \sqrt{2}\cos(p) + 1$

Er geldt $f(\frac{1}{4}\pi - p) = f(\frac{1}{4}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{4}\pi$.

b $f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = \cos(\frac{3}{4}\pi - p) + \sin(\frac{3}{4}\pi - p) + 1 + \cos(\frac{3}{4}\pi + p) + \sin(\frac{3}{4}\pi + p) + 1$
 $= \cos(\frac{3}{4}\pi)\cos(p) + \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) - \cos(\frac{3}{4}\pi)\sin(p) + 1$
 $+ \cos(\frac{3}{4}\pi)\cos(p) - \sin(\frac{3}{4}\pi)\sin(p) + \sin(\frac{3}{4}\pi)\cos(p) + \cos(\frac{3}{4}\pi)\sin(p) + 1$
 $= 2\cos(\frac{3}{4}\pi)\cos(p) + 2\sin(\frac{3}{4}\pi)\cos(p) + 2 = 2 \cdot -\frac{1}{2}\sqrt{2}\cos(p) + 2 \cdot \frac{1}{2}\sqrt{2}\cos(p) + 2 = 2$

Er geldt $f(\frac{3}{4}\pi - p) + f(\frac{3}{4}\pi + p) = 2y_A$, dus de grafiek van f is symmetrisch in het punt $A(\frac{3}{4}\pi, 1)$.

28 $f(\frac{1}{6}\pi - p) = \sin(\frac{1}{6}\pi - p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi - p)$
 $= \sin(\frac{1}{6}\pi)\cos(p) - \cos(\frac{1}{6}\pi)\sin(p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi)\cos(p) + \sqrt{3} \cdot \sin(\frac{1}{6}\pi)\sin(p)$
 $= \frac{1}{2}\cos(p) - \frac{1}{2}\sqrt{3}\sin(p) + \sqrt{3} \cdot \frac{1}{2}\sqrt{3}\cos(p) + \sqrt{3} \cdot \frac{1}{2}\sin(p) = 2\cos(p)$
 $f(\frac{1}{6}\pi + p) = \sin(\frac{1}{6}\pi + p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi + p)$
 $= \sin(\frac{1}{6}\pi)\cos(p) + \cos(\frac{1}{6}\pi)\sin(p) + \sqrt{3} \cdot \cos(\frac{1}{6}\pi)\cos(p) - \sqrt{3} \cdot \sin(\frac{1}{6}\pi)\sin(p)$
 $= \frac{1}{2}\cos(p) + \frac{1}{2}\sqrt{3}\sin(p) + \sqrt{3} \cdot \frac{1}{2}\sqrt{3}\cos(p) - \sqrt{3} \cdot \frac{1}{2}\sin(p) = 2\cos(p)$
 $f(\frac{1}{6}\pi - p) = f(\frac{1}{6}\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \frac{1}{6}\pi$.

29 a $F(x) = \frac{1}{3}\sin^3(x)$ geeft $F'(x) = \sin^2(x) \cdot \cos(x)$ en dit is niet $f(x)$,
dus $y = \frac{1}{3}\sin^3(x)$ is geen primitieve van f .

b $\cos(2A) = 1 - 2\sin^2(A)$

$2\sin^2(A) = 1 - \cos(2A)$

$\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

Dus $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$.

c $f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x - \frac{1}{4}\sin(2x) + c$

Bladzijde 157

30 a $f(x) = \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x + \frac{1}{4}\sin(2x) + c$

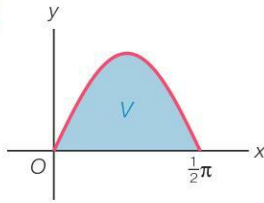
b $g(x) = \sin^2(3x) = \frac{1}{2} - \frac{1}{2}\cos(6x)$ geeft $G(x) = \frac{1}{2}x - \frac{1}{12}\sin(6x) + c$

c $h(x) = \sin(\frac{1}{2}x)\cos(\frac{1}{2}x) = \frac{1}{2}\sin(x)$ geeft $H(x) = -\frac{1}{2}\cos(x) + c$

31 a $\int_0^{\frac{1}{6}\pi} \sin(2x)\cos(2x) dx = \int_0^{\frac{1}{6}\pi} \frac{1}{2}\sin(4x) dx = \left[-\frac{1}{8}\cos(4x)\right]_0^{\frac{1}{6}\pi}$
 $= -\frac{1}{8}\cos(\frac{2}{3}\pi) + \frac{1}{8}\cos(0) = -\frac{1}{8} \cdot -\frac{1}{2} + \frac{1}{8} \cdot 1 = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$

b $\int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{2}\sin^2(x)) dx = \int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{4} + \frac{1}{4}\cos(2x)) dx = \int_{\frac{1}{3}\pi}^{\pi} (1\frac{3}{4} + \frac{1}{4}\cos(2x)) dx$
 $= \left[1\frac{3}{4}x + \frac{1}{8}\sin(2x)\right]_{\frac{1}{3}\pi}^{\pi} = 1\frac{3}{4}\pi + \frac{1}{8}\sin(2\pi) - (1\frac{3}{4} \cdot \frac{1}{3}\pi + \frac{1}{8}\sin(\frac{2}{3}\pi))$
 $= 1\frac{3}{4}\pi + 0 - (\frac{7}{12}\pi + \frac{1}{8} \cdot \frac{1}{2}\sqrt{3}) = 1\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$

32



$$\begin{aligned}
 I(L) &= \pi \int_0^{\frac{1}{2}\pi} (f(x))^2 dx = \pi \int_0^{\frac{1}{2}\pi} \sin^2(2x) dx = \pi \int_0^{\frac{1}{2}\pi} \left(\frac{1}{2} - \frac{1}{2} \cos(4x)\right) dx = \pi \left[\left(\frac{1}{2}x - \frac{1}{8} \sin(4x)\right)\right]_0^{\frac{1}{2}\pi} \\
 &= \pi \left(\frac{1}{4}\pi - \frac{1}{8} \sin(2\pi) - \left(0 - \frac{1}{8} \sin(0)\right)\right) = \frac{1}{4}\pi^2
 \end{aligned}$$

33

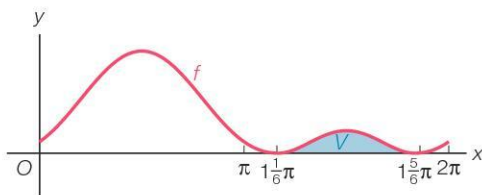
$$f(x) = 0 \text{ geeft } \sin^2(x) + \sin(x) + \frac{1}{4} = 0$$

$$(\sin(x) + \frac{1}{2})^2 = 0$$

$$\sin(x) = -\frac{1}{2}$$

$$x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$$



$$\begin{aligned}
 O(V) &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin^2(x) + \sin(x) + \frac{1}{4}) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \left(\frac{1}{2} - \frac{1}{2} \cos(2x) + \sin(x) + \frac{1}{4}\right) dx \\
 &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \left(\frac{3}{4} - \frac{1}{2} \cos(2x) + \sin(x)\right) dx = \left[\frac{3}{4}x - \frac{1}{4} \sin(2x) - \cos(x)\right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \\
 &= \frac{11}{8}\pi - \frac{1}{4} \sin\left(3\frac{2}{3}\pi\right) - \cos\left(1\frac{5}{6}\pi\right) - \left(\frac{7}{8}\pi - \frac{1}{4} \sin\left(2\frac{1}{3}\pi\right) - \cos\left(1\frac{1}{6}\pi\right)\right) \\
 &= \frac{11}{8}\pi - \frac{1}{4} \cdot -\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3} - \left(\frac{7}{8}\pi - \frac{1}{4} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}\right) \\
 &= \frac{11}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} - \frac{7}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} \\
 &= \frac{1}{2}\pi - \frac{3}{4}\sqrt{3}
 \end{aligned}$$

34

$$a \quad f(x) = 2 \sin^2(x) + \sin(x) - 1 \text{ geeft } f'(x) = 4 \sin(x) \cos(x) + \cos(x)$$

$$f'(x) = 0 \text{ geeft } 4 \sin(x) \cos(x) + \cos(x) = 0$$

$$\cos(x) (4 \sin(x) + 1) = 0$$

$$\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$$

$$\cos(x) = 0 \text{ geeft } x = \frac{1}{2}\pi + k \cdot \pi, \text{ dus } x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi.$$

$$f\left(\frac{1}{2}\pi\right) = 2 \sin^2\left(\frac{1}{2}\pi\right) + \sin\left(\frac{1}{2}\pi\right) - 1 = 2 \cdot 1^2 + 1 - 1 = 2$$

$$f\left(\frac{3}{2}\pi\right) = 2 \sin^2\left(\frac{3}{2}\pi\right) + \sin\left(\frac{3}{2}\pi\right) - 1 = 2 \cdot (-1)^2 + (-1) - 1 = 0$$

$$\sin(x) = -\frac{1}{4} \text{ geeft } f(x) = 2 \cdot \left(-\frac{1}{4}\right)^2 - \frac{1}{4} - 1 = -1\frac{1}{8}$$

$$\text{Dus } B_f = \left[-1\frac{1}{8}, 2\right].$$

b $f(x) = 0$ geeft $2\sin^2(x) + \sin(x) - 1 = 0$

Stel $\sin(x) = u$

$2u^2 + u - 1 = 0$

$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$

$u = \frac{-1 \pm 3}{4} = \frac{1}{2} \vee u = \frac{-1 - 3}{4} = -1$

$\sin(x) = \frac{1}{2} \vee \sin(x) = -1$

$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = 1\frac{1}{2}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{1}{2}\pi$

$O(V) = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (2\sin^2(x) + \sin(x) - 1) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (1 - \cos(2x) + \sin(x) - 1) dx$

$= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (-\cos(2x) + \sin(x)) dx = \left[-\frac{1}{2}\sin(2x) - \cos(x) \right]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$

$= -\frac{1}{2}\sin\left(\frac{5}{3}\pi\right) - \cos\left(\frac{5}{6}\pi\right) - \left(-\frac{1}{2}\sin\left(\frac{1}{3}\pi\right) - \cos\left(\frac{1}{6}\pi\right)\right)$

$= -\frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - \left(-\frac{1}{2}\sqrt{3} - \left(-\frac{1}{2} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}\right)\right)$

$= \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

35 Domein $[0, 1\frac{1}{2}\pi]$ en $p > 0$, dus $f_p(x) = p \sin(\frac{1}{3}x) \cos(\frac{1}{3}x) \geq 0$.

$f_p(x) = 0$ geeft $p \sin(\frac{1}{3}x) \cos(\frac{1}{3}x) = 0$

$\sin(\frac{1}{3}x) = 0 \vee \cos(\frac{1}{3}x) = 0$

$\frac{1}{3}x = k \cdot \pi \vee \frac{1}{3}x = \frac{1}{2}\pi + k \cdot \pi$

$x = k \cdot 3\pi \vee x = 1\frac{1}{2}\pi + k \cdot 3\pi$

x in $[0, 1\frac{1}{2}\pi]$ geeft $x = 0 \vee x = 1\frac{1}{2}\pi$

$O(V_p) = \int_0^{1\frac{1}{2}\pi} p \sin(\frac{1}{3}x) \cos(\frac{1}{3}x) dx = \int_0^{1\frac{1}{2}\pi} \frac{1}{2}p \sin(\frac{2}{3}x) dx = \left[\frac{1}{2}p \cdot -\frac{3}{2}\cos(\frac{2}{3}x) \right]_0^{1\frac{1}{2}\pi} = -\frac{3}{4}p \cos(\pi) + \frac{3}{4}p \cos(0)$

$= \frac{3}{4}p + \frac{3}{4}p = 1\frac{1}{2}p$

$O(V_p) = 10$ geeft $1\frac{1}{2}p = 10$

$p = 6\frac{2}{3}$

12.3 Eenparige cirkelbewegingen en harmonische trillingen

Bladzijde 159

36 P bevindt zich voor het eerst weer in $(1, 0)$ als t gelijk is aan 2π .
Dus na 2π seconden.

Bladzijde 160

37 a De formules $\sin^2(A) + \cos^2(A) = 1$ met $A = 1\frac{1}{2}t$ en met $A = t$
en $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ met $A = 1\frac{1}{2}t$ en $B = t$.

b Voor $t = \frac{1}{6}\pi$ is $PQ = \sqrt{2 - 2\cos(\frac{1}{12}\pi)} \approx 0,26$.

c $PQ = 1$ geeft $\sqrt{2 - 2\cos(\frac{1}{2}t)} = 1$

$2 - 2\cos(\frac{1}{2}t) = 1$

$-2\cos(\frac{1}{2}t) = -1$

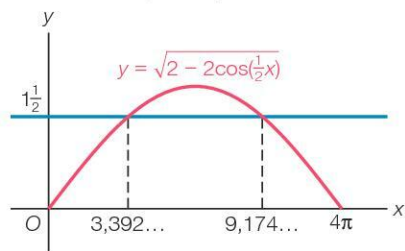
$\cos(\frac{1}{2}t) = \frac{1}{2}$

$\frac{1}{2}t = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}t = -\frac{1}{3}\pi + k \cdot 2\pi$

$t = \frac{2}{3}\pi + k \cdot 4\pi \vee t = -\frac{2}{3}\pi + k \cdot 4\pi$

t in $[0, 4\pi]$ geeft $t = \frac{2}{3}\pi \vee t = 3\frac{1}{3}\pi$

- d Voer in $y_1 = \sqrt{2 - 2\cos(\frac{1}{2}x)}$ en $y_2 = 1\frac{1}{2}$.
De optie snijpunt geeft $x = 3,392\dots$ en $x = 9,174\dots$

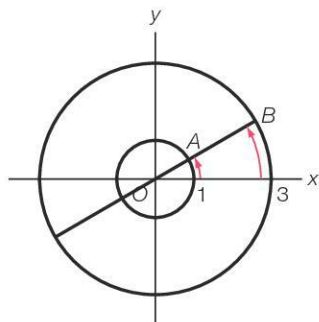


$PQ > 1\frac{1}{2}$ geeft $3,39 < t < 9,17$

- 38 a $t = \frac{1}{4}\pi$ geeft $A(\cos(\frac{1}{2}\pi), \sin(\frac{1}{2}\pi)) = A(0, 1)$ en $B(3\cos(\frac{1}{4}\pi), 3\sin(\frac{1}{4}\pi)) = B(1\frac{1}{2}\sqrt{2}, 1\frac{1}{2}\sqrt{2})$

$$AB = \sqrt{(1\frac{1}{2}\sqrt{2} - 0)^2 + (1\frac{1}{2}\sqrt{2} - 1)^2} = \sqrt{4\frac{1}{2} + 4\frac{1}{2} - 3\sqrt{2} + 1} = \sqrt{10 - 3\sqrt{2}}$$

b



De kleinste afstand tussen A en B is 2 en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B .

Dit geeft $2t = t + k \cdot 2\pi$

$$t = k \cdot 2\pi$$

$$t = 0 \vee t = 2\pi$$

De grootste afstand tussen A en B is 4 en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B plus π .

Dit geeft $2t = t + \pi + k \cdot 2\pi$

$$t = \pi + k \cdot 2\pi$$

$$t = \pi$$

- c $AB^2 = (3\cos(t) - \cos(2t))^2 + (3\sin(t) - \sin(2t))^2$
 $= 9\cos^2(t) - 6\cos(t)\cos(2t) + \cos^2(2t) + 9\sin^2(t) - 6\sin(t)\sin(2t) + \sin^2(2t)$
 $= 9 + 1 - 6(\cos(2t)\cos(t) + \sin(2t)\sin(t)) = 10 - 6\cos(2t - t)$
 $= 10 - 6\cos(t)$

$$\text{Dus } AB = \sqrt{10 - 6\cos(t)}.$$

- d $\sqrt{10 - 6\cos(t)} = \sqrt{7}$

$$10 - 6\cos(t) = 7$$

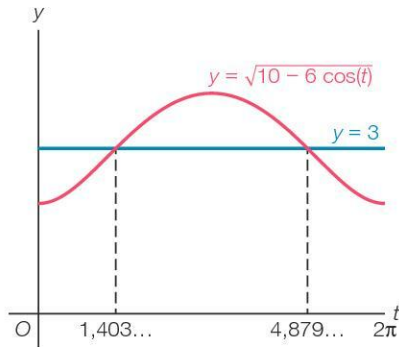
$$6\cos(t) = 3$$

$$\cos(t) = \frac{1}{2}$$

$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi$$

- e Voer in $y_1 = \sqrt{10 - 6 \cos(x)}$ en $y_2 = 3$.
De optie snijpunt geeft $x = 1,403...$ en $x = 4,879...$



$AB < 3$ geeft $0 \leq t < 1,40 \vee 4,88 < t \leq 2\pi$

Bladzijde 161

- 39 a $y = \frac{1}{2}\sqrt{2}$ geeft op de eenheidscirkel punten met richtingshoek $\frac{1}{4}\pi$ en met richtingshoek $\frac{3}{4}\pi$.
Tussen deze twee punten ligt precies een kwart van de eenheidscirkel. De lengte van de baan is dus $\frac{1}{4} \cdot 2\pi \cdot 1 = \frac{1}{2}\pi$.

- b Voor A geldt $\sin(2t) = \frac{2}{3}\cos^2(2t)$
 $\sin(2t) = \frac{2}{3}(1 - \sin^2(2t))$
 $\sin(2t) = \frac{2}{3} - \frac{2}{3}\sin^2(2t)$
 $1\frac{1}{2}\sin(2t) = 1 - \sin^2(2t)$
 $\sin^2(2t) + 1\frac{1}{2}\sin(2t) - 1 = 0$
 $(\sin(2t) - \frac{1}{2})(\sin(2t) + 2) = 0$
 $\sin(2t) = \frac{1}{2} \vee \sin(2t) = -2$
 $2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$
 $t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$

$P(\cos(\frac{1}{6}\pi), \sin(\frac{1}{6}\pi))$ oftewel $P(\frac{1}{2}\sqrt{3}, \frac{1}{2})$
 $Q(\cos(\frac{5}{6}\pi), \sin(\frac{5}{6}\pi))$ oftewel $Q(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$

- c Voor B geldt $3 \sin(t) = \frac{2}{3} \cdot (3 \cos(t))^2$
 $3 \sin(t) = 6 \cos^2(t)$
 $\sin(t) = 2 \cos^2(t)$

Voer in $y_1 = \sin(x)$ en $y_2 = 2 \cos^2(x)$.

De optie snijpunt geeft $x = 0,895...$ en $x = 2,245...$

Dus $t = 0,895...$ en $t = 2,245...$

$R(3 \cos(0,895...), 3 \sin(0,895...))$ oftewel $R(1,874; 2,342)$

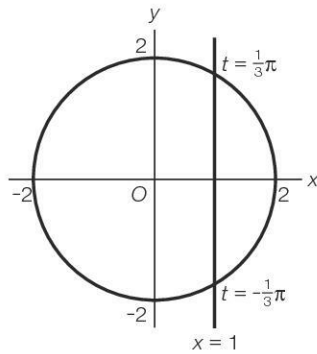
$S(3 \cos(2,245...), 3 \sin(2,245...))$ oftewel $S(-1,874; 2,342)$

- 40 a Substitutie van $x = 2 \cos(t)$ en $y = 2 \sin(t)$ in $y = x + 1$ geeft $2 \sin(t) = 2 \cos(t) + 1$.
Voer in $y_1 = 2 \sin(x)$ en $y_2 = 2 \cos(x) + 1$.
De optie snijpunt geeft $x \approx 1,15$ en $y \approx 1,82$ en $x \approx 3,57$ en $y \approx -0,82$.
Voor $t \approx 1,15$ is $y \approx 1,82$ en $x = y - 1 \approx 0,82$.
Voor $t \approx 3,57$ is $y \approx -0,82$ en $x = y - 1 \approx -1,82$.
Dit geeft de snijpunten $(0,82; 1,82)$ en $(-1,82; -0,82)$.

- b** $x = 1$ geeft $2 \cos(t) = 1$

$$\cos(t) = \frac{1}{2}$$

$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$$



De baan ligt rechts van de lijn $x = 1$ indien $2 \cos(t) > 1$, dus als $\cos(t) > \frac{1}{2}$ en dit geeft $-\frac{1}{3}\pi < t < \frac{1}{3}\pi$.

De baan is de cirkel met middelpunt $(0, 0)$ en straal 2.

Dus ligt $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$ deel van de cirkel rechts van de lijn $x = 1$.

De lengte van dit deel is $\frac{1}{3} \cdot 2\pi \cdot 2 = 1\frac{1}{3}\pi$.

- c** $P(2 \cos(t), 2 \sin(t))$ en $Q(\cos(2t), \sin(2t))$ geeft

$$PQ^2 = (2 \cos(t) - \cos(2t))^2 + (2 \sin(t) - \sin(2t))^2$$

$$= 4 \cos^2(t) - 4 \cos(t) \cos(2t) + \cos^2(2t) + 4 \sin^2(t) - 4 \sin(t) \sin(2t) + \sin^2(2t)$$

$$= 4 \cos^2(t) + 4 \sin^2(t) + \cos^2(2t) + \sin^2(2t) - 4 \cos(t) \cos(2t) - 4 \sin(t) \sin(2t)$$

$$= 4 + 1 - 4(\cos(2t) \cos(t) + \sin(2t) \sin(t))$$

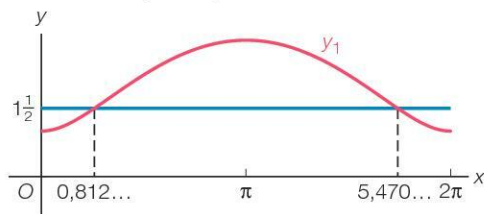
$$= 5 - 4 \cos(2t - t)$$

$$= 5 - 4 \cos(t)$$

$$\text{Dus } PQ = \sqrt{5 - 4 \cos(t)}.$$

- d** Voer in $y_1 = \sqrt{5 - 4 \cos(x)}$ en $y_2 = 1,5$.

De optie snijpunt geeft $x = 0,812...$ en $x = 5,470...$



Dus gedurende $5,470... - 0,812... \approx 4,66$ seconde per rondgang.

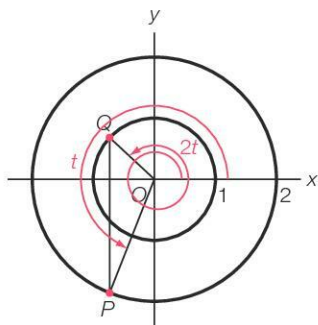
- e** $x_P = x_Q$ geeft $2 \cos(t) = \cos(2t)$

Voer in $y_1 = 2 \cos(x)$ en $y_2 = \cos(2x)$.

De optie snijpunt geeft $x = 1,9455...$ en $x = 4,3376...$

Voor de getekende situatie geldt $t \approx 1,946$.

- f** Voor de andere situatie geldt $t \approx 4,338$.



41 $\left. \begin{array}{l} y_{P'} = y_P \\ y_P = 3 \sin(\frac{2}{3}\pi t) \end{array} \right\} y_{P'} = 3 \sin(\frac{2}{3}\pi t)$

42 a De frequentie is $\frac{50\pi}{2\pi} = 25$ Hz.

b Per trilling is de afgelegde afstand $4 \cdot 5 = 20$ cm.

Er zijn 25 trillingen per seconde, dus per kwartier zijn er $15 \cdot 60 \cdot 25 = 22\,500$ trillingen.

De afgelegde afstand in één kwartier is $22\,500 \cdot 20 = 450\,000$ cm = 4,5 km.

43 amplitude = 10 geeft $b = 10$

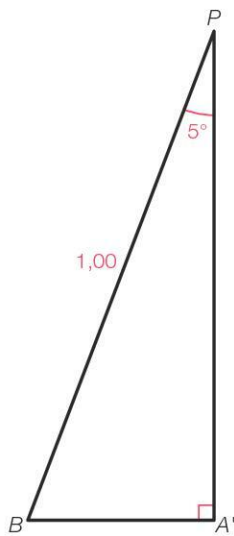
frequentie = 3 Hz geeft $c = 2\pi \cdot 3 = 6\pi$

Dus $u = 10 \sin(6\pi t)$.

44 a De omtrek van de cirkel met middelpunt P en straal 1,00 m is $2\pi \cdot 1 = 2\pi$ m.

De lengte van boog BC is $\frac{10}{360} \cdot 2\pi \approx 0,1745$ m.

b



$$\sin(5^\circ) = \frac{A'B}{1}, \text{ dus } A'B = \sin(5^\circ).$$

$$BC = 2 A'B = 2 \sin(5^\circ) \approx 0,1743 \text{ m}$$

c $b = A'B = \sin(5^\circ) \approx 0,09$

$$c = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}} = \sqrt{\frac{9,81}{1}} \approx 3,13$$

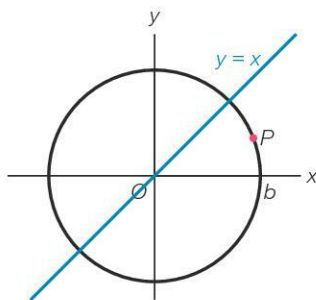
d $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9,81}} \approx 2,01$ seconde, dus de klok geeft ongeveer 1 tik per seconde.

45 a Door de figuur een kwartslag te draaien, zie je dat de projectie van P op de x -as op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y -as.

Dus P'' voert een harmonische trilling uit.

$$x_{P''} = x_P = b \cos(ct)$$

b



Door de figuur een achtste slag te draaien, zie je dat de projectie van P op de lijn $y = x$ op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y -as.

Dus de projectie van P op de lijn $y = x$ voert een harmonische trilling uit.

12.4 Bewegingsvergelijkingen met goniometrische formules

Bladzijde 165

- 46 a** Snijden met de x -as, dus $y = 0$.
 $y = 0$ geeft $\sin(4t) = 0$
 $4t = k \cdot \pi$
 $t = k \cdot \frac{1}{4}\pi$
 t in $[0, 2\pi]$ geeft $t = 0, t = \frac{1}{4}\pi, t = \frac{1}{2}\pi, t = \frac{3}{4}\pi, t = \pi, t = 1\frac{1}{4}\pi, t = 1\frac{1}{2}\pi, t = 1\frac{3}{4}\pi$ en $t = 2\pi$
 Het punt P gaat door de oorsprong voor $t = 0, t = \pi$ en $t = 2\pi$
 en snijdt de x -as in $(-1, 0)$ voor $t = 1\frac{1}{2}\pi$ en in $(1, 0)$ voor $t = \frac{1}{2}\pi$.
 $t = \frac{1}{4}\pi$ en $t = \frac{3}{4}\pi$ geven $x = \sin(\frac{1}{4}\pi) = \sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}$, dus $x_B = \frac{1}{2}\sqrt{2}$.
 $t = 1\frac{1}{4}\pi$ en $t = 1\frac{3}{4}\pi$ geven $x = \sin(1\frac{1}{4}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$, dus $x_A = -\frac{1}{2}\sqrt{2}$.
- b** C is een top, dus $y_C = 1$.
 $y = 1$ geeft $\sin(4t) = 1$
 $4t = \frac{1}{2}\pi + k \cdot 2\pi$
 $t = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$
 $t = \frac{1}{8}\pi$ geeft $x = \sin(\frac{1}{8}\pi) = 0,3826\dots$, dus $x_C \approx 0,383$.

Bladzijde 166

- 47 a** $x = \sin(2t)$ en $y = \sin(t)$ substitueren in $y = -x + 1$ geeft $\sin(t) = -\sin(2t) + 1$.
 Voer in $y_1 = \sin(x)$ en $y_2 = -\sin(2x) + 1$.
 De optie snijpunt geeft $x = 0,3552\dots$ en $x = 1,5707\dots$
 $t = 1,5707\dots$ geeft $\sin(2t) = 0$ en $\sin(t) = 1$, dus het punt $(0, 1)$.
 $t = 0,3552\dots$ geeft $\sin(2t) = 0,6521\dots$ en $\sin(t) = 0,3478\dots$, dus het punt $S(0,652; 0,348)$.
- b** De baansnelheid in O is $v(0) = \sqrt{(2 \cos(0))^2 + (\cos(0))^2} = \sqrt{(2 \cdot 1)^2 + 1^2} = \sqrt{5}$.
- c** $\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
 $\tan(\varphi) = \frac{1}{2}$, dus $\varphi \approx 26,6^\circ$.

Bladzijde 167

- 48 a** $y = \frac{1}{2}$ geeft $\sin(t) = \frac{1}{2}$
 $t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$
 t in $[0, 2\pi]$ geeft $t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$
 $x(\frac{1}{6}\pi) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$, dus $x_D = \frac{1}{2}\sqrt{3}$.
 $x(\frac{5}{6}\pi) = \sin(1\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}$, dus $x_C = -\frac{1}{2}\sqrt{3}$.
 $CD = x_D - x_C = \frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3}) = \sqrt{3}$
- b** De baansnelheid in D is
 $v(\frac{1}{6}\pi) = \sqrt{(2 \cos(\frac{1}{3}\pi))^2 + (\cos(\frac{1}{6}\pi))^2} = \sqrt{(2 \cdot \frac{1}{2})^2 + (\frac{1}{2}\sqrt{3})^2} = \sqrt{1 + \frac{3}{4}} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}$.
- c** $\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix}$ en $\vec{r}_{y=\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
 $\cos(\alpha) = \frac{\left| \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{|1 \cdot 1 + \frac{1}{2}\sqrt{3} \cdot 0|}{\sqrt{1^2 + (\frac{1}{2}\sqrt{3})^2} \cdot 1} = \frac{1}{\sqrt{1\frac{3}{4}}}$
 Dus $\alpha \approx 40,9^\circ$.

d $x(t) = 0$ geeft $\sin(2t) = 0$

$$2t = k \cdot \pi$$

$$t = k \cdot \frac{1}{2}\pi$$

$y(t) = 0$ geeft $\sin(t) = 0$

$$t = k \cdot \pi$$

Dus de baan gaat in $[0, 2\pi]$ door de oorsprong voor $t = 0$, $t = \pi$ en $t = 2\pi$.

$$\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ en } \vec{v}(\pi) = \begin{pmatrix} 2 \cos(2\pi) \\ \cos(\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\cos(\beta) = \frac{\left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right|} = \frac{|2 \cdot 2 + 1 \cdot (-1)|}{\sqrt{2^2 + 1^2} \cdot \sqrt{2^2 + (-1)^2}} = \frac{|4 - 1|}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5}$$

Dus $\beta \approx 53,1^\circ$.

49 a $y = 1$ geeft $\sin(3t) = 1$

$$3t = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

$t = \frac{5}{6}\pi$ geeft $A(\sin(1\frac{2}{3}\pi), \sin(2\frac{1}{2}\pi)) = A(-\frac{1}{2}\sqrt{3}, 1)$

$x = -1$ geeft $\sin(2t) = -1$

$$2t = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{3}{4}\pi + k \cdot \pi$$

$t = \frac{3}{4}\pi$ geeft $B(\sin(1\frac{1}{2}\pi), \sin(2\frac{1}{4}\pi)) = B(-1, \frac{1}{2}\sqrt{2})$

b $x(t) = \sin(2t)$ en $y(t) = \sin(3t)$ substitueren in $y = x$ geeft

$$\sin(3t) = \sin(2t)$$

$$3t = 2t + k \cdot 2\pi \vee 3t = \pi - 2t + k \cdot 2\pi$$

$$t = k \cdot 2\pi \vee 5t = \pi + k \cdot 2\pi$$

$$t = k \cdot 2\pi \vee t = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$$

t in $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{5}\pi \vee t = \frac{3}{5}\pi \vee t = \pi \vee t = 1\frac{2}{5}\pi \vee t = 1\frac{4}{5}\pi \vee t = 2\pi$

Bij de oorsprong horen $t = 0$, $t = \pi$ en $t = 2\pi$.

Bij de andere punten horen $t = \frac{1}{5}\pi$, $t = \frac{3}{5}\pi$, $t = 1\frac{2}{5}\pi$ en $t = 1\frac{4}{5}\pi$.

c $x(t) = \frac{1}{2}$ geeft $\sin(2t) = \frac{1}{2}$

$$2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{12}\pi \vee t = 1\frac{1}{12}\pi \vee t = \frac{5}{12}\pi \vee t = 1\frac{5}{12}\pi$

$y(t) = \frac{1}{2}\sqrt{2}$ geeft $\sin(3t) = \frac{1}{2}\sqrt{2}$

$$3t = \frac{1}{4}\pi + k \cdot 2\pi \vee 3t = \frac{3}{4}\pi + k \cdot 2\pi$$

$$t = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi \vee t = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{12}\pi \vee t = \frac{3}{4}\pi \vee t = 1\frac{5}{12}\pi \vee t = \frac{1}{4}\pi \vee t = 1\frac{7}{12}\pi$

Dus voor $t = \frac{1}{12}\pi$ en $t = 1\frac{5}{12}\pi$ passeert P het punt $(\frac{1}{2}, \frac{1}{2}\sqrt{2})$,

dus de baan snijdt zichzelf in $C(\frac{1}{2}, \frac{1}{2}\sqrt{2})$.

d $x(t) = \sin(2t)$ geeft $x'(t) = 2 \cos(2t)$

$y(t) = \sin(3t)$ geeft $y'(t) = 3 \cos(3t)$

P is voor het eerst in C op $t = \frac{1}{12}\pi$.

De baansnelheid in C is dan

$$v(\frac{1}{12}\pi) = \sqrt{(2 \cos(\frac{1}{6}\pi))^2 + (3 \cos(\frac{1}{4}\pi))^2} = \sqrt{(2 \cdot \frac{1}{2}\sqrt{3})^2 + (3 \cdot \frac{1}{2}\sqrt{2})^2} = \sqrt{3 + 4\frac{1}{2}} = \sqrt{7\frac{1}{2}} = \frac{1}{2}\sqrt{30}.$$

e $\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ 3 \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ en $\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(2\pi) \\ 3 \cos(3\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|} = \frac{|4 - 9|}{\sqrt{2^2 + 3^2} \cdot \sqrt{2^2 + (-3)^2}} = \frac{|-5|}{\sqrt{13} \cdot \sqrt{13}} = \frac{5}{13}$$

Dus $\varphi \approx 67^\circ$.

50 a $x = 1$ geeft $\sin(t - \frac{1}{6}\pi) = 1$

$$t - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{2}{3}\pi + k \cdot 2\pi$$

t in $[0, 2\pi]$ geeft $t = \frac{2}{3}\pi$

$x(\frac{2}{3}\pi) = \sin(\frac{1}{2}\pi) = 1$ en $y(\frac{2}{3}\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$, dus $A(1, -\frac{1}{2}\sqrt{3})$.

b $x = \frac{1}{2}$ geeft $\sin(t - \frac{1}{6}\pi) = \frac{1}{2}$

$$t - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee t - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = \pi + k \cdot 2\pi$$

t in $[0, 2\pi]$ geeft $t = \frac{1}{3}\pi \vee t = \pi$

$y(\frac{1}{3}\pi) = \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3}$ geeft $B(\frac{1}{2}, \frac{1}{2}\sqrt{3})$

$y(\pi) = \sin(2\pi) = 0$ geeft $C(\frac{1}{2}, 0)$

Dus $BC = \frac{1}{2}\sqrt{3}$.

c $x(t) = \sin(t - \frac{1}{6}\pi)$ en $y(t) = \sin(2t)$ substitueren in $y = x$ geeft

$$\sin(2t) = \sin(t - \frac{1}{6}\pi)$$

$$2t = t - \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - (t - \frac{1}{6}\pi) + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - t + \frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 3t = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$$

t in $[0, 2\pi]$ geeft $t = 1\frac{5}{6}\pi \vee t = \frac{7}{18}\pi \vee t = 1\frac{1}{18}\pi \vee t = 1\frac{13}{18}\pi$

d $x(t) = 0$ geeft $\sin(t - \frac{1}{6}\pi) = 0$

$$t - \frac{1}{6}\pi = k \cdot \pi$$

$$t = \frac{1}{6}\pi + k \cdot \pi$$

Dus de baan snijdt zichzelf voor $t = \frac{1}{6}\pi$ en voor $t = 1\frac{1}{6}\pi$ in $(0, \frac{1}{2}\sqrt{3})$.

$x(t) = \sin(t - \frac{1}{6}\pi)$ geeft $x'(t) = \cos(t - \frac{1}{6}\pi)$

$y(t) = \sin(2t)$ geeft $y'(t) = 2\cos(2t)$

$$\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(0) \\ 2\cos(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ en } \vec{v}(1\frac{1}{6}\pi) = \begin{pmatrix} x'(1\frac{1}{6}\pi) \\ y'(1\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(\pi) \\ 2\cos(2\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0, \text{ dus de hoek waaronder de baan zichzelf snijdt is } 90^\circ.$$

e De baansnelheid op $t = \frac{1}{3}\pi$ is

$$v(\frac{1}{3}\pi) = \sqrt{(\cos(\frac{1}{6}\pi))^2 + (2\cos(\frac{2}{3}\pi))^2} = \sqrt{(\frac{1}{2}\sqrt{3})^2 + (2 \cdot -\frac{1}{2})^2} = \sqrt{\frac{3}{4} + 1} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}.$$

Bladzijde 168

51 a $x = 0$ geeft $\sin(2t) = 0$

$$2t = k \cdot \pi$$

$$t = k \cdot \frac{1}{2}\pi$$

t in $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{2}\pi \vee t = \pi \vee t = 1\frac{1}{2}\pi \vee t = 2\pi$

$y(0) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$

$y(\frac{1}{2}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$

$y(\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$

$y(1\frac{1}{2}\pi) = \sin(1\frac{5}{6}\pi) = -\frac{1}{2}$

$y(2\pi) = \sin(2\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$

Dus $A(0, \frac{1}{2}\sqrt{3})$, $B(0, \frac{1}{2})$, $C(0, -\frac{1}{2}\sqrt{3})$ en $D(0, -\frac{1}{2}\sqrt{3})$.

b $x = -\frac{1}{2}$ geeft $\sin(2t) = -\frac{1}{2}$

$$2t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi + \frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{12}\pi + k \cdot \pi \vee t = \frac{7}{12}\pi + k \cdot \pi$$

t in $[0, 2\pi]$ geeft $t = \frac{7}{12}\pi \vee t = \frac{11}{12}\pi \vee t = 1\frac{7}{12}\pi \vee t = 1\frac{11}{12}\pi$

$$y(\frac{7}{12}\pi) = \sin(\frac{11}{12}\pi) \approx 0,26$$

$$y(\frac{11}{12}\pi) = \sin(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$$

$$y(1\frac{7}{12}\pi) = \sin(1\frac{11}{12}\pi) \approx -0,26$$

$$y(1\frac{11}{12}\pi) = \sin(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

Dus $E(-\frac{1}{2}, \frac{1}{2}\sqrt{2})$ en $H(-\frac{1}{2}, -\frac{1}{2}\sqrt{2})$ en dit geeft $EH = \frac{1}{2}\sqrt{2} - (-\frac{1}{2}\sqrt{2}) = \sqrt{2}$.

c $x(t) = \sin(2t)$ geeft $x'(t) = 2\cos(2t)$

$$y(t) = \sin(t + \frac{1}{3}\pi) \text{ geeft } y'(t) = \cos(t + \frac{1}{3}\pi)$$

$$\vec{v}(\frac{1}{2}\pi) = \begin{pmatrix} x'(\frac{1}{2}\pi) \\ y'(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2\cos(\pi) \\ \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \text{ en } \vec{r}_{y\text{-as}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|} = \frac{\left| -\frac{1}{2}\sqrt{3} \right|}{\sqrt{(-2)^2 + (-\frac{1}{2}\sqrt{3})^2} \cdot 1} = \frac{\frac{1}{2}\sqrt{3}}{\sqrt{4 + \frac{3}{4}}} = \frac{\frac{1}{2}\sqrt{3}}{\sqrt{4\frac{3}{4}}}$$

Dus $\varphi \approx 67^\circ$.

d De baansnelheid in E is

$$v(1\frac{11}{12}\pi) = \sqrt{(2\cos(3\frac{5}{6}\pi))^2 + (\cos(2\frac{1}{4}\pi))^2} = \sqrt{4 \cdot (\frac{1}{2}\sqrt{3})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{3 + \frac{1}{2}} = \frac{1}{2}\sqrt{14}.$$

52 a $x(a) = \sin(2a)$ en $y(a) = \sin(a + \frac{1}{3}\pi)$, dus $T(\sin(2a), \sin(a + \frac{1}{3}\pi))$.

$x(a + \pi) = \sin(2(a + \pi)) = \sin(2a + 2\pi) = \sin(2a)$ en $y(a + \pi) = \sin(a + \pi + \frac{1}{3}\pi) = \sin(a + 1\frac{1}{3}\pi)$,
dus $U(\sin(2a), \sin(a + 1\frac{1}{3}\pi))$.

$$TU = |y_T - y_U| = |\sin(a + \frac{1}{3}\pi) - \sin(a + 1\frac{1}{3}\pi)| = |\sin(a + \frac{1}{3}\pi) - (-\sin(a + \frac{1}{3}\pi))| = |2\sin(a + \frac{1}{3}\pi)|$$

b $|2\sin(a + \frac{1}{3}\pi)| = 1$

$$2\sin(a + \frac{1}{3}\pi) = 1 \vee 2\sin(a + \frac{1}{3}\pi) = -1$$

$$\sin(a + \frac{1}{3}\pi) = \frac{1}{2} \vee \sin(a + \frac{1}{3}\pi) = -\frac{1}{2}$$

$$a + \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$a = -\frac{1}{6}\pi + k \cdot 2\pi \vee a = \frac{1}{2}\pi + k \cdot 2\pi \vee a = -\frac{1}{2}\pi + k \cdot 2\pi \vee a = \frac{5}{6}\pi + k \cdot 2\pi$$

$0 \leq a \leq 2\pi$ en $0 \leq a + \pi \leq 2\pi$ oftewel $0 \leq a \leq \pi$ geeft $a = \frac{1}{2}\pi \vee a = \frac{5}{6}\pi$.

53

a

t	$\frac{1}{4}\pi$	$\frac{3}{4}\pi$	$1\frac{1}{4}\pi$
x	1	0	-1
y	1	-1	1

b $y = px^2 + q$ } $p \cdot 0^2 + q = -1$
door $(0, -1)$ } $q = -1$

$y = px^2 - 1$ } $p \cdot 1^2 - 1 = 1$
door $(1, 1)$ } $p = 2$

Dus $p = 2$ en $q = -1$.

Bladzijde 170**54** Substitutie van $x = \sin(t - \frac{1}{4}\pi)$ en $y = \sin(2t)$ in $y = -2x^2 + 1$ geeft

$$\sin(2t) = -2 \sin^2(t - \frac{1}{4}\pi) + 1$$

$$\sin(2t) = 1 - 2 \sin^2(t - \frac{1}{4}\pi)$$

$$\sin(2t) = \cos(2(t - \frac{1}{4}\pi))$$

$$\sin(2t) = \cos(2t - \frac{1}{2}\pi)$$

$$\sin(2t) = \sin(2t - \frac{1}{2}\pi + \frac{1}{2}\pi)$$

$$\sin(2t) = \sin(2t)$$

Dit klopt voor elke t .

$$x = \sin(t - \frac{1}{4}\pi)$$

$$-1 \leq \sin(t - \frac{1}{4}\pi) \leq 1$$

Bij de parametervoorstelling hoort de formule $y = -2x^2 + 1$ met $-1 \leq x \leq 1$.**55 a** $-2 \leq 2 \sin(t) \leq 2$, dus $-2 \leq x \leq 2$.

$$-1 \leq \sin(2t - \frac{1}{2}\pi) \leq 1, \text{ dus } -1 \leq y \leq 1.$$

Dus de keerpunten zijn $(-2, 1)$ en $(2, 1)$.**b** Stel $K: y = ax^2 + b$.

$$x = 0 \text{ geeft } t = 0 \text{ en } y(0) = \sin(-\frac{1}{2}\pi) = -1, \text{ dus door } (0, -1).$$

$$\left. \begin{array}{l} y = ax^2 + b \\ \text{door } (0, -1) \end{array} \right\} \begin{array}{l} a \cdot 0^2 + b = -1 \\ b = -1 \end{array}$$

$$\left. \begin{array}{l} y = ax^2 - 1 \\ \text{door } (2, 1) \end{array} \right\} \begin{array}{l} a \cdot 2^2 - 1 = 1 \\ 4a = 2 \\ a = \frac{1}{2} \end{array}$$

Vermoedelijk hoort bij K de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \leq x \leq 2$.Substitutie van $x = 2 \sin(t)$ en $y = \sin(2t - \frac{1}{2}\pi)$ in $y = \frac{1}{2}x^2 - 1$ geeft

$$\sin(2t - \frac{1}{2}\pi) = \frac{1}{2}(2 \sin(t))^2 - 1$$

$$\cos(2t - \frac{1}{2}\pi - \frac{1}{2}\pi) = \frac{1}{2} \cdot 4 \sin^2(t) - 1$$

$$\cos(2t - \pi) = 2 \sin^2(t) - 1$$

$$\cos(2t + \pi) = -1 + 2 \sin^2(t)$$

$$-\cos(2t) = -(1 - 2 \sin^2(t))$$

$$\cos(2t) = \cos(2t)$$

Dit klopt voor elke t , dus bij K hoort de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \leq x \leq 2$.**56** Substitutie van $x = \sin(t)$ en $y = \sin(2t)$ in $y^2 = 4x^2 - 4x^4$ geeft

$$\sin^2(2t) = 4 \sin^2(t) - 4 \sin^4(t)$$

$$(2 \sin(t) \cos(t))^2 = 4 \sin^2(t) (1 - \sin^2(t))$$

$$4 \sin^2(t) \cos^2(t) = 4 \sin^2(t) \cos^2(t)$$

Dit klopt voor elke t .

$$x = \sin(t)$$

$$-1 \leq \sin(t) \leq 1$$

Bij de baan van P hoort de formule $y^2 = 4x^2 - 4x^4$ met $-1 \leq x \leq 1$.**57 a** $-1 \leq \sin(t) \leq 1$, dus $-1 \leq x \leq 1$.

$$-1 \leq \sin(3t) \leq 1, \text{ dus } -1 \leq y \leq 1.$$

Dus de keerpunten van K zijn $(-1, 1)$ en $(1, -1)$.**b** Substitutie van $x = \sin(t)$ en $y = \sin(3t)$ in $y = 3x - 4x^3$ geeft

$$\sin(3t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t + 2t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) \cos(2t) + \cos(t) \sin(2t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) (1 - 2 \sin^2(t)) + \cos(t) \cdot 2 \sin(t) \cos(t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) - 2 \sin^3(t) + 2 \sin(t) \cos^2(t) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) - 2 \sin^3(t) + 2 \sin(t) \cdot (1 - \sin^2(t)) = 3 \sin(t) - 4 \sin^3(t)$$

$$\sin(t) - 2 \sin^3(t) + 2 \sin(t) - 2 \sin^3(t) = 3 \sin(t) - 4 \sin^3(t)$$

$$3 \sin(t) - 4 \sin^3(t) = 3 \sin(t) - 4 \sin^3(t)$$

Dit klopt voor elke t , dus alle punten van K liggen op de grafiek van $y = 3x - 4x^3$.

58 $-2 \leq 2 \cos(t) \leq 2$, dus $-2 \leq x \leq 2$ en dit geeft $c = -2$ en $d = 2$.

$-1 \leq \cos(3t) \leq 1$, dus $-1 \leq y \leq 1$.

Dus de keerpunten zijn $(-2, -1)$ en $(2, 1)$.

$x(\frac{1}{3}\pi) = 2 \cos(\frac{1}{3}\pi) = 2 \cdot \frac{1}{2} = 1$ en $y(\frac{1}{3}\pi) = \cos(\pi) = -1$, dus door $(1, -1)$.

$$\left. \begin{array}{l} y = ax^3 + bx \\ \text{door } (2, 1) \end{array} \right\} \begin{array}{l} a \cdot 2^3 + b \cdot 2 = 1 \\ 8a + 2b = 1 \end{array}$$

$$\left. \begin{array}{l} y = ax^3 + bx \\ \text{door } (1, -1) \end{array} \right\} \begin{array}{l} a \cdot 1^3 + b \cdot 1 = -1 \\ a + b = -1 \end{array}$$

$$\left\{ \begin{array}{l} 8a + 2b = 1 \\ a + b = -1 \end{array} \right| \begin{array}{l} 1 \\ 2 \end{array} \text{ geeft } \left\{ \begin{array}{l} 8a + 2b = 1 \\ 2a + 2b = -2 \end{array} \right. -$$

$$\begin{array}{l} 6a = 3 \\ a = \frac{1}{2} \end{array} \left. \begin{array}{l} \\ a + b = -1 \end{array} \right\} \begin{array}{l} \frac{1}{2} + b = -1 \\ b = -1\frac{1}{2} \end{array}$$

Vermoedelijk hoort bij de baan van P de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \leq x \leq 2$.

Substitutie van $x = 2 \cos(t)$ en $y = \cos(3t)$ in $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ geeft

$$\cos(3t) = \frac{1}{2}(2 \cos(t))^3 - 1\frac{1}{2} \cdot 2 \cos(t)$$

$$\cos(3t) = \frac{1}{2} \cdot 8 \cos^3(t) - 3 \cos(t)$$

$$\cos(2t) \cos(t) - \sin(2t) \sin(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$(2 \cos^2(t) - 1) \cos(t) - 2 \sin(t) \cos(t) \sin(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$2 \cos^3(t) - \cos(t) - 2 \sin^2(t) \cos(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$2 \cos^3(t) - \cos(t) - 2(1 - \cos^2(t)) \cos(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$2 \cos^3(t) - \cos(t) - 2 \cos(t) + 2 \cos^3(t) = 4 \cos^3(t) - 3 \cos(t)$$

$$4 \cos^3(t) - 3 \cos(t) = 4 \cos^3(t) - 3 \cos(t)$$

Dit klopt voor elke t , dus bij de baan van P hoort de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \leq x \leq 2$.

59 Noem M het midden van OP .

Er geldt $MS \perp OP$ en $MS = OM = \frac{1}{2}OP$

Dus $\vec{s} = \vec{m} + \vec{MS} = \frac{1}{2}\vec{p} + \vec{m}_L = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_L$.

Bladzijde 172

60 $\vec{t} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_R = \frac{1}{2} \begin{pmatrix} 2 \sin(t) \\ 2 \sin(2t) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(2t) \\ -2 \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(t) + \sin(2t) \\ \sin(2t) - \sin(t) \end{pmatrix}$

Dus $\begin{cases} x_T(t) = \sin(t) + \sin(2t) \\ y_T(t) = \sin(2t) - \sin(t) \end{cases}$

61 a $\vec{q} = \vec{a} + \vec{AQ} = \vec{a} + \frac{1}{2}\vec{AP} + \frac{1}{2}\vec{AP}_R$

$$\vec{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 2 \sin(t) \\ 2 \sin(2t) \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \sin(t) - 2 \\ 2 \sin(2t) \end{pmatrix}$$

$$\vec{AP}_R = \begin{pmatrix} 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix}$$

$$\vec{q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(t) - 2 \\ 2 \sin(2t) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix} = \begin{pmatrix} 2 + \sin(t) - 1 + \sin(2t) \\ \sin(2t) + 1 - \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t) + 1 \\ \sin(2t) - \sin(t) + 1 \end{pmatrix}$$

Dus $\begin{cases} x_Q(t) = \sin(2t) + \sin(t) + 1 \\ y_Q(t) = \sin(2t) - \sin(t) + 1 \end{cases}$

$$\mathbf{b} \quad \vec{r} = \vec{a} + \overrightarrow{AR} = \vec{a} + \overrightarrow{AP_R} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix} = \begin{pmatrix} 2 + 2 \sin(2t) \\ 2 - 2 \sin(t) \end{pmatrix}$$

$$y = 1 \text{ geeft } 2 - 2 \sin(t) = 1$$

$$-2 \sin(t) = -1$$

$$\sin(t) = \frac{1}{2}$$

$$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t \text{ in } \langle 0, \pi \rangle \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$$

$$x(\frac{1}{6}\pi) = 2 + 2 \sin(\frac{1}{3}\pi) = 2 + 2 \cdot \frac{1}{2}\sqrt{3} = 2 + \sqrt{3}$$

$$x(\frac{5}{6}\pi) = 2 + 2 \sin(1\frac{2}{3}\pi) = 2 + 2 \cdot -\frac{1}{2}\sqrt{3} = 2 - \sqrt{3}$$

Dus de snijpunten zijn $(2 + \sqrt{3}, 1)$ en $(2 - \sqrt{3}, 1)$.

$$\mathbf{62} \quad \mathbf{a} \quad \vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{PA_L}$$

$$\overrightarrow{PA} = \vec{a} - \vec{p} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \sin(t) \\ 2 \sin(t - \frac{1}{4}\pi) \end{pmatrix} = \begin{pmatrix} 4 - 2 \sin(t) \\ -2 \sin(t - \frac{1}{4}\pi) \end{pmatrix}$$

$$\overrightarrow{PA_L} = \begin{pmatrix} 2 \sin(t - \frac{1}{4}\pi) \\ 4 - 2 \sin(t) \end{pmatrix}$$

$$\vec{q} = \begin{pmatrix} 2 \sin(t) \\ 2 \sin(t - \frac{1}{4}\pi) \end{pmatrix} + \begin{pmatrix} 2 \sin(t - \frac{1}{4}\pi) \\ 4 - 2 \sin(t) \end{pmatrix} = \begin{pmatrix} 2 \sin(t) + 2 \sin(t - \frac{1}{4}\pi) \\ 2 \sin(t - \frac{1}{4}\pi) + 4 - 2 \sin(t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = 2 \sin(t) + 2 \sin(t - \frac{1}{4}\pi) \\ y_Q(t) = 2 \sin(t - \frac{1}{4}\pi) - 2 \sin(t) + 4 \end{cases}$$

$$\mathbf{b} \quad \text{Voer in } y_1 = 2 \sin(x - \frac{1}{4}\pi) - 2 \sin(x) + 4.$$

De optie minimum geeft $x = 0,392\dots$ en $y = 2,469\dots$

Het maximum van $y_P(t) = 2 \sin(t - \frac{1}{4}\pi)$ is 2.

Dus het laagste punt van de baan van Q ligt hoger dan het hoogste punt van de baan van P .

Bladzijde 173

$$\mathbf{63} \quad \mathbf{a} \quad \vec{s} = \vec{a} + \overrightarrow{AS} = \vec{a} + \frac{1}{2}\overrightarrow{AP} + \frac{1}{2}\overrightarrow{AP_R}$$

$$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 4 \cos(t) \\ 6 \sin(t) \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \cos(t) - 4 \\ 6 \sin(t) \end{pmatrix} \text{ geeft } \overrightarrow{AP_R} = \begin{pmatrix} 6 \sin(t) \\ 4 - 4 \cos(t) \end{pmatrix}$$

$$\vec{s} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 4 \cos(t) - 4 \\ 6 \sin(t) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \sin(t) \\ 4 - 4 \cos(t) \end{pmatrix} = \begin{pmatrix} 4 + 2 \cos(t) - 2 + 3 \sin(t) \\ 3 \sin(t) + 2 - 2 \cos(t) \end{pmatrix} = \begin{pmatrix} 2 + 3 \sin(t) + 2 \cos(t) \\ 2 + 3 \sin(t) - 2 \cos(t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_S(t) = 2 + 3 \sin(t) + 2 \cos(t) \\ y_S(t) = 2 + 3 \sin(t) - 2 \cos(t) \end{cases}$$

$$\mathbf{b} \quad \text{Substitueren van } x = 4 \cos(t) \text{ en } y = 6 \sin(t) \text{ in } 9x^2 + 4y^2 = 144 \text{ geeft}$$

$$9 \cdot (4 \cos(t))^2 + 4 \cdot (6 \sin(t))^2 = 144$$

$$9 \cdot 16 \cos^2(t) + 4 \cdot 36 \sin^2(t) = 144$$

$$144 \cos^2(t) + 144 \sin^2(t) = 144$$

$$\cos^2(t) + \sin^2(t) = 1.$$

Dit klopt voor elke t , dus bij de baan van P hoort de formule $9x^2 + 4y^2 = 144$.

$$\mathbf{c} \quad x = 2 + 3 \sin(t) + 2 \cos(t) \text{ en } y = 2 + 3 \sin(t) - 2 \cos(t) \text{ substitueren in } 9x^2 + 4y^2 = 144 \text{ geeft}$$

$$9(2 + 3 \sin(t) + 2 \cos(t))^2 + 4(2 + 3 \sin(t) - 2 \cos(t))^2 = 144.$$

$$\text{Voer in } y_1 = 9(2 + 3 \sin(x) + 2 \cos(x))^2 + 4(2 + 3 \sin(x) - 2 \cos(x))^2 \text{ en } y_2 = 144.$$

De optie snijpunt geeft $x = 2,583\dots$

$$t = 2,583\dots \text{ geeft } x = 2 + 3 \sin(2,583\dots) + 2 \cos(2,583\dots) \approx 1,89 \text{ en}$$

$$y = 2 + 3 \sin(2,583\dots) - 2 \cos(2,583\dots) \approx 5,29.$$

Dus $B(1,89; 5,29)$.

$$64 \quad \vec{q} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{p}_R = \frac{1}{2} \begin{pmatrix} 2 \sin(2t) \\ 2 \sin(t + \frac{1}{4}\pi) \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2 \sin(t + \frac{1}{4}\pi) \\ -2 \sin(2t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{cases}$$

$x = \sin(2t) + \sin(t + \frac{1}{4}\pi)$ en $y = \sin(t + \frac{1}{4}\pi) - \sin(2t)$ substitueren in $y = -x$ geeft

$$\sin(t + \frac{1}{4}\pi) - \sin(2t) = -\sin(2t) - \sin(t + \frac{1}{4}\pi)$$

$$2 \sin(t + \frac{1}{4}\pi) = 0$$

$$\sin(t + \frac{1}{4}\pi) = 0$$

$$t + \frac{1}{4}\pi = k \cdot \pi$$

$$t = -\frac{1}{4}\pi + k \cdot \pi$$

$$\frac{1}{4}\pi \leq t \leq 1\frac{1}{4}\pi \text{ geeft } t = \frac{3}{4}\pi$$

$$t = \frac{3}{4}\pi \text{ geeft } x = \sin(\frac{3}{2}\pi) + \sin(\pi) = -1$$

Dus $A(-1, 1)$.

$$65 \quad x = 2 \sin(2t) \text{ en } y = 2 \sin(t + \frac{1}{4}\pi) \text{ substitueren in } y = -x \text{ geeft}$$

$$2 \sin(t + \frac{1}{4}\pi) = -2 \sin(2t)$$

$$\sin(t + \frac{1}{4}\pi) = -\sin(2t)$$

$$\sin(t + \frac{1}{4}\pi) = \sin(2t + \pi)$$

$$t + \frac{1}{4}\pi = 2t + \pi + k \cdot 2\pi \vee t + \frac{1}{4}\pi = \pi - (2t + \pi) + k \cdot 2\pi$$

$$-t = \frac{3}{4}\pi + k \cdot 2\pi \vee t + \frac{1}{4}\pi = \pi - 2t - \pi + k \cdot 2\pi$$

$$t = -\frac{3}{4}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$t = -\frac{3}{4}\pi + k \cdot 2\pi \vee t = -\frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$$

$$\frac{1}{4}\pi \leq t \leq 1\frac{1}{4}\pi \text{ geeft } t = \frac{7}{12}\pi \vee t = 1\frac{1}{4}\pi$$

$$x_P(\frac{7}{12}\pi) = 2 \sin(\frac{1}{6}\pi) = 2 \cdot \frac{1}{2} = 1 \text{ en } y_P(\frac{7}{12}\pi) = 2 \sin(\frac{5}{6}\pi) = 2 \cdot \frac{1}{2} = 1$$

Dus het punt $A(-1, 1)$ ligt op beide banen.

$$x_P(t) = 2 \sin(2t) \text{ geeft } x_P'(t) = 4 \cos(2t)$$

$$y_P(t) = 2 \sin(t + \frac{1}{4}\pi) \text{ geeft } y_P'(t) = 2 \cos(t + \frac{1}{4}\pi)$$

$$\vec{v}_P(\frac{7}{12}\pi) = \begin{pmatrix} x_P'(\frac{7}{12}\pi) \\ y_P'(\frac{7}{12}\pi) \end{pmatrix} = \begin{pmatrix} 4 \cos(\frac{1}{6}\pi) \\ 2 \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} 4 \cdot \frac{1}{2} \\ 2 \cdot (-\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \text{ geeft } x_Q'(t) = 2 \cos(2t) + \cos(t + \frac{1}{4}\pi)$$

$$y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \text{ geeft } y_Q'(t) = \cos(t + \frac{1}{4}\pi) - 2 \cos(2t)$$

$$\vec{v}_Q(\frac{3}{4}\pi) = \begin{pmatrix} x_Q'(\frac{3}{4}\pi) \\ y_Q'(\frac{3}{4}\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(\frac{3}{2}\pi) + \cos(\pi) \\ \cos(\pi) - 2 \cos(\frac{3}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2 \cdot 0 + (-1) \\ (-1) - 2 \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$\begin{pmatrix} -2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$ is niet evenwijdig met $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus de banen raken elkaar niet op de lijn $y = -x$.

Diagnostische toets

Bladzijde 176

$$1 \quad a \quad \sin(3x - \frac{1}{4}\pi) = \cos(2x)$$

$$\cos(3x - \frac{3}{4}\pi) = \cos(2x)$$

$$3x - \frac{3}{4}\pi = 2x + k \cdot 2\pi \vee 3x - \frac{3}{4}\pi = -2x + k \cdot 2\pi$$

$$x = \frac{3}{4}\pi + k \cdot 2\pi \vee 5x = \frac{3}{4}\pi + k \cdot 2\pi$$

$$x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{20}\pi + k \cdot \frac{2}{5}\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = \frac{3}{20}\pi \vee x = \frac{11}{20}\pi \vee x = \frac{19}{20}\pi$$

b $2 \cos^2(2x) + \sin(2x) = 1$
 $2(1 - \sin^2(2x)) + \sin(2x) = 1$
 $2 - 2 \sin^2(2x) + \sin(2x) = 1$
 $2 \sin^2(2x) - \sin(2x) - 1 = 0$
 Stel $\sin(2x) = u$.
 $2u^2 - u - 1 = 0$
 $D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$
 $u = \frac{1+3}{4} = 1 \vee u = \frac{1-3}{4} = -\frac{1}{2}$
 $\sin(2x) = 1 \vee \sin(2x) = -\frac{1}{2}$
 $2x = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{4}\pi + k \cdot \pi \vee x = -\frac{1}{12}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{4}\pi \vee x = \frac{11}{12}\pi \vee x = \frac{7}{12}\pi \vee x = \frac{13}{12}\pi$

c $\cos(\frac{2}{3}\pi t) = -\sin(\frac{1}{6}\pi t)$
 $\cos(\frac{2}{3}\pi t) = \sin(\frac{1}{6}\pi t + \pi)$
 $\cos(\frac{2}{3}\pi t) = \cos(\frac{1}{6}\pi t + \frac{1}{2}\pi)$
 $\frac{2}{3}\pi t = \frac{1}{6}\pi t + \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{2}{3}\pi t = -\frac{1}{6}\pi t - \frac{1}{2}\pi + k \cdot 2\pi$
 $\frac{7}{30}\pi t = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{17}{30}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi$
 $t = 2\frac{1}{7} + k \cdot 8\frac{4}{7} \vee t = -\frac{15}{17} + k \cdot 3\frac{9}{17}$
 t in $[0, 10]$ geeft $t = 2\frac{1}{7} \vee t = 2\frac{11}{17} \vee t = 6\frac{3}{17} \vee t = 9\frac{12}{17}$

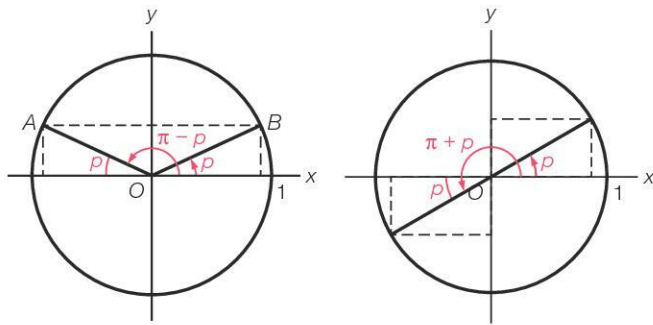
2 a $\sin(\frac{1}{2}x) \cos(\frac{1}{2}x) + \cos(\frac{1}{2}x) \sin(\frac{1}{2}x) = \sin(\frac{1}{2}x + \frac{1}{2}x) = \sin(x)$
b $\sin(3x) \sin(5x) + \cos(3x) \cos(5x) = \cos(3x - 5x) = \cos(-2x) = \cos(2x)$
c $2 \sin(3x) \cos(3x) = \sin(2 \cdot 3x) = \sin(6x)$
d $2 \cos^2(5x) - 1 = \cos(2 \cdot 5x) = \cos(10x)$

3 a $\sin(x + \frac{1}{3}\pi) = 2 \sin(2x) \cos(2x)$
 $\sin(x + \frac{1}{3}\pi) = \sin(4x)$
 $x + \frac{1}{3}\pi = 4x + k \cdot 2\pi \vee x + \frac{1}{3}\pi = \pi - 4x + k \cdot 2\pi$
 $-3x = -\frac{1}{3}\pi + k \cdot 2\pi \vee 5x = \frac{2}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{2}{15}\pi + k \cdot \frac{2}{5}\pi$
b $\sin^2(2x) + \frac{1}{4} = \cos(4x)$
 $\sin^2(2x) + \frac{1}{4} = 1 - 2 \sin^2(2x)$
 $3 \sin^2(2x) = \frac{3}{4}$
 $\sin^2(2x) = \frac{1}{4}$
 $\sin(2x) = \frac{1}{2} \vee \sin(2x) = -\frac{1}{2}$
 $2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{11}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi \vee x = -\frac{1}{12}\pi + k \cdot \pi \vee x = \frac{11}{12}\pi + k \cdot \pi$

4 $y = \sin^2(2x) + 2 \cos^2(2x) + 3 \cos(4x) = \sin^2(2x) + \cos^2(2x) + \cos^2(2x) + 3(2 \cos^2(2x) - 1)$
 $= 1 + \cos^2(2x) + 6 \cos^2(2x) - 3 = -2 + 7 \cos^2(2x) = -2 + 7 \cos^2(x) - 3\frac{1}{2} + 3\frac{1}{2}$
 $= 1\frac{1}{2} + 3\frac{1}{2}(2 \cos^2(x) - 1) = 1\frac{1}{2} + 3\frac{1}{2} \cos(4x)$

5 a $f(\frac{1}{2}\pi - p) + f(\frac{1}{2}\pi + p) = \sin(3\pi - 2p) + \cos(\frac{1}{2}\pi - p) + \sin(3\pi + 2p) + \cos(\frac{1}{2}\pi + p)$
 $= \sin(3\pi) \cos(2p) - \cos(3\pi) \sin(2p) + \cos(\frac{1}{2}\pi) \cos(p) + \sin(\frac{1}{2}\pi) \sin(p)$
 $+ \sin(3\pi) \cos(2p) + \cos(3\pi) \sin(2p) + \cos(\frac{1}{2}\pi) \cos(p) - \sin(\frac{1}{2}\pi) \sin(p)$
 $= 2 \sin(3\pi) \cos(2p) + 2 \cos(\frac{1}{2}\pi) \cos(p) = 2 \cdot 0 \cdot \cos(2p) + 2 \cdot 0 \cdot \cos(p) = 0$
 Er geldt $f(\frac{1}{2}\pi - p) + f(\frac{1}{2}\pi + p) = 0$.
 Dus de grafiek van f is symmetrisch in het punt $(\frac{1}{2}\pi, 0)$.

b



$$f(\pi - p) = 2 \sin^2(\pi - p) \cos(\pi - p) = 2 \sin^2(p) \cdot -\cos(p) = -2 \sin^2(p) \cos(p)$$

$$f(\pi + p) = 2 \sin^2(\pi + p) \cos(\pi + p) = 2(-\sin(p))^2 \cdot -\cos(p) = -2 \sin^2(p) \cos(p)$$

Er geldt $f(\pi - p) = f(\pi + p)$, dus de grafiek van f is symmetrisch in de lijn $x = \pi$.

6 $f(x) = 0$ geeft $2 \sin(x) = 0$

$$\sin(x) = 0$$

$$x = k \cdot \pi$$

$$\begin{aligned} I(L) &= \pi \int_0^\pi (2 \sin(x))^2 dx = \pi \int_0^\pi 4 \sin^2(x) dx = \pi \int_0^\pi 2(1 - \cos(2x)) dx = \pi \int_0^\pi (2 - 2 \cos(2x)) dx \\ &= \pi [2x - \sin(2x)]_0^\pi = \pi(2\pi - 0 - 0) = 2\pi^2 \end{aligned}$$

7 $f(x) = 0$ geeft $1 - 2 \cos^2(x) - \cos(x) = 0$

$$\text{Stel } \cos(x) = u.$$

$$1 - 2u^2 - u = 0$$

$$2u^2 + u - 1 = 0$$

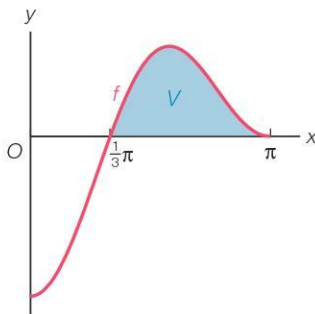
$$D = 1^2 - 4 \cdot 2 \cdot -1 = 9$$

$$u = \frac{-1 + 3}{4} = \frac{1}{2} \vee u = \frac{-1 - 3}{4} = -1$$

$$\cos(x) = \frac{1}{2} \vee \cos(x) = -1$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \pi + k \cdot 2\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{3}\pi \vee x = \pi$$



$$\begin{aligned} O(V) &= \int_{\frac{1}{3}\pi}^\pi (1 - 2 \cos^2(x) - \cos(x)) dx = \int_{\frac{1}{3}\pi}^\pi (-\cos(2x) - \cos(x)) dx \\ &= \left[-\frac{1}{2} \sin(2x) - \sin(x) \right]_{\frac{1}{3}\pi}^\pi = -\frac{1}{2} \sin(2\pi) - \sin(\pi) - \left(-\frac{1}{2} \sin\left(\frac{2}{3}\pi\right) - \sin\left(\frac{1}{3}\pi\right) \right) \\ &= 0 - 0 + \frac{1}{2} \cdot \frac{1}{2} \sqrt{3} + \frac{1}{2} \sqrt{3} = \frac{3}{4} \sqrt{3} \end{aligned}$$

- 8 a** Het punt A doorloopt de cirkel met middelpunt O en straal 2.
 Het punt B doorloopt de cirkel met middelpunt O en straal 3.
 De minimale afstand is $3 - 2 = 1$ voor $\frac{1}{2}t = 2t + k \cdot 2\pi$

$$-1\frac{1}{2}t = k \cdot 2\pi$$

$$t = k \cdot 1\frac{1}{3}\pi$$

Dus de afstand is minimaal voor $t = 0$, $t = 1\frac{1}{3}\pi$, $t = 2\frac{2}{3}\pi$ en $t = 4\pi$.

De maximale afstand is $3 + 2 = 5$ voor $\frac{1}{2}t = 2t + \pi + k \cdot 2\pi$

$$-1\frac{1}{2}t = \pi + k \cdot 2\pi$$

$$t = -\frac{2}{3}\pi + k \cdot 1\frac{1}{3}\pi$$

Dus de afstand is maximaal voor $t = \frac{2}{3}\pi$, $t = 2\pi$ en $t = 3\frac{1}{3}\pi$.

- b** $AB^2 = (3 \cos(2t) - 2 \cos(\frac{1}{2}t))^2 + (3 \sin(2t) - 2 \sin(\frac{1}{2}t))^2$
 $= 9 \cos^2(2t) - 12 \cos(2t) \cos(\frac{1}{2}t) + 4 \cos^2(\frac{1}{2}t) + 9 \sin^2(2t) - 12 \sin(2t) \sin(\frac{1}{2}t) + 4 \sin^2(\frac{1}{2}t)$
 $= 9 + 4 - 12(\cos(2t) \cos(\frac{1}{2}t) + \sin(2t) \sin(\frac{1}{2}t))$
 $= 13 - 12 \cos(2t - \frac{1}{2}t)$
 $= 13 - 12 \cos(\frac{3}{2}t)$

$$\text{Dus } AB = \sqrt{13 - 12 \cos(\frac{3}{2}t)}.$$

- c** $AB = \sqrt{7}$ geeft $\sqrt{13 - 12 \cos(\frac{3}{2}t)} = \sqrt{7}$

$$13 - 12 \cos(\frac{3}{2}t) = 7$$

$$12 \cos(\frac{3}{2}t) = 6$$

$$\cos(\frac{3}{2}t) = \frac{1}{2}$$

$$1\frac{1}{2}t = \frac{1}{3}\pi + k \cdot 2\pi \vee 1\frac{1}{2}t = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$t = \frac{2}{9}\pi + k \cdot 1\frac{1}{3}\pi \vee t = -\frac{2}{9}\pi + k \cdot 1\frac{1}{3}\pi$$

t in $[0, 4\pi]$ geeft $t = \frac{2}{9}\pi \vee t = \frac{5}{9}\pi \vee t = 2\frac{8}{9}\pi \vee t = 1\frac{1}{9}\pi \vee t = 2\frac{4}{9}\pi \vee t = 3\frac{7}{9}\pi$

- 9 a** $u = b \sin(2\pi ft)$
 $b = \text{amplitude} = 5$
 $f = 4 \text{ Hz}$
 Dus $u = 5 \sin(8\pi t)$.

- b** Per trilling is de afgelegde afstand $4 \cdot 5 = 20 \text{ dm} = 2 \text{ m}$.
 Er zijn 4 trillingen per seconde, dus per uur zijn er $3600 \cdot 4 = 14400$ trillingen.
 Dus in een uur is de afgelegde afstand $14400 \cdot 2 \text{ m} = 28800 \text{ m} = 28,8 \text{ km}$.

- 10 a** $y = -\frac{1}{2}\sqrt{3}$ geeft $\cos(2t - \frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$
 $2t - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$
 $2t = 1\frac{1}{6}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{2}\pi + k \cdot 2\pi$
 $t = \frac{7}{12}\pi + k \cdot \pi \vee t = -\frac{1}{4}\pi + k \cdot \pi$

t in $[0, 1\frac{1}{2}\pi]$ geeft $t = \frac{7}{12}\pi \vee t = \frac{3}{4}\pi$

$$x(\frac{7}{12}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$$

$$x(\frac{3}{4}\pi) = \sin(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

$$\text{Dus } AB = \frac{1}{2}\sqrt{2} - (-\frac{1}{2}\sqrt{2}) = \sqrt{2}.$$

- b** $-1 \leq \sin(3t) \leq 1$, dus $-1 \leq x \leq 1$.
 $-1 \leq \cos(2t - \frac{1}{3}\pi) \leq 1$, dus $-1 \leq y \leq 1$.
 Dus de keerpunten zijn $(-1, 1)$ en $(1, 1)$.

c $x = 0$ geeft $\sin(3t) = 0$

$$3t = k \cdot \pi$$

$$t = k \cdot \frac{1}{3}\pi$$

x in $[0, 1\frac{1}{2}\pi]$ geeft $t = 0 \vee t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi \vee t = \pi \vee t = 1\frac{1}{3}\pi$

$$y(0) = \cos(-\frac{1}{3}\pi) = \frac{1}{2}$$

$$y(\frac{1}{3}\pi) = \cos(\frac{1}{3}\pi) = \frac{1}{2}$$

$$y(\frac{2}{3}\pi) = \cos(\pi) = -1$$

$$y(\pi) = \cos(1\frac{2}{3}\pi) = \frac{1}{2}$$

$$y(1\frac{1}{3}\pi) = \cos(2\frac{1}{3}\pi) = \frac{1}{2}$$

$$x(t) = \sin(3t) \text{ geeft } x'(t) = 3 \cos(3t)$$

$$y(t) = \cos(2t - \frac{1}{3}\pi) \text{ geeft } y'(t) = -2 \sin(2t - \frac{1}{3}\pi)$$

$$\vec{v}(\frac{1}{3}\pi) = \begin{pmatrix} x'(\frac{1}{3}\pi) \\ y'(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cos(\pi) \\ -2 \sin(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cdot -1 \\ -2 \cdot \frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \text{ en}$$

$$\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 3 \cos(3\pi) \\ -2 \sin(1\frac{2}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cdot -1 \\ -2 \cdot -\frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|}{\left| \begin{pmatrix} -3 \\ -\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|} = \frac{|9 - 3|}{\sqrt{(-3)^2 + (-\sqrt{3})^2} \cdot \sqrt{(-3)^2 + (\sqrt{3})^2}} = \frac{6}{\sqrt{12} \cdot \sqrt{12}} = \frac{6}{12} = \frac{1}{2}$$

Dus $\varphi = 60^\circ$.

$$\begin{aligned} \text{d } v(\frac{2}{3}\pi) &= \sqrt{(x'(\frac{2}{3}\pi))^2 + (y'(\frac{2}{3}\pi))^2} = \sqrt{(3 \cos(2\pi))^2 + (-2 \sin(\pi))^2} = \sqrt{(3 \cdot 1)^2 + (-2 \cdot 0)^2} \\ &= \sqrt{3^2 + 0^2} = 3 \end{aligned}$$

Dus de baansnelheid is 3.

$$\text{e } \vec{q} = \overrightarrow{OP} + \overrightarrow{PQ} = \vec{p} + \vec{p}_L = \begin{pmatrix} \sin(3t) \\ \cos(2t - \frac{1}{3}\pi) \end{pmatrix} + \begin{pmatrix} -\cos(2t - \frac{1}{3}\pi) \\ \sin(3t) \end{pmatrix} = \begin{pmatrix} \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{pmatrix}$$

$$\text{Dus } \begin{cases} x_Q(t) = \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ y_Q(t) = \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{cases}$$

$$y = 0 \text{ geeft } \cos(2t - \frac{1}{3}\pi) + \sin(3t) = 0$$

$$\cos(2t - \frac{1}{3}\pi) = -\sin(3t)$$

$$\cos(2t - \frac{1}{3}\pi) = -\cos(3t - \frac{1}{2}\pi)$$

$$\cos(2t - \frac{1}{3}\pi) = \cos(3t + \frac{1}{2}\pi)$$

$$2t - \frac{1}{3}\pi = 3t + \frac{1}{2}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -3t - \frac{1}{2}\pi + k \cdot 2\pi$$

$$-t = \frac{5}{6}\pi + k \cdot 2\pi \vee 5t = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{5}{6}\pi + k \cdot 2\pi \vee t = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$$

t in $[0, 1\frac{1}{2}\pi]$ geeft $t = 1\frac{1}{6}\pi \vee t = \frac{11}{30}\pi \vee t = \frac{23}{30}\pi$

$$x_Q(1\frac{1}{6}\pi) = -2$$

$$x_Q(\frac{11}{30}\pi) \approx -0,618$$

$$x_Q(\frac{23}{30}\pi) \approx 1,618$$

Dus voor $t = \frac{23}{30}\pi$.

K Voortgezette integraalrekening

Voorkennis Afgeleiden en primitieven

Bladzijde 181

- 1 a** $f(x) = \sqrt{6x+1}$ geeft $f'(x) = \frac{1}{2\sqrt{6x+1}} \cdot 6 = \frac{3}{\sqrt{6x+1}}$
- b** $f(x) = \frac{4}{\sqrt{2x-1}} = 4(2x-1)^{-\frac{1}{2}}$ geeft $f'(x) = -2(2x-1)^{-\frac{1}{2}} \cdot 2 = -\frac{4}{(2x-1)\sqrt{2x-1}}$
- c** $f(x) = \ln(x^2+2x)$ geeft $f'(x) = \frac{1}{x^2+2x} \cdot (2x+2) = \frac{2x+2}{x^2+2x}$
- d** $f(x) = 2^{4x-1}$ geeft $f'(x) = 2^{4x-1} \cdot \ln(2) \cdot 4 = 4 \ln(2) \cdot 2^{4x-1}$
- e** $f(x) = \sin(x^2-x)$ geeft $f'(x) = \cos(x^2-x) \cdot (2x-1) = (2x-1)\cos(x^2-x)$
- f** $f(x) = \tan(4x - \frac{1}{3}\pi)$ geeft $f'(x) = \frac{1}{\cos^2(4x - \frac{1}{3}\pi)} \cdot 4 = \frac{4}{\cos^2(4x - \frac{1}{3}\pi)}$
- 2 a** $f(x) = e^x \sin(2x)$ geeft $f'(x) = e^x \sin(2x) + e^x \cdot 2 \cos(2x) = (\sin(2x) + 2 \cos(2x)) e^x$
- b** $f(x) = \frac{\ln(x)}{x^2+1}$ geeft $f'(x) = \frac{(x^2+1) \cdot \frac{1}{x} - \ln(x) \cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2 \ln(x)}{x(x^2+1)^2}$
- c** $f(x) = \frac{1}{2}x^2 \ln(x)$ geeft $f'(x) = x \cdot \ln(x) + \frac{1}{2}x^2 \cdot \frac{1}{x} = x \ln(x) + \frac{1}{2}x$
- d** $f(x) = \frac{2^x+1}{2^x} = 1 + 2^{-x}$ geeft $f'(x) = 2^{-x} \cdot \ln(2) \cdot -1 = -\ln(2) \cdot \frac{1}{2^x} = -\frac{\ln(2)}{2^x}$
- e** $f(x) = 2x \tan(x)$ geeft $f'(x) = 2 \cdot \tan(x) + 2x \cdot \frac{1}{\cos^2(x)} = 2 \tan(x) + \frac{2x}{\cos^2(x)}$
- f** $f(x) = \frac{4x+2}{\sqrt{2x+1}} = \frac{2(2x+1)}{\sqrt{2x+1}} = 2\sqrt{2x+1}$ geeft $f'(x) = 2 \cdot \frac{1}{2\sqrt{2x+1}} \cdot 2 = \frac{2}{\sqrt{2x+1}}$

Bladzijde 182

- 3 a** $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3}x^3 - \cos(x) + c$
- b** $f(x) = \frac{x^2-2}{x^3} = \frac{1}{x} - 2x^{-3}$ geeft $F(x) = \ln|x| + x^{-2} + c = \ln|x| + \frac{1}{x^2} + c$
- c** $f(x) = 4 \cdot 3^x$ geeft $F(x) = 4 \cdot \frac{3^x}{\ln(3)} + c = \frac{4 \cdot 3^x}{\ln(3)} + c$
- d** $f(x) = \ln\left(\frac{2}{\sqrt{x}}\right) = \ln(2) - \ln(x^{\frac{1}{2}}) = \ln(2) - \frac{1}{2}\ln(x)$ geeft
 $F(x) = x \ln(2) - \frac{1}{2}(x \ln(x) - x) + c = x \ln(2) - \frac{1}{2}x \ln(x) + \frac{1}{2}x + c$
- e** $f(x) = 6 \cdot \log(3x)$ geeft $F(x) = 6 \cdot \frac{1}{3} \cdot \frac{1}{\ln(10)} (3x \ln(3x) - 3x) + c = \frac{6x(\ln(3x) - 1)}{\ln(10)} + c$
- f** $f(x) = \frac{4^x - 2^x + 1}{2^x} = 2^x - 1 + 2^{-x}$ geeft $F(x) = \frac{2^x}{\ln(2)} - x - \frac{2^{-x}}{\ln(2)} + c = \frac{2^x}{\ln(2)} - x - \frac{1}{2^x \ln(2)} + c$
- 4 a** $f(x) = e^{\frac{1}{2}x-1}$ geeft $F(x) = 2 e^{\frac{1}{2}x-1} + c$
- b** $f(x) = \frac{3}{2x-1}$ geeft $F(x) = 3 \cdot \frac{1}{2} \ln|2x-1| + c = \frac{3}{2} \ln|2x-1| + c$
- c** $f(x) = 4 \sin(\pi x)$ geeft $F(x) = 4 \cdot \frac{1}{\pi} \cdot -\cos(\pi x) + c = -\frac{4 \cos(\pi x)}{\pi} + c$
- d** $f(x) = \frac{4x+1}{2\sqrt{4x+1}} = \frac{1}{2} \sqrt{4x+1} = \frac{1}{2} (4x+1)^{\frac{1}{2}}$ geeft
 $F(x) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{2}{3} (4x+1)^{\frac{3}{2}} + c = \frac{1}{12} (4x+1) \sqrt{4x+1} + c$
- e** $f(x) = (\frac{1}{2})^{2x-3}$ geeft $F(x) = \frac{1}{2} \cdot \frac{(\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \frac{\frac{1}{2} \cdot (\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \frac{(\frac{1}{2})^{2x-2}}{\ln(\frac{1}{2})} + c$
- f** $f(x) = 4 \ln(x-1)$ geeft $F(x) = 4((x-1) \ln(x-1) - (x-1)) + c = 4(x-1) \ln(x-1) - 4(x-1) + c$

- 5 a** $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin(2x) dx = \left[-\frac{1}{2} \cos(2x)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = -\frac{1}{2} \cos\left(\frac{2}{3}\pi\right) + \frac{1}{2} \cos\left(\frac{1}{3}\pi\right) = -\frac{1}{2} \cdot -\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$
- b** $\int_1^4 \frac{6}{2x-1} dx = \left[6 \cdot \frac{1}{2} \ln|2x-1|\right]_1^4 = 3 \ln(7) - 3 \ln(1) = 3 \ln(7)$
- c** $\int_e^{e^2} 10 \ln(\sqrt[4]{x}) dx = \int_e^{e^2} 10 \ln(x^{\frac{1}{4}}) dx = \int_e^{e^2} 2\frac{1}{2} \ln(x) dx = \left[2\frac{1}{2}(x \ln(x) - x)\right]_e^{e^2}$
 $= 2\frac{1}{2}(e^2 \ln(e^2) - e^2 - (e \ln(e) - e)) = 2\frac{1}{2}(e^2 \cdot 2 - e^2 - (e \cdot 1 - e)) = 2\frac{1}{2}e^2$
- d** $\int_0^4 6e^{\frac{1}{2}x-3} dx = \left[6 \cdot 2e^{\frac{1}{2}x-3}\right]_0^4 = 12e^{-1} - 12e^{-3} = \frac{12}{e} - \frac{12}{e^3}$

K.1 De substitutiemethode

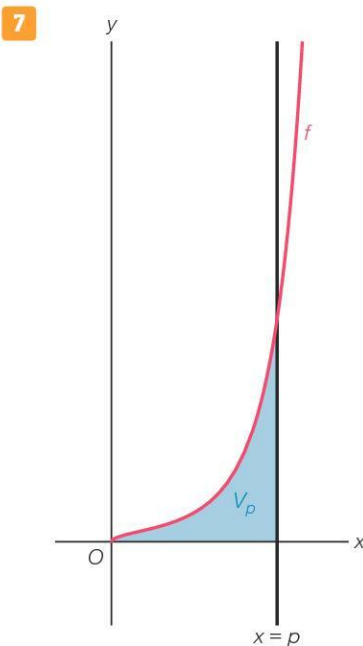
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- 1 a** $f(x) = \sin(x^2 + x)$ geeft $f'(x) = \cos(x^2 + x) \cdot (2x + 1) = (2x + 1) \cos(x^2 + x)$
- b** $G'(x) = (2x + 1) \cos(x^2 + x)$, dus $G(x) = \sin(x^2 + x) + 3$ is een primitieve van $g(x) = (2x + 1) \cos(x^2 + x)$.

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- 2 a** Omdat $\frac{d(x^2 + 1)}{dx} = 2x$ en $6x = 3 \cdot 2x$ lukt het primitiveren van $f(x)$ wel op deze manier.
- Omdat $\frac{d(x^3 + 1)}{dx} = 3x^2$ en $6x$ niet geschreven kan worden als het product van een constante en $3x^2$ lukt het primitiveren van $h(x)$ niet op deze manier.
- b** Omdat $\frac{d(x^4 + 7)}{dx} = 4x^3$ en $10x^3 = 2\frac{1}{2} \cdot 4x^3$ lukt het primitiveren van $g(x)$ wel op deze manier.
- Omdat $\frac{d(x^4 + x)}{dx} = 4x^3 + 1$ en $10x^3$ niet geschreven kan worden als het product van een constante en $4x^3 + 1$ lukt het primitiveren van $k(x)$ niet op deze manier.
- c** $\frac{d(x^4 + x)}{dx} = 4x^3 + 1$
 $10x^3 + a = 2\frac{1}{2}(4x^3 + \frac{2}{5}a)$ moet gelijk zijn aan $2\frac{1}{2}(4x^3 + 1)$, dus $\frac{2}{5}a = 1$ en dit geeft $a = 2\frac{1}{2}$.
- 3 a** $3x^2 dx = d(x^3 + 5)$
- b** $5x^4 dx = d(x^5 - 3)$
- c** $\cos(2x) dx = d(\frac{1}{2} \sin(2x) + \pi)$
- d** $\frac{1}{x} dx = d \ln(x)$
- e** $(5 - 2x) dx = d(5x - x^2 + 5)$
- f** $2 \cos(4x) dx = d(\frac{1}{2} \sin(4x))$
- 4 a** $F(x) = \int 2x(x^2 + 4)^3 dx = \int (x^2 + 4)^3 \cdot 2x dx = \int (x^2 + 4)^3 d(x^2 + 4) = \int u^3 du = \frac{1}{4} u^4 + c$
 $= \frac{1}{4} (x^2 + 4)^4 + c$
- b** $G(x) = \int 6x \sqrt{x^2 + 1} dx = \int 3 \sqrt{x^2 + 1} \cdot 2x dx = \int 3 \sqrt{x^2 + 1} d(x^2 + 1) = \int 3u^{\frac{1}{2}} du = \frac{3}{\frac{1}{2}} u^{\frac{1}{2}+1} + c$
 $= 2u \sqrt{u} + c = 2(x^2 + 1) \sqrt{x^2 + 1} + c$
- c** $H(x) = \int 6x^2(x^3 - 1)^4 dx = \int 2(x^3 - 1)^4 \cdot 3x^2 dx = \int 2(x^3 - 1)^4 d(x^3 - 1) = \int 2u^4 du = \frac{2}{5} u^5 + c$
 $= \frac{2}{5} (x^3 - 1)^5 + c$
- d** $J(x) = \int 3x^2 \sin(x^3 - 1) dx = \int \sin(x^3 - 1) \cdot 3x^2 dx = \int \sin(x^3 - 1) d(x^3 - 1) = \int \sin(u) du$
 $= -\cos(u) + c = -\cos(x^3 - 1) + c$

- 5 a** $F(x) = \int x(3x^2 - 4)^3 dx = \int \frac{1}{6}(3x^2 - 4)^3 \cdot 6x dx = \int \frac{1}{6}(3x^2 - 4)^3 d(3x^2 - 4) = \int \frac{1}{6}u^3 du = \frac{1}{24}u^4 + c$
 $= \frac{1}{24}(3x^2 - 4)^4 + c$
- b** $F(x) = \int 2x\sqrt{x^2 - 3} dx = \int \sqrt{x^2 - 3} d(x^2 - 3) = \int \sqrt{u} du = \frac{2}{3}u^{\frac{1}{2}} + c = \frac{2}{3}(x^2 - 3)\sqrt{x^2 - 3} + c$
- c** $F(x) = \int \frac{6x}{3x^2 + 2} dx = \int \frac{1}{3x^2 + 2} \cdot 6x dx = \int \frac{1}{3x^2 + 2} d(3x^2 + 2) = \int \frac{1}{u} du = \ln|u| + c = \ln|3x^2 + 2| + c = \ln(3x^2 + 2) + c$
- d** $F(x) = \int x \ln(x^2 + 1) dx = \int \frac{1}{2} \ln(x^2 + 1) \cdot 2x dx = \int \frac{1}{2} \ln(x^2 + 1) d(x^2 + 1) = \int \frac{1}{2} \ln(u) du$
 $= \frac{1}{2}(u \ln(u) - u) + c = \frac{1}{2}((x^2 + 1) \ln(x^2 + 1) - (x^2 + 1)) + c$
- 6 a** $F(x) = \int \frac{\ln(x)}{x} dx = \int \ln(x) \cdot \frac{1}{x} dx = \int \ln(x) d \ln(x) = \int u du = \frac{1}{2}u^2 + c = \frac{1}{2} \ln^2(x) + c$
- b** $G(x) = \int x e^{-x^2} dx = \int -\frac{1}{2} e^{-x^2} \cdot -2x dx = \int -\frac{1}{2} e^{-x^2} d(-x^2) = \int -\frac{1}{2} e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2} + c$
- c** $H(x) = \int x \sqrt{5 - x^2} dx = \int -\frac{1}{2} \sqrt{5 - x^2} \cdot -2x dx = \int -\frac{1}{2} \sqrt{5 - x^2} d(5 - x^2) = \int -\frac{1}{2} u^{\frac{1}{2}} du$
 $= -\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} u^{\frac{1}{2} + 1} + c = -\frac{1}{3} u \sqrt{u} + c = -\frac{1}{3} (5 - x^2) \sqrt{5 - x^2} + c$
- d** $J(x) = \int \frac{x}{\sqrt{x^2 + 1}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + 1}} \cdot 2x dx = \int \frac{1}{2\sqrt{x^2 + 1}} d(x^2 + 1) = \int \frac{1}{2\sqrt{u}} du = \sqrt{u} + c = \sqrt{x^2 + 1} + c$



$$\int (f(x))^2 dx = \int \frac{1}{100} \cdot 3x \cdot (e^{x^2})^2 dx = \int \frac{3}{400} \cdot e^{2x^2} \cdot 4x dx = \int \frac{3}{400} \cdot e^{2x^2} d 2x^2 = \int \frac{3}{400} \cdot e^u du = \frac{3}{400} e^u = \frac{3}{400} e^{2x^2}$$

$$I(L_p) = \pi \int_0^p (f(x))^2 dx = \pi \left[\frac{3}{400} e^{2x^2} \right]_0^p = \pi \left(\frac{3}{400} \cdot e^{2p^2} - \frac{3}{400} \cdot 1 \right) = \frac{3}{400} \pi (e^{2p^2} - 1)$$

$$I(L_p) = 3 \frac{3}{10} \pi \text{ geeft } \frac{3}{400} \pi (e^{2p^2} - 1) = 3 \frac{3}{10} \pi$$

$$e^{2p^2} - 1 = 440$$

$$e^{2p^2} = 441$$

$$2p^2 = \ln(441)$$

$$p^2 = \frac{1}{2} \ln(441)$$

$$p^2 = \ln(\sqrt{441})$$

$$p^2 = \ln(21)$$

$$p = \sqrt{\ln(21)}$$

8 a $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{1}{\cos(x)} \cdot \sin(x)$

b $\int \tan(x) dx = \int \frac{\sin(x)}{\cos(x)} dx = \int \frac{-1}{\cos(x)} \cdot -\sin(x) dx = \int \frac{-1}{\cos(x)} d\cos(x)$

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9 a $\int_0^{\frac{1}{6}\pi} \tan(2x) dx = \left[-\frac{1}{2} \ln|\cos(2x)| \right]_0^{\frac{1}{6}\pi} = -\frac{1}{2} \ln|\cos(\frac{1}{3}\pi)| + \frac{1}{2} \ln|\cos(0)| = -\frac{1}{2} \ln(\frac{1}{2}) + \frac{1}{2} \ln(1)$
 $= -\frac{1}{2} \ln(2^{-1}) + \frac{1}{2} \cdot 0 = \frac{1}{2} \ln(2)$

b $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \sin^3(x) dx = \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \sin^2(x) \cdot \sin(x) dx = \int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} -(1 - \cos^2(x)) \cdot -\sin(x) dx = \int_{x=\frac{1}{4}\pi}^{x=\frac{1}{3}\pi} -(1 - \cos^2(x)) d\cos(x)$
 $= \int_{\frac{1}{2}\sqrt{2}}^{\frac{1}{2}} (-1 + u^2) du = \left[-u + \frac{1}{3} u^3 \right]_{\frac{1}{2}\sqrt{2}}^{\frac{1}{2}} = -\frac{1}{2} + \frac{1}{3} \cdot (\frac{1}{2})^3 - (-\frac{1}{2}\sqrt{2} + \frac{1}{3} \cdot (\frac{1}{2}\sqrt{2})^3)$
 $= -\frac{1}{2} + \frac{1}{24} + \frac{1}{2}\sqrt{2} - \frac{1}{12}\sqrt{2} = -\frac{11}{24} + \frac{5}{12}\sqrt{2}$

c $\int_0^{\frac{1}{6}\pi} \frac{\sin(x)}{\cos^3(x)} dx = \int_0^{\frac{1}{6}\pi} \frac{1}{\cos^3(x)} \cdot -\sin(x) dx = \int_{x=0}^{x=\frac{1}{6}\pi} -\frac{1}{\cos^3(x)} d\cos(x) = \int_1^{\frac{1}{2}\sqrt{3}} -\frac{1}{u^3} du = \int_1^{\frac{1}{2}\sqrt{3}} -u^{-3} du$
 $= \left[\frac{1}{2} u^{-2} \right]_1^{\frac{1}{2}\sqrt{3}} = \left[\frac{1}{2u^2} \right]_1^{\frac{1}{2}\sqrt{3}} = \frac{1}{2 \cdot \frac{3}{4}} - \frac{1}{2 \cdot 1} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

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10 a $\int \frac{\cos(x)}{\sin^2(x)} dx = \int \frac{1}{\sin^2(x)} \cdot \cos(x) dx = \int \frac{1}{\sin^2(x)} d\sin(x) = \int \frac{1}{u^2} du = \int u^{-2} du = -u^{-1} + c = -\frac{1}{u} + c$
 $= -\frac{1}{\sin(x)} + c$

$f(x) = \cos(x) - \frac{\cos(x)}{\sin^2(x)}$ geeft $F(x) = \sin(x) + \frac{1}{\sin(x)} + c$

b $G(x) = \int \sin^5(x) dx = \int -\sin^4(x) \cdot -\sin(x) dx = \int -(\sin^2(x))^2 \cdot -\sin(x) dx = \int -(1 - \cos^2(x))^2 d\cos(x)$
 $= \int -(1 - u^2)^2 du = \int -(1 - 2u^2 + u^4) du = \int (-1 + 2u^2 - u^4) du = -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + c$
 $= -\frac{1}{5}u^5 + \frac{2}{3}u^3 - u + c = -\frac{1}{5}\cos^5(x) + \frac{2}{3}\cos^3(x) - \cos(x) + c$

11 a $\int_1^2 \frac{\ln^2(x)}{x} dx = \int_1^2 \ln^2(x) \cdot \frac{1}{x} dx = \int_{x=1}^{x=2} \ln^2(x) d\ln(x) = \int_0^{\ln(2)} u^2 du = \left[\frac{1}{3} u^3 \right]_0^{\ln(2)} = \frac{1}{3} \ln^3(2)$

b $\int_e^{e^2} \frac{1}{x \ln(x)} dx = \int_e^{e^2} \frac{1}{\ln(x)} \cdot \frac{1}{x} dx = \int_{x=e}^{x=e^2} \frac{1}{\ln(x)} d\ln(x) = \int_1^2 \frac{1}{u} du = [\ln|u|]_1^2 = \ln(2) - \ln(1) = \ln(2)$

c $\int_0^1 \frac{x^2}{e^{2x^3}} dx = \int_0^1 -\frac{1}{6} \cdot e^{-2x^3} \cdot -6x^2 dx = \int_{x=0}^{x=1} -\frac{1}{6} \cdot e^{-2x^3} d(-2x^3) = \int_0^{-2} -\frac{1}{6} e^u du = \left[-\frac{1}{6} e^u \right]_0^{-2} = -\frac{1}{6} e^{-2} + \frac{1}{6} e^0$
 $= -\frac{1}{6e^2} + \frac{1}{6} = \frac{e^2 - 1}{6e^2}$

12 a $x = 0$ geeft $u = \sqrt{0+4} = 2$
 $x = 5$ geeft $u = \sqrt{5+4} = 3$

b $\sqrt{x+4} = u$
kwadrateren geeft
 $x+4 = u^2$
 $x = u^2 - 4$

Omdat $x = u^2 - 4$ en $\sqrt{x+4} = u$ krijg je $\int x\sqrt{x+4} dx = \int (u^2 - 4) \cdot u d(u^2 - 4)$.

c $\int (u^2 - 4) \cdot u d(u^2 - 4) = \int (u^2 - 4) \cdot u \cdot 2u du = \int (2u^4 - 8u^2) du$

$$\begin{aligned} \text{d} \quad \int_0^5 x\sqrt{x+4} \, dx &= \int_2^3 (2u^4 - 8u^2) \, du = \left[\frac{2}{5}u^5 - \frac{8}{3}u^3 \right]_2^3 = \frac{2}{5} \cdot 3^5 - \frac{8}{3} \cdot 3^3 - \frac{2}{5} \cdot 2^5 + \frac{8}{3} \cdot 2^3 \\ &= 97\frac{1}{5} - 72 - 12\frac{4}{5} + 21\frac{1}{3} = 33\frac{11}{15} \end{aligned}$$

13 a $\sqrt{x+1} = u$
kwadrateren geeft
 $x+1 = u^2$
 $x = u^2 - 1$

$$\begin{aligned} \int_0^3 x\sqrt{x+1} \, dx &= \int_{u=1}^{u=2} (u^2 - 1) \cdot u \, d(u^2 - 1) = \int_1^2 (u^2 - 1) \cdot u \cdot 2u \, du = \int_1^2 (2u^4 - 2u^2) \, du = \left[\frac{2}{5}u^5 - \frac{2}{3}u^3 \right]_1^2 \\ &= \frac{2}{5} \cdot 2^5 - \frac{2}{3} \cdot 2^3 - \left(\frac{2}{5} \cdot 1^5 - \frac{2}{3} \cdot 1^3 \right) = 12\frac{4}{5} - 5\frac{1}{3} - \frac{2}{5} + \frac{2}{3} = 7\frac{11}{15} \end{aligned}$$

b $\sqrt{x+9} = u$
kwadrateren geeft
 $x+9 = u^2$
 $x = u^2 - 9$

$$\begin{aligned} \int_0^7 \frac{5x}{\sqrt{x+9}} \, dx &= \int_{u=3}^{u=4} \frac{5(u^2 - 9)}{u} \, d(u^2 - 9) = \int_3^4 \frac{5(u^2 - 9)}{u} \cdot 2u \, du = \int_3^4 (10u^2 - 90) \, du = \left[\frac{10}{3}u^3 - 90u \right]_3^4 \\ &= \frac{10}{3} \cdot 4^3 - 90 \cdot 4 - \left(\frac{10}{3} \cdot 3^3 - 90 \cdot 3 \right) = 213\frac{1}{3} - 360 - 90 + 270 = 33\frac{1}{3} \end{aligned}$$

14 a $f(x) = 0$ geeft $\frac{2 + \ln(x)}{x} = 0$
 $2 + \ln(x) = 0$
 $\ln(x) = -2$
 $x = e^{-2}$

$$\begin{aligned} O(V) &= \int_{e^{-2}}^e \frac{2 + \ln(x)}{x} \, dx = \int_{e^{-2}}^e (2 + \ln(x)) \cdot \frac{1}{x} \, dx = \int_{x=e^{-2}}^{x=e} (2 + \ln(x)) \, d\ln(x) = \int_{-2}^1 (2 + u) \, du \\ &= \left[2u + \frac{1}{2}u^2 \right]_{-2}^1 = 2 + \frac{1}{2} - (-4 + 2) = 4\frac{1}{2} \end{aligned}$$

b $\int_1^p \frac{2 + \ln(x)}{x} \, dx = \int_1^p (2 + \ln(x)) \cdot \frac{1}{x} \, dx = \int_{x=1}^{x=p} (2 + \ln(x)) \, d\ln(x) = \int_0^{\ln(p)} (2 + u) \, du = \left[2u + \frac{1}{2}u^2 \right]_0^{\ln(p)}$
 $= 2\ln(p) + \frac{1}{2}\ln^2(p)$

$$\begin{aligned} \int_1^p f(x) \, dx &= 6 \text{ geeft } 2\ln(p) + \frac{1}{2}\ln^2(p) = 6 \\ \ln^2(p) + 4\ln(p) - 12 &= 0 \\ (\ln(p) - 2)(\ln(p) + 6) &= 0 \\ \ln(p) &= 2 \vee \ln(p) = -6 \\ p &= e^2 \vee p = e^{-6} \\ &\text{vold. niet} \end{aligned}$$

Dus $p = e^2$.

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15 a $f(x) = g(x)$ geeft $\frac{4\ln^2(x)}{x} = \frac{1}{x}$
 $4\ln^2(x) = 1$

$$\ln^2(x) = \frac{1}{4}$$

$$\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$$

$$x = e^{\frac{1}{2}} \vee x = e^{-\frac{1}{2}}$$

$$x = \sqrt{e} \vee x = \frac{1}{\sqrt{e}}$$

$$f(x) \leq g(x) \text{ geeft } \frac{1}{\sqrt{e}} \leq x \leq \sqrt{e}$$

$$\begin{aligned}
 \text{b } O(V) &= \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} (g(x) - f(x)) dx = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} \left(\frac{1}{x} - \frac{4 \ln^2(x)}{x} \right) dx = \int_{\frac{1}{\sqrt{e}}}^{\sqrt{e}} (1 - 4 \ln^2(x)) \cdot \frac{1}{x} dx \\
 &= \int_{x=\frac{1}{\sqrt{e}}}^{x=\sqrt{e}} (1 - 4 \ln^2(x)) d \ln(x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - 4u^2) du = \left[u - \frac{4}{3} u^3 \right]_{-\frac{1}{2}}^{\frac{1}{2}} \\
 &= \frac{1}{2} - \frac{4}{3} \cdot \frac{1}{8} - \left(-\frac{1}{2} - \frac{4}{3} \cdot -\frac{1}{8} \right) = \frac{1}{2} - \frac{1}{6} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}
 \end{aligned}$$

16 a $f(x) = \frac{3x^2}{x^3 + 4}$ geeft $f'(x) = \frac{(x^3 + 4) \cdot 6x - 3x^2 \cdot 3x^2}{(x^3 + 4)^2} = \frac{6x^4 + 24x - 9x^4}{(x^3 + 4)^2} = \frac{-3x^4 + 24x}{(x^3 + 4)^2}$
 $f'(x) = 0$ geeft $-3x^4 + 24x = 0$
 $-3x(x^3 - 8) = 0$
 $x = 0 \vee x^3 = 8$
 $x = 0 \vee x = 2$

$$x^3 + 4 = 0$$

$$x^3 = -4$$

$$x = \sqrt[3]{-4}$$

$$\text{min. is } f(0) = 0$$

$$\text{max. is } f(2) = 1$$

$$f(x) = p \text{ heeft drie oplossingen voor } 0 < p < 1.$$

b $\int_0^p \frac{3x^2}{x^3 + 4} dx = \int_0^p \frac{1}{x^3 + 4} \cdot 3x^2 dx = \int_{x=0}^{x=p} \frac{1}{x^3 + 4} d(x^3 + 4) = \int_4^{p^3+4} \frac{1}{u} du = [\ln|u|]_4^{p^3+4} = \ln(p^3 + 4) - \ln(4)$
 $\int_0^p f(x) dx = 2$ geeft $\ln(p^3 + 4) - \ln(4) = 2$
 $\ln\left(\frac{p^3 + 4}{4}\right) = 2$
 $\frac{p^3 + 4}{4} = e^2$
 $p^3 + 4 = 4e^2$
 $p^3 = 4e^2 - 4$
 $p = \sqrt[3]{4e^2 - 4}$

K.2 Partieel integreren

Bladzijde 190

17 a $k(x) = (2x + 3) \sin(x)$ differentiëren met de productregel geeft $k'(x) = 2 \sin(x) + (2x + 3) \cos(x)$. Dus is k geen primitieve van h .

b Integreren van het linker- en rechterlid van de vergelijking $(2x + 3) \cos(x) = k'(x) - 2 \sin(x)$ geeft

$$\int (2x + 3) \cos(x) dx = \int (k'(x) - 2 \sin(x)) dx$$

$$\int (2x + 3) \cos(x) dx = (2x + 3) \sin(x) - \int 2 \sin(x) dx.$$

c $H(x) = \int (2x + 3) \cos(x) dx = (2x + 3) \sin(x) - \int 2 \sin(x) dx = (2x + 3) \sin(x) + 2 \cos(x) + c$

Bladzijde 192

18 a $\int \frac{\ln(x)}{\sqrt{x}} dx = \int \ln(x) \cdot x^{-\frac{1}{2}} dx = \int \ln(x) d 2\sqrt{x} = \ln(x) \cdot 2\sqrt{x} - \int 2\sqrt{x} d \ln(x)$
 $= 2\sqrt{x} \cdot \ln(x) - \int 2\sqrt{x} \cdot \frac{1}{x} dx = 2\sqrt{x} \cdot \ln(x) - \int 2x^{-\frac{1}{2}} dx$
 $= 2\sqrt{x} \cdot \ln(x) - 4x^{\frac{1}{2}} + c = 2\sqrt{x}(\ln(x) - 2) + c$

b $\int x \sin(x) dx = \int \sin(x) d \frac{1}{2} x^2 = \frac{1}{2} x^2 \sin(x) - \int \frac{1}{2} x^2 \cos(x) dx = \dots$

Het wordt op deze manier alleen maar ingewikkelder, dus het lukt zo niet.

c $F(x) = \int \ln(x) dx = x \ln(x) - \int x d \ln(x) = x \ln(x) - \int x \cdot \frac{1}{x} dx = x \ln(x) - \int 1 dx = x \ln(x) - x + c$

- 19 a** $F(x) = \int x e^{2x} dx = \int x d\frac{1}{2}e^{2x} = x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} dx = \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + c$
- b** $F(x) = \int 2x \cos(x) dx = \int 2x d \sin(x) = 2x \sin(x) - \int \sin(x) d 2x = 2x \sin(x) - \int 2 \sin(x) dx$
 $= 2x \sin(x) + 2 \cos(x) + c$
- c** $F(x) = \int x \ln(x) dx = \int \ln(x) d\frac{1}{2}x^2 = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 d \ln(x) = \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx$
 $= \frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + c$
- d** $\int x^3 \ln(x) dx = \int \ln(x) d\frac{1}{4}x^4 = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 d \ln(x) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x} dx$
 $= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^3 dx = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + c$
Dus $f(x) = x^3 \ln(x) + 3$ geeft $F(x) = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + 3x + c$
- 20 a** $\int 2x e^{x+1} dx = \int 2x d e^{x+1} = 2x e^{x+1} - \int e^{x+1} d 2x = 2x e^{x+1} - \int e^{x+1} \cdot 2 dx = 2x e^{x+1} - 2e^{x+1} + c$
 $\int_0^1 2x e^{x+1} dx = [2x e^{x+1} - 2e^{x+1}]_0^1 = 2e^2 - 2e^2 - (0 - 2e) = 2e$
- b** $\int (3x+1) \sin(x) dx = \int -(3x+1) d \cos(x) = -(3x+1) \cdot \cos(x) - \int -\cos(x) d(3x+1)$
 $= -(3x+1) \cos(x) + \int 3 \cos(x) dx = -(3x+1) \cos(x) + 3 \sin(x) + c$
 $\int_0^\pi (3x+1) \sin(x) dx = [-(3x+1) \cos(x) + 3 \sin(x)]_0^\pi$
 $= -(3\pi+1) \cos(\pi) + 3 \sin(\pi) - (-\cos(0) + 3 \sin(0)) = 3\pi + 1 + 1 = 3\pi + 2$
- 21 a** $f(x) = x^2 \ln(x)$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x \ln(x) + x$
 $f'(x) = 0$ geeft $2x \ln(x) + x = 0$
 $x(2 \ln(x) + 1) = 0$
 $x = 0 \vee 2 \ln(x) + 1 = 0$
 $x = 0 \vee \ln(x) = -\frac{1}{2}$
 $x = 0 \vee x = e^{-\frac{1}{2}}$
vold. niet
 $f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \cdot -\frac{1}{2} = -\frac{1}{2}e^{-1}$
Dus $A(e^{-\frac{1}{2}}, -\frac{1}{2}e^{-1}) = A(\frac{1}{\sqrt{e}}, -\frac{1}{2e})$.
- b** $\int x^2 \ln(x) dx = \int \ln(x) d\frac{1}{3}x^3 = \frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^3 d \ln(x) = \frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$
 $= \frac{1}{3}x^3 \ln(x) - \int \frac{1}{3}x^2 dx = \frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + c$
 $O(V) = \int_1^e x^2 \ln(x) dx = [\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3]_1^e = \frac{1}{3}e^3 \ln(e) - \frac{1}{9}e^3 - (\frac{1}{3} \ln(1) - \frac{1}{9}) = \frac{2}{9}e^3 + \frac{1}{9}$

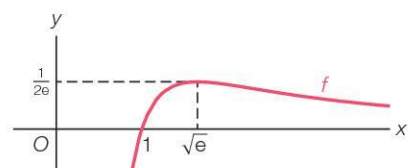
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- 22 a** $f(x) = \frac{\ln(x)}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot \frac{1}{x} - \ln(x) \cdot 2x}{x^4} = \frac{x - 2x \ln(x)}{x^4} = \frac{1 - 2 \ln(x)}{x^3}$
 $f'(x) = 0$ geeft $1 - 2 \ln(x) = 0$
 $\ln(x) = \frac{1}{2}$
 $x = e^{\frac{1}{2}}$
 $x = \sqrt{e}$

Zie de figuur hiernaast.

$$\text{max. is } f(\sqrt{e}) = \frac{\frac{1}{2}}{(\sqrt{e})^2} = \frac{\frac{1}{2}}{e} = \frac{1}{2e}$$

$$\text{Dus } B_f = \left\langle \leftarrow, \frac{1}{2e} \right\rangle.$$



$$\begin{aligned}
 \text{b} \quad \int \frac{\ln(x)}{x^2} dx &= \int \ln(x) \cdot x^{-2} dx = \int \ln(x) d(-x^{-1}) = -x^{-1} \ln(x) - \int -x^{-1} d \ln(x) = -\frac{1}{x} \ln(x) + \int \frac{1}{x} \cdot \frac{1}{x} dx \\
 &= -\frac{\ln(x)}{x} + \int x^{-2} dx = -\frac{\ln(x)}{x} - x^{-1} + c = -\frac{\ln(x) + 1}{x} + c \\
 O(V) &= \int_1^e \frac{\ln(x)}{x^2} dx = \left[-\frac{\ln(x) + 1}{x} \right]_1^e = -\frac{1+1}{e} + \frac{0+1}{1} = -\frac{2}{e} + 1
 \end{aligned}$$

- 23 a** $\int x^2 e^x dx = \int x^2 d e^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - \int e^x \cdot 2x dx = x^2 e^x - \int 2x e^x dx$
b $\int 2x e^x dx = 2x e^x - \int e^x d 2x = 2x e^x - \int e^x \cdot 2 dx = 2x e^x - 2 e^x + c$
c $\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - (2x e^x - 2 e^x + c) = x^2 e^x - 2x e^x + 2 e^x + c = (x^2 - 2x + 2) e^x + c$
 Dus de primitieven van f zijn $F(x) = (x^2 - 2x + 2) e^x + c$

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- 24 a** $F(x) = (ax^2 + bx + c) e^x$ geeft
 $F'(x) = (2ax + b) e^x + (ax^2 + bx + c) e^x = (ax^2 + (2a + b)x + b + c) e^x$
 Dit moet gelijk zijn aan $f(x) = (ax^2 + px + q) e^x$.
 Dit geeft $p = 2a + b$ en $q = b + c$.
b $f(x) = (4x^2 - 5x + 6) e^x$, dus $a = 4$, $p = -5$ en $q = 6$.
 $a = 4$ en $2a + b = -5$ geeft $8 + b = -5$, dus $b = -13$.
 $b = -13$ en $b + c = 6$ geeft $-13 + c = 6$, dus $c = 19$.
 Dus de primitieven van $f(x) = (4x^2 - 5x + 6) e^x$ zijn $F(x) = (4x^2 - 13x + 19) e^x + c$.

- 25 a** $\int e^x \sin(x) dx = \int \sin(x) d e^x = e^x \sin(x) - \int e^x d \sin(x) = e^x \sin(x) - \int e^x \cos(x) dx$
 $= e^x \sin(x) - \int \cos(x) d e^x = e^x \sin(x) - (e^x \cos(x) - \int e^x d \cos(x))$
 $= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx$
 Uit $\int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx$ volgt $2 \int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x)$
 dus $\int e^x \sin(x) dx = \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x)$.

De primitieven zijn $G(x) = \frac{1}{2} e^x (\sin(x) - \cos(x)) + c$.

- b** $F(x) = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + c$ geeft
 $F'(x) = -2x \cdot \cos(x) - x^2 \cdot -\sin(x) + 2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \sin(x) = x^2 \sin(x)$

- 26 a** $F(x) = \int \frac{1}{4} x^2 \cos(x) dx = \int \frac{1}{4} x^2 d \sin(x) = \frac{1}{4} x^2 \sin(x) - \int \sin(x) d \frac{1}{4} x^2 = \frac{1}{4} x^2 \sin(x) - \int \frac{1}{2} x \sin(x) dx$
 $= \frac{1}{4} x^2 \sin(x) + \int \frac{1}{2} x d \cos(x) = \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \int \cos(x) d \frac{1}{2} x$
 $= \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \int \frac{1}{2} \cos(x) dx = \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \frac{1}{2} \sin(x) + c$

- b** $\int e^{-x} \cos(x) dx = \int -\cos(x) d e^{-x} = -\cos(x) e^{-x} + \int e^{-x} d \cos(x) = -e^{-x} \cos(x) - \int e^{-x} \sin(x) dx$
 $= -e^{-x} \cos(x) + \int \sin(x) d e^{-x} = -e^{-x} \cos(x) + e^{-x} \sin(x) - \int e^{-x} d \sin(x)$
 $= -e^{-x} \cos(x) + e^{-x} \sin(x) - \int e^{-x} \cos(x) dx$

Uit $\int e^{-x} \cos(x) dx = -e^{-x} \cos(x) + e^{-x} \sin(x) - \int e^{-x} \cos(x) dx$ volgt $2 \int e^{-x} \cos(x) dx = -e^{-x} \cos(x) + e^{-x} \sin(x)$

dus $\int e^{-x} \cos(x) dx = \frac{1}{2} e^{-x} (\sin(x) - \cos(x))$.

De primitieven zijn $G(x) = \frac{1}{2} e^{-x} (\sin(x) - \cos(x)) + c$.

$$\begin{aligned}
\text{c} \quad \int e^{2x} \sin(x) dx &= \int e^{2x} d(-\cos(x)) = -e^{2x} \cos(x) + \int \cos(x) d e^{2x} = -e^{2x} \cos(x) + \int \cos(x) \cdot 2 e^{2x} dx \\
&= -e^{2x} \cos(x) + \int 2 e^{2x} d \sin(x) = -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - \int \sin(x) d 2 e^{2x} \\
&= -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - \int 4 e^{2x} \sin(x) dx \\
&= -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - 4 \int e^{2x} \sin(x) dx
\end{aligned}$$

Uit $\int e^{2x} \sin(x) dx = -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx - 4 \int e^{2x} \sin(x) dx$ volgt $5 \int e^{2x} \sin(x) dx = -e^{2x} \cos(x) + 2 \int e^{2x} \sin(x) dx$

$$\text{dus } \int e^{2x} \sin(x) dx = -\frac{1}{5} e^{2x} \cos(x) + \frac{2}{5} e^{2x} \sin(x).$$

De primitieven zijn $H(x) = \frac{2}{5} e^{2x} \sin(x) - \frac{1}{5} e^{2x} \cos(x) + c = \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + c$.

$$\begin{aligned}
\text{d} \quad K(x) &= \int \ln^2(x) dx = x \ln^2(x) - \int x d \ln^2(x) = x \ln^2(x) - \int x \cdot 2 \ln(x) \cdot \frac{1}{x} dx = x \ln^2(x) - \int 2 \ln(x) dx \\
&= x \ln^2(x) - 2(x \ln(x) - x) + c = x \ln^2(x) - 2x \ln(x) + 2x + c
\end{aligned}$$

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$$\begin{aligned}
\text{27 a} \quad \int (x^2 - x) e^x dx &= \int (x^2 - x) d e^x = (x^2 - x) e^x - \int e^x d(x^2 - x) = (x^2 - x) e^x - \int (2x - 1) e^x dx \\
&= (x^2 - x) e^x - \int (2x - 1) d e^x = (x^2 - x) e^x - (2x - 1) e^x + \int e^x d(2x - 1) \\
&= (x^2 - x) e^x - (2x - 1) e^x + \int 2 e^x dx = (x^2 - x) e^x - (2x - 1) e^x + 2 e^x + c \\
&= (x^2 - 3x + 3) e^x + c
\end{aligned}$$

$$\int_1^3 (x^2 - x) e^x dx = [(x^2 - 3x + 3) e^x]_1^3 = (9 - 9 + 3) e^3 - (1 - 3 + 3) e^1 = 3e^3 - e$$

$$\begin{aligned}
\text{b} \quad \int x \ln^2(x) dx &= \int \ln^2(x) d \frac{1}{2} x^2 = \frac{1}{2} x^2 \ln^2(x) - \int \frac{1}{2} x^2 d \ln^2(x) = \frac{1}{2} x^2 \ln^2(x) - \int \frac{1}{2} x^2 \cdot 2 \ln(x) \cdot \frac{1}{x} dx \\
&= \frac{1}{2} x^2 \ln^2(x) - \int x \ln(x) dx = \frac{1}{2} x^2 \ln^2(x) - \int \ln(x) d \frac{1}{2} x^2 \\
&= \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x^2 d \ln(x) = \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx \\
&= \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x dx = \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2 + c \\
\int_1^e x \ln^2(x) dx &= \left[\frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2 \right]_1^e \\
&= \frac{1}{2} e^2 \ln^2(e) - \frac{1}{2} e^2 \ln(e) + \frac{1}{4} e^2 - \left(\frac{1}{2} \ln^2(1) - \frac{1}{2} \ln(1) + \frac{1}{4} \right) = \frac{1}{4} e^2 - \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
\text{28 a} \quad f(x) &= (2x^2 + x - 1) e^x \text{ geeft } f'(x) = (4x + 1) \cdot e^x + (2x^2 + x - 1) \cdot e^x = (2x^2 + 5x) e^x \\
f'(x) &= 0 \text{ geeft } (2x^2 + 5x) e^x = 0 \\
2x^2 + 5x &= 0 \\
x(2x + 5) &= 0 \\
x &= 0 \vee x = -2\frac{1}{2}
\end{aligned}$$

$$\text{max. is } f(-2\frac{1}{2}) = (2 \cdot 6\frac{1}{4} - 2\frac{1}{2} - 1) e^{-2\frac{1}{2}} = \frac{9}{e^2 \sqrt{e}}$$

$$\text{min. is } f(0) = -1$$

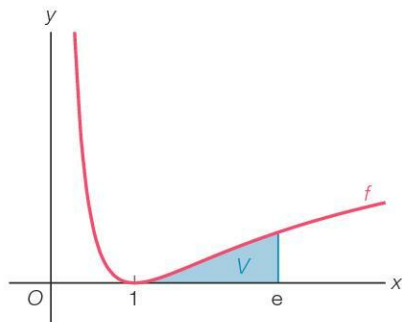
$$\begin{aligned}
\text{b} \quad f(x) &= 0 \text{ geeft } (2x^2 + x - 1) e^x = 0 \\
2x^2 + x - 1 &= 0 \\
D &= 1^2 - 4 \cdot 2 \cdot -1 = 9 \\
x &= \frac{-1 \pm 3}{4} = \frac{1}{2} \vee x = \frac{-1 - 3}{4} = -1
\end{aligned}$$

$$\begin{aligned}
\int (2x^2 + x - 1) e^x dx &= \int (2x^2 + x - 1) d e^x = (2x^2 + x - 1) e^x - \int e^x d(2x^2 + x - 1) \\
&= (2x^2 + x - 1) e^x - \int (4x + 1) e^x dx = (2x^2 + x - 1) e^x - \int (4x + 1) d e^x \\
&= (2x^2 + x - 1) e^x - (4x + 1) e^x + \int e^x d(4x + 1) \\
&= (2x^2 + x - 1) e^x - (4x + 1) e^x + \int 4 e^x dx \\
&= (2x^2 + x - 1) e^x - (4x + 1) e^x + 4 e^x + c = (2x^2 - 3x + 2) e^x + c
\end{aligned}$$

$$\int_{-1}^{\frac{1}{2}} (2x^2 + x - 1) e^x dx = [(2x^2 - 3x + 2) e^x]_{-1}^{\frac{1}{2}} = (\frac{1}{2} - 1\frac{1}{2} + 2) e^{\frac{1}{2}} - (2 + 3 + 2) e^{-1} = \sqrt{e} - \frac{7}{e}$$

$$\text{Dus } O(V) = \frac{7}{e} - \sqrt{e}.$$

- 29 a $f(x) = 0$ geeft $\ln^2(x) = 0$
 $\ln(x) = 0$
 $x = 1$



$$\begin{aligned}\int \frac{\ln^2(x)}{\sqrt{x}} dx &= \int 2 \ln^2(x) \cdot \frac{1}{2\sqrt{x}} dx = \int 2 \ln^2(x) d\sqrt{x} = 2\sqrt{x} \cdot \ln^2(x) - \int 2\sqrt{x} d\ln^2(x) \\ &= 2\sqrt{x} \cdot \ln^2(x) - \int 2\sqrt{x} \cdot 2 \ln(x) \cdot \frac{1}{x} dx = 2\sqrt{x} \cdot \ln^2(x) - \int 4 \ln(x) \cdot x^{-\frac{1}{2}} dx \\ &= 2\sqrt{x} \cdot \ln^2(x) - \int 8 \ln(x) dx^{\frac{1}{2}} = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8\sqrt{x} d\ln(x) \\ &= 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8x^{\frac{1}{2}} \cdot \frac{1}{x} dx = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + \int 8x^{-\frac{1}{2}} dx \\ &= 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16x^{\frac{1}{2}} + c = 2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16\sqrt{x} + c\end{aligned}$$

$$\begin{aligned}O(V) &= \int_1^e \frac{\ln^2(x)}{\sqrt{x}} dx = [2\sqrt{x} \cdot \ln^2(x) - 8\sqrt{x} \cdot \ln(x) + 16\sqrt{x}]_1^e \\ &= 2\sqrt{e} \cdot \ln^2(e) - 8\sqrt{e} \cdot \ln(e) + 16\sqrt{e} - (2\ln^2(1) - 8\ln(1) + 16) \\ &= 2\sqrt{e} - 8\sqrt{e} + 16\sqrt{e} - 16 = 10\sqrt{e} - 16\end{aligned}$$

b $I(L) = \pi \int_1^e \frac{\ln^4(x)}{x} dx = \pi \int_1^e \ln^4(x) \cdot \frac{1}{x} dx = \pi \int_{x=1}^{x=e} \ln^4(x) d\ln(x) = \pi \int_0^1 u^4 du = \pi \left[\frac{1}{5} u^5 \right]_0^1 = \frac{1}{5} \pi$

K.3 Cyclometrische functies

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30 $\int_0^1 \frac{1}{x^2 + 1} dx = \int_{t=0}^{t=\frac{1}{4}\pi} \frac{1}{\tan^2(t) + 1} d\tan(t) = \int_0^{\frac{1}{4}\pi} \frac{1}{\tan^2(t) + 1} \cdot (\tan^2(t) + 1) dt = \int_0^{\frac{1}{4}\pi} 1 dt = [t]_0^{\frac{1}{4}\pi} = \frac{1}{4}\pi$

Bladzijde 199

- 31 a Het bereik van $f(x) = \arctan(x)$ is $\left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$.
 $\frac{1}{6}\pi$ zit niet in het bereik, dus $\arctan\left(\frac{1}{3}\sqrt{3}\right) \neq \frac{1}{6}\pi$.
b $\sqrt{3} > \frac{1}{2}\pi$, dus de vergelijking $\arctan(x) = \sqrt{3}$ heeft geen oplossing.
c Een primitieve van $f(ax + b)$ is $\frac{1}{a}F(ax + b)$.

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x	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
$\arctan(x)$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$

- 33 a** $\arctan(x) = \frac{1}{3}\pi$
 $x = \tan(\frac{1}{3}\pi)$
 $x = \sqrt{3}$
- b** $\arctan(x-2) = -\frac{1}{4}\pi$
 $x-2 = \tan(-\frac{1}{4}\pi)$
 $x-2 = -1$
 $x = 1$
- c** $\arctan(x^2-1) = \frac{1}{4}\pi$
 $x^2-1 = \tan(\frac{1}{4}\pi)$
 $x^2-1 = 1$
 $x^2 = 2$
 $x = \sqrt{2} \vee x = -\sqrt{2}$
- d** $\arctan(x) = \frac{2}{3}\pi$
geen oplossing, want $\frac{2}{3}\pi > \frac{1}{2}\pi$
- e** $\arctan(x) = \sqrt{2}$
 $x = \tan(\sqrt{2})$
 $x \approx 6,334$
- f** $\arctan(x^2-1) = 1$
 $x^2-1 = \tan(1)$
 $x^2-1 = 1,557\dots$
 $x^2 = 2,557\dots$
 $x = \sqrt{2,557\dots} \vee x = -\sqrt{2,557\dots}$
 $x \approx 1,599 \vee x = -1,599$

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- 34 a** $f(x) = 2 \arctan(\frac{1}{2}x)$ geeft $f'(x) = 2 \cdot \frac{1}{(\frac{1}{2}x)^2 + 1} \cdot \frac{1}{2} = \frac{1}{\frac{1}{4}x^2 + 1} = \frac{4}{x^2 + 4}$
- b** $g(x) = \arctan(x-2)$ geeft $g'(x) = \frac{1}{(x-2)^2 + 1} = \frac{1}{x^2 - 4x + 4 + 1} = \frac{1}{x^2 - 4x + 5}$
- c** $h(x) = \arctan(x^2)$ geeft $h'(x) = \frac{1}{(x^2)^2 + 1} \cdot 2x = \frac{2x}{x^4 + 1}$
- 35 a** $\int_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} \frac{1}{x^2 + 1} dx = [\arctan(x)]_{-\frac{1}{3}\sqrt{3}}^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(-\frac{1}{3}\sqrt{3}) = \frac{1}{3}\pi - -\frac{1}{6}\pi = \frac{1}{2}\pi$
- b** $\int_{-2}^{-1} \frac{1}{(x+1)^2 + 1} dx = [\arctan(x+1)]_{-2}^{-1} = \arctan(0) - \arctan(-1) = 0 - -\frac{1}{4}\pi = \frac{1}{4}\pi$
- c** $\int_0^{\frac{1}{3}\sqrt{3}} \frac{3}{9x^2 + 1} dx = \int_0^{\frac{1}{3}\sqrt{3}} \frac{3}{(3x)^2 + 1} dx = [3 \cdot \frac{1}{3} \arctan(3x)]_0^{\frac{1}{3}\sqrt{3}} = [\arctan(3x)]_0^{\frac{1}{3}\sqrt{3}}$
 $= \arctan(\sqrt{3}) - \arctan(0) = \frac{1}{3}\pi - 0 = \frac{1}{3}\pi$
- 36 a** $\arctan(2x-1) = \frac{1}{6}\pi$
 $2x-1 = \tan(\frac{1}{6}\pi)$
 $2x-1 = \frac{1}{3}\sqrt{3}$
 $2x = 1 + \frac{1}{3}\sqrt{3}$
 $x = \frac{1}{2} + \frac{1}{6}\sqrt{3}$
- b** $\arctan(1\frac{1}{2}\pi + \sqrt{x}) = \sqrt{2}$
 $1\frac{1}{2}\pi + \sqrt{x} = \tan(\sqrt{2})$
 $\sqrt{x} = \tan(\sqrt{2}) - 1\frac{1}{2}\pi$
 $x = (\tan(\sqrt{2}) - 1\frac{1}{2}\pi)^2 \approx 2,63$
- c** $f(x) = \arctan(2x^2+1)$ geeft $f'(x) = \frac{1}{(2x^2+1)^2 + 1} \cdot 4x = \frac{4x}{4x^4 + 4x^2 + 1 + 1} = \frac{2x}{2x^4 + 2x^2 + 1}$
- d** $\int_{\frac{1}{5}}^{\frac{1}{3}\sqrt{3}} \frac{10}{25x^2 + 1} dx = \int_{\frac{1}{5}}^{\frac{1}{3}\sqrt{3}} \frac{10}{(5x)^2 + 1} dx = [10 \cdot \frac{1}{5} \arctan(5x)]_{\frac{1}{5}}^{\frac{1}{3}\sqrt{3}} = 2 \arctan(\sqrt{3}) - 2 \arctan(1) = 2 \cdot \frac{1}{3}\pi - 2 \cdot \frac{1}{4}\pi = \frac{1}{6}\pi$
- 37 a** $f(x) = \frac{10}{x^2 + 4} = \frac{2\frac{1}{2}}{\frac{1}{4}x^2 + 1} = \frac{2\frac{1}{2}}{(\frac{1}{2}x)^2 + 1}$
Dus $a = 2\frac{1}{2}$ en $b = \frac{1}{2}$.
- b** $F(x) = 2\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} \cdot \arctan(\frac{1}{2}x) + c = 5 \arctan(\frac{1}{2}x) + c$

$$38 \quad \mathbf{a} \quad F(x) = \int \frac{5}{32x^2 + 2} dx = \int \frac{2\frac{1}{2}}{16x^2 + 1} dx = \int \frac{2\frac{1}{2}}{(4x)^2 + 1} dx = 2\frac{1}{2} \cdot \frac{1}{4} \arctan(4x) + c = \frac{5}{8} \arctan(4x) + c$$

$$\mathbf{b} \quad G(x) = \int \frac{4}{x^2 + 4} dx = \int \frac{1}{\frac{1}{4}x^2 + 1} dx = \int \frac{1}{(\frac{1}{2}x)^2 + 1} dx = 2 \arctan(\frac{1}{2}x) + c$$

$$\mathbf{c} \quad H(x) = \int \frac{5}{x^2 + 6x + 10} dx = \int \frac{5}{(x+3)^2 - 9 + 10} dx = \int \frac{5}{(x+3)^2 + 1} dx = 5 \arctan(x+3) + c$$

$$\mathbf{d} \quad J(x) = \int \frac{3}{x^2 + 4x + 5} dx = \int \frac{3}{(x+2)^2 - 4 + 5} dx = \int \frac{3}{(x+2)^2 + 1} dx = 3 \arctan(x+2) + c$$

$$39 \quad \mathbf{a} \quad \int_0^1 \frac{3}{x^2 + 3} dx = \int_0^1 \frac{1}{\frac{1}{3}x^2 + 1} dx = \int_0^1 \frac{1}{\left(\frac{x}{\sqrt{3}}\right)^2 + 1} dx = \left[\sqrt{3} \cdot \arctan\left(\frac{x}{\sqrt{3}}\right) \right]_0^1$$

$$= \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan\left(\frac{0}{\sqrt{3}}\right) = \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan(0)$$

$$= \sqrt{3} \cdot \frac{1}{6}\pi - 0 = \frac{1}{6}\pi\sqrt{3}$$

$$\mathbf{b} \quad \int_0^1 \frac{2x}{x^4 + 1} dx = \int_{x=0}^{x=1} \frac{1}{(x^2)^2 + 1} dx^2 = [\arctan(x^2)]_0^1 = \arctan(1) - \arctan(0) = \frac{1}{4}\pi$$

$$\mathbf{c} \quad \int_3^6 \frac{5}{x^2 - 6x + 18} dx = \int_3^6 \frac{5}{(x-3)^2 - 9 + 18} dx = \int_3^6 \frac{5}{(x-3)^2 + 9} dx = \int_3^6 \frac{\frac{5}{9}}{\frac{1}{9}(x-3)^2 + 1} dx = \int_3^6 \frac{\frac{5}{9}}{(\frac{1}{3}(x-3))^2 + 1} dx$$

$$= \left[\frac{5}{9} \cdot 3 \arctan\left(\frac{1}{3}(x-3)\right) \right]_3^6 = \frac{5}{3} \arctan(1) - \frac{5}{3} \arctan(0) = \frac{5}{3} \cdot \frac{1}{4}\pi = \frac{5}{12}\pi$$

$$\mathbf{d} \quad \int_0^{\sqrt{3}} \frac{\arctan(x)}{x^2 + 1} dx = \int_0^{\sqrt{3}} \arctan(x) \cdot \frac{1}{x^2 + 1} dx = \int_{x=0}^{x=\sqrt{3}} \arctan(x) d\arctan(x) = \int_0^{\frac{1}{2}\pi} u du = \left[\frac{1}{2}u^2 \right]_0^{\frac{1}{2}\pi}$$

$$= \frac{1}{2} \cdot \frac{1}{9}\pi^2 - 0 = \frac{1}{18}\pi^2$$

$$40 \quad \mathbf{a} \quad \int_0^{\frac{1}{3}} \frac{2}{4x^2 + 9} dx = \int_0^{\frac{1}{3}} \frac{\frac{2}{9}}{\frac{4}{9}x^2 + 1} dx = \int_0^{\frac{1}{3}} \frac{\frac{2}{9}}{(\frac{2}{3}x)^2 + 1} dx = \left[\frac{2}{9} \cdot \frac{3}{2} \arctan\left(\frac{2}{3}x\right) \right]_0^{\frac{1}{3}} = \frac{1}{3} \arctan(1) - \frac{1}{3} \arctan(0)$$

$$= \frac{1}{3} \cdot \frac{1}{4}\pi = \frac{1}{12}\pi$$

$$\mathbf{b} \quad \int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x} + 1} dx = \int_{x=0}^{x=\frac{1}{2}\ln(3)} \frac{1}{(e^x)^2 + 1} de^x = \int_1^{\sqrt{3}} \frac{1}{u^2 + 1} du = [\arctan(u)]_1^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(1) = \frac{1}{3}\pi - \frac{1}{4}\pi = \frac{1}{12}\pi$$

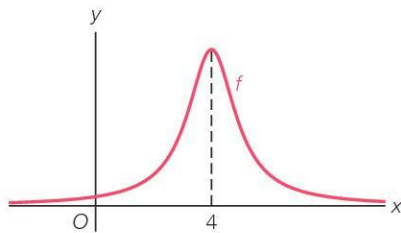
$$\mathbf{c} \quad \int_0^1 \frac{1}{4x^2 - 4x + 2} dx = \int_0^1 \frac{1}{(2x-1)^2 - 1 + 2} dx = \int_0^1 \frac{1}{(2x-1)^2 + 1} dx = \left[\frac{1}{2} \arctan(2x-1) \right]_0^1$$

$$= \frac{1}{2} \arctan(1) - \frac{1}{2} \arctan(-1) = \frac{1}{2} \cdot \frac{1}{4}\pi - \frac{1}{2} \cdot -\frac{1}{4}\pi = \frac{1}{4}\pi$$

$$\mathbf{d} \quad \int_0^{\frac{1}{2}\pi} \frac{\sin(x)}{\cos^2(x) + 1} dx = \int_{x=0}^{x=\frac{1}{2}\pi} \frac{1}{\cos^2(x) + 1} d\cos(x) = \int_1^0 \frac{1}{u^2 + 1} du = [-\arctan(u)]_1^0$$

$$= -\arctan(0) + \arctan(1) = \frac{1}{4}\pi$$

41 a $f(x) = \frac{10}{x^2 - 8x + 17}$ geeft $f'(x) = \frac{(x^2 - 8x + 17) \cdot 0 - 10(2x - 8)}{(x^2 - 8x + 17)^2} = \frac{-20x + 80}{(x^2 - 8x + 17)^2}$
 $f'(x) = 0$ geeft $-20x + 80 = 0$
 $-20x = -80$
 $x = 4$



max. is $f(4) = \frac{10}{16 - 32 + 17} = 10$

b $O(V) = \int_4^5 \frac{10}{x^2 - 8x + 17} dx = \int_4^5 \frac{10}{(x-4)^2 - 16 + 17} dx = \int_4^5 \frac{10}{(x-4)^2 + 1} dx = [10 \arctan(x-4)]_4^5$
 $= 10 \arctan(1) - 10 \arctan(0) = 10 \cdot \frac{1}{4} \pi = 2\frac{1}{2} \pi$

c $O(W_p) = 10$ geeft $\int_4^p f(x) dx = 10$
 $[10 \arctan(x-4)]_4^p = 10$
 $10 \arctan(p-4) - 10 \arctan(0) = 10$
 $\arctan(p-4) = 1$
 $p-4 = \tan(1)$
 $p = 4 + \tan(1)$

42 $\int \frac{4}{x \ln^2(x) + x} dx = \int \frac{1}{x} \cdot \frac{4}{\ln^2(x) + 1} dx = \int \frac{4}{\ln^2(x) + 1} d \ln(x) = \int \frac{4}{u^2 + 1} du = 4 \arctan(u) + c = 4 \arctan(\ln(x)) + c$

$O(V_p) = \int_1^p f(x) dx = [4 \arctan(\ln(x))]_1^p = 4 \arctan(\ln(p)) - 4 \arctan(0) = 4 \arctan(\ln(p))$

$O(V_p) = \pi$ geeft $4 \arctan(\ln(p)) = \pi$
 $\arctan(\ln(p)) = \frac{1}{4} \pi$
 $\ln(p) = \tan(\frac{1}{4} \pi)$
 $\ln(p) = 1$
 $p = e$

43 $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx = \int_{t=0}^{t=\frac{1}{6}\pi} \frac{1}{\sqrt{1-\sin^2(t)}} d \sin(t) = \int_0^{\frac{1}{6}\pi} \frac{1}{\sqrt{\cos^2(t)}} \cdot \cos(t) dt = \int_0^{\frac{1}{6}\pi} \frac{1}{\cos(t)} \cdot \cos(t) dt = \int_0^{\frac{1}{6}\pi} 1 dt = [t]_0^{\frac{1}{6}\pi} = \frac{1}{6} \pi$

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44

x	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\arcsin(x)$	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$

45 a $\arcsin(x) = \frac{1}{2} \pi$
 $x = \sin(\frac{1}{2} \pi)$
 $x = 1$

b $\arcsin(x) = -\frac{1}{6} \pi$
 $x = \sin(-\frac{1}{6} \pi)$
 $x = -\frac{1}{2}$

- c** $\arcsin(x) = 2$
geen oplossing, want $2 > \frac{1}{2}\pi$
- d** $3 \arcsin(x - \sqrt{3}) = \pi$
 $\arcsin(x - \sqrt{3}) = \frac{1}{3}\pi$
 $x - \sqrt{3} = \sin(\frac{1}{3}\pi)$
 $x - \sqrt{3} = \frac{1}{2}\sqrt{3}$
 $x = \frac{1}{2}\sqrt{3} \approx 2,598$

46 a $\int_{\frac{1}{6}}^{\frac{1}{3}\sqrt{3}} \frac{1}{\sqrt{1-9x^2}} dx = \int_{\frac{1}{6}}^{\frac{1}{3}\sqrt{3}} \frac{1}{\sqrt{1-(3x)^2}} dx = \left[\frac{1}{3} \arcsin(3x) \right]_{\frac{1}{6}}^{\frac{1}{3}\sqrt{3}} = \frac{1}{3} \arcsin(\frac{1}{2}\sqrt{3}) - \frac{1}{3} \arcsin(\frac{1}{2})$
 $= \frac{1}{3} \cdot \frac{1}{3}\pi - \frac{1}{3} \cdot \frac{1}{6}\pi = \frac{1}{9}\pi - \frac{1}{18}\pi = \frac{1}{18}\pi$

b $\int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{1}{\sqrt{9-x^2}} dx = \int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-\frac{1}{9}x^2}} dx = \int_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-(\frac{1}{3}x)^2}} dx = \left[\arcsin(\frac{1}{3}x) \right]_{-1\frac{1}{2}\sqrt{2}}^{1\frac{1}{2}\sqrt{2}}$
 $= \arcsin(\frac{1}{3} \cdot 1\frac{1}{2}\sqrt{2}) - \arcsin(\frac{1}{3} \cdot -1\frac{1}{2}\sqrt{2}) = \arcsin(\frac{1}{2}\sqrt{2}) - \arcsin(-\frac{1}{2}\sqrt{2})$
 $= \frac{1}{4}\pi - (-\frac{1}{4}\pi) = \frac{1}{2}\pi$

c $\int_0^{\frac{1}{2}\sqrt{2}} \frac{x}{\sqrt{1-x^4}} dx = \int_{x=0}^{x=\frac{1}{2}\sqrt{2}} \frac{1}{2} \cdot \frac{1}{\sqrt{1-(x^2)^2}} dx^2 = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}} du = \left[\frac{1}{2} \arcsin(u) \right]_0^{\frac{1}{2}}$
 $= \frac{1}{2} \arcsin(\frac{1}{2}) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - 0 = \frac{1}{12}\pi$

d $\int_0^1 \frac{x}{\sqrt{4-x^4}} dx = \int_0^1 \frac{\frac{1}{2}x}{\sqrt{1-\frac{1}{4}x^4}} dx = \int_{x=0}^{x=1} \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{2}x^2)^2}} d\frac{1}{2}x^2 = \int_0^{\frac{1}{2}} \frac{\frac{1}{2}}{\sqrt{1-u^2}} du = \left[\frac{1}{2} \arcsin(u) \right]_0^{\frac{1}{2}}$
 $= \frac{1}{2} \arcsin(\frac{1}{2}) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - 0 = \frac{1}{12}\pi$

47 a $F(x) = \int \arctan(x) dx = x \arctan(x) - \int x d \arctan(x) = x \arctan(x) - \int x \cdot \frac{1}{x^2+1} dx$
 $= x \arctan(x) - \int \frac{1}{2} \cdot \frac{1}{x^2+1} d(x^2+1) = x \arctan(x) - \frac{1}{2} \ln(x^2+1) + c$

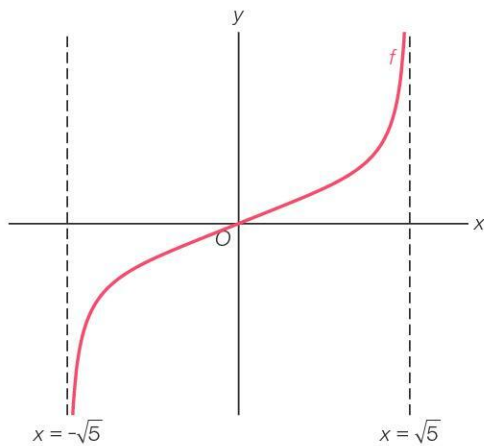
b $G(x) = \int \arcsin(x) dx = x \arcsin(x) - \int x d \arcsin(x) = x \arcsin(x) - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$
 $= x \arcsin(x) - \int -\frac{1}{2\sqrt{1-x^2}} d(1-x^2) = x \arcsin(x) + \sqrt{1-x^2} + c$

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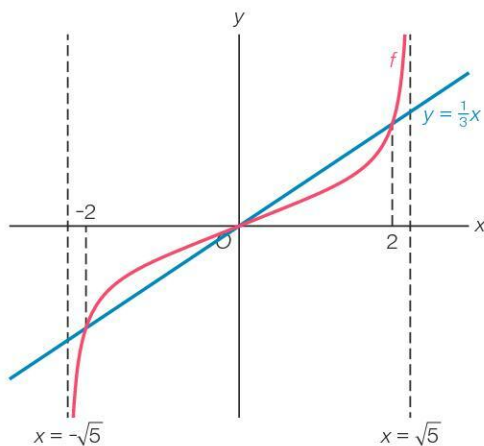
48 a $F(x) = \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx = \int \arcsin(x) \cdot \frac{1}{\sqrt{1-x^2}} dx = \int \arcsin(x) d \arcsin(x) = \frac{1}{2} \arcsin^2(x) + c$

b $G(x) = \int \frac{1}{x\sqrt{1-\ln^2(x)}} dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} \cdot \frac{1}{x} dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} d \ln(x) = \arcsin(\ln(x)) + c$

49 a $25 - x^4 > 0$
 $x^4 < 25$
 $-5 < x^2 < 5$
 $x^2 < 5$
 $-\sqrt{5} < x < \sqrt{5}$
Dus $D_f = \langle -\sqrt{5}, \sqrt{5} \rangle$.



b $f(x) = \frac{1}{3}x$ geeft $\frac{x}{\sqrt{25-x^4}} = \frac{x}{3}$
 $x = 0 \vee \sqrt{25-x^4} = 3$
 $x = 0 \vee 25 - x^4 = 9$
 $x = 0 \vee x^4 = 16$
 $x = 0 \vee x = 2 \vee x = -2$



$f(x) \leq \frac{1}{3}x$ geeft $-\sqrt{5} < x \leq -2 \vee 0 \leq x \leq 2$

c $O(V) = \int_0^{\frac{1}{2}\sqrt{10}} \frac{x}{\sqrt{25-x^4}} dx = \int_0^{\frac{1}{2}\sqrt{10}} \frac{\frac{1}{5}x}{\sqrt{1-\frac{1}{25}x^4}} dx = \int_0^{\frac{1}{2}\sqrt{10}} \frac{\frac{1}{5}x}{\sqrt{1-(\frac{1}{5}x^2)^2}} dx = \int_{x=0}^{x=\frac{1}{2}\sqrt{10}} \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{5}x^2)^2}} d\frac{1}{5}x^2$
 $= \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}} du = \left[\frac{1}{2} \arcsin(u) \right]_0^{\frac{1}{2}} = \frac{1}{2} \arcsin\left(\frac{1}{2}\right) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6} \pi = \frac{1}{12} \pi$

K.4 Breuksplitsen

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50 a $F(x) = \int \frac{2x}{x^2+1} dx = \int \frac{1}{x^2+1} d(x^2+1) = \ln(x^2+1)$

$G(x) = \int \frac{1}{x^2+1} dx = \arctan(x)$

b Uitdelen geeft $h(x) = \frac{2x+1}{x^2+1} = \frac{2x}{x^2+1} + \frac{1}{x^2+1} = f(x) + g(x)$.

Dus $H(x) = \ln(x^2+1) + \arctan(x) + c$.

51 a Uitdelen geeft $f(x) = \frac{2x+5}{x+1} = \frac{2x}{x+1} + \frac{5}{x+1}$.

f is zo niet te primitiveren, want er is geen primitieve te geven van $\frac{2x}{x+1}$.

b $f(x) = 2 + \frac{3}{x+1} = \frac{2x+2}{x+1} + \frac{3}{x+1} = \frac{2x+2+3}{x+1} = \frac{2x+5}{x+1}$

Dus de splitsing is correct.

$$F(x) = \int \left(2 + \frac{3}{x+1} \right) dx = 2x + 3 \ln|x+1| + c$$

Bladzijde 209

52 a $x+1 / 2x+1 \setminus 2$

$$\frac{2x+2}{-1} -$$

Dus $\frac{2x+1}{x+1} = 2 - \frac{1}{x+1}$.

$$F(x) = \int \frac{2x+1}{x+1} dx = \int \left(2 - \frac{1}{x+1} \right) dx = 2x - \ln|x+1| + c$$

b $x+1 / x \setminus 1$

$$\frac{x+1}{-1} -$$

Dus $\frac{x}{x+1} = 1 - \frac{1}{x+1}$.

$$F(x) = \int \frac{x}{x+1} dx = \int \left(1 - \frac{1}{x+1} \right) dx = x - \ln|x+1| + c$$

c $2x+1 / x+1 \setminus \frac{1}{2}$

$$\frac{x+\frac{1}{2}}{\frac{1}{2}} -$$

Dus $\frac{x+1}{2x+1} = \frac{1}{2} + \frac{\frac{1}{2}}{2x+1}$.

$$F(x) = \int \frac{x+1}{2x+1} dx = \int \left(\frac{1}{2} + \frac{\frac{1}{2}}{2x+1} \right) dx = \frac{1}{2}x + \frac{1}{4} \ln|2x+1| + c$$

d $f(x) = \frac{2-x}{x+1} = \frac{-x+2}{x+1}$

$$x+1 / -x+2 \setminus -1$$

$$\frac{-x-1}{3} -$$

Dus $\frac{2-x}{x+1} = -1 + \frac{3}{x+1}$.

$$F(x) = \int \frac{2-x}{x+1} dx = \int \left(-1 + \frac{3}{x+1} \right) dx = -x + 3 \ln|x+1| + c$$

53 a $x-1 / x^2-2x+3 \setminus x-1$

$$\frac{x^2-x}{-x+3} -$$

$$\frac{-x+1}{2} -$$

Dus $\frac{x^2-2x+3}{x-1} = x-1 + \frac{2}{x-1}$.

$$\begin{aligned} \int_2^3 \frac{x^2-2x+3}{x-1} dx &= \int_2^3 \left(x-1 + \frac{2}{x-1} \right) dx = \left[\frac{1}{2}x^2 - x + 2 \ln|x-1| \right]_2^3 = 4\frac{1}{2} - 3 + 2 \ln(2) - (2 - 2 + 2 \ln(1)) \\ &= 1\frac{1}{2} + 2 \ln(2) \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & x+1 \mid -2x^2-x \quad \setminus -2x+1 \\ & \frac{-2x^2-2x}{x} - \\ & \frac{x+1}{-1} - \end{aligned}$$

$$\text{Dus } \frac{-2x^2-x}{x+1} = -2x+1 - \frac{1}{x+1}.$$

$$\begin{aligned} \int_{-4}^{-2} \frac{-2x^2-x}{x+1} dx &= \int_{-4}^{-2} \left(-2x+1 - \frac{1}{x+1} \right) dx = \left[-x^2+x - \ln|x+1| \right]_{-4}^{-2} \\ &= -4-2-\ln(1) - (-16-4-\ln(3)) = 14+\ln(3) \end{aligned}$$

$$\begin{aligned} \mathbf{54} \quad \mathbf{a} \quad & f(x) = \frac{3-4x}{2x+1} = \frac{-4x+3}{2x+1} \\ & 2x+1 \mid -4x+3 \quad \setminus -2 \\ & \frac{-4x-2}{5} - \end{aligned}$$

$$\text{Dus } \frac{3-4x}{2x+1} = -2 + \frac{5}{2x+1}.$$

$$F(x) = \int \frac{3-4x}{2x+1} dx = \int \left(-2 + \frac{5}{2x+1} \right) dx = -2x + 2\frac{1}{2} \ln|2x+1| + c$$

$$\begin{aligned} g(x) &= \frac{6x-1}{1-2x} = \frac{6x-1}{-2x+1} \\ & -2x+1 \mid 6x-1 \quad \setminus -3 \\ & \frac{6x-3}{2} - \end{aligned}$$

$$\text{Dus } \frac{6x-1}{1-2x} = -3 + \frac{2}{1-2x}.$$

$$G(x) = \int \frac{6x-1}{1-2x} dx = \int \left(-3 + \frac{2}{1-2x} \right) dx = -3x - \ln|1-2x| + c$$

$$\begin{aligned} \mathbf{b} \quad & 2x-1 \mid 2x^2-5x \quad \setminus x-2 \\ & \frac{2x^2-x}{-4x} - \\ & \frac{-4x+2}{-2} - \end{aligned}$$

$$\text{Dus } \frac{2x^2-5x}{2x-1} = x-2 - \frac{2}{2x-1}.$$

$$\begin{aligned} \int_3^6 \frac{2x^2-5x}{2x-1} dx &= \int_3^6 \left(x-2 - \frac{2}{2x-1} \right) dx = \left[\frac{1}{2}x^2 - 2x - \ln|2x-1| \right]_3^6 = 18-12-\ln(11) - (4\frac{1}{2}-6-\ln(5)) \\ &= 6-\ln(11) + 1\frac{1}{2} + \ln(5) = 7\frac{1}{2} + \ln\left(\frac{5}{11}\right) \end{aligned}$$

$$\begin{aligned} \mathbf{55} \quad \mathbf{a} \quad & f(x) = \frac{x^3+x}{x+1} \text{ geeft} \\ f'(x) &= \frac{(x+1) \cdot (3x^2+1) - (x^3+x) \cdot 1}{(x+1)^2} = \frac{3x^3+x+3x^2+1-x^3-x}{(x+1)^2} = \frac{2x^3+3x^2+1}{(x+1)^2} \end{aligned}$$

$$f'(0) = 1, \text{ dus } k: y = x.$$

$$f(x) = x \text{ geeft } \frac{x^3+x}{x+1} = x$$

$$x \cdot \frac{x^2+1}{x+1} = x$$

$$x = 0 \vee \frac{x^2+1}{x+1} = 1$$

$$x = 0 \vee x^2+1 = x+1$$

$$x = 0 \vee x^2 = x$$

$$x = 0 \vee x = 0 \vee x = 1$$

$$f(1) = 1, \text{ dus } A(1, 1).$$

$$\begin{array}{r}
x + 1/x^3 + x \quad \setminus x^2 - x + 2 \\
\hline
x^3 + x^2 \\
-x^2 + x \\
\hline
-x^2 - x \\
\hline
2x \\
\hline
2x + 2 \\
\hline
-2
\end{array}$$

$$\text{Dus } \frac{x^3 + x}{x + 1} = x^2 - x + 2 - \frac{2}{x + 1}.$$

$$\begin{aligned}
O(V) &= \int_0^2 \frac{x^3 + x}{x + 1} dx = \int_0^2 \left(x^2 - x + 2 - \frac{2}{x + 1} \right) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x - 2\ln|x + 1| \right]_0^2 \\
&= \frac{8}{3} - 2 + 4 - 2\ln(3) - (0 - 0 + 0 - 2\ln(1)) = 4\frac{2}{3} - 2\ln(3)
\end{aligned}$$

56 a $f(x) = \frac{3x + 1}{x + 1} = \frac{3(x + 1) - 3 + 1}{x + 1} = 3 - \frac{2}{x + 1}$
Dus $F(x) = 3x - 2\ln|x + 1| + c$.

b $g(x) = \frac{4x}{2x - 1} = \frac{2(2x - 1) + 2}{2x - 1} = 2 + \frac{2}{2x - 1}$
Dus $G(x) = 2x + \ln|2x - 1| + c$.

Bladzijde 210

57 a $f(x)$ is voor elke x gedefinieerd, want de noemer $x^2 + 4x + 5$ heeft geen nulpunten.
 $x^2 + 4x + 4 = (x + 2)^2$ heeft één nulpunt, namelijk -2 , dus de grafiek van g heeft één verticale asymptoot.

$x^2 + 4x + 3 = (x + 1)(x + 3)$ heeft twee nulpunten, namelijk -1 en -3 , dus de grafiek van h heeft twee verticale asymptoten.

b $x^2 + 4x + 5 = (x + 2)^2 - 4 + 5 = (x + 2)^2 + 1$

c $\int \frac{2x + 4}{x^2 + 4x + 5} dx = \int \frac{1}{x^2 + 4x + 5} d(x^2 + 4x + 5) = \ln(x^2 + 4x + 5) + c$
 $\int \frac{c}{(x + 2)^2 + 1} dx = c \arctan(x + 2) + d$

d $\int \frac{a}{x + 2} dx = a \ln|x + 2| + c$
 $\int \frac{b}{(x + 2)^2} dx = \int b(x + 2)^{-2} dx = -b(x + 2)^{-1} + c = \frac{-b}{x + 2} + c$

e $\int \frac{a}{x + 1} dx = a \ln|x + 1| + c$
 $\int \frac{b}{x + 3} dx = b \ln|x + 3| + c$

Bladzijde 213

58 $p(x) = x^2 - 3x + 5$, $q(x) = 1$, $l(x) = x + 1$, $b = -4$ en $c = 4$.

59 a $x^2 + 1/x^2 + x \quad \setminus 1$
 $\frac{x^2 + 1}{x - 1} -$

$$f(x) = \frac{x^2 + x}{x^2 + 1} = 1 + \frac{x - 1}{x^2 + 1} = 1 + \frac{\frac{1}{2} \cdot 2x - 1}{x^2 + 1} = 1 + \frac{\frac{1}{2} \cdot 2x}{x^2 + 1} - \frac{1}{x^2 + 1}$$

$$\int \frac{\frac{1}{2} \cdot 2x}{x^2 + 1} dx = \int \frac{\frac{1}{2}}{x^2 + 1} d(x^2 + 1) = \frac{1}{2} \ln(x^2 + 1) + c$$

$$\text{Dus } F(x) = x + \frac{1}{2} \ln(x^2 + 1) - \arctan(x) + c.$$

b $x^2 - 6x + 9 / x^2 - 6x + 8 \setminus 1$

$$\frac{x^2 - 6x + 9}{-1} -$$

$$f(x) = \frac{x^2 - 6x + 8}{x^2 - 6x + 9} = 1 - \frac{1}{x^2 - 6x + 9} = 1 - \frac{1}{(x-3)^2} = 1 - (x-3)^{-2} \text{ geeft}$$

$$F(x) = x + (x-3)^{-1} + c = x + \frac{1}{x-3} + c$$

c $x^2 + 2x - 3 / x^3 \setminus x - 2$

$$\frac{x^3 + 2x^2 - 3x}{-2x^2 + 3x} -$$

$$\frac{-2x^2 - 4x + 6}{7x - 6} -$$

$$f(x) = \frac{x^3}{x^2 + 2x - 3} = x - 2 + \frac{7x - 6}{x^2 + 2x - 3} = x - 2 + \frac{7x - 6}{(x-1)(x+3)} = x - 2 + \frac{a}{x-1} + \frac{b}{x+3}$$

$$= x - 2 + \frac{a(x+3) + b(x-1)}{(x-1)(x+3)} = x - 2 + \frac{ax + 3a + bx - b}{(x-1)(x+3)} = x - 2 + \frac{(a+b)x + 3a - b}{(x-1)(x+3)}$$

$$\begin{cases} a + b = 7 \\ 3a - b = -6 \end{cases} +$$

$$\frac{4a = 1}{a = \frac{1}{4}}$$

$$\left. \begin{matrix} a = \frac{1}{4} \\ a + b = 7 \end{matrix} \right\} \begin{matrix} \frac{1}{4} + b = 7 \\ b = 6\frac{3}{4} \end{matrix}$$

$$\text{Dus } f(x) = x - 2 + \frac{\frac{1}{4}}{x-1} + \frac{6\frac{3}{4}}{x+3}.$$

$$\text{Dit geeft } F(x) = \frac{1}{2}x^2 - 2x + \frac{1}{4}\ln|x-1| + 6\frac{3}{4}\ln|x+3| + c.$$

60 a $f(x) = \frac{x^3 + x}{x^2 + 1} = \frac{x(x^2 + 1)}{x^2 + 1} = x$ geeft $F(x) = \frac{1}{2}x^2 + c$

b $x^2 - 6x + 9 / x^3 + x \setminus x + 6$

$$\frac{x^3 - 6x^2 + 9x}{6x^2 - 8x} -$$

$$\frac{6x^2 - 36x + 54}{28x - 54} -$$

$$f(x) = \frac{x^3 + x}{x^2 - 6x + 9} = x + 6 + \frac{28x - 54}{x^2 - 6x + 9} = x + 6 + \frac{28(x-3) + 84 - 54}{(x-3)^2} = x + 6 + \frac{28}{x-3} + \frac{30}{(x-3)^2}$$

$$= x + 6 + \frac{28}{x-3} + 30(x-3)^{-2} \text{ geeft}$$

$$F(x) = \frac{1}{2}x^2 + 6x + 28\ln|x-3| - 30(x-3)^{-1} + c = \frac{1}{2}x^2 + 6x + 28\ln|x-3| - \frac{30}{x-3} + c$$

c $2x^2 + x / 2x^3 + 1 \setminus x - \frac{1}{2}$

$$\frac{2x^3 + x^2}{-x^2} -$$

$$\frac{-x^2 - \frac{1}{2}x}{\frac{1}{2}x + 1} -$$

$$f(x) = \frac{2x^3 + 1}{2x^2 + x} = x - \frac{1}{2} + \frac{\frac{1}{2}x + 1}{2x^2 + x} = x - \frac{1}{2} + \frac{\frac{1}{2}x + 1}{x(2x + 1)} = x - \frac{1}{2} + \frac{a}{x} + \frac{b}{2x + 1}$$

$$= x - \frac{1}{2} + \frac{a(2x + 1) + bx}{x(2x + 1)} = x - \frac{1}{2} + \frac{2ax + a + bx}{x(2x + 1)} = x - \frac{1}{2} + \frac{(2a + b)x + a}{x(2x + 1)}$$

$$\begin{cases} 2a + b = \frac{1}{2} \\ a = 1 \end{cases} \left\{ \begin{matrix} 2 + b = \frac{1}{2} \\ b = -1\frac{1}{2} \end{matrix} \right.$$

$$\text{Dus } f(x) = x - \frac{1}{2} + \frac{1}{x} - \frac{1\frac{1}{2}}{2x + 1}.$$

$$\text{Dit geeft } F(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \ln|x| - \frac{3}{4}\ln|2x + 1| + c.$$

61 a $x^2 + 4 / x^3 \quad \setminus x$

$$\frac{x^3 + 4x}{-4x} -$$

$$\text{Dus } \frac{x^3}{x^2 + 4} = x - \frac{4x}{x^2 + 4}.$$

$$\int \frac{4x}{x^2 + 4} dx = \int \frac{2}{x^2 + 4} d(x^2 + 4) = 2 \ln(x^2 + 4) + c$$

$$\begin{aligned} \text{Dus } \int_0^2 \frac{x^3}{x^2 + 4} dx &= \int_0^2 \left(x - \frac{4x}{x^2 + 4} \right) dx = \left[\frac{1}{2}x^2 - 2 \ln(x^2 + 4) \right]_0^2 = 2 - 2 \ln(8) - (0 - 2 \ln(4)) \\ &= 2 - 2 \ln(8) + 2 \ln(4) = 2 - 2(\ln(8) - \ln(4)) = 2 - 2 \ln(2) \end{aligned}$$

b $x^2 + 4x + 4 / x^3 \quad \setminus x - 4$

$$\frac{x^3 + 4x^2 + 4x}{-4x^2 - 4x} - \frac{-4x^2 - 16x - 16}{12x + 16} -$$

$$\begin{aligned} \text{Dus } \frac{x^3}{x^2 + 4x + 4} &= x - 4 + \frac{12x + 16}{x^2 + 4x + 4} = x - 4 + \frac{12x + 16}{(x + 2)^2} = x - 4 + \frac{12(x + 2) - 8}{(x + 2)^2} \\ &= x - 4 + \frac{12}{x + 2} - \frac{8}{(x + 2)^2} = x - 4 + \frac{12}{x + 2} - 8(x + 2)^{-2}. \end{aligned}$$

$$\begin{aligned} \int_0^8 \frac{x^3}{x^2 + 4x + 4} dx &= \int_0^8 \left(x - 4 + \frac{12}{x + 2} - 8(x + 2)^{-2} \right) dx = \left[\frac{1}{2}x^2 - 4x + 12 \ln|x + 2| + 8(x + 2)^{-1} \right]_0^8 \\ &= \left[\frac{1}{2}x^2 - 4x + 12 \ln|x + 2| + \frac{8}{x + 2} \right]_0^8 \\ &= 32 - 32 + 12 \ln(10) + \frac{4}{5} - (0 - 0 + 12 \ln(2) + 4) = 12 \ln(10) + \frac{4}{5} - 12 \ln(2) - 4 \\ &= 12 \ln(5) - 3\frac{1}{5} \end{aligned}$$

c $\frac{4x - 8}{x^2 - 4x - 5} = \frac{4x - 8}{(x + 1)(x - 5)} = \frac{a}{x + 1} + \frac{b}{x - 5} = \frac{a(x - 5) + b(x + 1)}{(x + 1)(x - 5)} = \frac{ax - 5a + bx + b}{(x + 1)(x - 5)}$

$$= \frac{(a + b)x - 5a + b}{(x + 1)(x - 5)}$$

$$\begin{cases} a + b = 4 \\ -5a + b = -8 \\ 6a = 12 \\ a = 2 \\ a + b = 4 \end{cases} \Rightarrow \begin{cases} 2 + b = 4 \\ b = 2 \end{cases}$$

$$\begin{aligned} \int_0^2 \frac{4x - 8}{x^2 - 4x - 5} dx &= \int_0^2 \left(\frac{2}{x + 1} + \frac{2}{x - 5} \right) dx = [2 \ln|x + 1| + 2 \ln|x - 5|]_0^2 \\ &= 2 \ln(3) + 2 \ln(3) - (2 \ln(1) + 2 \ln(5)) = 4 \ln(3) - 2 \ln(5) \end{aligned}$$

62 a $x^2 + 1 / x^4 \quad + 1 \setminus x^2 - 1$

$$\frac{x^4 + x^2}{-x^2} - \frac{-x^2 - 1}{2} -$$

$$\begin{aligned} \int_0^1 \frac{x^4 + 1}{x^2 + 1} dx &= \int_0^1 \left(x^2 - 1 + \frac{2}{x^2 + 1} \right) dx = \left[\frac{1}{3}x^3 - x + 2 \arctan(x) \right]_0^1 \\ &= \frac{1}{3} - 1 + 2 \arctan(1) - (0 - 0 + 2 \arctan(0)) = -\frac{2}{3} + 2 \cdot \frac{1}{4} \pi = -\frac{2}{3} + \frac{1}{2} \pi \end{aligned}$$

$$\text{b } x^2 + 2x + 1 / x^4 \quad + 1 \setminus x^2 - 2x + 3$$

$$\frac{x^4 + 2x^3 + x^2}{-2x^3 - x^2} - \frac{-2x^3 - 4x^2 - 2x}{3x^2 + 2x} - \frac{3x^2 + 6x + 3}{-4x - 2}$$

$$\text{Dus } \frac{x^4 + 1}{x^2 + 2x + 1} = x^2 - 2x + 3 - \frac{4x + 2}{x^2 + 2x + 1} = x^2 - 2x + 3 - \frac{4x + 2}{(x + 1)^2} = x^2 - 2x + 3 - \frac{4(x + 1) - 2}{(x + 1)^2}$$

$$= x^2 - 2x + 3 - \frac{4}{x + 1} + \frac{2}{(x + 1)^2} = x^2 - 2x + 3 - \frac{4}{x + 1} + 2(x + 1)^{-2}$$

$$\int_0^1 \frac{x^4 + 1}{x^2 + 2x + 1} dx = \int_0^1 \left(x^2 - 2x + 3 - \frac{4}{x + 1} + 2(x + 1)^{-2} \right) dx$$

$$= \left[\frac{1}{3}x^3 - x^2 + 3x - 4 \ln|x + 1| - 2(x + 1)^{-1} \right]_0^1 = \left[\frac{1}{3}x^3 - x^2 + 3x - 4 \ln|x + 1| - \frac{2}{x + 1} \right]_0^1$$

$$= \frac{1}{3} - 1 + 3 - 4 \ln(2) - 1 - (0 - 0 + 0 - 4 \ln(1) - 2) = \frac{1}{3} - 4 \ln(2) + 2 = 3\frac{1}{3} - 4 \ln(2)$$

$$\text{c } x^2 + 4x + 3 / x^4 \quad \setminus x^2 - 4x + 13$$

$$\frac{x^4 + 4x^3 + 3x^2}{-4x^3 - 3x^2} - \frac{-4x^3 - 16x^2 - 12x}{13x^2 + 12x} - \frac{13x^2 + 52x + 39}{-40x - 39}$$

$$\frac{-40x - 39}{x^2 + 4x + 3} = \frac{-40x - 39}{(x + 1)(x + 3)} = \frac{a}{x + 1} + \frac{b}{x + 3} = \frac{a(x + 3) + b(x + 1)}{(x + 1)(x + 3)} = \frac{ax + 3a + bx + b}{(x + 1)(x + 3)}$$

$$= \frac{(a + b)x + 3a + b}{(x + 1)(x + 3)}$$

$$\begin{cases} a + b = -40 \\ 3a + b = -39 \end{cases} \quad \begin{cases} -2a = -1 \\ a = \frac{1}{2} \end{cases} \quad \begin{cases} \frac{1}{2} + b = -40 \\ b = -40\frac{1}{2} \end{cases}$$

$$\int_0^2 \frac{x^4}{x^2 + 4x + 3} dx = \int_0^2 \left(x^2 - 4x + 13 + \frac{\frac{1}{2}}{x + 1} - \frac{40\frac{1}{2}}{x + 3} \right) dx$$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 13x + \frac{1}{2} \ln|x + 1| - 40\frac{1}{2} \ln|x + 3| \right]_0^2$$

$$= \frac{8}{3} - 8 + 26 + \frac{1}{2} \ln(3) - 40\frac{1}{2} \ln(5) - (0 - 0 + 0 + \frac{1}{2} \ln(1) - 40\frac{1}{2} \ln(3))$$

$$= 20\frac{2}{3} + \frac{1}{2} \ln(3) - 40\frac{1}{2} \ln(5) + 40\frac{1}{2} \ln(3) = 20\frac{2}{3} + 41 \ln(3) - 40\frac{1}{2} \ln(5)$$

63

$$\text{I } \frac{2x - 5}{x^2 - 5x + 6} = \frac{2x - 5}{(x - 2)(x - 3)} = \frac{a}{x - 2} + \frac{b}{x - 3} = \frac{a(x - 3) + b(x - 2)}{(x - 2)(x - 3)} = \frac{ax - 3a + bx - 2b}{(x - 2)(x - 3)}$$

$$= \frac{(a + b)x - 3a - 2b}{(x - 2)(x - 3)}$$

$$\begin{cases} a + b = 2 \\ -3a - 2b = -5 \end{cases} \quad \begin{cases} 2a + 2b = 4 \\ -3a - 2b = -5 \\ -a = -1 \\ a = 1 \end{cases} \quad \begin{cases} 1 + b = 2 \\ b = 1 \end{cases}$$

$$f(x) = \frac{1}{x - 2} + \frac{1}{x - 3} \text{ geeft } F(x) = \ln|x - 2| + \ln|x - 3| + c$$

$$\text{II } F(x) = \int \frac{2x - 5}{x^2 - 5x + 6} dx = \int \frac{1}{x^2 - 5x + 6} d(x^2 - 5x + 6) = \ln|x^2 - 5x + 6| + c$$

$$\ln|x - 2| + \ln|x - 3| = \ln|(x - 2)(x - 3)| = \ln|x^2 - 5x + 6|$$

Dus de primitieven komen op hetzelfde neer.

64 a $f(x) = \frac{10x+5}{4x^2-4x+1}$ geeft

$$f'(x) = \frac{(4x^2-4x+1) \cdot 10 - (10x+5) \cdot (8x-4)}{(4x^2-4x+1)^2} = \frac{40x^2-40x+10 - (80x^2-40x+20)}{(2x-1)^4}$$

$$= \frac{40x^2-40x+10-80x^2+40x-20}{(2x-1)^4} = \frac{-40x^2-40x+30}{(2x-1)^4}$$

$$f'(x) = 0 \text{ geeft } -40x^2 - 40x + 30 = 0$$

$$4x^2 + 4x - 3 = 0$$

$$D = 4^2 - 4 \cdot 4 \cdot -3 = 64$$

$$x = \frac{-4 \pm 8}{8} = \frac{1}{2} \vee x = \frac{-4-8}{8} = -1\frac{1}{2}$$

vold. niet

$$\text{min. is } f(-1\frac{1}{2}) = \frac{-15+5}{9+6+1} = \frac{-10}{16} = -\frac{5}{8}$$

$$\text{Dus } B_f = [-\frac{5}{8}, \rightarrow)$$

b $O(V) = \int_1^3 \frac{10x+5}{4x^2-4x+1} dx = \int_1^3 \frac{5(2x-1)+10}{(2x-1)^2} dx = \int_1^3 \left(\frac{5}{2x-1} + \frac{10}{(2x-1)^2} \right) dx$

$$= \int_1^3 \left(\frac{5}{2x-1} + 10(2x-1)^{-2} \right) dx = \left[\frac{5}{2} \ln|2x-1| - 5(2x-1)^{-1} \right]_1^3 = \left[2\frac{1}{2} \ln|2x-1| - \frac{5}{2x-1} \right]_1^3$$

$$= 2\frac{1}{2} \ln(5) - \frac{5}{5} - \left(2\frac{1}{2} \ln(1) - \frac{5}{1} \right) = 2\frac{1}{2} \ln(5) - 1 + 5 = 4 + 2\frac{1}{2} \ln(5)$$

Bladzijde 214

65 a $f(x) = \frac{x^2+4}{x^2+5x+4}$ geeft

$$f'(x) = \frac{(x^2+5x+4) \cdot 2x - (x^2+4) \cdot (2x+5)}{(x^2+5x+4)^2} = \frac{2x^3+10x^2+8x - (2x^3+5x^2+8x+20)}{(x^2+5x+4)^2}$$

$$= \frac{2x^3+10x^2+8x-2x^3-5x^2-8x-20}{(x^2+5x+4)^2} = \frac{5x^2-20}{(x^2+5x+4)^2}$$

$$f'(x) = 0 \text{ geeft } 5x^2 - 20 = 0$$

$$5x^2 = 20$$

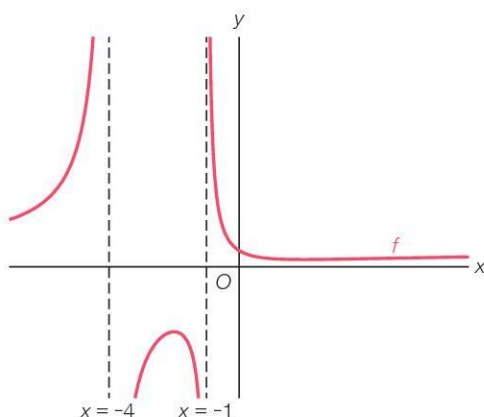
$$x^2 = 4$$

$$x = 2 \vee x = -2$$

$$x^2 + 5x + 4 = 0$$

$$(x+1)(x+4) = 0$$

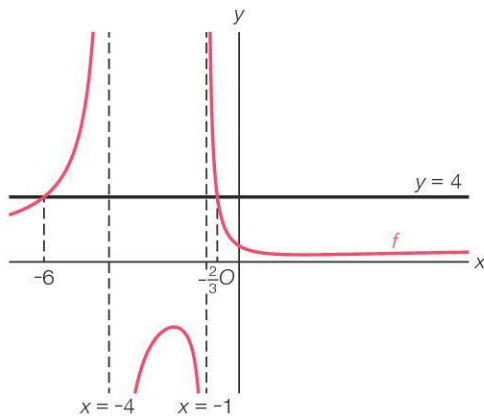
$$x = -1 \vee x = -4$$



$$\text{max. is } f(-2) = -4$$

$$\text{min. is } f(2) = \frac{4}{9}$$

b $f(x) = 4$ geeft $\frac{x^2 + 4}{x^2 + 5x + 4} = 4$
 $4x^2 + 20x + 16 = x^2 + 4$
 $3x^2 + 20x + 12 = 0$
 $D = 20^2 - 4 \cdot 3 \cdot 12 = 256$
 $x = \frac{-20 \pm 16}{6} = -\frac{2}{3} \vee x = \frac{-20 - 16}{6} = -6$



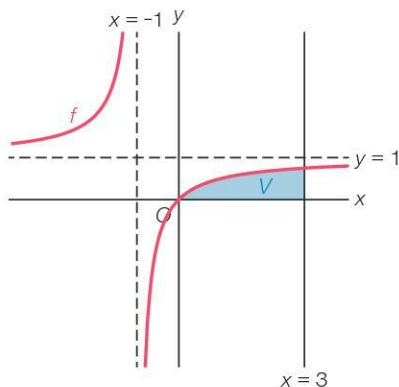
$f(x) \leq 4$ geeft $x \leq -6 \vee -4 < x < -1 \vee x \geq -\frac{2}{3}$

c $x^2 + 5x + 4 / x^2 + 4 \setminus 1$
 $\frac{x^2 + 5x + 4}{-5x} -$
 $f(x) = 1 - \frac{5x}{x^2 + 5x + 4} = 1 - \frac{5x}{(x+1)(x+4)} = 1 - \left(\frac{a}{x+1} + \frac{b}{x+4} \right) = 1 - \frac{a(x+4) + b(x+1)}{(x+1)(x+4)}$
 $= 1 - \frac{ax + 4a + bx + b}{(x+1)(x+4)} = 1 - \frac{(a+b)x + 4a + b}{(x+1)(x+4)}$

$\begin{cases} a + b = 5 \\ 4a + b = 0 \end{cases} \Rightarrow \begin{cases} -3a = 5 \\ a = -1\frac{2}{3} \end{cases} \Rightarrow \begin{cases} -1\frac{2}{3} + b = 5 \\ b = 6\frac{2}{3} \end{cases}$

$O(V) = \int_0^6 \left(1 - \frac{5x}{x^2 + 5x + 4} \right) dx = \int_0^6 \left(1 + \frac{1\frac{2}{3}}{x+1} - \frac{6\frac{2}{3}}{x+4} \right) dx = \left[x + 1\frac{2}{3} \ln|x+1| - 6\frac{2}{3} \ln|x+4| \right]_0^6$
 $= 6 + 1\frac{2}{3} \ln(7) - 6\frac{2}{3} \ln(10) - (0 + 1\frac{2}{3} \ln(1) - 6\frac{2}{3} \ln(4)) = 6 + 1\frac{2}{3} \ln(7) - 6\frac{2}{3} \ln(10) + 6\frac{2}{3} \ln(4)$
 $= 6 + 1\frac{2}{3} \ln(7) + 6\frac{2}{3} \ln(\frac{2}{5})$

66 a



$f(x) = \frac{x}{x+1} = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$

$O(V) = \int_0^3 \left(1 - \frac{1}{x+1} \right) dx = \left[x - \ln|x+1| \right]_0^3 = 3 - \ln(4) - (0 - \ln(1)) = 3 - \ln(4)$

$$\begin{aligned}
 \text{b } I(L) &= \pi \int_0^3 \left(\frac{x}{x+1} \right)^2 dx = \pi \int_0^3 \left(1 - \frac{1}{x+1} \right)^2 dx = \pi \int_0^3 \left(1 - \frac{2}{x+1} + \frac{1}{(x+1)^2} \right) dx \\
 &= \pi \int_0^3 \left(1 - \frac{2}{x+1} + (x+1)^{-2} \right) dx = \pi \left[x - 2 \ln|x+1| - (x+1)^{-1} \right]_0^3 = \pi \left[x - 2 \ln|x+1| - \frac{1}{x+1} \right]_0^3 \\
 &= \pi \left(3 - 2 \ln(4) - \frac{1}{4} - (0 - 2 \ln(1) - 1) \right) = \pi \left(3 - 2 \ln(4) - \frac{1}{4} + 1 \right) = 3\frac{3}{4}\pi - 2\pi \ln(4)
 \end{aligned}$$

67 a $f(x) = \ln(x^2 + 1)$ geeft $f'(x) = \frac{1}{x^2 + 1} \cdot 2x = \frac{2x}{x^2 + 1}$

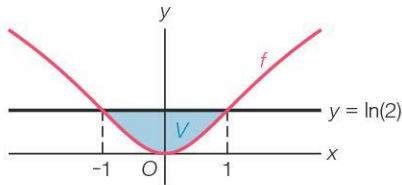
$$\begin{aligned}
 f'(x) &= \frac{3}{5} \text{ geeft } \frac{2x}{x^2 + 1} = \frac{3}{5} \\
 3x^2 + 3 &= 10x \\
 3x^2 - 10x + 3 &= 0 \\
 D &= (-10)^2 - 4 \cdot 3 \cdot 3 = 64 \\
 x &= \frac{10 \pm 8}{6} = 3 \vee x = \frac{10 - 8}{6} = \frac{1}{3}
 \end{aligned}$$

$$f(3) = \ln(10) \text{ en } f\left(\frac{1}{3}\right) = \ln\left(1\frac{1}{9}\right)$$

Dus de raakpunten zijn $(3, \ln(10))$ en $(\frac{1}{3}, \ln(1\frac{1}{9}))$.

b $f(x) = \ln(2)$ geeft $\ln(x^2 + 1) = \ln(2)$

$$\begin{aligned}
 x^2 + 1 &= 2 \\
 x^2 &= 1 \\
 x &= 1 \vee x = -1
 \end{aligned}$$



$$\begin{aligned}
 \int f(x) dx &= \int \ln(x^2 + 1) dx = x \ln(x^2 + 1) - \int x d \ln(x^2 + 1) = x \ln(x^2 + 1) - \int x \cdot \frac{1}{x^2 + 1} \cdot 2x dx \\
 &= x \ln(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} dx = x \ln(x^2 + 1) - \int \frac{2x^2 + 2 - 2}{x^2 + 1} dx \\
 &= x \ln(x^2 + 1) - \int \left(2 - \frac{2}{x^2 + 1} \right) dx = x \ln(x^2 + 1) - (2x - 2 \arctan(x)) + c \\
 &= x \ln(x^2 + 1) - 2x + 2 \arctan(x) + c \\
 O(V) &= \int_{-1}^1 (\ln(2) - f(x)) dx = [x \ln(2) - (x \ln(x^2 + 1) - 2x + 2 \arctan(x))]_{-1}^1 \\
 &= [x \ln(2) - x \ln(x^2 + 1) + 2x - 2 \arctan(x)]_{-1}^1 \\
 &= \ln(2) - \ln(2) + 2 - 2 \arctan(1) - (-\ln(2) + \ln(2) - 2 - 2 \arctan(-1)) \\
 &= 2 - 2 \cdot \frac{1}{4}\pi + 2 + 2 \cdot -\frac{1}{4}\pi = 4 - \pi
 \end{aligned}$$

K.5 Integralen bij parameterkrommen

Bladzijde 216

68 a $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) = -(F(a) - F(b)) = -[F(x)]_b^a = -\int_b^a f(x) dx$

b $\int_a^b f(x) dx + \int_b^c f(x) dx = [F(x)]_a^b + [F(x)]_b^c = F(b) - F(a) + F(c) - F(b) = F(c) - F(a)$

$$= [F(x)]_a^c = \int_a^c f(x) dx$$

- 69 a Bij A hoort $t = 0$, bij B hoort $t = 2$ en bij C hoort $t = 4$, dus

$$O(V) = \int_{x_A}^{x_B} y \, dx - \int_{x_C}^{x_B} y \, dx = \int_{t=0}^{t=2} y \, dx - \int_{t=4}^{t=2} y \, dx.$$

$$\text{Herleiden geeft } O(V) = \int_{t=0}^{t=2} y \, dx - \int_{t=4}^{t=2} y \, dx = \int_{t=0}^{t=2} y \, dx + \int_{t=2}^{t=4} y \, dx = \int_{t=0}^{t=4} y \, dx.$$

$$\begin{aligned} \text{b } O(V) &= \int_{t=0}^{t=4} \left(-\frac{1}{2}t^2 + 2\right) d\left(-\frac{1}{2}t^2 + 2t\right) = \int_0^4 \left(-\frac{1}{2}t^2 + 2\right)(-t + 2) \, dt = \int_0^4 \left(\frac{1}{2}t^3 - t^2 - 2t + 4\right) \, dt \\ &= \left[\frac{1}{8}t^4 - \frac{1}{3}t^3 - t^2 + 4t\right]_0^4 = 32 - \frac{64}{3} - 16 + 16 - 0 = 10\frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{c } O(V) &= \int_{t=4}^{t=0} x \, dy = \int_{t=4}^{t=0} \left(-\frac{1}{2}t^2 + 2t\right) d\left(-\frac{1}{2}t^2 + 2\right) = \int_{t=4}^{t=0} \left(-\frac{1}{2}t^2 + 2t\right) \cdot -t \, dt = \int_{t=4}^{t=0} \left(\frac{1}{2}t^3 - 2t^2\right) \, dt \\ &= \left[\frac{1}{8}t^4 - \frac{2}{3}t^3\right]_4^0 = \frac{1}{8} \cdot 0^4 - \frac{2}{3} \cdot 0^3 - \left(\frac{1}{8} \cdot 4^4 - \frac{2}{3} \cdot 4^3\right) = 0 - 32 + 42\frac{2}{3} = 10\frac{2}{3} \end{aligned}$$

Dus hetzelfde antwoord als bij b.

Bladzijde 217

$$70 \text{ a } O(V) = -\int_{t=2}^{t=-2} x \, dy + \int_{t=2}^{t=4} x \, dy = \int_{t=-2}^{t=2} x \, dy + \int_{t=2}^{t=4} x \, dy = \int_{t=-2}^{t=4} x \, dy$$

$$\begin{aligned} \text{b } I(L) &= \pi \int_{t=2}^{t=-2} x^2 \, dy = \pi \int_{t=2}^{t=-2} \left(\frac{1}{2}t^2 - 2\right)^2 d\left(\frac{1}{2}t^2 - 2t\right) = \pi \int_2^{-2} \left(\frac{1}{2}t^2 - 2\right)^2 (t - 2) \, dt \\ &= \pi \int_2^{-2} \left(\frac{1}{4}t^4 - 2t^2 + 4\right)(t - 2) \, dt = \pi \int_2^{-2} \left(\frac{1}{4}t^5 - \frac{1}{2}t^4 - 2t^3 + 4t^2 + 4t - 8\right) \, dt \\ &= \pi \left[\frac{1}{24}t^6 - \frac{1}{10}t^5 - \frac{1}{2}t^4 + \frac{1}{3}t^3 + 2t^2 - 8t\right]_2^{-2} \\ &= \pi \left(\frac{64}{24} + \frac{32}{10} - 8 - 10\frac{2}{3} + 8 + 16 - \left(\frac{64}{24} - \frac{32}{10} - 8 + 10\frac{2}{3} + 8 - 16\right)\right) \\ &= \pi \left(2 \cdot \frac{32}{10} - 2 \cdot 10\frac{2}{3} + 2 \cdot 16\right) = 17\frac{1}{15}\pi \end{aligned}$$

Bladzijde 218

$$71 \quad x = 0 \text{ geeft } t^2 - 2t = 0$$

$$t(t - 2) = 0$$

$$t = 0 \vee t = 2$$

$$t = 0 \text{ geeft } (0, 0)$$

$$t = 2 \text{ geeft } (0, 6)$$

$$y = 0 \text{ geeft } t^3 - t = 0$$

$$t(t^2 - 1) = 0$$

$$t = 0 \vee t^2 = 1$$

$$t = 0 \vee t = 1 \vee t = -1$$

$$t = 1 \text{ geeft } (-1, 0)$$

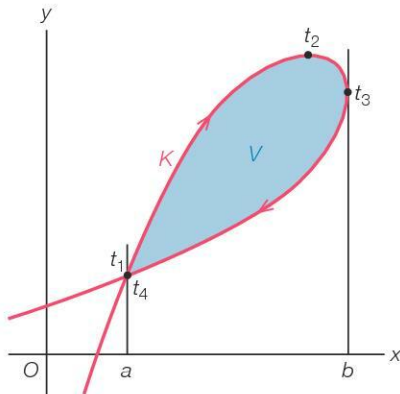
$$t = -1 \text{ geeft } (3, 0)$$

$$\begin{aligned} O(V) &= -\int_{t=1}^{t=0} y \, dx + \int_{t=1}^{t=2} y \, dx = \int_{t=0}^{t=1} y \, dx + \int_{t=1}^{t=2} y \, dx = \int_{t=0}^{t=2} y \, dx = \int_{t=0}^{t=2} (t^3 - t) d(t^2 - 2t) = \int_0^2 (t^3 - t)(2t - 2) \, dt \\ &= \int_0^2 (2t^4 - 2t^3 - 2t^2 + 2t) \, dt = \left[\frac{2}{5}t^5 - \frac{1}{2}t^4 - \frac{2}{3}t^3 + t^2\right]_0^2 = \frac{2}{5} \cdot 32 - \frac{1}{2} \cdot 16 - \frac{2}{3} \cdot 8 + 4 - 0 = 3\frac{7}{15} \end{aligned}$$

$$72 \quad O(V) = -\int_{t=3}^{t=1} y \, dx + \int_{t=0}^{t=1} y \, dx = \int_{t=1}^{t=3} y \, dx + \int_{t=0}^{t=1} y \, dx = \int_{t=0}^{t=3} y \, dx = \int_{t=0}^{t=3} \left(\frac{2}{3}t^2 - 2\frac{2}{3}t + 2\right) d(-t^2 + 2t)$$

$$\begin{aligned} &= \int_0^3 \left(\frac{2}{3}t^2 - 2\frac{2}{3}t + 2\right)(-2t + 2) \, dt = \int_0^3 \left(-\frac{4}{3}t^3 + \frac{20}{3}t^2 - \frac{28}{3}t + 4\right) \, dt = \left[-\frac{1}{3}t^4 + \frac{20}{9}t^3 - \frac{14}{3}t^2 + 4t\right]_0^3 \\ &= -27 + 60 - 42 + 12 - 0 = 3 \end{aligned}$$

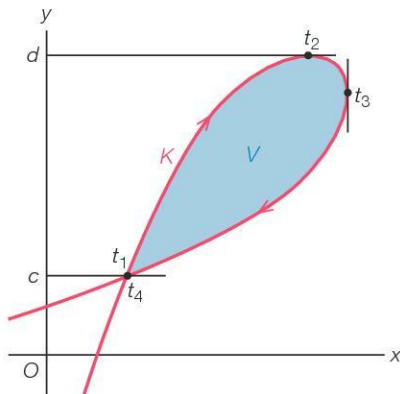
- a** Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_3$.



Noem V_1 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K en V_2 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het onderste deel van K .

$$\text{Dan geldt } O(V) = O(V_1) - O(V_2) = \int_{t=t_1}^{t=t_3} y \, dx - \int_{t=t_4}^{t=t_3} y \, dx = \int_{t=t_1}^{t=t_3} y \, dx + \int_{t=t_3}^{t=t_4} y \, dx = \int_{t=t_1}^{t=t_4} y \, dx.$$

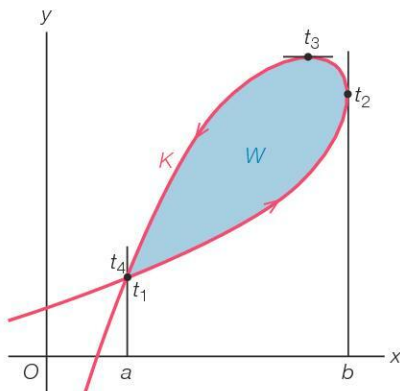
- b** Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_2$.



Noem V_3 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het rechterdeel van K en V_4 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het linkerdeel van K .

$$\text{Dan geldt } O(V) = O(V_3) - O(V_4) = \int_{t=t_4}^{t=t_2} x \, dy - \int_{t=t_1}^{t=t_2} x \, dy = \int_{t=t_4}^{t=t_2} x \, dy + \int_{t=t_2}^{t=t_1} x \, dy = \int_{t=t_4}^{t=t_1} x \, dy.$$

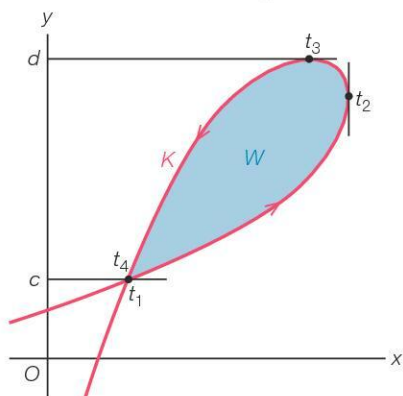
- c** Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_2$.



Noem W_1 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K en W_2 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het onderste deel van K .

$$\text{Dan geldt } O(W) = O(W_1) - O(W_2) = \int_{t=t_4}^{t=t_2} y \, dx - \int_{t=t_1}^{t=t_2} y \, dx = \int_{t=t_4}^{t=t_2} y \, dx + \int_{t=t_2}^{t=t_1} y \, dx = \int_{t=t_4}^{t=t_1} y \, dx.$$

Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_3$.



Noem W_3 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het rechterdeel van K en W_4 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het linkerdeel van K .

$$\text{Dan geldt } O(W) = O(W_3) - O(W_4) = \int_{t=t_1}^{t=t_3} x \, dy - \int_{t=t_4}^{t=t_3} x \, dy = \int_{t=t_1}^{t=t_3} x \, dy + \int_{t=t_3}^{t=t_4} x \, dy = \int_{t=t_1}^{t=t_4} x \, dy.$$

Bladzijde 220

74 a $y = 0$ geeft $4t - t^3 = 0$
 $t(4 - t^2) = 0$
 $t = 0 \vee t^2 = 4$
 $t = 0 \vee t = 2 \vee t = -2$

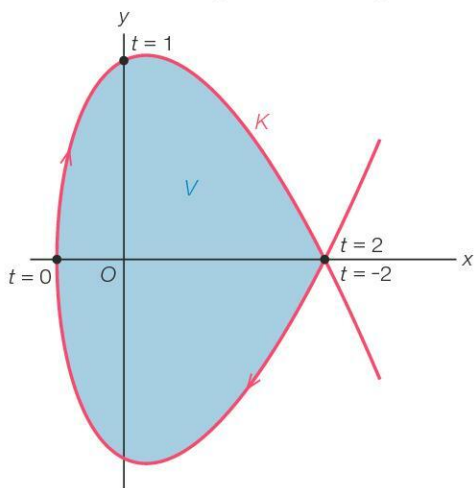
$t = -2$ geeft $(3, 0)$

$t = 0$ geeft $(-1, 0)$

$t = 2$ geeft $(3, 0)$.

$t = 1$ geeft $(0, 3)$

Dus V wordt in negatieve richting omlopen.



$$\begin{aligned} O(V) &= \int_{t=-2}^{t=2} y \, dx = \int_{t=-2}^{t=2} (4t - t^3) \, d(t^2 - 1) = \int_{t=-2}^{t=2} (4t - t^3) \cdot 2t \, dt = \int_{t=-2}^{t=2} (8t^2 - 2t^4) \, dt = \left[\frac{8}{3} t^3 - \frac{2}{5} t^5 \right]_{-2}^2 \\ &= \frac{8}{3} \cdot 2^3 - \frac{2}{5} \cdot 2^5 - \left(\frac{8}{3} \cdot (-2)^3 - \frac{2}{5} \cdot (-2)^5 \right) = \frac{8}{3} \cdot 8 - \frac{2}{5} \cdot 32 - \left(\frac{8}{3} \cdot -8 + \frac{2}{5} \cdot -32 \right) = 17 \frac{1}{15} \end{aligned}$$

b $I(L) = \pi \int_{t=0}^{t=2} y^2 \, dx = \pi \int_{t=0}^{t=2} (4t - t^3)^2 \, d(t^2 - 1) = \pi \int_{t=0}^{t=2} (16t^2 - 8t^4 + t^6) \cdot 2t \, dt$
 $= \pi \int_{t=0}^{t=2} (32t^3 - 16t^5 + 2t^7) \, dt = \pi \left[8t^4 - \frac{16}{6} t^6 + \frac{2}{8} t^8 \right]_0^2$
 $= \pi \left(8 \cdot 2^4 - \frac{16}{6} \cdot 2^6 + \frac{1}{4} \cdot 2^8 \right) - \pi(0 - 0 + 0) = 21 \frac{1}{3} \pi$

75 $x = 0$ geeft $-t^2 + 6t = 0$
 $-t(t - 6) = 0$
 $t = 0 \vee t = 6$

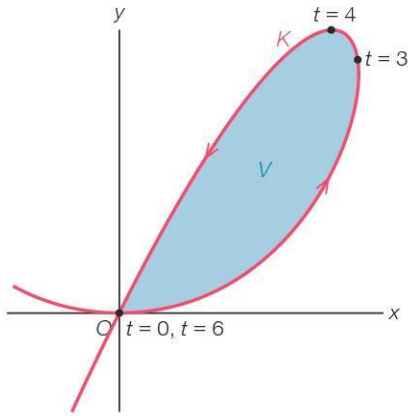
$t = 0$ geeft $(0, 0)$

$t = 6$ geeft $(0, 0)$

$t = 3$ geeft $(9, 9)$

$t = 4$ geeft $(8, 10\frac{2}{3})$

Dus V wordt in positieve richting omlopen.



$$\begin{aligned} O(V) &= \int_{t=6}^{t=0} y \, dx = \int_{t=6}^{t=0} (-\frac{1}{3}t^3 + 2t^2) d(-t^2 + 6t) = \int_{t=6}^0 (-\frac{1}{3}t^3 + 2t^2)(-2t + 6) dt = \int_6^0 (\frac{2}{3}t^4 - 6t^3 + 12t^2) dt \\ &= [\frac{2}{15}t^5 - 1\frac{1}{2}t^4 + 4t^3]_6^0 = 0 - (\frac{2}{15} \cdot 6^5 - 1\frac{1}{2} \cdot 6^4 + 4 \cdot 6^3) = 43\frac{1}{5} \end{aligned}$$

76 a $y = 0$ geeft $\sin(2t) = 0$

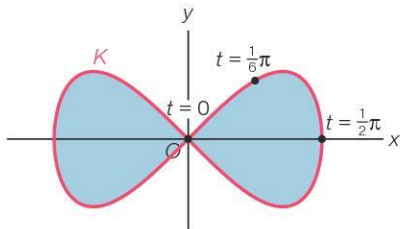
$$2t = k \cdot \pi$$

$$t = k \cdot \frac{1}{2}\pi$$

$t = 0$ geeft $(0, 0)$

$t = \frac{1}{2}\pi$ geeft $(2, 0)$

$t = \frac{1}{6}\pi$ geeft $(1, \frac{1}{2}\sqrt{3})$



Noem V het vlakdeel boven de x -as en rechts van de y -as.

$$\begin{aligned} O(V) &= \int_{t=0}^{t=\frac{1}{2}\pi} y \, dx = \int_{t=0}^{t=\frac{1}{2}\pi} \sin(2t) d(2\sin(t)) = \int_0^{\frac{1}{2}\pi} \sin(2t) \cdot 2\cos(t) dt = \int_0^{\frac{1}{2}\pi} 2\sin(t)\cos(t) \cdot 2\cos(t) dt \\ &= \int_0^{\frac{1}{2}\pi} 4\sin(t)\cos^2(t) dt = \int_{t=0}^{t=\frac{1}{2}\pi} -4\cos^2(t) d\cos(t) = \int_1^0 -4u^2 du = [-\frac{1}{3}u^3]_1^0 = 0 - (-\frac{1}{3} \cdot 1^3) = \frac{1}{3} \end{aligned}$$

Dus de totale oppervlakte van de vlakdelen is $4 \cdot \frac{1}{3} = \frac{4}{3}$.

$$\begin{aligned} \text{b } I(L) &= 2 \cdot \pi \int_{t=0}^{t=\frac{1}{2}\pi} y^2 dx = 2\pi \int_{t=0}^{t=\frac{1}{2}\pi} \sin^2(2t) d(2\sin(t)) = 2\pi \int_0^{\frac{1}{2}\pi} (2\sin(t)\cos(t))^2 \cdot 2\cos(t) dt \\ &= 2\pi \int_0^{\frac{1}{2}\pi} 4\sin^2(t)\cos^2(t) \cdot 2\cos(t) dt = 16\pi \int_{t=0}^{t=\frac{1}{2}\pi} \sin^2(t)(1 - \sin^2(t)) d\sin(t) = 16\pi \int_0^1 u^2(1 - u^2) du \\ &= 16\pi \int_0^1 (u^2 - u^4) du = 16\pi [\frac{1}{3}u^3 - \frac{1}{5}u^5]_0^1 = 16\pi(\frac{1}{3} - \frac{1}{5}) - 16\pi(0 - 0) = 2\frac{2}{15}\pi \end{aligned}$$

77 a $x = 0$ geeft $2 \sin(t) = 0$

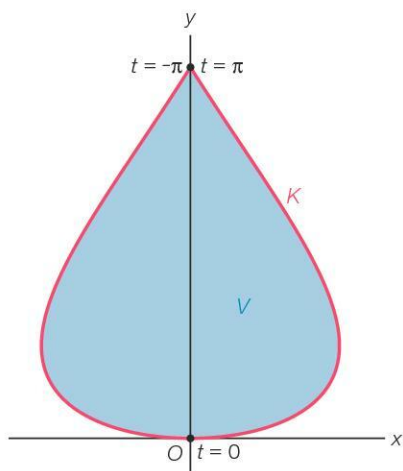
$$t = k \cdot \pi$$

$$t = -\pi \text{ geeft } (0, \frac{1}{2}\pi^2)$$

$$t = 0 \text{ geeft } (0, 0)$$

$$t = \pi \text{ geeft } (0, \frac{1}{2}\pi^2)$$

$$t = \frac{1}{2}\pi \text{ geeft } (2, \frac{1}{8}\pi^2)$$



Dus V wordt in positieve richting omlopen.

K is symmetrisch ten opzichte van de y -as, want

$$x(-t) = 2 \sin(-t) = -2 \sin(t) = -x(t) \text{ en } y(-t) = \frac{1}{2}(-t)^2 = \frac{1}{2}t^2 = y(t).$$

$$O(V) = 2 \cdot \int_{t=0}^{t=\pi} x dy = 2 \cdot \int_{t=0}^{t=\pi} 2 \sin(t) d\frac{1}{2}t^2 = 2 \cdot \int_0^{\pi} 2 \sin(t) \cdot t dt = \int_0^{\pi} 4t \sin(t) dt$$

$$\int 4t \sin(t) dt = \int 4t d(-\cos(t)) = 4t \cdot (-\cos(t)) + \int \cos(t) d(4t) = -4t \cos(t) + \int 4 \cos(t) dt \\ = -4t \cos(t) + 4 \sin(t) + c$$

$$\text{Dus } O(V) = [-4t \cos(t) + 4 \sin(t)]_0^{\pi} = -4\pi \cdot -1 + 0 - (0 + 0) = 4\pi.$$

$$\mathbf{b} \quad I(L) = \pi \int_{t=0}^{t=\pi} x^2 dy = \pi \int_{t=0}^{t=\pi} 4 \sin^2(t) d\frac{1}{2}t^2 = \pi \int_0^{\pi} 4 \sin^2(t) \cdot t dt = \pi \int_0^{\pi} 4t \sin^2(t) dt$$

$$\int 4t \sin^2(t) dt = \int 4t \left(\frac{1}{2} - \frac{1}{2} \cos(2t)\right) dt = \int (2t - 2t \cos(2t)) dt = \int 2t dt - \int 2t \cos(2t) dt =$$

$$t^2 - \int 2t d\frac{1}{2} \sin(2t) = t^2 - 2t \cdot \frac{1}{2} \sin(2t) + \int \frac{1}{2} \sin(2t) d(2t) = t^2 - t \sin(2t) + \int \sin(2t) dt =$$

$$t^2 - t \sin(2t) - \frac{1}{2} \cos(2t) + c$$

$$\text{Dus } I(L) = \pi \int_0^{\pi} 4t \sin^2(t) dt = \pi \left[t^2 - t \sin(2t) - \frac{1}{2} \cos(2t) \right]_0^{\pi} = \pi \left(\pi^2 - 0 - \frac{1}{2} - (0 - 0 - \frac{1}{2}) \right) = \pi^3.$$

Diagnostische toets

Bladzijde 224

$$\mathbf{1 a} \quad F(x) = \int 3x \sqrt{x^2 + 2} dx = \int \frac{1}{2} (x^2 + 2)^{\frac{1}{2}} d(x^2 + 2) = \int \frac{1}{2} u^{\frac{1}{2}} du = \frac{2}{3} \cdot \frac{2}{3} u^{\frac{3}{2}} + c = (x^2 + 2) \sqrt{x^2 + 2} + c$$

$$\mathbf{b} \quad F(x) = \int 3x^2 \cos(x^3 + 2) dx = \int \cos(x^3 + 2) d(x^3 + 2) = \int \cos(u) du = \sin(u) + c = \sin(x^3 + 2) + c$$

$$\mathbf{c} \quad F(x) = \int (4x + 6) \ln(x^2 + 3x) dx = \int \ln(x^2 + 3x) \cdot 2 d(x^2 + 3x) = \int 2 \ln(u) du = 2(u \ln(u) - u) + c \\ = 2u \ln(u) - 2u + c = 2(x^2 + 3x) \ln(x^2 + 3x) - 2(x^2 + 3x) + c$$

2 a $\int_1^e \frac{\sqrt{\ln(x)}}{x} dx = \int_{x=1}^{x=e} \sqrt{\ln(x)} d \ln(x) = \int_0^1 \sqrt{u} du = \int_0^1 u^{\frac{1}{2}} du = \left[\frac{2}{3} u^{\frac{3}{2}} \right]_0^1 = \frac{2}{3} \cdot 1 - \frac{2}{3} \cdot 0 = \frac{2}{3}$

b $\int_0^{\frac{1}{2}\pi} \cos(x) \sin^3(x) dx = \int_{x=0}^{x=\frac{1}{2}\pi} \sin^3(x) d \sin(x) = \int_0^1 u^3 du = \left[\frac{1}{4} u^4 \right]_0^1 = \frac{1}{4} \cdot 1 - \frac{1}{4} \cdot 0 = \frac{1}{4}$

c $\int_1^3 \frac{6x}{x^2+1} dx = \int_{x=1}^{x=3} \frac{3}{x^2+1} d(x^2+1) = \int_2^{10} \frac{3}{u} du = \left[3 \ln|u| \right]_2^{10} = 3 \ln(10) - 3 \ln(2) = 3 \ln(5)$

3 $f(x) = 0$ geeft $-2x \sin(x^2) = 0$
 $x = 0 \vee \sin(x^2) = 0$
 $x = 0 \vee x^2 = k \cdot \pi$
 $x = 0 \vee x = \sqrt{k \cdot \pi} \vee x = -\sqrt{k \cdot \pi}$
 x in $[0, \sqrt{2\pi}]$ geeft $x = 0 \vee x = \sqrt{\pi} \vee x = \sqrt{2\pi}$
 $\int -2x \sin(x^2) dx = \int -\sin(x^2) \cdot 2x dx = \int -\sin(x^2) d(x^2) = \int -\sin(u) du = \cos(u) + c = \cos(x^2) + c$
 $O(V) = \int_0^{\sqrt{\pi}} 2x \sin(x^2) dx = [-\cos(x^2)]_0^{\sqrt{\pi}} = -\cos(\pi) + \cos(0) = 1 + 1 = 2$
 $O(W) = \int_{\sqrt{\pi}}^{\sqrt{2\pi}} -2x \sin(x^2) dx = [\cos(x^2)]_{\sqrt{\pi}}^{\sqrt{2\pi}} = \cos(2\pi) - \cos(\pi) = 1 + 1 = 2$

Dus de oppervlakten van de twee vlakdelen zijn gelijk.

4 a $F(x) = \int 4x \sin(2x) dx = \int -2x \cdot -2 \sin(2x) dx = \int -2x d \cos(2x) = -2x \cos(2x) + \int 2 \cos(2x) dx$
 $= -2x \cos(2x) + \sin(2x) + c$

b $G(x) = \int x^2 \ln(x) dx = \int \ln(x) d \frac{1}{3} x^3 = \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^3 d \ln(x) = \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^3 \cdot \frac{1}{x} dx$
 $= \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^2 dx = \frac{1}{3} x^3 \ln(x) - \frac{1}{9} x^3 + c$

5 a $f(x) = (2x - x^2) e^x$ geeft $f'(x) = (2 - 2x) \cdot e^x + (2x - x^2) \cdot e^x = (2 - x^2) e^x$
 $f'(x) = 0$ geeft $(2 - x^2) e^x = 0$
 $2 - x^2 = 0$
 $x^2 = 2$
 $x = \sqrt{2} \vee x = -\sqrt{2}$

min. is $f(-\sqrt{2}) = (-2\sqrt{2} - 2) e^{-\sqrt{2}}$

max. is $f(\sqrt{2}) = (2\sqrt{2} - 2) e^{\sqrt{2}}$

b $f(x) = 0$ geeft $2x - x^2 = 0$
 $x(2 - x) = 0$
 $x = 0 \vee x = 2$
 $\int (2x - x^2) e^x dx = \int (2x - x^2) d e^x = (2x - x^2) e^x - \int e^x d(2x - x^2)$
 $= (2x - x^2) e^x - \int (2 - 2x) e^x dx$
 $= (2x - x^2) e^x - \int (2 - 2x) d e^x$
 $= (2x - x^2) e^x - (2 - 2x) e^x + \int e^x d(2 - 2x)$
 $= (2x - x^2) e^x - (2 - 2x) e^x - \int 2 e^x dx$
 $= (2x - x^2) e^x - (2 - 2x) e^x - 2 e^x = (-x^2 + 4x - 4) e^x$
 $O(V) = \int_0^2 f(x) dx = \int_0^2 (2x - x^2) e^x dx = [(-x^2 + 4x - 4) e^x]_0^2 = 0 + 4 = 4$

6 a $F(x) = \int 4x^2 \sin(2x) dx = \int 4x^2 d(-\frac{1}{2} \cos(2x)) = -2x^2 \cos(2x) - \int -\frac{1}{2} \cos(2x) d4x^2$
 $= -2x^2 \cos(2x) + \int 4x \cos(2x) dx = -2x^2 \cos(2x) + \int 4x d(\frac{1}{2} \sin(2x))$
 $= -2x^2 \cos(2x) + 2x \sin(2x) - \int \frac{1}{2} \sin(2x) d4x$
 $= -2x^2 \cos(2x) + 2x \sin(2x) - \int 2 \sin(2x) dx$
 $= -2x^2 \cos(2x) + 2x \sin(2x) + \cos(2x) + c$

b $\int e^x \sin(2x) dx = \int \sin(2x) d e^x = e^x \sin(2x) - \int e^x d \sin(2x) = e^x \sin(2x) - \int 2 \cos(2x) e^x dx$
 $= e^x \sin(2x) - \int 2 \cos(2x) d e^x = e^x \sin(2x) - 2 e^x \cos(2x) + \int 2 e^x d \cos(2x)$
 $= e^x \sin(2x) - 2 e^x \cos(2x) - \int 4 e^x \sin(2x) dx$

Dus $\int e^x \sin(2x) dx = e^x \sin(2x) - 2 e^x \cos(2x) - 4 \int e^x \sin(2x) dx$

$5 \int e^x \sin(2x) dx = e^x \sin(2x) - 2 e^x \cos(2x)$

$\int e^x \sin(2x) dx = \frac{1}{5} e^x \sin(2x) - \frac{2}{5} e^x \cos(2x)$

Dus de primitieven zijn $G(x) = \frac{1}{5} e^x (\sin(2x) - 2 \cos(2x)) + c$.

7 a $\int_0^{2\sqrt{3}} \frac{4}{x^2+4} dx = \int_0^{2\sqrt{3}} \frac{1}{\frac{1}{4}x^2+1} dx = \int_{x=0}^{x=2\sqrt{3}} 2 \cdot \frac{1}{(\frac{1}{2}x)^2+1} d\frac{1}{2}x = [2 \arctan(\frac{1}{2}x)]_0^{2\sqrt{3}}$
 $= 2 \arctan(\sqrt{3}) - 2 \arctan(0) = 2 \cdot \frac{1}{3} \pi - 0 = \frac{2}{3} \pi$

b $\int_1^2 \frac{1}{x^2-2x+2} dx = \int_1^2 \frac{1}{(x-1)^2-1+2} dx = \int_1^2 \frac{1}{(x-1)^2+1} dx = [\arctan(x-1)]_1^2 = \arctan(1) - \arctan(0) = \frac{1}{4} \pi$

c $\int_{-1}^1 \frac{3x^2}{x^6+1} dx = \int_{x=-1}^{x=1} \frac{1}{(x^3)^2+1} dx^3 = [\arctan(x^3)]_{-1}^1 = \arctan(1) - \arctan(-1) = \frac{1}{4} \pi - (-\frac{1}{4} \pi) = \frac{1}{2} \pi$

d $\int_0^1 \frac{\arctan^2(x)}{x^2+1} dx = \int_0^1 \arctan^2(x) \cdot \frac{1}{x^2+1} dx = \int_{x=0}^{x=1} \arctan^2(x) d \arctan(x) = \int_0^{\frac{1}{4}\pi} u^2 du = [\frac{1}{3} u^3]_0^{\frac{1}{4}\pi} = \frac{1}{3} \cdot (\frac{1}{4}\pi)^3 - 0 = \frac{1}{192} \pi^3$

Bladzijde 225

8 a $f(x) = \frac{1}{2}$ geeft $\frac{1}{x^2-4x+5} = \frac{1}{2}$
 $x^2-4x+5 = 2$
 $x^2-4x+3 = 0$
 $(x-1)(x-3) = 0$
 $x = 1 \vee x = 3$

$\int_1^3 f(x) dx = \int_1^3 \frac{1}{x^2-4x+5} dx = \int_1^3 \frac{1}{(x-2)^2-4+5} dx = \int_1^3 \frac{1}{(x-2)^2+1} dx = [\arctan(x-2)]_1^3$

$= \arctan(1) - \arctan(-1) = \frac{1}{4} \pi + \frac{1}{4} \pi = \frac{1}{2} \pi$

$O(V) = \int_1^3 f(x) dx - 2 \cdot \frac{1}{2} = \frac{1}{2} \pi - 1$

b $\int_2^p f(x) dx = [\arctan(x-2)]_2^p = \arctan(p-2) - \arctan(0) = \arctan(p-2)$

$O(W) = \frac{1}{3} \pi$ geeft $\arctan(p-2) = \frac{1}{3} \pi$

$p-2 = \sqrt{3}$
 $p = 2 + \sqrt{3}$

9 a $F(x) = \int \frac{1}{\sqrt{1-4x^2}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{1-(2x)^2}} d2x = \frac{1}{2} \arcsin(2x) + c$

b $G(x) = \int \frac{5x^4}{\sqrt{1-x^{10}}} dx = \int \frac{1}{\sqrt{1-(x^5)^2}} d(x^5) = \arcsin(x^5) + c$

10 a $\int_{-1}^1 \frac{3x+4}{x+4} dx = \int_{-1}^1 \frac{3(x+4)-8}{x+4} dx = \int_{-1}^1 \left(3 - \frac{8}{x+4}\right) dx = [3x - 8 \ln|x+4|]_{-1}^1$
 $= 3 - 8 \ln(5) - (-3 - 8 \ln(3)) = 6 - 8 \ln(5) + 8 \ln(3) = 6 + 8 \ln\left(\frac{3}{5}\right)$

b $x+1/x^3 \quad -x+1/x^2-x$

$$\frac{x^3+x^2}{-x^2-x} = x+1$$

$$\frac{-x^2-x}{1} = -x-1$$

$$\int_1^3 \frac{x^3-x+1}{x+1} dx = \int_1^3 \left(x^2 - x + \frac{1}{x+1}\right) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + \ln|x+1|\right]_1^3$$

$$= 9 - 4\frac{1}{2} + \ln(4) - \left(\frac{1}{3} - \frac{1}{2} + \ln(2)\right) = 4\frac{2}{3} + \ln(4) - \ln(2) = 4\frac{2}{3} + \ln(2)$$

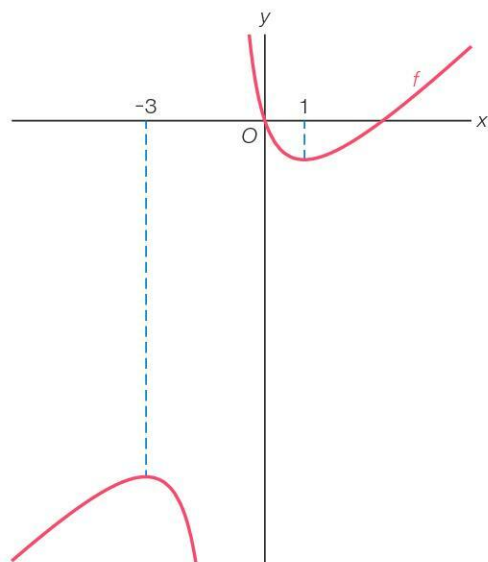
11 a $f(x) = \frac{x^2-3x}{x+1}$ geeft

$$f'(x) = \frac{(x+1) \cdot (2x-3) - (x^2-3x) \cdot 1}{(x+1)^2} = \frac{2x^2-3x+2x-3-x^2+3x}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2}$$

$$f'(x) = 0 \text{ geeft } x^2+2x-3=0$$

$$(x-1)(x+3)=0$$

$$x=1 \vee x=-3$$



max. is $f(-3) = -9$

min. is $f(1) = -1$

b $f(x) = 0$ geeft $x^2-3x=0$
 $x(x-3)=0$
 $x=0 \vee x=3$

$$x+1/x^2-3x/x-4$$

$$\frac{x^2+x}{-4x} = -\frac{x+1}{4}$$

$$O(V) = -\int_0^3 f(x) dx = -\int_0^3 \left(x-4 + \frac{4}{x+1}\right) dx = -\left[\frac{1}{2}x^2 - 4x + 4 \ln|x+1|\right]_0^3$$

$$= -(4\frac{1}{2} - 12 + 4 \ln(4)) + (0 - 0 + 4 \ln(1)) = 7\frac{1}{2} - 4 \ln(4)$$

12 a $f(x) = \frac{2x+3}{x^2-6x+10} = \frac{2x-6+9}{x^2-6x+10} = \frac{2x-6}{x^2-6x+10} + \frac{9}{x^2-6x+10}$

$$\int \frac{2x-6}{x^2-6x+10} dx = \int \frac{1}{x^2-6x+10} \cdot (2x-6) dx = \int \frac{1}{x^2-6x+10} d(x^2-6x+10) = \ln|x^2-6x+10| + c$$

$$\int \frac{9}{x^2-6x+10} dx = \int 9 \cdot \frac{1}{(x-3)^2-9+10} dx = \int 9 \cdot \frac{1}{(x-3)^2+1} dx = 9 \arctan(x-3) + c$$

Dus $F(x) = \ln|x^2-6x+10| + 9 \arctan(x-3) + c$

b $x^2 - 4x + 4 / x^2 + 4x \quad \setminus 1$

$$\frac{x^2-4x+4}{8x-4}$$

$$G(x) = \int \frac{x^2+4x}{x^2-4x+4} dx = \int \left(1 + \frac{8x-4}{x^2-4x+4} \right) dx = \int \left(1 + \frac{8(x-2)+12}{(x-2)^2} \right) dx$$

$$= \int \left(1 + \frac{8}{x-2} + \frac{12}{(x-2)^2} \right) dx = \int \left(1 + \frac{8}{x-2} + 12(x-2)^{-2} \right) dx$$

$$= x + 8 \ln|x-2| - 12(x-2)^{-1} + c = x + 8 \ln|x-2| - \frac{12}{x-2} + c$$

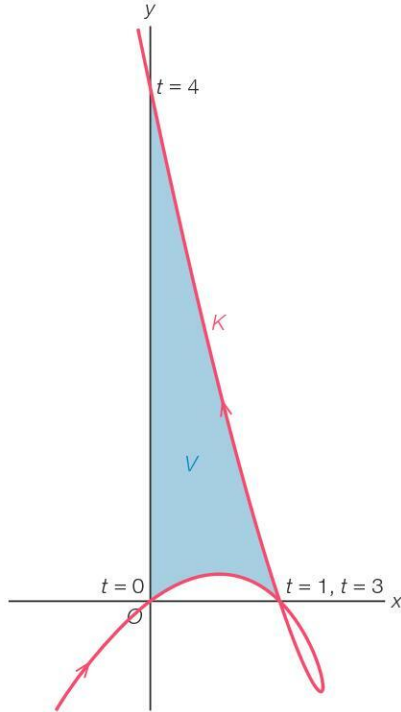
c $h(x) = \frac{6}{x^2-1} = \frac{6}{(x-1)(x+1)} = \frac{a}{x-1} + \frac{b}{x+1} = \frac{a(x+1)+b(x-1)}{(x-1)(x+1)}$

$$= \frac{ax+a+bx-b}{(x-1)(x+1)} = \frac{(a+b)x+a-b}{(x-1)(x+1)}$$

$$\begin{cases} a+b=0 \\ a-b=6 \\ 2a=6 \\ a=3 \\ a+b=0 \end{cases} \quad b=-3$$

Dus $h(x) = \frac{3}{x-1} - \frac{3}{x+1}$ en dit geeft $H(x) = 3 \ln|x-1| - 3 \ln|x+1| + c$.

13 a $x(t) = 0$ geeft $4t - t^2 = 0$
 $t(4 - t) = 0$
 $t = 0 \vee t = 4$
 $y(t) = 0$ geeft $t^3 - 4t^2 + 3t = 0$
 $t(t^2 - 4t + 3) = 0$
 $t(t - 1)(t - 3) = 0$
 $t = 0 \vee t = 1 \vee t = 3$
 $t = 0$ geeft $(0, 0)$



$$O(V) = \int_{t=4}^{t=3} y \, dx - \int_{t=0}^{t=1} y \, dx$$

$$\int y \, dx = \int (t^3 - 4t^2 + 3t) d(4t - t^2) = \int (t^3 - 4t^2 + 3t)(4 - 2t) \, dt = \int (-2t^4 + 12t^3 - 22t^2 + 12t) \, dt$$

$$= -\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 + c$$

$$O(V) = \left[-\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 \right]_4^3 - \left[-\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 \right]_0^1 = 1\frac{4}{5} - 14\frac{14}{15} - (1\frac{4}{15} - 0) = 15\frac{7}{15}$$

b $O(W) = \int_{t=1}^{t=3} y \, dx = \left[-\frac{2}{5}t^5 + 3t^4 - \frac{22}{3}t^3 + 6t^2 \right]_1^3 = 1\frac{4}{5} - 1\frac{4}{15} = \frac{8}{15}$

Gemengde opgaven

9 Exponentiële en logaritmische functies

Bladzijde 226

- 1 a $x + 1 > 0$ geeft $x > -1$, dus $D_f = \langle -1, \rightarrow \rangle$.

$$f(x) = g(x) \text{ geeft } 2 - {}^3\log(x+1) = {}^{\frac{1}{3}}\log(5-x)$$

$${}^{\frac{1}{3}}\log\left(\frac{1}{9}\right) + {}^{\frac{1}{3}}\log(x+1) = {}^{\frac{1}{3}}\log(5-x)$$

$${}^{\frac{1}{3}}\log\left(\frac{1}{9}(x+1)\right) = {}^{\frac{1}{3}}\log(5-x)$$

$$\frac{1}{9}(x+1) = 5-x$$

$$x+1 = 45-9x$$

$$10x = 44$$

$$x = 4\frac{2}{5}$$

$$f(x) \geq g(x) \text{ geeft } -1 < x \leq 4\frac{2}{5}$$

- b $f(x) = 3$ geeft $2 - {}^3\log(x+1) = 3$

$$-{}^3\log(x+1) = 1$$

$${}^3\log(x+1) = -1$$

$$x+1 = 3^{-1}$$

$$x+1 = \frac{1}{3}$$

$$x = -\frac{2}{3}$$

$$AB = 4\frac{26}{27} - \frac{2}{3} = 4\frac{26}{27} + \frac{18}{27} = 5\frac{17}{27}$$

$$f(2) = 2 - {}^3\log(3) = 2 - 1 = 1 \text{ en } g(2) = {}^{\frac{1}{3}}\log(3) = {}^{\frac{1}{3}}\log\left(\left(\frac{1}{3}\right)^{-1}\right) = -1$$

$$CD = 1 - (-1) = 2$$

Dus AB is niet meer dan drie keer zo lang als CD .

- c $f(x) = 2 - {}^3\log(x+1)$ geeft $f'(x) = -\frac{1}{(x+1)\ln(3)}$

$$rc_{\text{raaklijn}} = -1, \text{ dus } f'(x) = -1$$

$$-\frac{1}{(x+1)\ln(3)} = -1$$

$$\frac{1}{(x+1)\ln(3)} = 1$$

$$x+1 = \frac{1}{\ln(3)}$$

$$x = \frac{1}{\ln(3)} - 1$$

$$x = \frac{1 - \ln(3)}{\ln(3)}$$

$$\text{Dus } x_P = \frac{1 - \ln(3)}{\ln(3)}.$$

- 2 a $4^{\log(x)} = 2^{3+\log(x)}$

$$(2^2)^{\log(x)} = 2^{3+\log(x)}$$

$$2^{2\log(x)} = 2^{3+\log(x)}$$

$$2\log(x) = 3 + \log(x)$$

$$\log(x) = 3$$

$$x = 1000$$

- b ${}^8\log(2x+1) = {}^4\log(25)$

$$\frac{{}^2\log(2x+1)}{{}^2\log(8)} = \frac{{}^2\log(25)}{{}^2\log(4)}$$

$$\frac{{}^2\log(2x+1)}{3} = \frac{{}^2\log(25)}{2}$$

$${}^2\log(2x+1) = 1\frac{1}{2} \cdot {}^2\log(25)$$

$${}^2\log(2x+1) = {}^2\log(25^{1\frac{1}{2}})$$

$$2x+1 = 125$$

$$2x = 124$$

$$x = 62$$

$$\begin{array}{ll} \text{c} & {}^5\log^2(x+2) = 6 \cdot {}^5\log(x+2) + 7 \\ & \text{Stel } {}^5\log(x+2) = u. \\ & u^2 = 6u + 7 \\ & u^2 - 6u - 7 = 0 \\ & (u+1)(u-7) = 0 \\ & u = -1 \vee u = 7 \\ & {}^5\log(x+2) = -1 \vee {}^5\log(x+2) = 7 \\ & x+2 = 5^{-1} \vee x+2 = 5^7 \\ & x = -2 + \frac{1}{5} \vee x = 5^7 - 2 \\ & x = -1\frac{4}{5} \vee x = 78123 \\ \text{d} & 2 \cdot {}^3\log(2x-3) + \frac{1}{3}\log(2x+1) = 2 \\ & {}^3\log((2x-3)^2) - {}^3\log(2x+1) = {}^3\log(9) \\ & {}^3\log((2x-3)^2) = {}^3\log(9) + {}^3\log(2x+1) \\ & {}^3\log((2x-3)^2) = {}^3\log(9(2x+1)) \\ & (2x-3)^2 = 9(2x+1) \\ & 4x^2 - 12x + 9 = 18x + 9 \\ & 4x^2 - 30x = 0 \\ & 4x(x - 7\frac{1}{2}) = 0 \\ & x = 0 \vee x = 7\frac{1}{2} \\ & \text{vold. niet} \end{array}$$

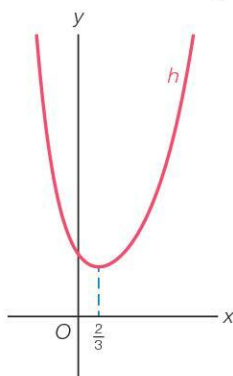
$$\begin{array}{ll} \text{3 a} & \ln^2(x) + 1 = 2\frac{1}{2}\ln(x) \\ & \text{Stel } \ln(x) = u. \\ & u^2 + 1 = 2\frac{1}{2}u \\ & u^2 - 2\frac{1}{2}u + 1 = 0 \\ & (u - \frac{1}{2})(u - 2) = 0 \\ & u = \frac{1}{2} \vee u = 2 \\ & \ln(x) = \frac{1}{2} \vee \ln(x) = 2 \\ & x = \sqrt{e} \vee x = e^2 \\ \text{b} & \frac{e^x}{e^x - 2} = 2 \\ & e^x = 2(e^x - 2) \\ & e^x = 2e^x - 4 \\ & -e^x = -4 \\ & e^x = 4 \\ & x = \ln(4) \\ \text{c} & \ln(3x+2) = \frac{1}{2} \\ & 3x+2 = e^{\frac{1}{2}} \\ & 3x+2 = \sqrt{e} \\ & 3x = -2 + \sqrt{e} \\ & x = -\frac{2}{3} + \frac{1}{3}\sqrt{e} \\ \text{d} & \ln(4x) - \ln(x+4) = 1 \\ & \ln\left(\frac{4x}{x+4}\right) = 1 \\ & \frac{4x}{x+4} = e \\ & 4x = e(x+4) \\ & 4x = ex + 4e \\ & 4x - ex = 4e \\ & (4-e)x = 4e \\ & x = \frac{4e}{4-e} \\ \text{e} & \ln^2(x-2) = 4 \\ & \ln(x-2) = 2 \vee \ln(x-2) = -2 \\ & x-2 = e^2 \vee x-2 = e^{-2} \\ & x = 2 + e^2 \vee x = 2 + \frac{1}{e^2} \\ \text{f} & 3e^{2x} + 2 = 5e^x \\ & 3(e^x)^2 + 2 = 5e^x \\ & \text{Stel } e^x = u. \\ & 3u^2 + 2 = 5u \\ & 3u^2 - 5u + 2 = 0 \\ & D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1 \\ & u = \frac{5 \pm 1}{6} = 1 \vee u = \frac{5-1}{6} = \frac{2}{3} \\ & e^x = 1 \vee e^x = \frac{2}{3} \\ & x = 0 \vee x = \ln(\frac{2}{3}) \end{array}$$

$$\begin{array}{ll} \text{4 a} & g_{\text{jaar}} = 1,096 \\ & 1,096^T = 2 \\ & T = {}^{1,096}\log(2) = 7,561... \\ & \text{De verdubbelingstijd is 7 jaar en } 0,561... \cdot 12 \approx 7 \text{ maanden.} \\ \text{b} & g_{\text{dag}} = 0,83 \\ & 0,83^T = \frac{1}{2} \\ & T = {}^{0,83}\log(\frac{1}{2}) = 3,720... \\ & \text{De halveringstijd is } 3,720... \cdot 24 \approx 89 \text{ uren.} \\ \text{c} & g_{\text{maand}} = 2 \\ & g_{\text{dag}} = 2^{\frac{1}{30}} = 1,023... \\ & \text{De toename per dag is 2,3\%.} \\ \text{d} & g_{8,3 \text{ dagen}} = \frac{1}{2} \\ & g_{\text{dag}} = (\frac{1}{2})^{\frac{1}{8,3}} = 0,919... \\ & 0,919...^t = 0,01 \\ & t = {}^{0,919...}\log(0,01) = 55,1... \\ & \text{Dus na 55 dagen is er nog 1\% over.} \end{array}$$

5 a $f(x) = g(x)$ geeft $3 \cdot 2^x = 6 \cdot (\frac{1}{4})^x$
 $3 \cdot 2^x = 6 \cdot (2^{-2})^x$
 $3 \cdot 2^x = 6 \cdot (2^x)^{-2}$
 Stel $2^x = u$.
 $3u = 6u^{-2}$
 $3u^3 = 6$
 $u^3 = 2$
 $u = 2^{\frac{1}{3}}$
 $2^x = 2^{\frac{1}{3}}$
 $x = \frac{1}{3}$

$f(x) \leq g(x)$ geeft $x \leq \frac{1}{3}$

b $h(x) = f(x) + g(x) = 3 \cdot 2^x + 6 \cdot (\frac{1}{4})^x$ geeft $h'(x) = 3 \cdot 2^x \cdot \ln(2) + 6 \cdot (\frac{1}{4})^x \cdot \ln(\frac{1}{4})$
 $h'(x) = 0$ geeft $3 \cdot 2^x \cdot \ln(2) + 6 \cdot (\frac{1}{4})^x \cdot \ln(\frac{1}{4}) = 0$
 $3 \cdot 2^x \cdot \ln(2) + 6 \cdot (2^{-2})^x \cdot \ln(2^{-2}) = 0$
 $3 \cdot 2^x \cdot \ln(2) + 6 \cdot 2^{-2x} \cdot -2 \ln(2) = 0$
 $3 \cdot 2^x \cdot \ln(2) - 12 \cdot 2^{-2x} \cdot \ln(2) = 0$
 $3 \cdot 2^x \cdot \ln(2) = 12 \cdot 2^{-2x} \cdot \ln(2)$
 $2^x = 4 \cdot 2^{-2x}$
 $2^x = 2^2 \cdot 2^{-2x}$
 $2^x = 2^{2-2x}$
 $x = 2 - 2x$
 $3x = 2$
 $x = \frac{2}{3}$



Dus $h(x) = f(x) + g(x)$ heeft een minimum voor $x = \frac{2}{3}$.

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6 a $f(x) = x^2 e^{x-1}$ geeft $f'(x) = 2x \cdot e^{x-1} + x^2 \cdot e^{x-1} = (x^2 + 2x) e^{x-1}$

b $g(x) = \ln^2(x) + \ln(x^2) = (\ln(x))^2 + \ln(x^2)$ geeft
 $g'(x) = 2 \ln(x) \cdot \frac{1}{x} + \frac{1}{x^2} \cdot 2x = \frac{2 \ln(x)}{x} + \frac{2}{x} = \frac{2 \ln(x) + 2}{x}$

c $h(x) = {}^2\log(x^3 - x^2)$ geeft $h'(x) = \frac{1}{(x^3 - x^2) \ln(2)} \cdot (3x^2 - 2x) = \frac{3x^2 - 2x}{(x^3 - x^2) \ln(2)}$

d $j(x) = \ln(\ln(2x))$ geeft $j'(x) = \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot 2 = \frac{1}{x \ln(2x)}$

7 a $y = e^{3x-1} \xrightarrow{\text{translatie } (2, 0)} y = e^{3(x-2)-1}$

Er geldt $e^{3(x-2)-1} = e^{3x-6-1} = e^{3x-1-6} = e^{3x-1} \cdot e^{-6} = \frac{1}{e^6} \cdot e^{3x-1}$.

Dus de vermenigvuldiging ten opzichte van de x -as met $\frac{1}{e^6}$ levert dezelfde beeldfiguur op.

b $y = \ln(x^2) \xrightarrow{\text{verm. } y\text{-as, } 3} y = \ln((\frac{1}{3}x)^2) = \ln(\frac{1}{9}x^2)$

Er geldt $\ln(\frac{1}{9}x^2) = \ln(\frac{1}{9}) + \ln(x^2) = \ln(3^{-2}) + \ln(x^2) = -2 \ln(3) + \ln(x^2)$.

Dus de translatie $(0, -2 \ln(3))$ levert dezelfde beeldfiguur op.

8 a $f(x) = (x-3)^2 e^x = (x^2 - 6x + 9)e^x$ geeft $f'(x) = (2x-6) \cdot e^x + (x^2 - 6x + 9) \cdot e^x = (x^2 - 4x + 3)e^x$
 $f'(x) = 0$ geeft $(x^2 - 4x + 3)e^x = 0$
 $x^2 - 4x + 3 = 3$
 $(x-1)(x-3) = 0$
 $x = 1 \vee x = 3$

$f(1) = (1-3)^2 e^1 = 4e$ en $f(3) = (3-3)^2 e^3 = 0$

De toppen zijn $(1, 4e)$ en $(3, 0)$.

b $f'(x) = (x^2 - 4x + 3)e^x$ geeft $f''(x) = (2x-4) \cdot e^x + (x^2 - 4x + 3) \cdot e^x = (x^2 - 2x - 1)e^x$
 $f''(x) = 0$ geeft $(x^2 - 2x - 1)e^x = 0$
 $x^2 - 2x - 1 = 0$
 $D = (-2)^2 - 4 \cdot 1 \cdot -1 = 8$, dus $\sqrt{D} = \sqrt{8} = 2\sqrt{2}$
 $x = \frac{2 + 2\sqrt{2}}{2} \vee x = \frac{2 - 2\sqrt{2}}{2}$
 $x = 1 + \sqrt{2} \vee x = 1 - \sqrt{2}$

De x -coördinaten van de buigpunten zijn $x = 1 + \sqrt{2}$ en $x = 1 - \sqrt{2}$.

c Stel $x_A = a$, dan is $x_B = a + 3$.

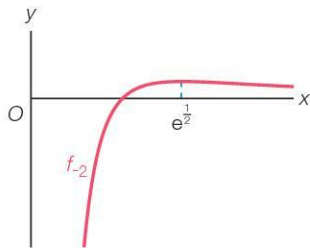
$p = f(x_A) = f(x_B)$ geeft $(a-3)^2 e^a = a^2 e^{a+3}$

Voer in $y_1 = (x-3)^2 e^x$ en $y_2 = x^2 e^{x+3}$.

De optie snijpunt geeft $x = -0,861...$ en $y = 6,299...$ en $x = 0,547...$ en $y = 10,398...$

Dus $p \approx 6,30$.

9 a $f_{-2}(x) = x^{-2} \cdot \ln(x)$ geeft $f_{-2}'(x) = -2x^{-3} \cdot \ln(x) + x^{-2} \cdot \frac{1}{x} = \frac{-2 \ln(x)}{x^3} + \frac{1}{x^2} \cdot \frac{1}{x} = \frac{-2 \ln(x)}{x^3} + \frac{1}{x^3} = \frac{-2 \ln(x) + 1}{x^3}$
 $f_{-2}'(x) = 0$ geeft $\frac{-2 \ln(x) + 1}{x^3} = 0$
 $-2 \ln(x) + 1 = 0$
 $-2 \ln(x) = -1$
 $\ln(x) = \frac{1}{2}$
 $x = e^{\frac{1}{2}}$



max. is $f_{-2}(e^{\frac{1}{2}}) = (e^{\frac{1}{2}})^{-2} \cdot \frac{1}{2} = e^{-1} \cdot \frac{1}{2} = \frac{1}{2e}$

b $f_3(x) = x^3 \cdot \ln(x)$ geeft $f_3'(x) = 3x^2 \cdot \ln(x) + x^3 \cdot \frac{1}{x} = 3x^2 \ln(x) + x^2$

$f_3'(x) = 3x^2 \ln(x) + x^2$ geeft $f_3''(x) = 6x \cdot \ln(x) + 3x^2 \cdot \frac{1}{x} + 2x = 6x \ln(x) + 3x + 2x = 6x \ln(x) + 5x$

$f_3''(x) = 0$ geeft $6x \ln(x) + 5x = 0$

$x(6 \ln(x) + 5) = 0$

$x = 0 \vee 6 \ln(x) + 5 = 0$

vold. niet $6 \ln(x) = -5$

$\ln(x) = -\frac{5}{6}$

$x = e^{-\frac{5}{6}}$

De x -coördinaat van het buigpunt van de grafiek van f_3 is $x = e^{-\frac{5}{6}}$.

c $f_p(x) = x^p \cdot \ln(x)$ geeft $f'_p(x) = px^{p-1} \cdot \ln(x) + x^p \cdot \frac{1}{x} = px^{p-1} \ln(x) + x^{p-1} = px^{p-1} \ln(x) + x^{p-1}$

$f'_p(x) = 0$ geeft $px^{p-1} \ln(x) + x^{p-1} = 0$

$$x^{p-1}(p \ln(x) + 1) = 0$$

$x = 0 \quad \vee \quad p \ln(x) + 1 = 0$

vold. niet $p \ln(x) = -1$

$$\ln(x) = -\frac{1}{p}$$

$$x = e^{-\frac{1}{p}}$$

$f_p(e^{-\frac{1}{p}}) = (e^{-\frac{1}{p}})^p \cdot -\frac{1}{p} = e^{-1} \cdot -\frac{1}{p} = -\frac{1}{pe}$

$y_{\text{top}} = \frac{1}{3}e$ geeft $-\frac{1}{pe} = \frac{1}{3}e$

$$-\frac{1}{pe} = \frac{e}{3}$$

$$pe^2 = -3$$

$$p = \frac{-3}{e^2}$$

10 a $f(x) = g(x)$ geeft $\ln(4x) = \ln\left(\frac{1}{x}\right)$

$$4x = \frac{1}{x}$$

$$4x^2 = 1$$

$$x^2 = \frac{1}{4}$$

$$x = \frac{1}{2} \vee x = -\frac{1}{2}$$

vold. niet

$f\left(\frac{1}{2}\right) = \ln(2)$ geeft $A\left(\frac{1}{2}, \ln(2)\right)$

$f(x) = \ln(4x) = \ln(4) + \ln(x)$ geeft $f'(x) = \frac{1}{x}$

Stel $y = ax + b$ met $a = f'\left(\frac{1}{2}\right) = 2$.

$$\left. \begin{array}{l} y = 2x + b \\ \text{door } A\left(\frac{1}{2}, \ln(2)\right) \end{array} \right\} \begin{array}{l} 2 \cdot \frac{1}{2} + b = \ln(2) \\ b = \ln(2) - 1 \end{array}$$

De lijn $y = 2x + \ln(2) - 1$ snijdt de y -as in $(0, \ln(2) - 1)$.

$g(x) = \ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = -\ln(x)$ geeft $g'(x) = -\frac{1}{x}$

Stel $y = ax + b$ met $a = g'\left(\frac{1}{2}\right) = -2$.

$$\left. \begin{array}{l} y = -2x + b \\ \text{door } A\left(\frac{1}{2}, \ln(2)\right) \end{array} \right\} \begin{array}{l} -2 \cdot \frac{1}{2} + b = \ln(2) \\ b = \ln(2) + 1 \end{array}$$

De lijn $y = -2x + \ln(2) + 1$ snijdt de y -as in $(0, \ln(2) + 1)$.

De lengte van het gevraagde lijnstuk is $\ln(2) + 1 - (\ln(2) - 1) = 2$.

b Stel $x_p = p$.

$PS = g(p) = \ln\left(\frac{1}{p}\right)$

$PQRS$ is een vierkant, dus $PQ = PS = \ln\left(\frac{1}{p}\right)$

$$\left. \begin{array}{l} PQ = PS \\ x_Q = p \end{array} \right\} x_Q = p + \ln\left(\frac{1}{p}\right)$$

$x_Q = p + \ln\left(\frac{1}{p}\right)$ geeft $QR = f\left(p + \ln\left(\frac{1}{p}\right)\right) = \ln\left(4\left(p + \ln\left(\frac{1}{p}\right)\right)\right) = \ln\left(4p + 4\ln\left(\frac{1}{p}\right)\right)$

$PQRS$ is een vierkant, dus $PS = QR$

$$\ln\left(\frac{1}{p}\right) = \ln\left(4p + 4\ln\left(\frac{1}{p}\right)\right)$$

$$\frac{1}{p} = 4p + 4\ln\left(\frac{1}{p}\right)$$

Voer in $y_1 = \frac{1}{x}$ en $y_2 = 4x + 4\ln\left(\frac{1}{x}\right)$.

De optie snijpunt geeft $x = 0,1065\dots$

Dus $x_p \approx 0,107$.

11

a $p = 10 e^{-\frac{1}{2}t+1}$

$$e^{-\frac{1}{2}t+1} = \frac{1}{10}p$$

$$-\frac{1}{2}t + 1 = \ln\left(\frac{1}{10}p\right)$$

$$-\frac{1}{2}t = -1 + \ln\left(\frac{1}{10}p\right)$$

$$t = 2 - 2 \ln\left(\frac{1}{10}p\right)$$

$$t = \ln(e^2) + \ln\left(\left(\frac{1}{10}p\right)^{-2}\right)$$

$$t = \ln(e^2) + \ln(100p^{-2})$$

$$t = \ln(100e^2p^{-2})$$

b $y = 3 \ln\left(\frac{2}{x}\right) + 2 \ln(5x)$

$$y = 3(\ln(2) - \ln(x)) + 2(\ln(5) + \ln(x))$$

$$y = 3 \ln(2) - 3 \ln(x) + 2 \ln(5) + 2 \ln(x)$$

$$y = \ln(2^3) + \ln(5^2) - \ln(x)$$

$$\ln(x) = \ln(2^3 \cdot 5^2) - y$$

$$\ln(x) = \ln(200) + \ln(e^{-y})$$

$$\ln(x) = \ln(200 e^{-y})$$

$$x = 200 e^{-y}$$

c $y = e^x + e^{x+1} + e^{x+2} + e^{x+3}$

$$e^x + e^{x+1} + e^{x+2} + e^{x+3} = y$$

$$e^x + e^x \cdot e + e^x \cdot e^2 + e^x \cdot e^3 = y$$

$$e^x(1 + e + e^2 + e^3) = y$$

$$e^x = \frac{y}{1 + e + e^2 + e^3}$$

$$x = \ln\left(\frac{y}{1 + e + e^2 + e^3}\right)$$

$$\frac{e-1}{e^4-1} = \frac{e-1}{(e^2+1)(e^2-1)} = \frac{e-1}{(e^2+1)(e+1)(e-1)} = \frac{1}{(e^2+1)(e+1)} = \frac{1}{e^3+e^2+e+1}$$

Dus $x = \ln\left(\frac{y}{1 + e + e^2 + e^3}\right)$ kan worden geschreven als $x = \ln\left(\frac{e-1}{e^4-1}y\right)$.

10 Meetkunde met vectoren

12

a Substitutie van $x = -1 + 5t$ en $y = -t$ in $3x + 2y = 10$ geeft $3(-1 + 5t) + 2 \cdot -t = 10$

$$-3 + 15t - 2t = 10$$

$$13t = 13$$

$$t = 1$$

$t = 1$ geeft $x = -1 + 5 = 4$ en $y = -1$, dus $S(4, -1)$.

b $m \perp k$, dus $\vec{n}_m = \vec{r}_k = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

$$m: 5x - y = c \quad \left. \begin{array}{l} \text{door } A(1, 3) \end{array} \right\} c = 5 \cdot 1 - 3 = 2$$

Dus $m: 5x - y = 2$.

c $\vec{n}_l = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left| \begin{pmatrix} 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|}{\left| \begin{pmatrix} 5 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|} = \frac{|10 + 3|}{\sqrt{26} \cdot \sqrt{13}} = \frac{13}{13\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\angle(k, l) = 45^\circ$$

d P op k , dus $\vec{p} = \begin{pmatrix} -1 + 5t \\ -t \end{pmatrix}$.

$$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} -1 + 5t \\ -t \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5t - 2 \\ -t - 3 \end{pmatrix}$$

De lijnen AP en l zijn evenwijdig en $\vec{n}_l = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, dus $\begin{pmatrix} 5t - 2 \\ -t - 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 0$

$$15t - 6 - 2t - 6 = 0$$

$$13t = 12$$

$$t = \frac{12}{13}$$

$t = \frac{12}{13}$ geeft $P(-1 + 5 \cdot \frac{12}{13}, -\frac{12}{13}) = P(3\frac{8}{13}, -\frac{12}{13})$

13 a $\vec{r}_m = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$$\cos(\angle(k, m)) = \frac{\left| \begin{pmatrix} -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right|}{\left| \begin{pmatrix} -2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right|} = \frac{|-4 - 15|}{\sqrt{13} \cdot \sqrt{29}} = \frac{19}{\sqrt{377}}$$

$\angle(k, m) \approx 11,9^\circ$

b $\vec{r}_k = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$k: 3x + 2y = c$
 door $(1, 2)$ $\left. \vphantom{\begin{matrix} k: 3x + 2y = c \\ \text{door } (1, 2) \end{matrix}} \right\} c = 3 \cdot 1 + 2 \cdot 2 = 7$

Dus $k: 3x + 2y = 7$.

$x = 2u - 6$ en $y = -5u + 4$ substitueren in $3x + 2y = 7$ geeft $3(2u - 6) + 2(-5u + 4) = 7$

$$6u - 18 - 10u + 8 = 7$$

$$-4u = 17$$

$$u = -4\frac{1}{4}$$

$u = -4\frac{1}{4}$ geeft $x = 2 \cdot -4\frac{1}{4} - 6 = -14\frac{1}{2}$ en $y = -5 \cdot -4\frac{1}{4} + 4 = 25\frac{1}{4}$

Dus $S(-14\frac{1}{2}, 25\frac{1}{4})$.

c $m: x = 2u - 6 \wedge y = -5u + 4$ geeft $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix} + u \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

$n: 2x - 5y = c$
 door $O(0, 0)$ $\left. \vphantom{\begin{matrix} n: 2x - 5y = c \\ \text{door } O(0, 0) \end{matrix}} \right\} n: 2x - 5y = 0$

d $p \perp l$, dus $p: 3x - y = c$
 door $A(6, 8)$ $\left. \vphantom{\begin{matrix} p: 3x - y = c \\ \text{door } A(6, 8) \end{matrix}} \right\} c = 3 \cdot 6 - 8 = 10$

Dus $p: 3x - y = 10$.

$x = 1 - 2t$ en $y = 2 + 3t$ substitueren in $3x - y = 10$ geeft $3(1 - 2t) - (2 + 3t) = 10$

$$3 - 6t - 2 - 3t = 10$$

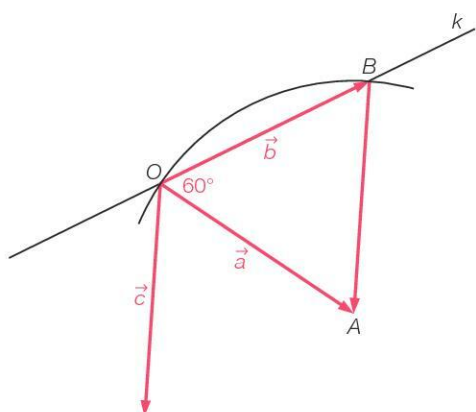
$$-9t = 9$$

$$t = -1$$

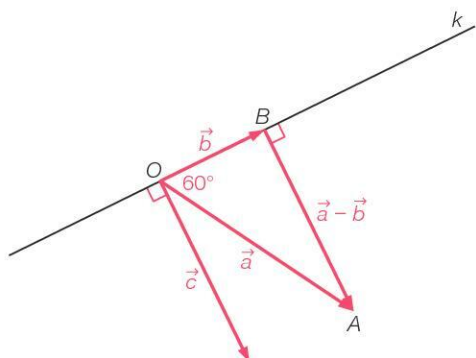
$t = -1$ geeft $x = 1 - 2 \cdot -1 = 3$ en $y = 2 + 3 \cdot -1 = -1$

Dus $T(3, -1)$.

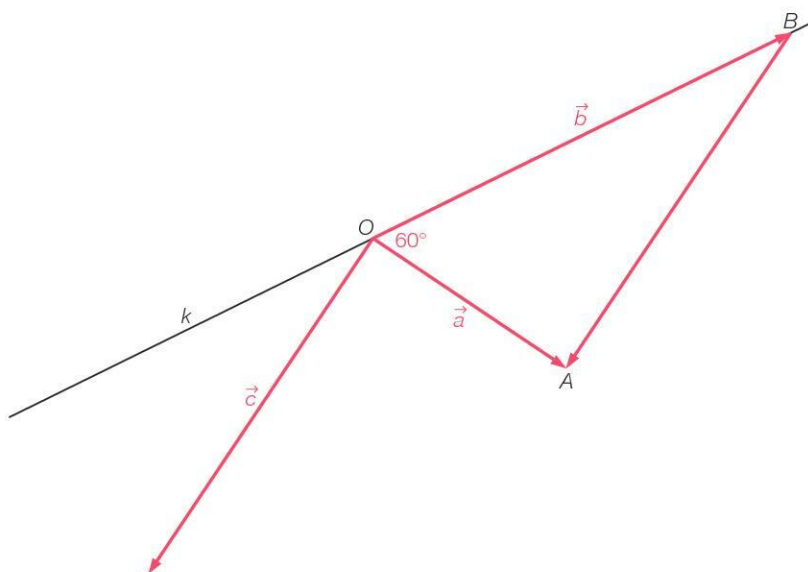
- 14 a** A is het eindpunt van $\vec{a}$ en B is het eindpunt van $\vec{b}$.
 Teken een cirkelboog van de cirkel met middelpunt A en straal OA . Deze cirkelboog
 snijdt k in O en B . Je hebt nu $\vec{b}$ en $\overrightarrow{BA} = \vec{a} - \vec{b}$.
 Teken $\vec{c}$ evenwijdig met $\overrightarrow{BA}$ en even lang als $\overrightarrow{BA}$.



- b** A is het eindpunt van $\vec{a}$ en B is het eindpunt van $\vec{b}$.
 $|\vec{c}| = \sqrt{3} \cdot |\vec{b}|$ betekent dat driehoek OBA een bijzondere rechthoekige driehoek is met
 zijden waarvan de lengten zich verhouden als $1 : 2 : \sqrt{3}$.
 Teken lijnstuk AB loodrecht op k . Je hebt nu $\vec{b}$ en $\overrightarrow{BA} = \vec{a} - \vec{b}$.
 Teken $\vec{c}$ evenwijdig met $\overrightarrow{BA}$ en even lang als $\overrightarrow{BA}$.



- c** Zie de figuur hieronder.
 A is het eindpunt van $\vec{a}$, B is het eindpunt van $\vec{b}$, $|\vec{b}| = 2 \cdot |\vec{a}|$ oftewel $OB = 2OA$,
 $\overrightarrow{BA} = \vec{a} - \vec{b}$ en $\vec{c} = \vec{a} - \vec{b}$.



De cosinusregel in $\triangle OAB$ geeft $AB^2 = OA^2 + (2OA)^2 - 2 \cdot OA \cdot 2OA \cdot \cos(60^\circ)$

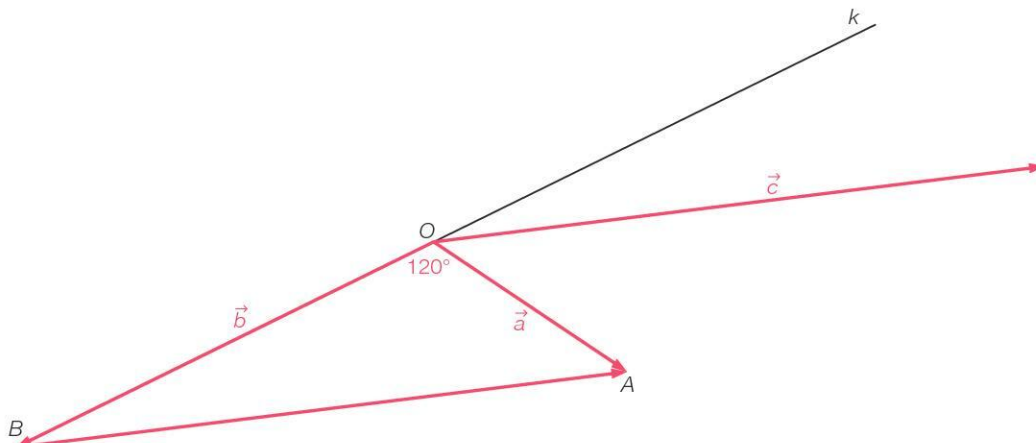
$$AB^2 = OA^2 + 4OA^2 - 4OA^2 \cdot \frac{1}{2}$$

$$AB^2 = 3OA^2$$

$AB = |\overrightarrow{BA}| = |\vec{c}|$ en $OA = |\vec{a}|$, dus $|\vec{c}| = \sqrt{3} \cdot |\vec{a}|$.

Zie de figuur hieronder.

A is het eindpunt van $\vec{a}$, B is het eindpunt van $\vec{b}$, $|\vec{b}| = 2 \cdot |\vec{a}|$ oftewel $OB = 2OA$,
 $\vec{BA} = \vec{a} - \vec{b}$ en $\vec{c} = \vec{a} - \vec{b}$.



De cosinusregel in $\triangle OAB$ geeft $AB^2 = OA^2 + (2OA)^2 - 2 \cdot OA \cdot 2OA \cdot \cos(120^\circ)$

$$AB^2 = OA^2 + 4OA^2 - 4OA^2 \cdot -\frac{1}{2}$$

$$AB^2 = 7OA^2$$

$AB = |\vec{BA}| = |\vec{c}|$ en $OA = |\vec{a}|$, dus $|\vec{c}| = \sqrt{7} \cdot |\vec{a}|$.

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- 15 a** E is het snijpunt van de diagonalen DH en CI van het vierkant $DCHI$ en is dus het midden van diagonaal DH .

$$\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\vec{d} = \vec{a} + \vec{AD} = \vec{a} + \vec{AB}_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\vec{h} = \vec{b} + 2\vec{BC} = \vec{b} + 2\vec{AD} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$\vec{e} = \frac{1}{2}(\vec{d} + \vec{h}) = \frac{1}{2}\left(\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \text{ dus } E(3, 5).$$

- b** $\vec{c} = \vec{b} + \vec{BC} = \vec{b} + \vec{AD} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, dus $C(4, 3)$.

$$\vec{f} = \frac{1}{2}(\vec{d} + \vec{e}) = \frac{1}{2}\left(\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 4 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 4\frac{1}{2} \end{pmatrix}, \text{ dus } F(2, 4\frac{1}{2}).$$

$$\text{Stel } CF: y = ax + b \text{ met } a = \frac{3 - 4\frac{1}{2}}{4 - 2} = -\frac{3}{4}.$$

$$\left. \begin{array}{l} y = -\frac{3}{4}x + b \\ \text{door } C(4, 3) \end{array} \right\} \begin{array}{l} -\frac{3}{4} \cdot 4 + b = 3 \\ -3 + b = 3 \\ b = 6 \end{array}$$

$$\text{Dus } CF: y = -\frac{3}{4}x + 6 \text{ en } G(0, 6).$$

$$CG = \sqrt{(0-4)^2 + (6-3)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$$

- 16 a** $\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} c-a \\ d \end{pmatrix}$

$$\vec{e} = \vec{a} + \vec{AE} = \vec{a} + \vec{AC}_L = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -d \\ c-a \end{pmatrix} = \begin{pmatrix} a-d \\ c-a \end{pmatrix}$$

$$\vec{BC} = \vec{c} - \vec{b} = \begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} c-b \\ d \end{pmatrix}$$

$$\vec{f} = \vec{b} + \vec{BF} = \vec{b} + \vec{BC}_R = \begin{pmatrix} b \\ 0 \end{pmatrix} + \begin{pmatrix} d \\ b-c \end{pmatrix} = \begin{pmatrix} b+d \\ b-c \end{pmatrix}$$

$$\vec{m} = \frac{1}{2}(\vec{e} + \vec{f}) = \frac{1}{2}\left(\begin{pmatrix} a-d \\ c-a \end{pmatrix} + \begin{pmatrix} b+d \\ b-c \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} a+b \\ -a+b \end{pmatrix}, \text{ dus } M(\frac{1}{2}a + \frac{1}{2}b, -\frac{1}{2}a + \frac{1}{2}b).$$

$$\begin{aligned}
\mathbf{b} \quad \vec{n} &= \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 0 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} a+b \\ 0 \end{pmatrix} \\
\vec{d} &= \vec{c} + \overrightarrow{CD} = \vec{c} + \overrightarrow{AE} = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} -d \\ c-a \end{pmatrix} = \begin{pmatrix} c-d \\ -a+c+d \end{pmatrix} \\
\vec{p} &= \frac{1}{2}(\vec{a} + \vec{d}) = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} c-d \\ -a+c+d \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} a+c-d \\ -a+c+d \end{pmatrix} \\
\overrightarrow{NP} &= \vec{p} - \vec{n} = \frac{1}{2}\begin{pmatrix} a+c-d \\ -a+c+d \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -b+c-d \\ -a+c+d \end{pmatrix} \\
\vec{g} &= \vec{c} + \overrightarrow{CG} = \vec{c} + \overrightarrow{BF} = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} d \\ b-c \end{pmatrix} = \begin{pmatrix} c+d \\ b-c+d \end{pmatrix} \\
\vec{q} &= \frac{1}{2}(\vec{b} + \vec{g}) = \frac{1}{2}\left(\begin{pmatrix} b \\ 0 \end{pmatrix} + \begin{pmatrix} c+d \\ b-c+d \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} b+c+d \\ b-c+d \end{pmatrix} \\
\overrightarrow{NQ} &= \vec{q} - \vec{n} = \frac{1}{2}\begin{pmatrix} b+c+d \\ b-c+d \end{pmatrix} - \frac{1}{2}\begin{pmatrix} a+b \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a+c+d \\ b-c+d \end{pmatrix} \\
\overrightarrow{NQ} &= \overrightarrow{NP}_R, \text{ dus } NP = NQ \text{ en } NP \perp NQ, \text{ dus driehoek } PQN \text{ is een gelijkbenige} \\
&\text{rechthoekige driehoek.}
\end{aligned}$$

$$\begin{aligned}
\mathbf{17} \quad \mathbf{a} \quad \overrightarrow{DA} &= \vec{a} - \vec{d} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ d \end{pmatrix} = \begin{pmatrix} a \\ -d \end{pmatrix} \\
\vec{c} &= \vec{d} + \overrightarrow{DA}_L = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} d \\ a+d \end{pmatrix} \\
\overrightarrow{AC} &= \vec{c} - \vec{a} = \begin{pmatrix} d \\ a+d \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} d-a \\ d+a \end{pmatrix} \\
\vec{p} &= \vec{a} + \overrightarrow{AP} = \vec{a} + \overrightarrow{AC}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} a+d \\ a-d \end{pmatrix} = \begin{pmatrix} 2a+d \\ a-d \end{pmatrix}
\end{aligned}$$

$$P(10, -1) \text{ geeft } \begin{cases} 2a+d=10 \\ a-d=-1 \\ 3a=9 \\ a=3 \\ a-d=-1 \end{cases} \Rightarrow \begin{cases} 3-d=-1 \\ d=4 \end{cases}$$

Dus $a=3$ en $d=4$.

$$\begin{aligned}
\mathbf{b} \quad \left. \begin{aligned} x+y &= 6 \\ P(2a+d, a-d) \end{aligned} \right\} \begin{aligned} 2a+d+a-d &= 6 \\ 3a &= 6 \\ a &= 2 \end{aligned} \\
a=2 \text{ geeft } \vec{p} &= \begin{pmatrix} 4+d \\ 2-d \end{pmatrix} \text{ en } \overrightarrow{AC} = \begin{pmatrix} d-2 \\ d+2 \end{pmatrix} \\
\vec{q} &= \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{AC} = \begin{pmatrix} 4+d \\ 2-d \end{pmatrix} + \begin{pmatrix} d-2 \\ d+2 \end{pmatrix} = \begin{pmatrix} 2d+2 \\ 4 \end{pmatrix} \\
y = \frac{1}{2}x^2 \quad \left. \begin{aligned} Q(2d+2, 4) \end{aligned} \right\} \begin{aligned} \frac{1}{2}(2d+2)^2 &= 4 \\ (2d+2)^2 &= 8 \\ 2d+2 &= 2\sqrt{2} \vee 2d+2 = -2\sqrt{2} \\ d+1 &= \sqrt{2} \vee d+1 = -\sqrt{2} \\ d &= -1 + \sqrt{2} \vee d = -1 - \sqrt{2} \\ &\text{vold. niet} \end{aligned}
\end{aligned}$$

Dus $d = -1 + \sqrt{2}$.

$$\begin{aligned}
\mathbf{18} \quad \mathbf{a} \quad AB: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{a} + t \cdot (\vec{b} - \vec{a}) \\
\vec{a} &= \begin{pmatrix} -1 \\ 5 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix} \left. \right\} AB: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} \\
x &= 99 \text{ oftewel } -1 + t = 99 \text{ geeft } t = 100 \\
t &= 100 \text{ geeft } y = 5 + 100 \cdot -2 = -195 \\
&\text{Dus } D(99, -195) \text{ op de lijn } AB.
\end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \left. \begin{aligned} BC: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + t \cdot (\vec{c} - \vec{b}) \\ \vec{b} &= \begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ en } \vec{c} - \vec{b} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{aligned} \right\} BC: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} \\ & \left. \begin{aligned} k: x + 3y &= c \\ \text{door } A(-1, 5) \end{aligned} \right\} c &= -1 + 3 \cdot 5 = 14 \\ & \text{Dus } k: x + 3y = 14. \\ & \text{Substitutie van } x = 2 + t \text{ en } y = -1 + 3t \text{ in } x + 3y = 14 \text{ geeft } 2 + t + 3(-1 + 3t) = 14 \\ & \qquad \qquad \qquad 2 + t - 3 + 9t = 14 \\ & \qquad \qquad \qquad 10t = 15 \\ & \qquad \qquad \qquad t = 1\frac{1}{2} \\ & t = 1\frac{1}{2} \text{ geeft } x = 2 + 1\frac{1}{2} = 3\frac{1}{2} \text{ en } y = -1 + 3 \cdot 1\frac{1}{2} = 3\frac{1}{2}, \text{ dus } S(3\frac{1}{2}, 3\frac{1}{2}). \end{aligned}$$

c $AP = BP$, dus P ligt op de middelloodlijn m van AB .

$$\begin{aligned} m \perp AB, \text{ dus } \vec{r}_m &= \vec{n}_{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \\ \vec{s}_m &= \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \left(\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right) = \frac{1}{2} \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} \\ \text{Dus } m: \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \\ P \text{ op } m, \text{ dus } \vec{p} &= \begin{pmatrix} \frac{1}{2} + 2t \\ 2 + t \end{pmatrix}. \\ \overrightarrow{AP} &= \vec{p} - \vec{a} = \begin{pmatrix} \frac{1}{2} + 2t \\ 2 + t \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2t + 1\frac{1}{2} \\ t - 3 \end{pmatrix} \\ \overrightarrow{BP} &= \vec{p} - \vec{b} = \begin{pmatrix} \frac{1}{2} + 2t \\ 2 + t \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2t - 1\frac{1}{2} \\ t + 3 \end{pmatrix} \\ AP \perp BP, \text{ dus } \overrightarrow{AP} \cdot \overrightarrow{BP} &= 0 \\ \begin{pmatrix} 2t + 1\frac{1}{2} \\ t - 3 \end{pmatrix} \cdot \begin{pmatrix} 2t - 1\frac{1}{2} \\ t + 3 \end{pmatrix} &= 0 \\ 4t^2 - 2\frac{1}{4} + t^2 - 9 &= 0 \\ 5t^2 &= 11\frac{1}{4} \\ t^2 &= 2\frac{1}{4} \\ t &= 1\frac{1}{2} \vee t = -1\frac{1}{2} \end{aligned}$$

$$\begin{aligned} t = 1\frac{1}{2} \text{ geeft } P(\frac{1}{2} + 2 \cdot 1\frac{1}{2}, 2 + 1\frac{1}{2}) &= P(3\frac{1}{2}, 3\frac{1}{2}) \\ t = -1\frac{1}{2} \text{ geeft } P(\frac{1}{2} + 2 \cdot -1\frac{1}{2}, 2 - 1\frac{1}{2}) &= P(-2\frac{1}{2}, \frac{1}{2}) \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad & \left. \begin{aligned} BC: \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ (zie b)} \\ BC: \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} p \\ 38 \end{pmatrix} + t \cdot \begin{pmatrix} q \\ 3 \end{pmatrix} \end{aligned} \right\} q = 1 \\ (2, -1) \text{ op } BC, \text{ dus } 38 + 3t &= -1 \\ 3t &= -39 \\ t &= -13 \\ q = 1 \text{ en } t = -13 \text{ geeft } p + -13 \cdot 1 &= 2 \\ p - 13 &= 2 \\ p &= 15 \end{aligned}$$

19 a $\overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix}$
 $\vec{c} = \vec{b} + \overrightarrow{BC} = \vec{b} + \overrightarrow{BA}_R = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 16 \\ 8 \end{pmatrix} = \begin{pmatrix} 24 \\ 8 \end{pmatrix}$
 $\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{b} + \overrightarrow{BA} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 16 \end{pmatrix} = \begin{pmatrix} 16 \\ 24 \end{pmatrix}$
 $CD: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t \cdot (\vec{d} - \vec{c})$
 $\vec{c} = \begin{pmatrix} 24 \\ 8 \end{pmatrix}$ en $\vec{d} - \vec{c} = \begin{pmatrix} 16 \\ 24 \end{pmatrix} - \begin{pmatrix} 24 \\ 8 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} CD: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
 $x_P = 22$ en P op CD geeft $24 + t = 22$, dus $t = -2$.
 $t = -2$ geeft $y = 8 + -2 \cdot -2 = 12$ en dit is y_P .
 Verder geldt $x_D \leq x_P \leq x_C$ en $y_C \leq y_P \leq y_D$.
 Dus P ligt op de zijde CD .

b $AP: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{p} - \vec{a})$
 $\vec{a} = \begin{pmatrix} 0 \\ 16 \end{pmatrix}$ en $\vec{p} - \vec{a} = \begin{pmatrix} 22 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 16 \end{pmatrix} = \begin{pmatrix} 22 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 11 \\ -2 \end{pmatrix}$ $\left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} AP: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} + t \cdot \begin{pmatrix} 11 \\ -2 \end{pmatrix}$
 Q op AP , dus $\vec{q} = \begin{pmatrix} 11t \\ 16 - 2t \end{pmatrix}$.
 $\overrightarrow{DQ} = \vec{q} - \vec{d} = \begin{pmatrix} 11t \\ 16 - 2t \end{pmatrix} - \begin{pmatrix} 16 \\ 24 \end{pmatrix} = \begin{pmatrix} 11t - 16 \\ -2t - 8 \end{pmatrix}$
 DQ loodrecht op AP , dus $\begin{pmatrix} 11t - 16 \\ -2t - 8 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ -2 \end{pmatrix} = 0$
 $121t - 176 + 4t + 16 = 0$
 $125t = 160$
 $t = 1\frac{7}{25}$
 $t = 1\frac{7}{25}$ geeft $Q(11 \cdot 1\frac{7}{25}, 16 - 2 \cdot 1\frac{7}{25}) = Q(14\frac{2}{25}, 13\frac{11}{25})$

20 a $x(t) = t^2 - 4t$ geeft $x'(t) = 2t - 4$
 $y(t) = t^4 - 4t^3 + 4t^2$ geeft $y'(t) = 4t^3 - 12t^2 + 8t$
 Evenwijdig met de x -as als $y'(t) = 0 \wedge x'(t) \neq 0$
 $4t^3 - 12t^2 + 8t = 0 \wedge 2t - 4 \neq 0$
 $4t(t^2 - 3t + 2) = 0 \wedge 2t \neq 4$
 $4t(t-1)(t-2) = 0 \wedge t \neq 2$
 $(t=0 \vee t=1 \vee t=2) \wedge t \neq 2$
 $t=0 \vee t=1$

$t = 0$ geeft het punt $(0, 0)$.

$t = 1$ geeft het punt $(-3, 1)$.

Dus evenwijdig met de x -as in de punten $(0, 0)$ en $(-3, 1)$.

Evenwijdig met de y -as als $x'(t) = 0 \wedge y'(t) \neq 0$

$$t = 2 \wedge t \neq 0 \wedge t \neq 1 \wedge t \neq 2$$

Er is dus geen waarde van t die voldoet, dus er zijn geen punten waarin de raaklijn evenwijdig is met de y -as.

b Naar links betekent $x'(t) < 0$

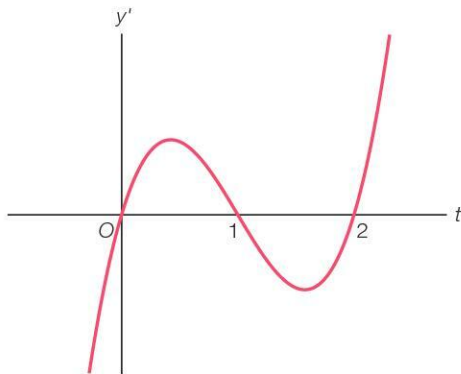
$$2t - 4 < 0$$

$$2t < 4$$

$$t < 2$$

$y'(t) = 0$ geeft $t = 0 \vee t = 1 \vee t = 2$ (zie a)

Omhoog betekent $y'(t) > 0$.



$y'(t) > 0$ geeft $0 < t < 1 \vee t > 2$

$$x'(t) < 0 \wedge y'(t) > 0$$

$$t < 2 \wedge (0 < t < 1 \vee t > 2)$$

$$0 < t < 1$$

Dus naar links en omhoog voor $0 < t < 1$.

c $x(t) = -3$ geeft $t^2 - 4t = -3$

$$t^2 - 4t + 3 = 0$$

$$(t - 1)(t - 3) = 0$$

$$t = 1 \vee t = 3$$

$t = 1$ geeft het punt $(-3, 1)$, dus voldoet niet.

$t = 3$ geeft het punt $(-3, 9)$, dus voldoet.

$$x'(3) = 2 \cdot 3 - 4 = 2 \text{ en } y'(3) = 4 \cdot 3^3 - 12 \cdot 3^2 + 8 \cdot 3 = 24$$

$$\text{De baansnelheid is } v'(3) = \sqrt{2^2 + 24^2} = \sqrt{580} = 2\sqrt{145}.$$

d $v(t) = \sqrt{(2t - 4)^2 + (4t^3 - 12t^2 + 8t)^2}$

$$\text{Voer in } y_1 = \sqrt{(2x - 4)^2 + (4x^3 - 12x^2 + 8x)^2}.$$

De optie minimum geeft $x = 2$ en $y = 0$.

De minimale baansnelheid is 0 voor $t = 2$.

e $x'(t) = 2t - 4$ geeft $x''(t) = 2$

$$y'(t) = 4t^3 - 12t^2 + 8t \text{ geeft } y''(t) = 12t^2 - 24t + 8$$

De versnellingsvector is evenwijdig met de lijn $y = x$ als $y''(t) = x''(t)$

$$12t^2 - 24t + 8 = 2$$

$$12t^2 - 24t + 6 = 0$$

$$2t^2 - 4t + 1 = 0$$

$$D = (-4)^2 - 4 \cdot 2 \cdot 1 = 8, \text{ dus } \sqrt{D} = \sqrt{8} = 2\sqrt{2}$$

$$t = \frac{4 + 2\sqrt{2}}{4} \vee t = \frac{4 - 2\sqrt{2}}{4}$$

$$t = 1 + \frac{1}{2}\sqrt{2} \vee t = 1 - \frac{1}{2}\sqrt{2}$$

21 a $y(t) = 0$ geeft $t^2 - 1 = 0$

$$t^2 = 1$$

$$t = 1 \vee t = -1$$

$t = 1$ geeft het punt $(0, 0)$.

$t = -1$ geeft het punt $(6, 0)$.

Dus de snijpunten met de x -as zijn $(0, 0)$ en $(6, 0)$.

$$x(t) = 0 \text{ geeft } -t^3 + 3t^2 - 2t = 0$$

$$-t(t^2 - 3t + 2) = 0$$

$$-t(t - 1)(t - 2) = 0$$

$$t = 0 \vee t = 1 \vee t = 2$$

$t = 0$ geeft het punt $(0, -1)$.

$t = 1$ geeft het punt $(0, 0)$.

$t = 2$ geeft het punt $(0, 3)$.

Dus de snijpunten met de y -as zijn $(0, -1)$, $(0, 0)$ en $(0, 3)$.

- b** $x(t) = -t^3 + 3t^2 - 2t$ geeft $x'(t) = -3t^2 + 6t - 2$
 $y(t) = t^2 - 1$ geeft $y'(t) = 2t$
 Evenwijdig met de x -as als $y'(t) = 0 \wedge x'(t) \neq 0$ en evenwijdig met de y -as als $x'(t) = 0 \wedge y'(t) \neq 0$.
 $x'(t) = 0$ geeft $-3t^2 + 6t - 2 = 0$

$$D = 6^2 - 4 \cdot (-3) \cdot (-2) = 12, \text{ dus } \sqrt{D} = \sqrt{12} = 2\sqrt{3}$$

$$t = \frac{-6 + 2\sqrt{3}}{-6} \vee t = \frac{-6 - 2\sqrt{3}}{-6}$$

$$t = 1 - \frac{1}{3}\sqrt{3} \vee t = 1 + \frac{1}{3}\sqrt{3}$$

$$y'(t) = 0 \text{ geeft } 2t = 0$$

$$t = 0$$

Dus raaklijn evenwijdig met de x -as of met de y -as voor $t = 0 \vee t = 1 - \frac{1}{3}\sqrt{3} \vee t = 1 + \frac{1}{3}\sqrt{3}$.

- c** Evenwijdig met de lijn $y = 2x + 3$ geeft $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$2 \cdot x'(t) = 1 \cdot y'(t)$$

$$2(-3t^2 + 6t - 2) = 2t$$

$$-6t^2 + 12t - 4 = 2t$$

$$-6t^2 + 10t - 4 = 0$$

$$3t^2 - 5t + 2 = 0$$

$$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1$$

$$t = \frac{5+1}{6} = 1 \vee t = \frac{5-1}{6} = \frac{2}{3}$$

$t = 1$ geeft het punt $(0, 0)$.

$t = \frac{2}{3}$ geeft het punt $(-\frac{8}{27}, -\frac{5}{9})$.

Dus de punten zijn $(0, 0)$ en $(-\frac{8}{27}, -\frac{5}{9})$.

- d** $y(t) = 8$ geeft $t^2 - 1 = 8$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$t = 3$ geeft het punt $(-6, 8)$, dus voldoet.

$t = -3$ geeft het punt $(60, 8)$, dus voldoet niet.

$$x'(3) = -3 \cdot 3^2 + 6 \cdot 3 - 2 = -11 \text{ en } y'(3) = 2 \cdot 3 = 6$$

De baansnelheid is $v(3) = \sqrt{(-11)^2 + 6^2} = \sqrt{157}$.

- e** $y(t) = -\frac{3}{4}$ geeft $t^2 - 1 = -\frac{3}{4}$

$$t^2 = \frac{1}{4}$$

$$t = \frac{1}{2} \vee t = -\frac{1}{2}$$

$x(\frac{1}{2}) = -\frac{1}{8} + 3 \cdot \frac{1}{4} - 2 \cdot \frac{1}{2} = -\frac{3}{8}$, dus $t = \frac{1}{2}$ voldoet.

$x(-\frac{1}{2}) = \frac{1}{8} + 3 \cdot \frac{1}{4} - 2 \cdot -\frac{1}{2} = \frac{7}{8}$, dus $t = -\frac{1}{2}$ voldoet niet.

$$v(t) = \sqrt{(-3t^2 + 6t - 2)^2 + (2t)^2} = \sqrt{9t^4 - 18t^3 + 6t^2 - 18t^3 + 36t^2 - 12t + 6t^2 - 12t + 4 + 4t^2}$$

$$= \sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}} \cdot (36t^3 - 108t^2 + 104t - 24) = \frac{18t^3 - 54t^2 + 52t - 12}{\sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}}$$

$$a(\frac{1}{2}) = \frac{18 \cdot \frac{1}{8} - 54 \cdot \frac{1}{4} + 52 \cdot \frac{1}{2} - 12}{\sqrt{9 \cdot \frac{1}{16} - 36 \cdot \frac{1}{8} + 52 \cdot \frac{1}{4} - 24 \cdot \frac{1}{2} + 4}} = \frac{\frac{23}{4}}{\sqrt{1\frac{1}{16}}} = \frac{11}{17}\sqrt{17}$$

Dus de baanversnelling is $\frac{11}{17}\sqrt{17}$.

11 Integraalrekening

Bladzijde 231

- 22 a** $F(x) = x + \ln\left(\frac{x-1}{x+1}\right)$ geeft
- $$F'(x) = 1 + \frac{1}{\frac{x-1}{x+1}} \cdot \frac{(x+1) \cdot 1 - (x-1) \cdot 1}{(x+1)^2} = 1 + \frac{x+1}{x-1} \cdot \frac{x+1-x+1}{(x+1)^2} = 1 + \frac{2}{(x-1)(x+1)} = \frac{x^2-1+2}{x^2-1}$$
- $$= \frac{x^2+1}{x^2-1}$$
- Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- b** $F(x) = x \ln\left(\frac{x+1}{x-1}\right) + \ln(x^2-1)$ geeft
- $$F'(x) = 1 \cdot \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{1}{\frac{x+1}{x-1}} \cdot \frac{(x-1) \cdot 1 - (x+1) \cdot 1}{(x-1)^2} + \frac{1}{x^2-1} \cdot 2x$$
- $$= \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{x-1}{x+1} \cdot \frac{x-1-x-1}{(x-1)^2} + \frac{2x}{x^2-1} = \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{x-1}{x+1} \cdot \frac{-2}{(x-1)^2} + \frac{2x}{x^2-1}$$
- $$= \ln\left(\frac{x+1}{x-1}\right) + x \cdot \frac{1}{x+1} \cdot \frac{-2}{x-1} + \frac{2x}{x^2-1} = \ln\left(\frac{x+1}{x-1}\right) - \frac{2x}{x^2-1} + \frac{2x}{x^2-1} = \ln\left(\frac{x+1}{x-1}\right)$$
- Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- c** $F(x) = -x^3 \cos(x) + 3x^2 \sin(x) + 6x \cos(x) - 6 \sin(x)$ geeft
- $$F'(x) = -3x^2 \cdot \cos(x) - x^3 \cdot (-\sin(x)) + 6x \cdot \sin(x) + 3x^2 \cdot \cos(x) + 6 \cdot \cos(x) + 6x \cdot (-\sin(x)) - 6 \cos(x)$$
- $$= -3x^2 \cos(x) + x^3 \sin(x) + 6x \sin(x) + 3x^2 \cos(x) + 6 \cos(x) - 6x \sin(x) - 6 \cos(x)$$
- $$= x^3 \sin(x)$$
- Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- d** $F(x) = (6x^2 - 2x - 4)\sqrt{x-1}$ geeft
- $$F'(x) = (12x - 2) \cdot \sqrt{x-1} + (6x^2 - 2x - 4) \cdot \frac{1}{2\sqrt{x-1}} = (12x - 2)\sqrt{x-1} + \frac{3x^2 - x - 2}{\sqrt{x-1}}$$
- $$= \frac{(12x - 2)(x - 1)}{\sqrt{x-1}} + \frac{3x^2 - x - 2}{\sqrt{x-1}} = \frac{12x^2 - 12x - 2x + 2 + 3x^2 - x - 2}{\sqrt{x-1}}$$
- $$= \frac{15x^2 - 15x}{\sqrt{x-1}} = \frac{15x(x-1)}{\sqrt{x-1}} = 15x\sqrt{x-1}$$
- Dus $F'(x) = f(x)$ oftewel F is een primitieve van f .
- 23 a** $f(x) = 7 \cdot \log(5x)$ geeft $F(x) = 7 \cdot \frac{1}{\ln(10)} \cdot (5x \ln(5x) - 5x) + c = \frac{7(x \ln(5x) - x)}{\ln(10)} + c$
- b** $f(x) = e^{\frac{1}{2}x-1}$ geeft $F(x) = 2e^{\frac{1}{2}x-1} + c$
- c** $f(x) = 10 \ln(2x-4)$ geeft
- $$F(x) = 10 \cdot \frac{1}{2}((2x-4) \ln(2x-4) - (2x-4)) + c = 5(2x-4) \ln(2x-4) - 5(2x-4) + c$$
- d** $f(x) = \frac{x^4 - 6x^2\sqrt{x} + 8x}{x^3} = \frac{x^4}{x^3} - \frac{6x^{\frac{5}{2}}}{x^3} + \frac{8x}{x^3} = x - 6x^{-\frac{1}{2}} + 8x^{-2}$ geeft
- $$F(x) = \frac{1}{2}x^2 - \frac{6}{\frac{1}{2}} \cdot x^{\frac{1}{2}} + \frac{8}{-1} \cdot x^{-1} + c = \frac{1}{2}x^2 - 12\sqrt{x} - \frac{8}{x} + c$$
- e** $f(x) = (x^2 + 3)^2 = x^4 + 6x^2 + 9$ geeft $F(x) = \frac{1}{5}x^5 + 2x^3 + 9x + c$
- f** $f(x) = 3 \sin(4x)$ geeft $F(x) = 3 \cdot -\frac{1}{4} \cos(4x) + c = -\frac{3}{4} \cos(4x) + c$
- 24 a** $f(x) = \sqrt{6x+3} = (6x+3)^{\frac{1}{2}}$ geeft $F(x) = \frac{1}{6} \cdot \frac{2}{\frac{1}{2}} (6x+3)^{\frac{1}{2}} + c = \frac{1}{9} (6x+3) \sqrt{6x+3} + c$
- b** $f(x) = \frac{10}{2x-1}$ geeft $F(x) = 10 \cdot \frac{1}{2} \ln|2x-1| + c = 5 \ln|2x-1| + c$
- c** $f(x) = (3x-6)^{-2}$ geeft $F(x) = \frac{1}{3} \cdot -\frac{1}{-1} (3x-6)^{-1} + c = -\frac{1}{3} (3x-6)^{-1} + c$
- d** $f(x) = (2x+5)^{-1} = \frac{1}{2x+5}$ geeft $F(x) = \frac{1}{2} \ln|2x+5| + c$
- e** $f(x) = 10^{2x-3}$ geeft $F(x) = \frac{1}{2} \cdot \frac{10^{2x-3}}{\ln(10)} + c = \frac{10^{2x-3}}{2 \ln(10)} + c$
- f** $f(x) = \frac{8^x - 1}{2^x} = \frac{8^x}{2^x} - \frac{1}{2^x} = 4^x - 2^{-x}$ geeft $F(x) = \frac{4^x}{\ln(4)} + \frac{2^{-x}}{\ln(2)} + c$

25 $F(x) = (ax^2 + bx + c)e^x$ geeft
 $F'(x) = (2ax + b) \cdot e^x + (ax^2 + bx + c) \cdot e^x = (ax^2 + (2a + b)x + b + c)e^x$
Dit moet gelijk zijn aan $f(x) = (x^2 + 1)e^x$, dus $a = 1$, $2a + b = 0$ en $b + c = 1$.

$$\left. \begin{array}{l} a = 1 \\ 2a + b = 0 \end{array} \right\} \begin{array}{l} 2 + b = 0 \\ b = -2 \end{array}$$

$$\left. \begin{array}{l} b = -2 \\ b + c = 1 \end{array} \right\} \begin{array}{l} -2 + c = 1 \\ c = 3 \end{array}$$

Dus $a = 1$, $b = -2$ en $c = 3$.

26 $f(x) = px$ geeft $x^3 - 3x = px$
 $x^3 = px + 3x$
 $x^3 = (p + 3)x$
 $x = 0 \vee x^2 = p + 3$
 $x = 0 \vee x = \sqrt{p + 3} \vee x = -\sqrt{p + 3}$
vold. niet

$$O(V) = \int_0^{\sqrt{p+3}} (px - f(x)) dx = \int_0^{\sqrt{p+3}} (px - (x^3 - 3x)) dx = \int_0^{\sqrt{p+3}} (-x^3 + (p + 3)x) dx$$

$$= \left[-\frac{1}{4}x^4 + (p + 3) \cdot \frac{1}{2}x^2 \right]_0^{\sqrt{p+3}} = -\frac{1}{4} \cdot (p + 3)^2 + (p + 3) \cdot \frac{1}{2} \cdot (p + 3) - 0 = \frac{1}{4}(p + 3)^2$$

$O(V) = 9$ geeft $\frac{1}{4}(p + 3)^2 = 9$
 $(p + 3)^2 = 36$
 $p + 3 = 6 \vee p + 3 = -6$
 $p = 3 \vee p = -9$
vold. niet

Dus $p = 3$.

27 $O(ABCD) = 5 \cdot (2 - p) = 10 - 5p$
 V is het vlakdeel dat wordt ingesloten door de grafiek van f en het lijnstuk CD .

$$O(V) = \int_0^5 (2 - f(x)) dx = \int_0^5 \left(2 - \frac{1}{2}x + 1 - \frac{3}{x+1} \right) dx = \int_0^5 \left(3 - \frac{1}{2}x - \frac{3}{x+1} \right) dx = \left[3x - \frac{1}{4}x^2 - 3 \ln|x+1| \right]_0^5$$

$$= 15 - 6\frac{1}{4} - 3 \ln(6) - (0 - 0 - 0) = 8\frac{3}{4} - 3 \ln(6)$$

$O(V) = \frac{1}{2}O(ABCD)$ geeft $8\frac{3}{4} - 3 \ln(6) = 5 - 2\frac{1}{2}p$
 $2\frac{1}{2}p = 3 \ln(6) - 3\frac{3}{4}$
 $p = 1\frac{1}{5} \ln(6) - 1\frac{1}{2}$

Bladzijde 232

28 a $54 \text{ km/uur} = 15 \text{ m/s}$
Versnelling van de wielrenner constant a en beginsnelheid 15 geeft snelheid $v_w(t) = at + 15$.
 $v_w(t) = at + 15$ geeft $s_w(t) = \frac{1}{2}at^2 + 15t + c$
 $s_w(t) = \frac{1}{2}at^2 + 15t + c$
 $s_w(0) = 0$ } $s_w(t) = \frac{1}{2}at^2 + 15t$
 $v_w(t) = 0$ geeft $at + 15 = 0$
 $at = -15$
 $t = -\frac{15}{a}$

$$\begin{aligned}
s_w\left(-\frac{15}{a}\right) &= 40 \text{ geeft } \frac{1}{2}a \cdot \left(-\frac{15}{a}\right)^2 + 15 \cdot -\frac{15}{a} = 40 \\
\frac{1}{2}a \cdot \frac{225}{a^2} - \frac{225}{a} &= 40 \\
\frac{112\frac{1}{2}}{a} - \frac{225}{a} &= 40 \\
-\frac{112\frac{1}{2}}{a} &= 40 \\
a &= -\frac{112\frac{1}{2}}{40} = -2,8125
\end{aligned}$$

Dus de versnelling van de wielrenner is $-2,8125 \text{ m/s}^2$.

- b** $a = -2,5$, $v_w(0) = 15$ en $s_w(0) = 0$ geeft $v_w(t) = -2,5t + 15$ en $s_w(t) = -1,25t^2 + 15t$

$6 \text{ km/uur} = 1\frac{2}{3} \text{ m/s}$

Voor de auto geldt $v_a(t) = 1\frac{2}{3}$.

$$\left. \begin{aligned} v_a(t) &= 1\frac{2}{3} \text{ geeft } s_a(t) = 1\frac{2}{3}t + b \\ s_a(0) &= 40 \end{aligned} \right\} s_a(t) = 1\frac{2}{3}t + 40$$

$$v_w(t) = v_a(t)$$

$$-2,5t + 15 = 1\frac{2}{3}$$

$$-2,5t = -13\frac{1}{3}$$

$$t = 5\frac{1}{3}$$

$$s_w(5\frac{1}{3}) = -1,25 \cdot (5\frac{1}{3})^2 + 15 \cdot 5\frac{1}{3} = 44\frac{4}{9}$$

$$s_a(5\frac{1}{3}) = 1\frac{2}{3} \cdot 5\frac{1}{3} + 40 = 48\frac{8}{9}$$

Dus op het moment dat de wielrenner en de auto dezelfde snelheid hebben, rijdt de auto $48\frac{8}{9} - 44\frac{4}{9} = 4\frac{4}{9}$ meter voor de wielrenner.

De wielrenner kan dus op tijd stoppen.

29 a $O(V) = \int_0^1 (3^x + 1) dx = \left[\frac{3^x}{\ln(3)} + x \right]_0^1 = \frac{3}{\ln(3)} + 1 - \frac{1}{\ln(3)} = \frac{2}{\ln(3)} + 1$

b $I(L) = \pi \int_0^1 (3^x + 1)^2 dx = \pi \int_0^1 (3^{2x} + 2 \cdot 3^x + 1) dx = \pi \left[\frac{1}{2} \cdot \frac{3^{2x}}{\ln(3)} + 2 \cdot \frac{3^x}{\ln(3)} + x \right]_0^1$
 $= \pi \left(\frac{4\frac{1}{2}}{\ln(3)} + \frac{6}{\ln(3)} + 1 - \left(\frac{\frac{1}{2}}{\ln(3)} + \frac{2}{\ln(3)} + 0 \right) \right) = \pi \left(\frac{4\frac{1}{2} + 6 - \frac{1}{2} - 2}{\ln(3)} + 1 \right) = \frac{8\pi}{\ln(3)} + \pi$

c $y = 3^x + 1$ geeft $3^x + 1 = y$
 $3^x = y - 1$
 $x = {}^3\log(y - 1)$

$$f(0) = 2 \text{ en } f(1) = 4$$

$$I(M) = \pi \cdot 1^2 \cdot 4 - \pi \int_2^4 ({}^3\log(y - 1))^2 dy \approx 9,89$$

30 a $I = \pi \int_{\frac{1}{3}r}^{\frac{1}{2}r} y^2 dx = \pi \int_{\frac{1}{3}r}^{\frac{1}{2}r} (r^2 - x^2) dx = \pi \left[r^2 x - \frac{1}{3} x^3 \right]_{\frac{1}{3}r}^{\frac{1}{2}r} = \pi \left(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot \left(\frac{1}{2}r \right)^3 \right) - \pi \left(r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot \left(\frac{1}{3}r \right)^3 \right)$
 $= \pi \left(\frac{1}{2}r^3 - \frac{1}{24}r^3 \right) - \pi \left(\frac{1}{3}r^3 - \frac{1}{81}r^3 \right) = \frac{11}{24}\pi r^3 - \frac{26}{81}\pi r^3 = \frac{89}{648}\pi r^3$

b $I_{\text{bol}} = \frac{4}{3}\pi r^3$

$$\begin{aligned}
I &= \pi \int_{-pr}^{pr} y^2 dx = \pi \int_{-pr}^{pr} (r^2 - x^2) dx = \pi \left[r^2 x - \frac{1}{3} x^3 \right]_{-pr}^{pr} = \pi \left(r^2 \cdot pr - \frac{1}{3} (pr)^3 \right) - \pi \left(r^2 \cdot -pr - \frac{1}{3} (-pr)^3 \right) \\
&= \pi (pr^3 - \frac{1}{3} p^3 r^3) + \pi (pr^3 - \frac{1}{3} p^3 r^3) = \pi (2pr^3 - \frac{2}{3} p^3 r^3) = (2p - \frac{2}{3} p^3) \pi r^3
\end{aligned}$$

$$I = \frac{1}{2} I_{\text{bol}} \text{ geeft } (2p - \frac{2}{3} p^3) \pi r^3 = \frac{2}{3} \pi r^3$$

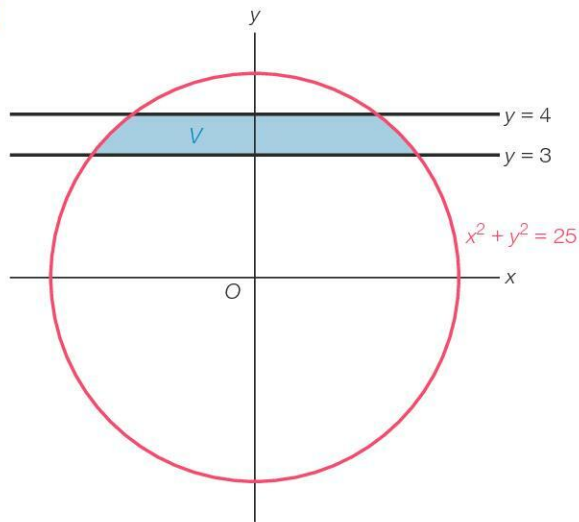
$$2p - \frac{2}{3} p^3 = \frac{2}{3}$$

$$\text{Voer in } y_1 = 2x - \frac{2}{3} x^3 \text{ en } y_2 = \frac{2}{3}.$$

De optie snijpunt geeft $x = 0,347...$

Dus $p \approx 0,35$.

31



Substitutie van $y = 3$ in $x^2 + y^2 = 25$ geeft $x^2 + 9 = 25$

$$x^2 = 16$$

$$x = 4 \vee x = -4$$

Substitutie van $y = 4$ in $x^2 + y^2 = 25$ geeft $x^2 + 16 = 25$

$$x^2 = 9$$

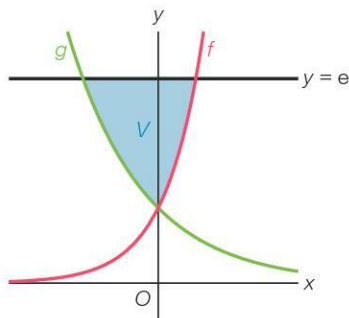
$$x = 3 \vee x = -3$$

$$\begin{aligned} I(L) &= \pi \int_{-4}^{-3} y^2 dx + \pi \int_{-3}^3 4^2 dx + \pi \int_3^4 y^2 dx - \pi \int_{-4}^4 3^2 dx \\ &= \pi \int_{-4}^{-3} (25 - x^2) dx + \pi [16x]_{-3}^3 + \pi \int_3^4 (25 - x^2) dx - \pi [9x]_{-4}^4 \\ &= \pi \left[25x - \frac{1}{3}x^3 \right]_{-4}^{-3} + \pi(48 - -48) + \pi \left[25x - \frac{1}{3}x^3 \right]_3^4 - \pi(36 - -36) \\ &= \pi(-75 + 9) - \pi(-100 + \frac{64}{3}) + 96\pi + \pi(100 - \frac{64}{3}) - \pi(75 - 9) - 72\pi \\ &= -66\pi + 78\frac{2}{3}\pi + 96\pi + 78\frac{2}{3}\pi - 66\pi - 72\pi = 49\frac{1}{3}\pi \end{aligned}$$

Bladzijde 233

32

a



$e^{2x} = e$	$e^{-x} = e$	$e^{2x} = e^{-x}$
$2x = 1$	$-x = 1$	$2x = -x$
$x = \frac{1}{2}$	$x = -1$	$x = 0$

e omlaag schuiven geeft

$$\begin{aligned} I(L) &= \pi \int_{-1}^0 (e^{-x} - e)^2 dx + \pi \int_0^{\frac{1}{2}} (e^{2x} - e)^2 dx = \pi \int_{-1}^0 (e^{-2x} - 2e^{-x+1} + e^2) dx + \pi \int_0^{\frac{1}{2}} (e^{4x} - 2e^{2x+1} + e^2) dx \\ &= \pi \left[-\frac{1}{2}e^{-2x} + 2e^{-x+1} + e^2 x \right]_{-1}^0 + \pi \left[\frac{1}{4}e^{4x} - e^{2x+1} + e^2 x \right]_0^{\frac{1}{2}} \\ &= \pi(-\frac{1}{2} + 2e) - \pi(-\frac{1}{2}e^2 + 2e^2 - e^2) + \pi(\frac{1}{4}e^2 - e^2 + \frac{1}{2}e^2) - \pi(\frac{1}{4} - e) = (-\frac{3}{4}e^2 + 3e - \frac{3}{4})\pi \end{aligned}$$

$$\text{b} \quad \int_{-1}^0 g(x) dx = \int_{-1}^0 e^{-x} dx = [-e^{-x}]_{-1}^0 = -1 + e$$

$$\int_0^{\frac{1}{2}} f(x) dx = \int_0^{\frac{1}{2}} e^{2x} dx = \left[\frac{1}{2} e^{2x} \right]_0^{\frac{1}{2}} = \frac{1}{2} e - \frac{1}{2}$$

$$\text{Dus } O(W) = -1 + e + \frac{1}{2} e - \frac{1}{2} = 1\frac{1}{2} e - 1\frac{1}{2}.$$

$$\int_{-1}^{\ln(\frac{4}{e+3})} e^{-x} dx = [-e^{-x}]_{-1}^{\ln(\frac{4}{e+3})} = -e^{-\ln(\frac{4}{e+3})} + e = -e^{\ln(\frac{e+3}{4})} + e = -\frac{e+3}{4} + e = -\frac{1}{4} e - \frac{3}{4} + e = \frac{3}{4} e - \frac{3}{4}$$

$$\frac{1}{2} O(W) = \frac{1}{2} (1\frac{1}{2} e - 1\frac{1}{2}) = \frac{3}{4} e - \frac{3}{4}$$

Dus de lijn $x = \ln\left(\frac{4}{e+3}\right)$ verdeelt W in twee delen met gelijke oppervlakte.

12 Goniometrische formules

$$\begin{aligned} \text{33 a} \quad & \sin\left(x + \frac{1}{3}\pi\right) = -\cos\left(x - \frac{2}{3}\pi\right) \\ & \cos\left(x - \frac{1}{6}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right) \\ & x - \frac{1}{6}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -x - \frac{1}{3}\pi + k \cdot 2\pi \\ & \text{geen oplossing} \quad 2x = -\frac{1}{6}\pi + k \cdot 2\pi \\ & \quad \quad \quad x = -\frac{1}{12}\pi + k \cdot \pi \end{aligned}$$

$$\begin{aligned} \text{b} \quad & 2 \sin(x) \cos(x) = \cos\left(x + \frac{1}{6}\pi\right) \\ & \sin(2x) = \sin\left(x + \frac{2}{3}\pi\right) \\ & 2x = x + \frac{2}{3}\pi + k \cdot 2\pi \vee 2x = \pi - x - \frac{2}{3}\pi + k \cdot 2\pi \\ & x = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi \\ & x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \end{aligned}$$

$$\begin{aligned} \text{c} \quad & \cos\left(x + \frac{1}{2}\pi\right) \cdot \cos(x) = \frac{1}{4}\sqrt{2} \\ & \cos\left(x + \frac{1}{2}\pi\right) \cdot \sin\left(x + \frac{1}{2}\pi\right) = \frac{1}{4}\sqrt{2} \\ & \frac{1}{2} \sin(2x + \pi) = \frac{1}{4}\sqrt{2} \\ & \sin(2x + \pi) = \frac{1}{2}\sqrt{2} \\ & 2x + \pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x + \pi = \frac{3}{4}\pi + k \cdot 2\pi \\ & 2x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{4}\pi + k \cdot 2\pi \\ & x = -\frac{3}{8}\pi + k \cdot \pi \vee x = -\frac{1}{8}\pi + k \cdot \pi \end{aligned}$$

$$\begin{aligned} \text{d} \quad & \sqrt{3} \cdot \sin(2x) + \cos(2x) = 1 \\ & 2\sqrt{3} \sin(x) \cos(x) + 1 - 2\sin^2(x) = 1 \\ & 2\sqrt{3} \sin(x) \cos(x) - 2\sin^2(x) = 0 \\ & 2\sin(x)(\sqrt{3} \cdot \cos(x) - \sin(x)) = 0 \\ & 2\sin(x) = 0 \vee \sqrt{3} \cdot \cos(x) - \sin(x) = 0 \\ & \sin(x) = 0 \vee \sin(x) = \sqrt{3} \cdot \cos(x) \\ & x = k \cdot \pi \vee \frac{\sin(x)}{\cos(x)} = \sqrt{3} \\ & x = k \cdot \pi \vee \tan(x) = \sqrt{3} \\ & x = k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot \pi \end{aligned}$$

$$\begin{aligned} \text{34 a} \quad & f(x) = (2 - \sin(x))^2 = 4 - 4\sin(x) + \sin^2(x) = 4 - 4\sin(x) + \frac{1}{2} - \frac{1}{2}\cos(2x) \\ & = 4\frac{1}{2} - 4\sin(x) - \frac{1}{2}\cos(2x) \end{aligned}$$

$$F(x) = 4\frac{1}{2}x + 4\cos(x) - \frac{1}{4}\sin(2x) + c$$

$$\text{b} \quad g(x) = \sin(3x) \cos\left(\frac{1}{2}x\right) + \cos(3x) \sin\left(\frac{1}{2}x\right) = \sin\left(3\frac{1}{2}x\right)$$

$$G(x) = -\frac{2}{7}\cos\left(3\frac{1}{2}x\right) + c$$

$$\begin{aligned} \text{c} \quad & h(x) = 4\cos^4(x) - \cos^2(2x) = (2\cos^2(x))^2 - \cos^2(2x) = (1 + \cos(2x))^2 - \cos^2(2x) \\ & = 1 + 2\cos(2x) + \cos^2(2x) - \cos^2(2x) = 1 + 2\cos(2x) \end{aligned}$$

$$H(x) = x + \sin(2x) + c$$

35 a $f(x) = \sin(x) - \cos(x) + \sqrt{2}$ geeft $f'(x) = \cos(x) + \sin(x)$
 $f'(x) = 0$ geeft $\cos(x) + \sin(x) = 0$
 $\sin(x) = -\cos(x)$
 $\tan(x) = -1$
 $x = -\frac{1}{4}\pi + k \cdot \pi$

$$f(-\frac{1}{4}\pi) = \sin(-\frac{1}{4}\pi) - \cos(-\frac{1}{4}\pi) + \sqrt{2} = -\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} + \sqrt{2} = 0$$

$f(-\frac{1}{4}\pi) = 0$ en $f'(-\frac{1}{4}\pi) = 0$, dus de grafiek van f raakt de x -as.

b $f(\frac{1}{4}\pi - p) + f(\frac{1}{4}\pi + p) = \sin(\frac{1}{4}\pi - p) - \cos(\frac{1}{4}\pi - p) + \sqrt{2} + \sin(\frac{1}{4}\pi + p) - \cos(\frac{1}{4}\pi + p) + \sqrt{2}$
 $= \sin(\frac{1}{4}\pi)\cos(p) - \cos(\frac{1}{4}\pi)\sin(p) - (\cos(\frac{1}{4}\pi)\cos(p) + \sin(\frac{1}{4}\pi)\sin(p)) + \sqrt{2}$
 $+ \sin(\frac{1}{4}\pi)\cos(p) + \cos(\frac{1}{4}\pi)\sin(p) - (\cos(\frac{1}{4}\pi)\cos(p) - \sin(\frac{1}{4}\pi)\sin(p)) + \sqrt{2}$
 $= \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) - \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2}$
 $+ \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) - \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2} = 2\sqrt{2} = 2v_A$

Dus de grafiek van f is puntsymmetrisch in $A(\frac{1}{4}\pi, \sqrt{2})$.

c $f(\frac{3}{4}\pi - p) = \sin(\frac{3}{4}\pi - p) - \cos(\frac{3}{4}\pi - p) + \sqrt{2}$
 $= \sin(\frac{3}{4}\pi)\cos(p) - \cos(\frac{3}{4}\pi)\sin(p) - (\cos(\frac{3}{4}\pi)\cos(p) + \sin(\frac{3}{4}\pi)\sin(p)) + \sqrt{2}$
 $= \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) + \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2}$
 $= \sqrt{2} \cdot \cos(p) + \sqrt{2}$
 $f(\frac{3}{4}\pi + p) = \sin(\frac{3}{4}\pi + p) - \cos(\frac{3}{4}\pi + p) + \sqrt{2}$
 $= \sin(\frac{3}{4}\pi)\cos(p) + \cos(\frac{3}{4}\pi)\sin(p) - (\cos(\frac{3}{4}\pi)\cos(p) - \sin(\frac{3}{4}\pi)\sin(p)) + \sqrt{2}$
 $= \frac{1}{2}\sqrt{2} \cdot \cos(p) - \frac{1}{2}\sqrt{2} \cdot \sin(p) + \frac{1}{2}\sqrt{2} \cdot \cos(p) + \frac{1}{2}\sqrt{2} \cdot \sin(p) + \sqrt{2}$
 $= \sqrt{2} \cdot \cos(p) + \sqrt{2}$

$$f(\frac{3}{4}\pi - p) = f(\frac{3}{4}\pi + p)$$

Dus de lijn $x = \frac{3}{4}\pi$ is symmetrieas van de grafiek van f .

36 a $\vec{v}(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} -ab \sin(bt) \\ ab \cos(bt) \end{pmatrix}$
 $v(t) = \sqrt{(-ab \sin(bt))^2 + (ab \cos(bt))^2} = \sqrt{a^2 b^2 \sin^2(bt) + a^2 b^2 \cos^2(bt)}$
 $= \sqrt{a^2 b^2 (\sin^2(bt) + \cos^2(bt))} = \sqrt{a^2 b^2} = ab$

b $\vec{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix} = \begin{pmatrix} -ab^2 \cos(bt) \\ -ab^2 \sin(bt) \end{pmatrix}$
 $\vec{v}(t) \cdot \vec{a}(t) = \begin{pmatrix} -ab \sin(bt) \\ ab \cos(bt) \end{pmatrix} \cdot \begin{pmatrix} -ab^2 \cos(bt) \\ -ab^2 \sin(bt) \end{pmatrix} = a^2 b^3 \sin(bt) \cos(bt) - a^2 b^3 \cos(bt) \sin(bt) = 0$

Dus $\vec{v}(t) \perp \vec{a}(t)$.

c $|\vec{a}(t)| = \sqrt{(-ab^2 \cos(bt))^2 + (-ab^2 \sin(bt))^2} = \sqrt{a^2 b^4 \cos^2(bt) + a^2 b^4 \sin^2(bt)}$
 $= \sqrt{a^2 b^4 (\cos^2(bt) + \sin^2(bt))} = \sqrt{a^2 b^4} = ab^2$

$$v = 10 \text{ en } |\vec{a}(t)| = 5 \text{ geeft } ab = 10 \text{ en } ab^2 = 5$$

$$\frac{ab^2}{ab} = \frac{5}{10}$$

$$b = \frac{1}{2}$$

$$b = \frac{1}{2} \text{ en } ab = 10 \text{ geeft } a = 20$$

Dus de straal van de cirkel is 20.

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37 a $f(x) = 2 \cos^2(x) + \sin(2x) = 2 \cos^2(x) - 1 + 1 + \sin(2x) = \cos(2x) + \sin(2x) + 1$
 $f(x) = \cos(2x) + \sin(2x) + 1$ geeft $f'(x) = -2 \sin(2x) + 2 \cos(2x)$
 $f'(x) = 0$ geeft $-2 \sin(2x) + 2 \cos(2x) = 0$
 $\sin(2x) = \cos(2x)$
 $\tan(2x) = 1$
 $2x = \frac{1}{4}\pi + k \cdot \pi$
 $x = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$
 x in $[0, \pi]$ geeft $x = \frac{1}{8}\pi \vee x = \frac{5}{8}\pi$

$$\text{max. is } f(\frac{1}{8}\pi) = \cos(\frac{1}{4}\pi) + \sin(\frac{1}{4}\pi) + 1 = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2} + 1 = 1 + \sqrt{2}$$

$$\text{min. is } f(\frac{5}{8}\pi) = \cos(\frac{1}{4}\pi) + \sin(\frac{1}{4}\pi) + 1 = -\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2} + 1 = 1 - \sqrt{2}$$

$$\text{Dus } B_f = [1 - \sqrt{2}, 1 + \sqrt{2}].$$

b $f(x) = g(x)$ geeft $2 \cos^2(x) + \sin(2x) = 2 \sin(2x)$

$$2 \cos^2(x) = \sin(2x)$$

$$2 \cos^2(x) = 2 \sin(x) \cos(x)$$

$$2 \cos(x) = 0 \vee \cos(x) = \sin(x)$$

$$\cos(x) = 0 \vee \tan(x) = 1$$

$$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{4}\pi + k \cdot \pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{4}\pi \vee x = \frac{1}{2}\pi$$

$$O(V) = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2 \sin(2x) - (2 \cos^2(x) + \sin(2x))) dx = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2 \sin(2x) - (\cos(2x) + \sin(2x) + 1)) dx$$

$$= \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (2 \sin(2x) - \cos(2x) - \sin(2x) - 1) dx = \int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} (\sin(2x) - \cos(2x) - 1) dx$$

$$= \left[-\frac{1}{2} \cos(2x) - \frac{1}{2} \sin(2x) - x \right]_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} = -\frac{1}{2} \cos(\pi) - \frac{1}{2} \sin(\pi) - \frac{1}{2} \pi - \left(-\frac{1}{2} \cos\left(\frac{1}{2}\pi\right) - \frac{1}{2} \sin\left(\frac{1}{2}\pi\right) - \frac{1}{4}\pi \right)$$

$$= \frac{1}{2} - 0 - \frac{1}{2}\pi + 0 + \frac{1}{2} + \frac{1}{4}\pi = 1 - \frac{1}{4}\pi$$

c C is het midden van AB als $g(p) = \frac{1}{2}f(p)$.

Dit geeft $2 \sin(2p) = \cos^2(p) + \frac{1}{2} \sin(2p)$

$$\frac{3}{2} \sin(2p) = \cos^2(p)$$

$$3 \sin(p) \cos(p) = \cos^2(p)$$

$$\cos(p) = 0 \vee 3 \sin(p) = \cos(p)$$

$$p = \frac{1}{2}\pi + k \cdot \pi \vee 3 \cdot \frac{\sin(p)}{\cos(p)} = 1$$

$$\text{vold. niet} \quad \tan(p) = \frac{1}{3}$$

38 a $u_P = 25 \sin(8\pi t)$

Per trilling wordt $4 \cdot 25 = 100$ mm afgelegd.

Er zijn $\frac{8\pi}{2\pi} = 4$ trillingen per seconde,

dus P legt per minuut $60 \cdot 4 \cdot 100 = 24\,000$ mm = 24 m af.

$$u_Q = 40 \sin(6\pi t).$$

Per trilling wordt $4 \cdot 40 = 160$ mm afgelegd.

Er zijn $\frac{6\pi}{2\pi} = 3$ trillingen per seconde,

dus Q legt per minuut $60 \cdot 3 \cdot 160 = 28\,800$ mm = 28,8 m af.

Het punt Q legt de grootste afstand per minuut af. Het verschil is 4,8 m.

b Op $t = 0$ beginnen P en Q beide aan een nieuwe trilling.

P heeft trillingstijd $\frac{1}{4}$ seconde en Q heeft trillingstijd $\frac{1}{3}$ seconde. Dus na 1 seconde heeft P 4 trillingen gemaakt en Q 3 trillingen, dus beginnen P en Q na 1 seconde beide de tweede keer aan een nieuwe trilling.

Dus op $t = 1$.

c De afstand tussen P en Q is $|25 \sin(8\pi t) - 40 \sin(6\pi t)|$.

Voer in $y_1 = |25 \sin(8\pi x) - 40 \sin(6\pi x)|$ en $y_2 = 30$.

De optie snijpunt geeft $x = 0,2360\dots$

Dus het eerste tijdstip is $t \approx 0,236$.

39 a $f(x) = 0$ geeft $2 \sin^2(x) + \sin(x) = 0$

$$\sin(x)(2 \sin(x) + 1) = 0$$

$$\sin(x) = 0 \vee \sin(x) = -\frac{1}{2}$$

$$x = k \cdot \pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \pi \vee x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = 2\pi$$

$$O = \int_0^\pi f(x) dx = \int_0^\pi (2 \sin^2(x) + \sin(x)) dx = \int_0^\pi (2 \sin^2(x) - 1 + 1 + \sin(x)) dx$$

$$= \int_0^\pi (-\cos(2x) + \sin(x) + 1) dx = \left[-\frac{1}{2} \sin(2x) - \cos(x) + x \right]_0^\pi$$

$$= -\frac{1}{2} \sin(2\pi) - \cos(\pi) + \pi - \left(-\frac{1}{2} \sin(0) - \cos(0) + 0 \right) = 0 - (-1) + \pi + 0 + 1 = 2 + \pi$$

- b** $f(x) = 2 \sin^2(x) + \sin(x)$ geeft $f'(x) = 4 \sin(x) \cos(x) + \cos(x)$
 $f'(x) = 0$ geeft $4 \sin(x) \cos(x) + \cos(x) = 0$
 $\cos(x)(4 \sin(x) + 1) = 0$
 $\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$
 $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee \sin(x) = -\frac{1}{4}$
 $f(\frac{1}{2}\pi) = 2 \sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) = 2 \cdot 1 + 1 = 3$
 $f(1\frac{1}{2}\pi) = 2 \sin^2(1\frac{1}{2}\pi) + \sin(1\frac{1}{2}\pi) = 2 \cdot 1 - 1 = 1$
 $\sin(x) = -\frac{1}{4}$ geeft $f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} = -\frac{1}{8}$
 $f(x) = p$ heeft precies vier oplossingen voor $-\frac{1}{8} < p < 0 \vee 0 < p < 1$.
- c** $f(\frac{1}{2}\pi - p) = 2 \sin^2(\frac{1}{2}\pi - p) + \sin(\frac{1}{2}\pi - p)$
 $= 2(\sin(\frac{1}{2}\pi) \cos(p) - \cos(\frac{1}{2}\pi) \sin(p))^2 + \sin(\frac{1}{2}\pi) \cos(p) - \cos(\frac{1}{2}\pi) \sin(p)$
 $= 2(1 \cdot \cos(p) - 0)^2 + 1 \cdot \cos(p) - 0$
 $= 2 \cos^2(p) + \cos(p)$
 $f(\frac{1}{2}\pi + p) = 2 \sin^2(\frac{1}{2}\pi + p) + \sin(\frac{1}{2}\pi + p)$
 $= 2(\sin(\frac{1}{2}\pi) \cos(p) + \cos(\frac{1}{2}\pi) \sin(p))^2 + \sin(\frac{1}{2}\pi) \cos(p) + \cos(\frac{1}{2}\pi) \sin(p)$
 $= 2(1 \cdot \cos(p) + 0)^2 + 1 \cdot \cos(p) + 0$
 $= 2 \cos^2(p) + \cos(p)$
 Er geldt $f(\frac{1}{2}\pi - p) = f(\frac{1}{2}\pi + p)$, dus de grafiek van f is symmetrisch op het interval $[0, \pi]$.
 $p = f(\frac{1}{2}\pi - \frac{1}{3}\pi) = f(\frac{1}{6}\pi) = 2 \cdot (\frac{1}{2})^2 + \frac{1}{2} = 1$

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- a** $\begin{cases} x(t) = \sin(t) + \cos(t) \\ y(t) = 1 + 2 \sin(2t + \pi) \end{cases}$
 $x^2 = (\sin(t) + \cos(t))^2 = \sin^2(t) + 2 \sin(t) \cos(t) + \cos^2(t) = 1 + \sin(2t)$
 $y = 1 + 2 \sin(2t + \pi) = 1 - 2 \sin(2t)$
 $x^2 = 1 + \sin(2t)$ geeft $\sin(2t) = x^2 - 1$ $\left. \begin{matrix} y = 1 - 2(x^2 - 1) \\ y = 1 - 2x^2 + 2 \\ y = -2x^2 + 3 \end{matrix} \right\}$

Dus de baan van P is een deel van de bergparabool $y = -2x^2 + 3$.

Dus $a = -2$ en $b = 3$.

- b** $\begin{cases} x^2 = 1 + \sin(2t) \\ -1 \leq \sin(2t) \leq 1 \end{cases} \left\{ \begin{matrix} 0 \leq x^2 \leq 2 \\ -\sqrt{2} \leq x \leq \sqrt{2} \end{matrix} \right.$
 $x = -\sqrt{2}$ en $y = -2x^2 + 3$ geeft $y = -1$
 $x = \sqrt{2}$ en $y = -2x^2 + 3$ geeft $y = -1$
 Dus de keerpunten zijn $(-\sqrt{2}, -1)$ en $(\sqrt{2}, -1)$.

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- a** De baan van P is de cirkel met middelpunt O en straal 4, de baan van Q is de cirkel met middelpunt O en straal 5.

De minimale afstand tussen P en Q is $5 - 4 = 1$ voor $5t = 3t + k \cdot 2\pi$

$$2t = k \cdot 2\pi$$

$$t = k \cdot \pi$$

Dus de minimale afstand wordt bereikt voor $t = 0$, $t = \pi$ en $t = 2\pi$.

De maximale afstand tussen P en Q is $5 + 4 = 9$ voor $5t = 3t + \pi + k \cdot 2\pi$

$$2t = \pi + k \cdot 2\pi$$

$$t = \frac{1}{2}\pi + k \cdot \pi$$

Dus de maximale afstand wordt bereikt voor $t = \frac{1}{2}\pi$ en $t = 1\frac{1}{2}\pi$.

- b** $PQ^2 = (5 \cos(3t) - 4 \cos(5t))^2 + (5 \sin(3t) - 4 \sin(5t))^2$
 $= 25 \cos^2(3t) - 40 \cos(3t) \cos(5t) + 16 \cos^2(5t) + 25 \sin^2(3t) - 40 \sin(3t) \sin(5t) + 16 \sin^2(5t)$
 $= 25(\cos^2(3t) + \sin^2(3t)) + 16(\cos^2(5t) + \sin^2(5t)) - 40(\cos(3t) \cos(5t) + \sin(3t) \sin(5t))$
 $= 25 + 16 - 40 \cos(3t - 5t) = 41 - 40 \cos(-2t) = 41 - 40 \cos(2t) = 41 - 40(1 - 2 \sin^2(t))$
 $= 41 - 40 + 80 \sin^2(t) = 1 + 80 \sin^2(t)$

Dus $PQ = \sqrt{1 + 80 \sin^2(t)}$.

c $PQ = \sqrt{41}$
 $\sqrt{1 + 80 \sin^2(t)} = \sqrt{41}$
 $1 + 80 \sin^2(t) = 41$
 $80 \sin^2(t) = 40$
 $\sin^2(t) = \frac{1}{2}$
 $\sin(t) = \frac{1}{\sqrt{2}} \vee \sin(t) = -\frac{1}{\sqrt{2}}$
 $t = \frac{1}{4}\pi + k \cdot 2\pi \vee t = \frac{3}{4}\pi + k \cdot 2\pi \vee t = -\frac{1}{4}\pi + k \cdot 2\pi \vee t = 1\frac{1}{4}\pi + k \cdot 2\pi$
 t in $[0, 2\pi]$ geeft $t = \frac{1}{4}\pi \vee t = \frac{3}{4}\pi \vee t = 1\frac{1}{4}\pi \vee t = 1\frac{3}{4}\pi$

d Evenwijdig met de y -as, dus $x_P = x_Q$
 $4 \cos(5t) = 5 \cos(3t)$
 Voer in $y_1 = 4 \cos(5x)$ en $y_2 = 5 \cos(3x)$.
 De optie snijpunt geeft $x = 0,756...$
 Dus voor $t \approx 0,76$.

42 a $y = 0$ geeft $\cos(3t) = 0$
 $3t = \frac{1}{2}\pi + k \cdot \pi$
 $t = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$
 $t = \frac{1}{6}\pi$ geeft het punt $(\frac{1}{2}, 0)$.
 $t = \frac{1}{2}\pi$ geeft het punt $(-1, 0)$.
 $t = \frac{5}{6}\pi$ geeft het punt $(\frac{1}{2}, 0)$.
 $\begin{cases} x(t) = \cos(2t) \\ y(t) = \cos(3t) \end{cases}$ geeft $\begin{cases} x'(t) = -2 \sin(2t) \\ y'(t) = -3 \sin(3t) \end{cases}$ en dus $\vec{v}(t) = \begin{pmatrix} -2 \sin(2t) \\ -3 \sin(3t) \end{pmatrix}$.
 $\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} -2 \sin(\frac{1}{3}\pi) \\ -3 \sin(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} -2 \cdot \frac{1}{2}\sqrt{3} \\ -3 \cdot 1 \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$
 $\vec{v}(\frac{5}{6}\pi) = \begin{pmatrix} -2 \sin(1\frac{2}{3}\pi) \\ -3 \sin(2\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} -2 \cdot -\frac{1}{2}\sqrt{3} \\ -3 \cdot 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$
 $\cos(\varphi) = \frac{\left| \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \right|} = \frac{|1 - 3|}{\sqrt{4} \cdot \sqrt{4}} = \frac{2}{4} = \frac{1}{2}$

Dus $\varphi = 60^\circ$.

b $x = 0$ geeft $\cos(2t) = 0$
 $2t = \frac{1}{2}\pi + k \cdot \pi$
 $t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$
 $t = \frac{1}{4}\pi$ geeft het punt $(0, -\frac{1}{2}\sqrt{2})$.
 $t = \frac{3}{4}\pi$ geeft het punt $(0, \frac{1}{2}\sqrt{2})$.
 $\vec{v}(\frac{3}{4}\pi) = \begin{pmatrix} -2 \cdot -1 \\ -3 \cdot \frac{1}{2}\sqrt{2} \end{pmatrix} = \begin{pmatrix} 2 \\ -1\frac{1}{2}\sqrt{2} \end{pmatrix}$
 Stel $k: y = ax + b$ met $a = \frac{-1\frac{1}{2}\sqrt{2}}{2} = -\frac{3}{4}\sqrt{2}$ en k door $(0, \frac{1}{2}\sqrt{2})$.
 Dus $k: y = -\frac{3}{4}\sqrt{2} \cdot x + \frac{1}{2}\sqrt{2}$.

c De baansnelheid is $v(\frac{1}{2}\pi) = \sqrt{(-2 \sin(\pi))^2 + (-3 \sin(1\frac{1}{2}\pi))^2} = \sqrt{0 + 9} = 3$.

d $d(A, B) = 1$, dus A op de lijn $y = \frac{1}{2}$, omdat de baan symmetrisch is in de x -as.
 $\cos(3t) = \frac{1}{2}$
 $3t = \frac{1}{3}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{3}\pi + k \cdot 2\pi$
 $t = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \vee t = -\frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$
 t in $[0, \pi]$ geeft $t = \frac{1}{9}\pi \vee t = \frac{7}{9}\pi \vee t = \frac{5}{9}\pi$
 $x(\frac{1}{9}\pi) = \cos(\frac{2}{9}\pi) = 0,766...$
 $x(\frac{7}{9}\pi) = \cos(\frac{14}{9}\pi) = 0,173...$
 $x(\frac{5}{9}\pi) = \cos(\frac{10}{9}\pi) = -0,939...$
 Dus $p \approx -0,94 \vee p \approx 0,17 \vee p \approx 0,77$.

43 a $y = 0$ geeft $2 \cos(2t) = 0$

$$\cos(2t) = 0$$

$$2t = \frac{1}{2}\pi + k \cdot \pi$$

$$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

$t = -\frac{1}{4}\pi$ geeft het punt $(\frac{1}{2}\sqrt{2}, 0)$.

$$\begin{cases} x(t) = 3 \cos(t + \frac{1}{2}\pi) \\ y(t) = 2 \cos(2t) \end{cases} \text{ geeft } \begin{cases} x'(t) = -3 \sin(t + \frac{1}{2}\pi) \\ y'(t) = -4 \sin(2t) \end{cases}$$

$$\text{Dus } \vec{v}(t) = \begin{pmatrix} -3 \sin(t + \frac{1}{2}\pi) \\ -4 \sin(2t) \end{pmatrix}.$$

$$\vec{v}(-\frac{1}{4}\pi) = \begin{pmatrix} -3 \cdot \frac{1}{2}\sqrt{2} \\ -4 \cdot -1 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2}\sqrt{2} \\ 4 \end{pmatrix}$$

$$\tan(\varphi) = \frac{4}{-1\frac{1}{2}\sqrt{2}} \text{ geeft } \varphi = -62,061\dots^\circ$$

Dus de hoek waaronder de baan de positieve x -as snijdt is ongeveer $62,1^\circ$.

b De baansnelheid is $v(-\frac{1}{4}\pi) = \sqrt{(-1\frac{1}{2}\sqrt{2})^2 + 4^2} = \sqrt{4\frac{1}{2} + 16} = \sqrt{\frac{41}{2}} = \frac{1}{2}\sqrt{82}$

c $x = 0$ geeft $3 \cos(t + \frac{1}{2}\pi) = 0$

$$\cos(t + \frac{1}{2}\pi) = 0$$

$$t + \frac{1}{2}\pi = \frac{1}{2}\pi + k \cdot \pi$$

$$t = k \cdot \pi$$

$t = 0$ geeft $y = 2$, dus de baan snijdt de y -as in het punt $(0, 2)$.

Dus de vergelijking van de parabool is van de vorm $y = ax^2 + 2$.

$$\begin{cases} y = ax^2 + 2 \\ t = \frac{1}{2}\pi \text{ geeft het punt } (-3, -2) \end{cases} \Rightarrow \begin{cases} a \cdot (-3)^2 + 2 = -2 \\ 9a = -4 \\ a = -\frac{4}{9} \end{cases}$$

$$\text{Dus } y = -\frac{4}{9}x^2 + 2.$$

Substitutie van $x = 3 \cos(t + \frac{1}{2}\pi)$ en $y = 2 \cos(2t)$ in $y = -\frac{4}{9}x^2 + 2$ geeft

$$2 \cos(2t) = -\frac{4}{9} \cdot (3 \cos(t + \frac{1}{2}\pi))^2 + 2$$

$$2 \cos(2t) = -\frac{4}{9} \cdot 9(\cos(t + \frac{1}{2}\pi))^2 + 2$$

$$2 \cos(2t) = -4(\sin(t + \pi))^2 + 2$$

$$\cos(2t) = -2(-\sin(t))^2 + 1$$

$$\cos(2t) = 1 - 2 \sin^2(t)$$

Dit klopt voor elke t , dus de baan van P is een deel van de parabool $y = -\frac{4}{9}x^2 + 2$.

d $\vec{p} = \begin{pmatrix} 3 \cos(t + \frac{1}{2}\pi) \\ 2 \cos(2t) \end{pmatrix}$

$$\vec{PQ} = \vec{p}_L = \begin{pmatrix} -2 \cos(2t) \\ 3 \cos(t + \frac{1}{2}\pi) \end{pmatrix}$$

$$\vec{q} = \vec{p} + \vec{p}_L = \begin{pmatrix} 3 \cos(t + \frac{1}{2}\pi) \\ 2 \cos(2t) \end{pmatrix} + \begin{pmatrix} -2 \cos(2t) \\ 3 \cos(t + \frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 3 \cos(t + \frac{1}{2}\pi) - 2 \cos(2t) \\ 2 \cos(2t) + 3 \cos(t + \frac{1}{2}\pi) \end{pmatrix}$$

$$\text{Dus de bewegingsvergelijkingen van } Q \text{ zijn } \begin{cases} x_Q = 3 \cos(t + \frac{1}{2}\pi) - 2 \cos(2t) \\ y_Q = 2 \cos(2t) + 3 \cos(t + \frac{1}{2}\pi) \end{cases}$$

e Q op de y -as geeft $x_Q = 0$

$$3 \cos(t + \frac{1}{2}\pi) - 2 \cos(2t) = 0$$

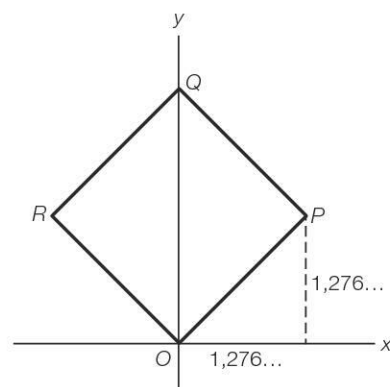
$$3 \cos(t + \frac{1}{2}\pi) = 2 \cos(2t)$$

Voer in $y_1 = 3 \cos(x + \frac{1}{2}\pi)$ en $y_2 = 2 \cos(2x)$.

De optie snijpunt geeft $x = -0,439\dots$

Dus $t = -0,439\dots$ en dit geeft $P(1,276\dots; 1,276\dots)$.

$$O(OPQR) = OP^2 = 1,276\dots^2 + 1,276\dots^2 \approx 3,26.$$



44 a $f(x) = \frac{x+2}{x^2-4x+8} = \frac{x-2+4}{x^2-4x+8} = \frac{\frac{1}{2}(2x-4)+4}{x^2-4x+8} = \frac{\frac{1}{2}(2x-4)}{x^2-4x+8} + \frac{4}{x^2-4x+8}$

$$\int \frac{\frac{1}{2}(2x-4)}{x^2-4x+8} dx = \int \frac{1}{2} \cdot \frac{1}{x^2-4x+8} (2x-4) dx = \int \frac{1}{2} \cdot \frac{1}{x^2-4x+8} d(x^2-4x+8) = \frac{1}{2} \ln|x^2-4x+8| + c$$

$$\int \frac{4}{x^2-4x+8} dx = \int \frac{4}{(x-2)^2-4+8} dx = \int \frac{1}{\frac{1}{4}(x-2)^2+1} dx = \int \frac{1}{(\frac{1}{2}(x-2))^2+1} dx$$

$$= \int \frac{1}{(\frac{1}{2}x-1)^2+1} dx = 2 \arctan(\frac{1}{2}x-1) + c$$

Dus $F(x) = \frac{1}{2} \ln|x^2-4x+8| + 2 \arctan(\frac{1}{2}x-1) + c$.

b $F(x) = \int 3x^2 \sin(x^3) dx = \int \sin(x^3) dx^3 = -\cos(x^3) + c$

c $F(x) = \int \cos(x) \cdot \sqrt[3]{1+\sin(x)} dx = \int \sqrt[3]{1+\sin(x)} d\sin(x) = \int \sqrt[3]{1+u} du = \int (1+u)^{\frac{1}{3}} du$

$$= \frac{3}{4}(1+u)^{\frac{4}{3}} + c = \frac{3}{4}(1+u) \cdot \sqrt[3]{1+u} + c = \frac{3}{4}(1+\sin(x)) \cdot \sqrt[3]{1+\sin(x)} + c$$

d $F(x) = \int (2x+4) \cos(2x) dx = \int (2x+4) d\frac{1}{2}\sin(2x) = (2x+4) \cdot \frac{1}{2}\sin(2x) - \int \frac{1}{2}\sin(2x) d(2x+4)$

$$= (x+2) \sin(2x) - \int \sin(2x) dx = (x+2) \sin(2x) + \frac{1}{2} \cos(2x) + c$$

e $F(x) = \int \frac{2x+1}{\sqrt{16-x^2}} dx = \int \left(\frac{2x}{\sqrt{16-x^2}} + \frac{1}{\sqrt{16-x^2}} \right) dx$

$$= \int -\frac{1}{\sqrt{16-x^2}} d(16-x^2) + \int \frac{\frac{1}{4}}{\sqrt{1-(\frac{1}{4}x)^2}} dx$$

$$= \int -\frac{1}{\sqrt{u}} du + \arcsin(\frac{1}{4}x) = -2\sqrt{u} + \arcsin(\frac{1}{4}x) + c = -2\sqrt{16-x^2} + \arcsin(\frac{1}{4}x) + c$$

f $F(x) = \int \frac{2x+4}{\sqrt{-x^2-4x-3}} dx = \int \frac{-1}{\sqrt{-x^2-4x-3}} d(-x^2-4x-3) = \int \frac{-1}{\sqrt{u}} du = -2\sqrt{u} + c$

$$= -2\sqrt{-x^2-4x-3} + c$$

g $f(x) = \frac{2x+7}{x^2+6x+8} = \frac{2x+7}{(x+2)(x+4)} = \frac{a}{x+2} + \frac{b}{x+4} = \frac{a(x+4)+b(x+2)}{(x+2)(x+4)} = \frac{(a+b)x+4a+2b}{(x+2)(x+4)}$

$$\begin{cases} a+b=2 \\ 4a+2b=7 \end{cases} \left| \begin{array}{r} 2 \\ 1 \end{array} \right| \text{ geeft } \begin{cases} 2a+2b=4 \\ 4a+2b=7 \\ -2a=-3 \\ a=1\frac{1}{2} \\ a+b=2 \end{cases} \left| \begin{array}{r} 1\frac{1}{2} \\ 2 \end{array} \right| \begin{cases} 1\frac{1}{2}+b=2 \\ b=\frac{1}{2} \end{cases}$$

$f(x) = \frac{1\frac{1}{2}}{x+2} + \frac{\frac{1}{2}}{x+4}$ geeft $F(x) = 1\frac{1}{2} \ln|x+2| + \frac{1}{2} \ln|x+4| + c$

45 a $f(x) = 4 \sin^2(x) \cos(x)$ geeft

$$f'(x) = 8 \sin(x) \cos(x) \cdot \cos(x) + 4 \sin^2(x) \cdot -\sin(x) = 8 \sin(x) \cos^2(x) - 4 \sin^3(x)$$

$$f(x) = 0 \text{ geeft } 4 \sin^2(x) \cdot \cos(x) = 0$$

$$\sin(x) = 0 \vee \cos(x) = 0$$

$$x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$$

$f'(0) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$

$f'(\pi) = 8 \cdot 0 \cdot (-1)^2 - 4 \cdot 0 = 0$

$f'(2\pi) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$

Dus de grafiek raakt de x -as in $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.

b Stel $k: y = ax + b$ met $a = f'(\frac{1}{2}\pi) = 8 \cdot 1 \cdot 0 - 4 \cdot 1 = -4$.

$$\left. \begin{array}{l} y = -4x + b \\ f(\frac{1}{2}\pi) = 0, \text{ dus } A(\frac{1}{2}\pi, 0) \end{array} \right\} \begin{array}{l} -4 \cdot \frac{1}{2}\pi + b = 0 \\ b = 2\pi \end{array}$$

Dus $k: y = -4x + 2\pi$.

$$\text{c } O(V) = \int_0^{\frac{1}{2}\pi} 4 \sin^2(x) \cos(x) dx = \int_{x=0}^{x=\frac{1}{2}\pi} 4 \sin^2(x) d \sin(x) = \left[\frac{4}{3} \sin^3(x) \right]_0^{\frac{1}{2}\pi} = \frac{4}{3} \cdot 1 - \frac{4}{3} \cdot 0 = 1\frac{1}{3}$$

46 a $f(x) = \frac{2}{3}$ geeft $\frac{2x}{x^2 - 4} = \frac{2}{3}$

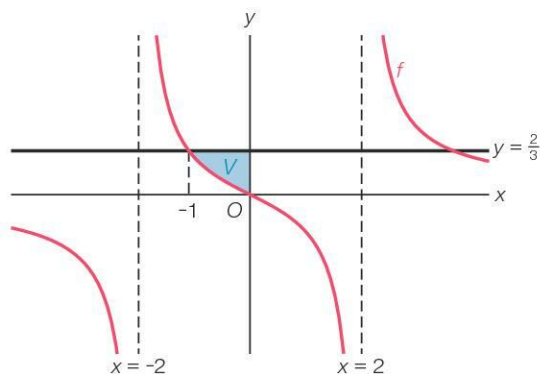
$$2x^2 - 8 = 6x$$

$$2x^2 - 6x - 8 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x + 1)(x - 4) = 0$$

$$x = -1 \vee x = 4$$



$$O(V) = \int_{-1}^4 \left(\frac{2}{3} - \frac{2x}{x^2 - 4} \right) dx$$

$$\int \frac{2x}{x^2 - 4} dx = \int \frac{1}{x^2 - 4} 2x dx = \int \frac{1}{x^2 - 4} d(x^2 - 4) = \ln|x^2 - 4| + c$$

$$\text{Dus } O(V) = \left[\frac{2}{3}x - \ln|x^2 - 4| \right]_{-1}^4 = 0 - \ln(4) - \left(-\frac{2}{3} - \ln(3) \right) = -\ln(4) + \frac{2}{3} + \ln(3) = \frac{2}{3} + \ln\left(\frac{3}{4}\right).$$

b $\int_p^{p+4} f(x) dx = \int_p^{p+4} \frac{2x}{x^2 - 4} dx = \left[\ln(x^2 - 4) \right]_p^{p+4} = \ln((p+4)^2 - 4) - \ln(p^2 - 4)$

$$= \ln(p^2 + 8p + 16 - 4) - \ln(p^2 - 4) = \ln\left(\frac{p^2 + 8p + 12}{p^2 - 4}\right)$$

Omdat $p^2 - 4 > 0$ als $p > 2$, kan de modulus wegblijven.

$$\int_p^{p+4} f(x) dx = \ln(2)$$

$$\ln\left(\frac{p^2 + 8p + 12}{p^2 - 4}\right) = \ln(2)$$

$$\frac{p^2 + 8p + 12}{p^2 - 4} = 2$$

$$p^2 + 8p + 12 = 2p^2 - 8$$

$$p^2 - 8p - 20 = 0$$

$$(p + 2)(p - 10) = 0$$

$$p = -2 \vee p = 10$$

vold. niet

Dus voor $p = 10$.

47

a $f(x) = \frac{x^3 + 10x}{x^2 + 1}$ geeft

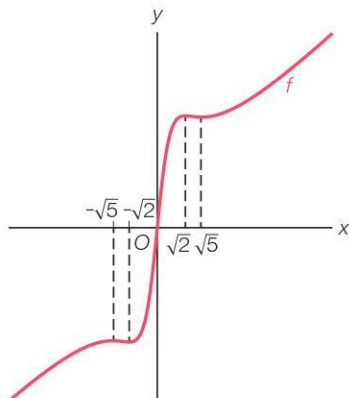
$$f'(x) = \frac{(x^2 + 1) \cdot (3x^2 + 10) - (x^3 + 10x) \cdot 2x}{(x^2 + 1)^2} = \frac{3x^4 + 10x^2 + 3x^2 + 10 - 2x^4 - 20x^2}{(x^2 + 1)^2} = \frac{x^4 - 7x^2 + 10}{(x^2 + 1)^2}$$

$$f'(x) = 0 \text{ geeft } x^4 - 7x^2 + 10 = 0$$

$$(x^2 - 2)(x^2 - 5) = 0$$

$$x^2 = 2 \vee x^2 = 5$$

$$x = \sqrt{2} \vee x = -\sqrt{2} \vee x = \sqrt{5} \vee x = -\sqrt{5}$$



$$f(\sqrt{2}) = \frac{(\sqrt{2})^3 + 10 \cdot \sqrt{2}}{(\sqrt{2})^2 + 1} = \frac{2\sqrt{2} + 10\sqrt{2}}{2 + 1} = \frac{12\sqrt{2}}{3} = 4\sqrt{2}$$

$$f(-\sqrt{2}) = \frac{(-\sqrt{2})^3 + 10 \cdot -\sqrt{2}}{(-\sqrt{2})^2 + 1} = \frac{-2\sqrt{2} - 10\sqrt{2}}{2 + 1} = \frac{-12\sqrt{2}}{3} = -4\sqrt{2}$$

$$f(\sqrt{5}) = \frac{(\sqrt{5})^3 + 10 \cdot \sqrt{5}}{(\sqrt{5})^2 + 1} = \frac{5\sqrt{5} + 10\sqrt{5}}{5 + 1} = \frac{15\sqrt{5}}{6} = 2\frac{1}{2}\sqrt{5}$$

$$f(-\sqrt{5}) = \frac{(-\sqrt{5})^3 + 10 \cdot -\sqrt{5}}{(-\sqrt{5})^2 + 1} = \frac{-5\sqrt{5} - 10\sqrt{5}}{5 + 1} = \frac{-15\sqrt{5}}{6} = -2\frac{1}{2}\sqrt{5}$$

De toppen zijn $(-\sqrt{5}, -2\frac{1}{2}\sqrt{5})$, $(-\sqrt{2}, -4\sqrt{2})$, $(\sqrt{2}, 4\sqrt{2})$ en $(\sqrt{5}, 2\frac{1}{2}\sqrt{5})$.

b Stel $k: y = ax + b$ met $a = f'(1) = \frac{1 - 7 + 10}{(1 + 1)^2} = 1$.

$$\left. \begin{array}{l} y = x + b \\ f(1) = 5\frac{1}{2}, \text{ dus } A(1, 5\frac{1}{2}) \end{array} \right\} \begin{array}{l} 1 + b = 5\frac{1}{2} \\ b = 4\frac{1}{2} \end{array}$$

Dus $k: y = x + 4\frac{1}{2}$.

c $f(x) = p$ heeft precies één oplossing voor $p < -4\sqrt{2} \vee -2\frac{1}{2}\sqrt{5} < p < 2\frac{1}{2}\sqrt{5} \vee p > 4\sqrt{2}$.

d $x^2 + 1 \mid x^3 + 10x \setminus x$

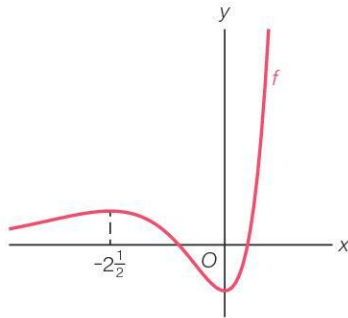
$$\frac{x^3 + x}{9x} -$$

$$f(x) = x + \frac{9x}{x^2 + 1}$$

$$\int \frac{9x}{x^2 + 1} dx = \int 4\frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot 2x dx = \int 4\frac{1}{2} \cdot \frac{1}{x^2 + 1} d(x^2 + 1) = 4\frac{1}{2} \ln|x^2 + 1| + c$$

$$\begin{aligned} O(V) &= \int_0^3 f(x) dx = \int_0^3 \left(x + \frac{9x}{x^2 + 1} \right) dx = \left[\frac{1}{2}x^2 + 4\frac{1}{2} \ln(x^2 + 1) \right]_0^3 \\ &= 4\frac{1}{2} + 4\frac{1}{2} \ln(10) - 0 = 4\frac{1}{2} + 4\frac{1}{2} \ln(10) \end{aligned}$$

- 48 a $f(x) = (2x^2 + x - 1)e^x$ geeft $f'(x) = (4x + 1) \cdot e^x + (2x^2 + x - 1) \cdot e^x = (2x^2 + 5x)e^x$
 $f'(x) = 0$ geeft $(2x^2 + 5x)e^x = 0$
 $2x^2 + 5x = 0$
 $x(2x + 5) = 0$
 $x = 0 \vee x = -2\frac{1}{2}$



$$f(0) = -1 \text{ en } f(-2\frac{1}{2}) = 9e^{-2\frac{1}{2}} = \frac{9}{e^{2\frac{1}{2}}} = \frac{9}{e^2 \cdot \sqrt{e}}$$

Dus de toppen zijn $(0, -1)$ en $(-2\frac{1}{2}, \frac{9}{e^2 \cdot \sqrt{e}})$.

- b $f(x) = 0$ geeft $(2x^2 + x - 1)e^x = 0$
 $2x^2 + x - 1 = 0$
 $D = 1^2 - 4 \cdot 2 \cdot (-1) = 9$
 $x = \frac{-1 \pm 3}{4} = \frac{1}{2} \vee x = \frac{-1 - 3}{4} = -1$

$$\begin{aligned} \int (2x^2 + x - 1)e^x dx &= \int (2x^2 + x - 1) d e^x = (2x^2 + x - 1)e^x - \int e^x d(2x^2 + x - 1) \\ &= (2x^2 + x - 1)e^x - \int (4x + 1)e^x dx = (2x^2 + x - 1)e^x - \int (4x + 1) d e^x \\ &= (2x^2 + x - 1)e^x - (4x + 1)e^x + \int e^x d(4x + 1) = (2x^2 + x - 1)e^x - (4x + 1)e^x + \int 4e^x dx \\ &= (2x^2 + x - 1)e^x - (4x + 1)e^x + 4e^x = (2x^2 - 3x + 2)e^x \end{aligned}$$

$$O(V) = -\int_{-1}^{\frac{1}{2}} f(x) dx = -[(2x^2 - 3x + 2)e^x]_{-1}^{\frac{1}{2}} = -(\frac{1}{2} - 1\frac{1}{2} + 2)e^{\frac{1}{2}} + (2 + 3 + 2)e^{-1} = -e^{\frac{1}{2}} + 7e^{-1} = \frac{7}{e} - \sqrt{e}$$

- c $-\int_{-1}^p f(x) dx = \frac{1}{2} \left(\frac{7}{e} - \sqrt{e} \right)$ geeft $-(2x^2 - 3x + 2)e^x \Big|_{-1}^p = \frac{7}{2e} - \frac{1}{2}\sqrt{e}$
 $-(2p^2 - 3p + 2)e^p + \frac{7}{e} = \frac{7}{2e} - \frac{1}{2}\sqrt{e}$
 $-(2p^2 - 3p + 2)e^p = -\frac{7}{2e} - \frac{1}{2}\sqrt{e}$
 $(2p^2 - 3p + 2)e^p = \frac{7}{2e} + \frac{1}{2}\sqrt{e}$

Voer in $y_1 = (2x^2 - 3x + 2)e^x$ en $y_2 = \frac{7}{2e} + \frac{1}{2}\sqrt{e}$.

De optie snijpunt geeft $x = -0,1130...$

Dus $p \approx -0,113$.

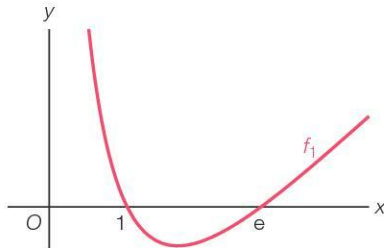
- 49 a $F(x) = \int x^3 \ln(x) dx = \int \ln(x) d(\frac{1}{4}x^4) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 d \ln(x) = \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x} dx$
 $= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^3 dx = \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + c$
- b $F(x) = \int x \ln(x^3) dx = \int 3x \ln(x) dx = \int 3 \ln(x) d(\frac{1}{2}x^2) = 3 \ln(x) \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 d 3 \ln(x)$
 $= 1\frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \frac{3}{x} dx = 1\frac{1}{2}x^2 \ln(x) - \int 1\frac{1}{2}x dx = 1\frac{1}{2}x^2 \ln(x) - \frac{3}{4}x^2 + c$
- c $F_n(x) = \int x^n \ln(x) dx = \int \ln(x) d(\frac{1}{n+1}x^{n+1}) = \frac{1}{n+1}x^{n+1} \ln(x) - \int \frac{1}{n+1}x^{n+1} d \ln(x)$
 $= \frac{1}{n+1}x^{n+1} \ln(x) - \int \frac{1}{n+1}x^{n+1} \cdot \frac{1}{x} dx = \frac{1}{n+1}x^{n+1} \ln(x) - \int \frac{1}{n+1}x^n dx$
 $= \frac{1}{n+1}x^{n+1} \ln(x) - \frac{1}{(n+1)^2}x^{n+1} + c$

d Er geldt $x > 0$, omdat anders x^m niet bestaat.

Uit c volgt $\int x \ln(x) dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + c$.

$$F_m(x) = \int x \ln(x^m) dx = \int mx \ln(x) dx = \frac{1}{2}mx^2 \ln(x) - \frac{1}{4}mx^2 + c$$

50 a $f_1(x) = 2 \ln^2(x) - 2 \ln(x)$
 $f_1(x) = 0$ geeft $2 \ln^2(x) - 2 \ln(x) = 0$
 $2 \ln(x)(\ln(x) - 1) = 0$
 $\ln(x) = 0 \vee \ln(x) = 1$
 $x = 1 \vee x = e$



$$\begin{aligned} \int 2 \ln^2(x) dx &= 2x \ln^2(x) - \int 2x d \ln^2(x) = 2x \ln^2(x) - \int 2x \cdot 2 \ln(x) \cdot \frac{1}{x} dx \\ &= 2x \ln^2(x) - \int 4 \ln(x) dx \\ &= 2x \ln^2(x) - 4(x \ln(x) - x) + c = 2x \ln^2(x) - 4x \ln(x) + 4x + c \\ \int 2 \ln(x) dx &= 2(x \ln(x) - x) + c = 2x \ln(x) - 2x + c \\ \int_1^e f_1(x) dx &= \int_1^e (2 \ln^2(x) - 2 \ln(x)) dx = [2x \ln^2(x) - 4x \ln(x) + 4x - (2x \ln(x) - 2x)]_1^e \\ &= [2x \ln^2(x) - 6x \ln(x) + 6x]_1^e = 2e - 6e + 6e - (0 - 0 + 6) = 2e - 6 \end{aligned}$$

De oppervlakte is $-(2e - 6) = 6 - 2e$.

b $f_p(x) = 0$ geeft $2 \ln^2(x) - 2p \ln(x) = 0$
 $2 \ln(x)(\ln(x) - p) = 0$
 $\ln(x) = 0 \vee \ln(x) = p$
 $x = 1 \vee x = e^p$

Uit a volgt $F_p(x) = 2x \ln^2(x) - 4x \ln(x) + 4x - 2px \ln(x) + 2px + c$.

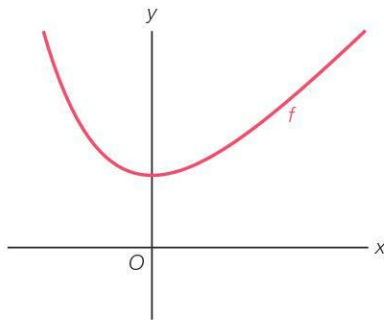
$$\begin{aligned} \int_1^{e^p} f_p(x) dx &= [2x \ln^2(x) - 4x \ln(x) + 4x - 2px \ln(x) + 2px]_1^{e^p} \\ &= 2e^p \cdot p^2 - 4e^p \cdot p + 4e^p - 2pe^p \cdot p + 2pe^p - (0 - 0 + 4 - 0 + 2p) \\ &= 2p^2e^p - 4pe^p + 4e^p - 2p^2e^p + 2pe^p - 4 - 2p \\ &= -2pe^p + 4e^p - 4 - 2p = (4 - 2p)e^p - 4 - 2p \end{aligned}$$

$f_p(x) \leq 0$ voor $1 \leq x \leq e^p$, dus opp $= (2p - 4)e^p + 2p + 4$.

opp $= 8$ geeft $(2p - 4)e^p + 2p + 4 = 8$
 $(2p - 4)e^p + 2p - 4 = 0$
 $(2p - 4)(e^p + 1) = 0$
 $2p - 4 = 0 \vee e^p + 1 = 0$
 $2p = 4$ geen opl.
 $p = 2$

Dus voor $p = 2$.

- 51 a** $f(x) = x + e^{-x}$ geeft $f'(x) = 1 - e^{-x}$
 $f'(x) = 0$ geeft $1 - e^{-x} = 0$
 $e^{-x} = 1$
 $-x = 0$
 $x = 0$



min. is $f(0) = 1$

$B_f = [1, \rightarrow)$

- b** $p > 0$

$$O(V_p) = 6 \text{ geeft } \int_{-p}^p (x + e^{-x}) dx = 6$$

$$\left[\frac{1}{2}x^2 - e^{-x} \right]_{-p}^p = 6$$

$$\frac{1}{2}p^2 - e^{-p} - \left(\frac{1}{2}p^2 - e^p \right) = 6$$

$$\frac{1}{2}p^2 - e^{-p} - \frac{1}{2}p^2 + e^p = 6$$

$$e^p - 6 - e^{-p} = 0$$

$$(e^p)^2 - 6e^p - 1 = 0$$

Stel $e^p = u$.

$$u^2 - 6u - 1 = 0$$

$$D = (-6)^2 - 4 \cdot 1 \cdot (-1) = 40$$

$$u = \frac{6 + 2\sqrt{10}}{2} \vee u = \frac{6 - 2\sqrt{10}}{2}$$

$$u = 3 + \sqrt{10} \vee u = 3 - \sqrt{10}$$

$$e^p = 3 + \sqrt{10} \vee e^p = 3 - \sqrt{10}$$

$$p = \ln(3 + \sqrt{10}) \text{ geen opl.}$$

Dus voor $p = \ln(3 + \sqrt{10})$ en voor $p = \ln(-3 + \sqrt{10})$.

c $I = \pi \int_{-1}^0 (f(x))^2 dx = \pi \int_{-1}^0 (x + e^{-x})^2 dx = \pi \int_{-1}^0 (x^2 + 2xe^{-x} + e^{-2x}) dx$

$$\int 2xe^{-x} dx = \int 2x d(-e^{-x}) = -2xe^{-x} + \int e^{-x} d2x = -2xe^{-x} + \int 2e^{-x} dx = -2xe^{-x} - 2e^{-x} + c$$

$$I = \pi \left[\frac{1}{3}x^3 - 2xe^{-x} - 2e^{-x} - \frac{1}{2}e^{-2x} \right]_{-1}^0$$

$$= \pi \left(0 - 0 - 2 - \frac{1}{2} \right) - \pi \left(-\frac{1}{3} + 2e - 2e - \frac{1}{2}e^2 \right)$$

$$= \pi \left(-2\frac{1}{2} + \frac{1}{3} + \frac{1}{2}e^2 \right) = \left(\frac{1}{2}e^2 - 2\frac{1}{6} \right) \pi$$

52 a $F(x) = \int \frac{1}{e^x + 1} dx = \int \frac{e^{-x}}{1 + e^{-x}} dx = \int \frac{-1}{1 + e^{-x}} d(1 + e^{-x}) = \int -\frac{1}{u} du = -\ln|u| + c = -\ln(1 + e^{-x}) + c$

b $F(x) = \int \frac{\ln(\sin(x))}{\cos^2(x)} dx = \int \ln(\sin(x)) d \tan(x) = \tan(x) \cdot \ln(\sin(x)) - \int \tan(x) d \ln(\sin(x))$

$$= \tan(x) \cdot \ln(\sin(x)) - \int \tan(x) \cdot \frac{1}{\sin(x)} \cdot \cos(x) dx$$

$$= \tan(x) \cdot \ln(\sin(x)) - \int \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\sin(x)} \cdot \cos(x) dx$$

$$= \tan(x) \cdot \ln(\sin(x)) - \int 1 dx$$

$$= \tan(x) \cdot \ln(\sin(x)) - x + c$$

$$\begin{aligned}
\text{c} \quad & \sqrt{2x+1} = u \\
& 2x+1 = u^2 \\
& 2x = u^2 - 1 \\
& x = \frac{1}{2}u^2 - \frac{1}{2} \\
& F(x) = \int x \sqrt{2x+1} \, dx = \int \left(\frac{1}{2}u^2 - \frac{1}{2}\right) \cdot u \, d\left(\frac{1}{2}u^2 - \frac{1}{2}\right) = \int \left(\frac{1}{2}u^2 - \frac{1}{2}\right) \cdot u \cdot u \, du = \int \left(\frac{1}{2}u^4 - \frac{1}{2}u^2\right) du \\
& = \frac{1}{10}u^5 - \frac{1}{6}u^3 + c = \frac{1}{10}(2x+1)^2 \cdot \sqrt{2x+1} - \frac{1}{6}(2x+1)\sqrt{2x+1} + c
\end{aligned}$$

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53 a $x = 11$ geeft $t^2 - 4 = 11$

$$t^2 = 15$$

$$t = \sqrt{15} \vee t = -\sqrt{15}$$

$$y(\sqrt{15}) = \sqrt{15} \ln(4 + \sqrt{15})$$

$$y(-\sqrt{15}) = -\sqrt{15} \ln(4 - \sqrt{15}) = \sqrt{15} \ln((4 - \sqrt{15})^{-1}) = \sqrt{15} \ln\left(\frac{1}{4 - \sqrt{15}}\right)$$

$$= \sqrt{15} \ln\left(\frac{1}{4 - \sqrt{15}} \cdot \frac{4 + \sqrt{15}}{4 + \sqrt{15}}\right) = \sqrt{15} \ln\left(\frac{4 + \sqrt{15}}{16 - 15}\right) = \sqrt{15} \ln(4 + \sqrt{15})$$

Dus $y(\sqrt{15}) = y(-\sqrt{15})$ en dus snijdt K zichzelf in het punt $(11, \sqrt{15} \ln(4 + \sqrt{15}))$.

Dit snijpunt ligt op de lijn $x = 11$, dus K snijdt zichzelf op de lijn $x = 11$.

b $\int y \, dx = \int t \ln(t+4) \, d(t^2 - 4) = \int t \ln(t+4) \cdot 2t \, dt = \int 2t^2 \ln(t+4) \, dt$

De GR geeft $\int_{-\sqrt{15}}^{\sqrt{15}} 2x^2 \ln(x+4) \, dx = 67,636\dots$

Dus oppervlakte $\approx 67,64$.

54 a $x = 0$ geeft $4 \sin(t) = 0$

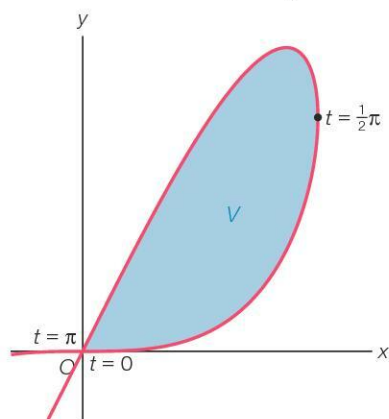
$$t = k \cdot \pi$$

$t = \frac{1}{2}\pi$ geeft het punt $(4, 4)$ en $t = \frac{3}{2}\pi$ geeft het punt $(2\sqrt{2}, 2\sqrt{2} - 2)$.

Dus V wordt in positieve richting omlopen.

$$\begin{aligned}
O(V) &= \int_{t=\pi}^{t=0} y \, dx = \int_{t=\pi}^{t=0} (4 \sin(t) - 2 \sin(2t)) \, d(4 \sin(t)) = \int_{\pi}^0 (4 \sin(t) - 4 \sin(t) \cos(t)) \cdot 4 \cos(t) \, dt \\
&= \int_{\pi}^0 (16 \sin(t) \cos(t) - 16 \sin(t) \cos^2(t)) \, dt = \int_{\pi}^0 16 \sin(t) \cos(t) \, dt - \int_{\pi}^0 16 \sin(t) \cos^2(t) \, dt \\
&= \int_{\pi}^0 8 \sin(2t) \, dt + \int_{t=\pi}^{t=0} 16 \cos^2(t) \, d \cos(t) = [-4 \cos(2t)]_{\pi}^0 + \left[5 \frac{1}{3} \cos^3(t)\right]_{\pi}^0 \\
&= -4 \cos(0) + 4 \cos(2\pi) + 5 \frac{1}{3} \cos^3(0) - 5 \frac{1}{3} \cos^3(\pi) = -4 + 4 + 5 \frac{1}{3} \cdot 1^3 - 5 \frac{1}{3} \cdot (-1)^3 = 10 \frac{2}{3}
\end{aligned}$$

b x is maximaal 4 voor $t = \frac{1}{2}\pi$.



$$I(L) = \pi \int_{t=\pi}^{t=\frac{1}{2}\pi} y^2 dx - \pi \int_{t=0}^{t=\frac{1}{2}\pi} y^2 dx = \pi \int_{t=\pi}^{t=\frac{1}{2}\pi} y^2 dx + \pi \int_{t=\frac{1}{2}\pi}^{t=0} y^2 dx = \pi \int_{t=\pi}^{t=0} y^2 dx$$

$$= \pi \int_{t=\pi}^{t=0} (4 \sin(t) - 2 \sin(2t))^2 d 4 \sin(t) = \pi \int_{\pi}^0 (4 \sin(t) - 2 \sin(2t))^2 \cdot 4 \cos(t) dt$$

De GR geeft $\pi \int_{\pi}^0 (4 \sin(x) - 2 \sin(2x))^2 \cdot 4 \cos(x) dx = 157,913...$

Dus $I(L) \approx 157,91$.

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