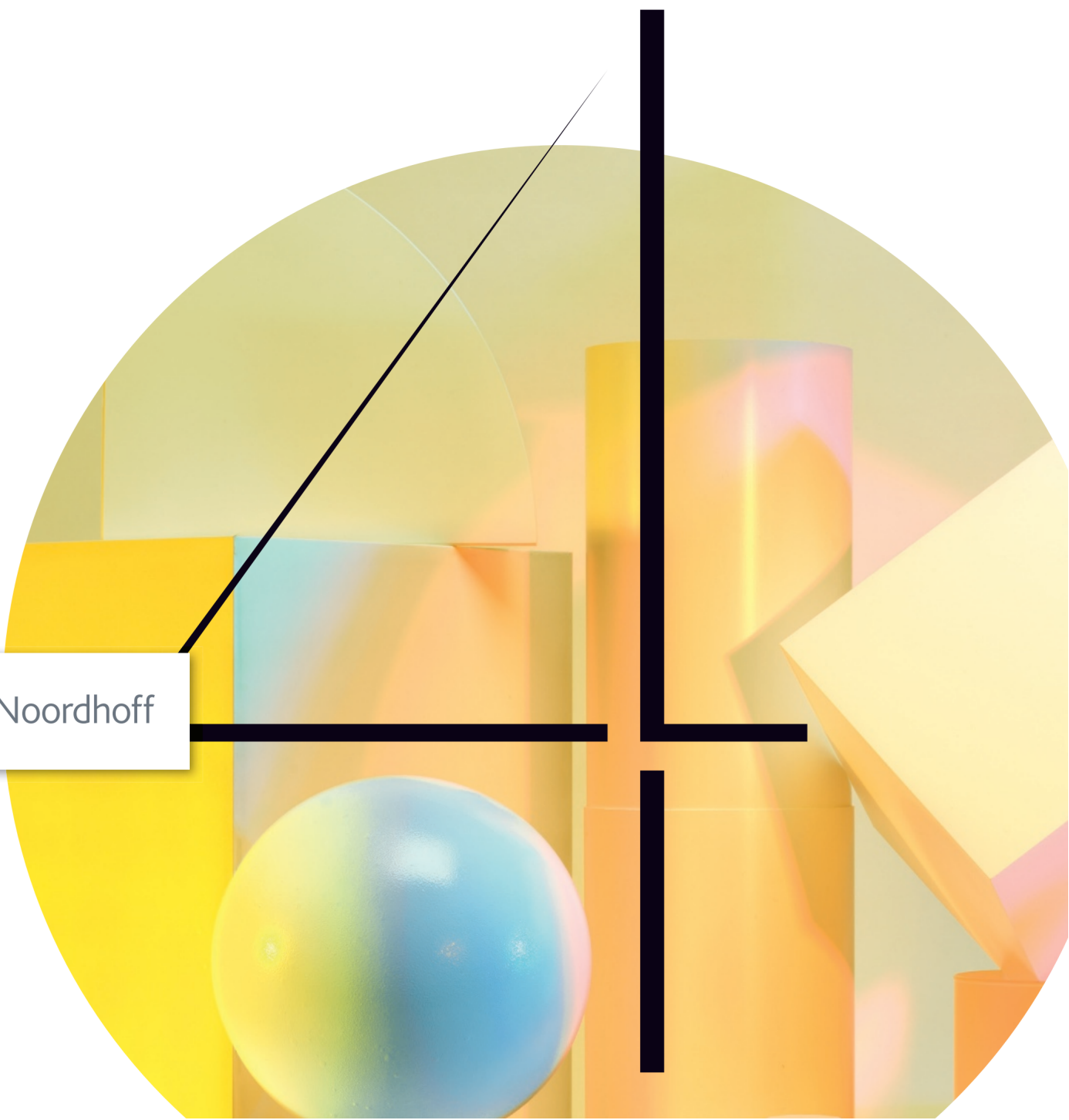


# getal & ruimte

## B



Noordhoff







# Getal & Ruimte

## Uitwerkingen vwo B deel 2

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# Inhoud

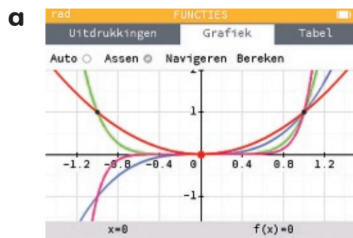
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# 5 Functies en vergelijkingen

## 5.1 Machtsfuncties

### Bladzijde 8

1



- b** De grafieken van  $f$  en  $h$  zijn lijnsymmetrisch.  
De grafieken van  $g$  en  $k$  zijn puntsymmetrisch.

### Bladzijde 10

2

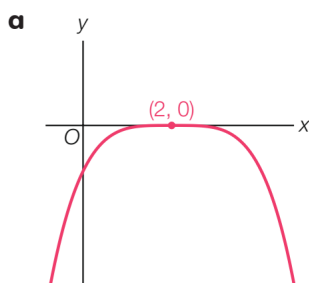
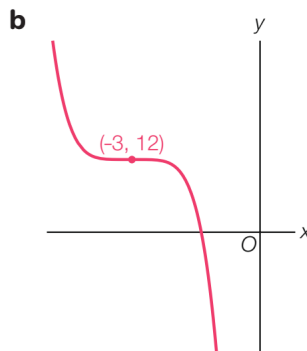
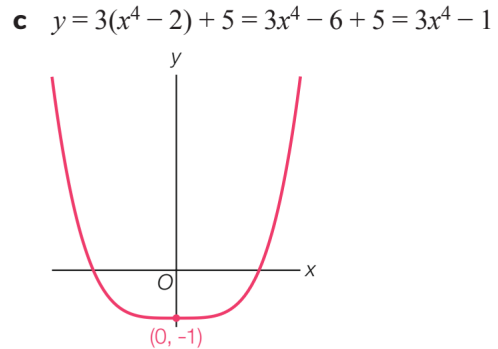
$y = -2x^3$   
 $\downarrow$  translatie  $(-3, -5)$   
 $y = -2(x + 3)^3 - 5$   
 $\downarrow \times -3$  t.o.v. de  $x$ -as  
 $y = -3(-2(x + 3)^3 - 5) = 6(x + 3)^3 + 15$   
 Het punt van symmetrie is  $(-3, 15)$ .

3

**a**  $y = 0,3x^4$   
 $\downarrow$  translatie  $(-5, 6)$   
 $y = 0,3(x + 5)^4 + 6$   
 $\downarrow \times -4$  t.o.v. de  $x$ -as  
 $y = -4(0,3(x + 5)^4 + 6) = -1,2(x + 5)^4 - 24$   
 De top is het punt  $(-5, -24)$ .

**b**  $y = 0,3x^4$   
 $\downarrow \times -4$  t.o.v. de  $x$ -as  
 $y = -1,2x^4$   
 $\downarrow$  translatie  $(-5, 6)$   
 $y = -1,2(x + 5)^4 + 6$   
 De top is het punt  $(-5, 6)$ .

4



5

$y = x^3$   
 $\downarrow$  translatie  $(5, -4)$   
 $y = (x - 5)^3 - 4$   
 $\downarrow \times -\frac{1}{2}$  t.o.v. de  $x$ -as  
 $f(x) = -\frac{1}{2}((x - 5)^3 - 4) = -\frac{1}{2}(x - 5)^3 + 2$   
 Voor  $f$  geldt  $y = -\frac{1}{2}(x - 5)^3 + 2$ , dus voor  $f^{\text{inv}}$  geldt  $x = -\frac{1}{2}(y - 5)^3 + 2$ .  
 $x = -\frac{1}{2}(y - 5)^3 + 2$  geeft  $-\frac{1}{2}(y - 5)^3 + 2 = x$   
 $-\frac{1}{2}(y - 5)^3 = x - 2$   
 $(y - 5)^3 = -2x + 4$   
 $y - 5 = \sqrt[3]{-2x + 4}$   
 $y = \sqrt[3]{-2x + 4} + 5$   
 Dus  $f^{\text{inv}}(x) = 5 + \sqrt[3]{4 - 2x}$ .

6

**a**  $f(x) = 3(x-2)^7 + 6$   
 $\downarrow \times a$  t.o.v. de  $x$ -as  
 $y = a(3(x-2)^7 + 6) = 3a(x-2)^7 + 6a$   
 $\downarrow$  translatie  $(m, n)$   
 $g(x) = 3a(x-m-2)^7 + 6a + n$   
 $g(x) = 3a(x-m-2)^7 + 6a + n$   
 $g(x) = -(x+5)^7 - 12$ 

$$\left. \begin{array}{l} 3a = -1, \text{ dus } a = -\frac{1}{3}. \\ -m-2 = 5, \text{ dus } m = -7. \\ -2+n = -12, \text{ dus } n = -10. \end{array} \right\}$$

**b**  $f(x) = 3(x-2)^7 + 6$   
 $\downarrow$  translatie  $(p, q)$   
 $y = 3(x-p-2)^7 + 6 + q$   
 $\downarrow \times c$  t.o.v. de  $x$ -as  
 $g(x) = c(3(x-p-2)^7 + 6 + q) = 3c(x-p-2)^7 + 6c + cq$   
 $g(x) = 3c(x-p-2)^7 + 6c + cq$   
 $g(x) = -(x+5)^7 - 12$ 

$$\left. \begin{array}{l} 3c = -1, \text{ dus } c = -\frac{1}{3}. \\ -p-2 = 5, \text{ dus } p = -7. \\ -2 - \frac{1}{3}q = -12 \text{ geeft } -\frac{1}{3}q = -10, \text{ dus } q = 30. \end{array} \right\}$$

7

**a**  $f(x) = \frac{1}{2}(x-3)^5 + 7$   
 $\downarrow$  translatie  $(1, 2)$   
 $y = \frac{1}{2}(x-1-3)^5 + 7 + 2 = \frac{1}{2}(x-4)^5 + 9$   
 $\downarrow \times -1$  t.o.v. de  $x$ -as  
 $y = -(\frac{1}{2}(x-4)^5 + 9) = -\frac{1}{2}(x-4)^5 - 9$   
 Het punt van symmetrie is  $(4, -9)$ .

**b**  $g(x) = -2\frac{1}{2}(x+4)^6 - 1$   
 $\downarrow \times -4$  t.o.v. de  $x$ -as  
 $y = -4(-2\frac{1}{2}(x+4)^6 - 1) = 10(x+4)^6 + 4$   
 $\downarrow$  translatie  $(-5, -4)$   
 $h(x) = 10(x+5+4)^6 + 4 - 4 = 10(x+9)^6$   
 min. is  $h(-9) = 0$

8

De top van de grafiek van  $f$  is het punt  $(1, 5)$ .

$$\left. \begin{array}{l} y = 3x^2 + bx + 6 \\ \text{door } (1, 5) \end{array} \right\} \begin{array}{l} 3 + b + 6 = 5 \\ b = -4 \end{array}$$

$f(x) = 2(x-1)^4 + 5$   
 $\downarrow \times -3$  t.o.v. de  $x$ -as  
 $y = -3(2(x-1)^4 + 5) = -6(x-1)^4 - 15$   
 $\downarrow$  translatie  $(a, 21)$

$g(x) = -6(x-a-1)^4 - 15 + 21 = -6(x-(a+1))^4 + 6$   
 De top van de grafiek van  $g$  is het punt  $(a+1, 6)$ .

$$\left. \begin{array}{l} y = 3x^2 - 4x + 6 \\ \text{door } (a+1, 6) \end{array} \right\} \begin{array}{l} 3(a+1)^2 - 4(a+1) + 6 = 6 \\ 3(a+1)^2 = 4(a+1) \\ a+1 = 0 \vee 3(a+1) = 4 \\ a = -1 \vee 3a + 3 = 4 \\ a = -1 \vee 3a = 1 \\ a = -1 \vee a = \frac{1}{3} \end{array}$$

Dus  $b = -4$  en  $a = -1$  of  $a = \frac{1}{3}$ .

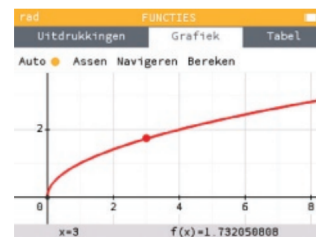
9  $f(x) = -\frac{1}{2}(x+2)^4 + 6$   
 $\downarrow$  translatie  $(p, q)$   
 $y = -\frac{1}{2}(x-p+2)^4 + 6 + q$   
 $\downarrow \times a$  t.o.v. de  $x$ -as  
 $g(x) = a(-\frac{1}{2}(x-p+2)^4 + 6 + q) = -\frac{1}{2}a(x-(p-2))^4 + 6a + aq$   
 De top van de grafiek van  $g$  is het punt  $(p-2, 6a+aq)$ .  
 Dit geeft  $p-2 = 5$ , dus  $p = 7$  en  $6a + aq = -10$ .  
 $g(x) = -\frac{1}{2}a(x-5)^4 - 10$   
 door  $A(1, 310)$   $\left. \begin{array}{l} -\frac{1}{2}a(1-5)^4 - 10 = 310 \\ -128a = 320 \end{array} \right\} \begin{array}{l} a = -2\frac{1}{2} \\ a = -2\frac{1}{2} \end{array}$   
 $a = -2\frac{1}{2}$  en  $6a + aq = -10$  geeft  $6 \cdot -2\frac{1}{2} - 2\frac{1}{2}q = -10$   
 $-2\frac{1}{2}q = 5$   
 $q = -2$   
 Dus  $a = -2\frac{1}{2}$ ,  $p = 7$  en  $q = -2$ .

C1  $f(x) = 0,2(x+1)^8 - 3$   
 $\downarrow$  translatie  $(-3, 2)$   
 $y = 0,2(x+3+1)^8 - 3 + 2 = 0,2(x+4)^8 - 1$   
 $\downarrow \times -10$  t.o.v. de  $x$ -as  
 $g(x) = -10(0,2(x+4)^8 - 1) = -2(x+4)^8 + 10$   
 max. is  $g(-4) = 10$

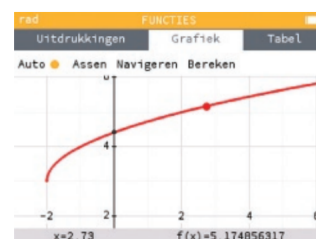
## 5.2 Wortelfuncties

### Bladzijde 12

- 10 a Zie de plot hiernaast.  
 $x \geq 0$  en  $y \geq 0$



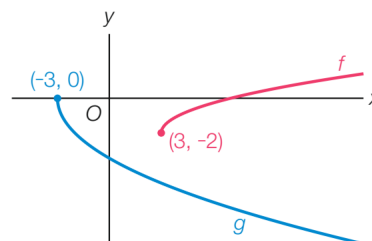
- b Zie de plot hiernaast.  
 $x \geq -2$  en  $y \geq 3$



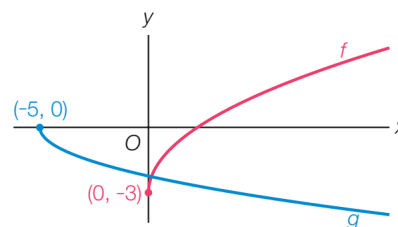
c  $y = \sqrt{x}$   
 $\downarrow$  translatie  $(-2, 3)$   
 $y = \sqrt{x+2} + 3$

### Bladzijde 13

- 11 a  $y = \sqrt{x}$   
 $\downarrow$  translatie  $(3, -2)$   
 $f(x) = \sqrt{x-3} - 2$   
 $y = \sqrt{x}$   
 $\downarrow \times -2$  t.o.v. de  $x$ -as  
 $y = -2\sqrt{x}$   
 $\downarrow$  translatie  $(-3, 0)$   
 $g(x) = -2\sqrt{x+3}$   
 b Zie de schets hiernaast.  
 c  $D_f = [3, \rightarrow)$ ,  $B_f = [-2, \rightarrow)$ ,  $D_g = [-3, \rightarrow)$  en  $B_g = \langle \leftarrow, 0]$



- 12 a**  $y = \sqrt{x}$   
 $\downarrow \times 2$  t.o.v. de  $x$ -as  
 $y = 2\sqrt{x}$   
 $\downarrow$  translatie  $(0, -3)$   
 $f(x) = 2\sqrt{x} - 3$
- $y = \sqrt{x}$   
 $\downarrow \times -1$  t.o.v. de  $x$ -as  
 $y = -\sqrt{x}$   
 $\downarrow$  translatie  $(-5, 0)$   
 $g(x) = -\sqrt{x+5}$
- b** Zie de schets hiernaast.
- c**  $D_f = [0, \rightarrow)$ ,  $B_f = [-3, \rightarrow)$ ,  $D_g = [-5, \rightarrow)$  en  $B_g = \langle \leftarrow, 0]$

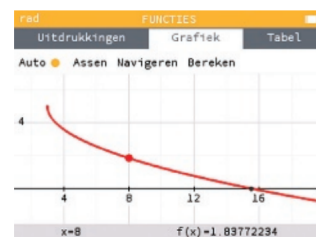


- 13 a** randpunt  $(-5, 3)$ ,  $D_f = [-5, \rightarrow)$  en  $B_f = [3, \rightarrow)$
- b** randpunt  $(-3, -7)$ ,  $D_g = [-3, \rightarrow)$  en  $B_g = [-7, \rightarrow)$
- c** randpunt  $(-1, 0)$ ,  $D_h = [-1, \rightarrow)$  en  $B_h = \langle \leftarrow, 0]$
- d** randpunt  $(0, 1)$ ,  $D_k = [0, \rightarrow)$  en  $B_k = [1, \rightarrow)$
- e** randpunt  $(1, -1)$ ,  $D_l = [1, \rightarrow)$  en  $B_l = \langle \leftarrow, -1]$
- f** randpunt  $(0, -3)$ ,  $D_m = [0, \rightarrow)$  en  $B_m = [-3, \rightarrow)$

#### Bladzijde 14

- 14**  $f(x) = \sqrt{4x - 12} - 5 = \sqrt{4(x - 3)} - 5 = 2\sqrt{x - 3} - 5$   
 $y = \sqrt{x}$   
 $\downarrow \times 2$  t.o.v. de  $x$ -as  
 $y = 2\sqrt{x}$   
 $\downarrow$  translatie  $(3, -5)$   
 $f(x) = 2\sqrt{x - 3} - 5$

- 15 a**  $2x - 6 = 0$   
 $2x = 6$   
 $x = 3$   
 $f(3) = 5$ , dus randpunt  $(3, 5)$ .
- b** Zie de plot hiernaast.  
 Hieruit volgt dat  $D_f = [3, \rightarrow)$  en  $B_f = \langle \leftarrow, 5]$ .

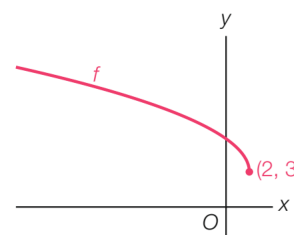
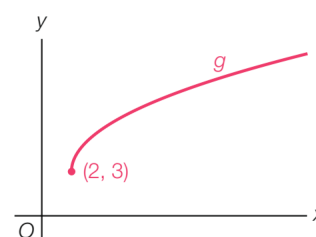


#### Bladzijde 15

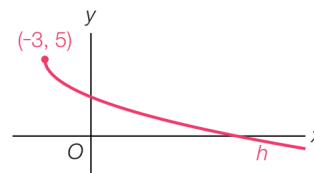
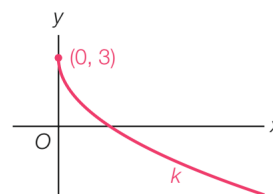
- 16 a** Bij voorbeeld a gaat het over  $x$ -waarden, dus bij voorbeeld a wordt er rekening gehouden met het domein.
- b** Bij voorbeeld b gaat het over functiewaarden oftewel  $y$ -waarden, dus bij voorbeeld b wordt er rekening gehouden met het bereik.

- 17 a**  $8 - 4x = 0$   
 $4x = 8$   
 $x = 2$   
 $f(2) = 3$ , dus randpunt  $(2, 3)$ .  
 Zie de schets hiernaast.  
 $D_f = \langle \leftarrow, 2]$  en  $B_f = [3, \rightarrow)$

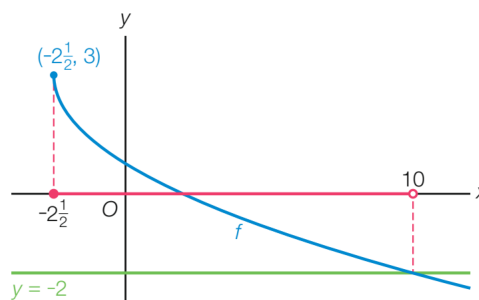
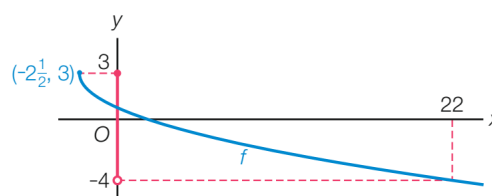
- b**  $4x - 8 = 0$   
 $4x = 8$   
 $x = 2$   
 $g(2) = 3$ , dus randpunt  $(2, 3)$ .  
 Zie de schets hiernaast.  
 $D_g = [2, \rightarrow)$  en  $B_g = [3, \rightarrow)$



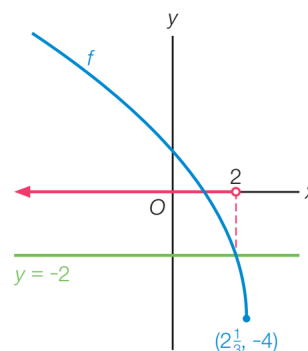
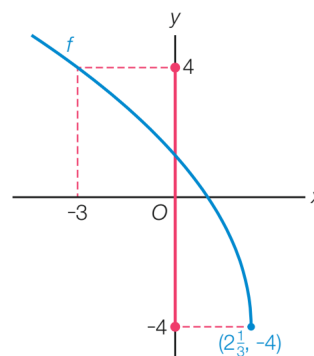
- c**  $2x + 6 = 0$   
 $2x = -6$   
 $x = -3$   
 $h(-3) = 5$ , dus randpunt  $(-3, 5)$ .  
 Zie de schets hiernaast.  
 $D_h = [-3, \rightarrow)$  en  $B_h = \langle \leftarrow, 5]$
- d**  $k(0) = 3$ , dus randpunt  $(0, 3)$ .  
 Zie de schets hiernaast.  
 $D_k = [0, \rightarrow)$  en  $B_k = \langle \leftarrow, 3]$



- 18 a**  $2x + 5 = 0$   
 $2x = -5$   
 $x = -2\frac{1}{2}$   
 $f(-2\frac{1}{2}) = 3$ , dus randpunt  $(-2\frac{1}{2}, 3)$ .  
 $f(x) = -2$  geeft  $3 - \sqrt{2x + 5} = -2$   
 $\sqrt{2x + 5} = 5$   
 $2x + 5 = 25$   
 $2x = 20$   
 $x = 10$   
 Zie de schets hiernaast.  
 $f(x) > -2$  geeft  $-2\frac{1}{2} \leq x < 10$
- b**  $f(22) = -4$   
 Zie de schets hiernaast.  
 $x < 22$  geeft  $-4 < f(x) \leq 3$



- 19 a**  $7 - 3x = 0$   
 $3x = 7$   
 $x = 2\frac{1}{3}$   
 $f(2\frac{1}{3}) = -4$ , dus randpunt  $(2\frac{1}{3}, -4)$ .  
 $f(x) = -2$  geeft  $2\sqrt{7 - 3x} - 4 = -2$   
 $2\sqrt{7 - 3x} = 2$   
 $\sqrt{7 - 3x} = 1$   
 $7 - 3x = 1$   
 $3x = 6$   
 $x = 2$   
 Zie de schets hiernaast.  
 $f(x) > -2$  geeft  $x < 2$
- b**  $f(-3) = 4$   
 Zie de schets hiernaast.  
 $x \geq -3$  geeft  $-4 \leq f(x) \leq 4$





**20**  $5 + ax = 0$

$$ax = -5$$

$$x = -\frac{5}{a}$$

$$f\left(-\frac{5}{a}\right) = 4, \text{ dus randpunt } \left(-\frac{5}{a}, 4\right).$$

$$g\left(-\frac{5}{a}\right) = 4 \text{ geeft } a \cdot \left(-\frac{5}{a}\right)^2 + 3 \cdot -\frac{5}{a} - 1 = 4$$

$$\frac{25}{a} - \frac{15}{a} = 5$$

$$\frac{10}{a} = 5$$

$$a = 2$$

$$a = 2 \text{ geeft randpunt } \left(-2\frac{1}{2}, 4\right)$$

Zie de schets hiernaast.

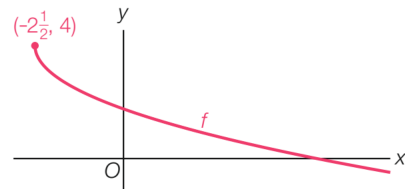
$$\text{Dus } D_f = \left[-2\frac{1}{2}, \rightarrow\right).$$

$$g(x) = 2x^2 + 3x - 1$$

$$x_{\text{top}} = -\frac{3}{2 \cdot 2} = -\frac{3}{4}$$

$$g\left(-\frac{3}{4}\right) = -2\frac{1}{8}$$

De grafiek van  $g$  is een dalparabool, dus  $B_g = \left[-2\frac{1}{8}, \rightarrow\right).$



**21 a**  $x^2 - 4 = 0$

$$x^2 = 4$$

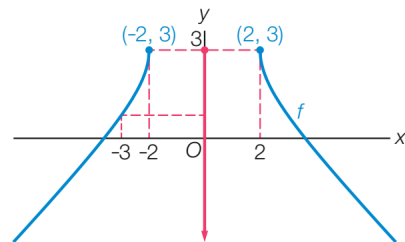
$$x = -2 \vee x = 2$$

$$f(-2) = 3 \text{ en } f(2) = 3$$

Dus randpunten  $(-2, 3)$  en  $(2, 3)$ .

**b** Zie de schets hiernaast.

$$x > -3 \text{ geeft } f(x) \leq 3$$



**c**  $f(x) = 0$  geeft  $3 - \sqrt{x^2 - 4} = 0$

$$\sqrt{x^2 - 4} = 3$$

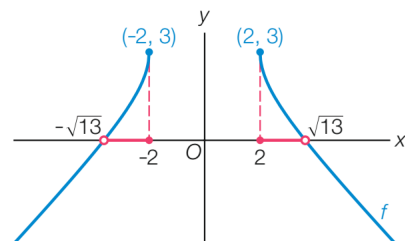
$$x^2 - 4 = 9$$

$$x^2 = 13$$

$$x = \sqrt{13} \vee x = -\sqrt{13}$$

Zie de schets hiernaast.

$$f(x) > 0 \text{ geeft } -\sqrt{13} < x \leq -2 \vee 2 \leq x < \sqrt{13}$$



**22 a**  $x^2 - 4x + 5 = 0$

$$D = (-4)^2 - 4 \cdot 1 \cdot 5 = -4 < 0, \text{ dus de vergelijking heeft geen oplossing.}$$

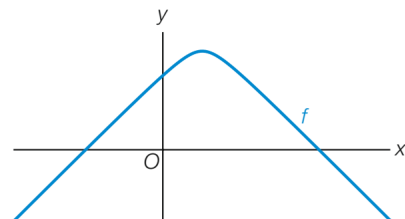
Dus de grafiek van  $f$  heeft geen randpunten.

**b** Zie de schets hiernaast.

$f(x)$  is maximaal als  $x^2 - 4x + 5$  minimaal is.

$$x_{\text{top}} = -\frac{-4}{2 \cdot 1} = 2$$

$$f(2) = 5, \text{ dus } B_f = \langle \leftarrow, 5 \rangle.$$



c  $f(x) = 3$  geeft  $6 - \sqrt{x^2 - 4x + 5} = 3$

$$\sqrt{x^2 - 4x + 5} = 3$$

$$x^2 - 4x + 5 = 9$$

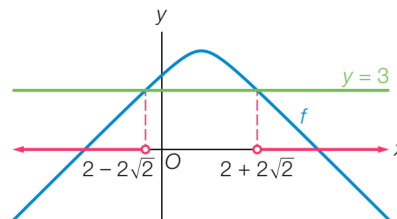
$$x^2 - 4x - 4 = 0$$

$$D = (-4)^2 - 4 \cdot 1 \cdot -4 = 32, \text{ dus } \sqrt{D} = 4\sqrt{2}$$

$$x = \frac{4 + 4\sqrt{2}}{2} = 2 + 2\sqrt{2} \vee x = \frac{4 - 4\sqrt{2}}{2} = 2 - 2\sqrt{2}$$

Zie de schets hiernaast.

$$f(x) < 3 \text{ geeft } x < 2 - 2\sqrt{2} \vee x > 2 + 2\sqrt{2}$$



23 a  $x^2 - 2x - 3 = 0$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \vee x = 3$$

$$f(-1) = 4 \text{ en } f(3) = 4$$

Dus randpunten  $(-1, 4)$  en  $(3, 4)$ .

Zie de schets hiernaast.

$$x < 5 \text{ geeft } f(x) \leq 4$$

b  $f(x) = 2$  geeft  $4 - \sqrt{x^2 - 2x - 3} = 2$

$$\sqrt{x^2 - 2x - 3} = 2$$

$$x^2 - 2x - 3 = 4$$

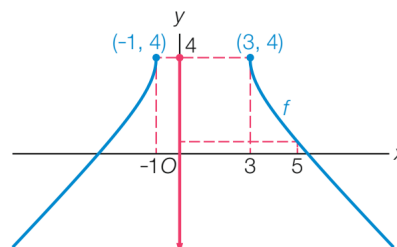
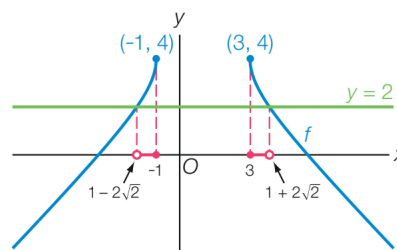
$$x^2 - 2x - 7 = 0$$

$$D = (-2)^2 - 4 \cdot 1 \cdot -7 = 32, \text{ dus } \sqrt{D} = 4\sqrt{2}$$

$$x = \frac{2 + 4\sqrt{2}}{2} = 1 + 2\sqrt{2} \vee x = \frac{2 - 4\sqrt{2}}{2} = 1 - 2\sqrt{2}$$

Zie de schets hiernaast.

$$f(x) > 2 \text{ geeft } 1 - 2\sqrt{2} < x \leq -1 \vee 3 \leq x < 1 + 2\sqrt{2}$$



24 a Knik voor  $x = 0$ .

$$f(0) = 4, \text{ dus } B(0, 4).$$

$$f(-3) = 1, \text{ dus } C(-3, 1).$$

$$k: y = ax + b$$

$$a = \frac{\Delta y}{\Delta x} = \frac{4 - 1}{0 - -3} = 1$$

$$\text{Dus } k: y = x + 4.$$

$$k \text{ snijden met de } x\text{-as geeft } x + 4 = 0, \text{ dus } x_D = -4.$$

b  $x + a = x\sqrt{x + a}$

$$(\sqrt{x + a})^2 = x\sqrt{x + a}$$

$$\sqrt{x + a} \cdot \sqrt{x + a} = \sqrt{x + a} \cdot x$$

$$A \cdot B = A \cdot C \text{ geeft } A = 0 \vee B = C$$

$$\text{Dus } \sqrt{x + a} = 0 \vee \sqrt{x + a} = x.$$

**c**  $x + 4 = 0$   
 $x = -4$   
 $f(-4) = 4$ , dus randpunt  $A(-4, 4)$ .  
 $l: y = ax + b$

$$a = \frac{\Delta y}{\Delta x} = \frac{0 - 4}{0 - -4} = -1$$

Dus  $l: y = -x$ .

$$f(x) = \begin{cases} 4 - x\sqrt{x+4} & \text{als } x \geq 0 \\ 4 + x\sqrt{x+4} & \text{als } x < 0 \end{cases}$$

De grafiek van  $f$  en  $l$  snijden geeft

$$4 - x\sqrt{x+4} = -x$$

$$x + 4 = x\sqrt{x+4}$$

$$\sqrt{x+4} = 0 \vee \sqrt{x+4} = x$$

kwadrateren geeft

$$x + 4 = 0 \vee x + 4 = x^2$$

$$x = -4 \vee x^2 - x - 4 = 0$$

$$D = (-1)^2 - 4 \cdot 1 \cdot -4 = 17$$

$$x = \frac{1 + \sqrt{17}}{2} = \frac{1}{2} + \frac{1}{2}\sqrt{17} \vee x = \frac{1 - \sqrt{17}}{2} = \frac{1}{2} - \frac{1}{2}\sqrt{17}$$

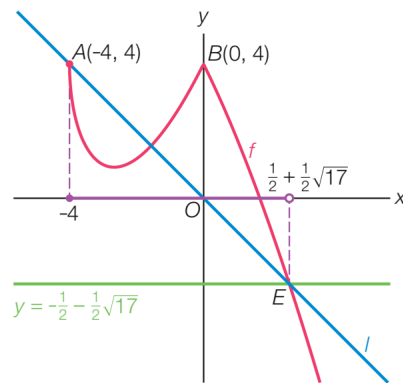
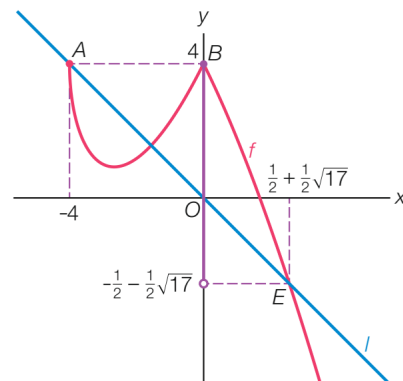
Dus  $E(\frac{1}{2} + \frac{1}{2}\sqrt{17}, -\frac{1}{2} - \frac{1}{2}\sqrt{17})$ .

Zie de schets hiernaast.

$$f(x) > -\frac{1}{2} - \frac{1}{2}\sqrt{17} \text{ geeft } -4 \leq x < \frac{1}{2} + \frac{1}{2}\sqrt{17}$$

**d** Zie de schets hiernaast.

$$x < \frac{1}{2} + \frac{1}{2}\sqrt{17} \text{ geeft } -\frac{1}{2} - \frac{1}{2}\sqrt{17} < f(x) \leq 4$$



**25 a**  $y = 2\sqrt{x}$  geeft  $2\sqrt{x} = y$   
kwadrateren geeft  
 $4x = y^2$   
 $x = \frac{1}{4}y^2$

**b**  $y = 2\sqrt{x-2}$  geeft  $2\sqrt{x-2} = y$   
kwadrateren geeft  
 $4(x-2) = y^2$   
 $x-2 = \frac{1}{4}y^2$   
 $x = \frac{1}{4}y^2 + 2$

### Bladzijde 17

**26 a**  $F = 3\sqrt{2t-1}$   
 $3\sqrt{2t-1} = F$

kwadrateren geeft  $9(2t-1) = F^2$   
 $18t-9 = F^2$   
 $18t = F^2 + 9$   
 $t = \frac{1}{18}F^2 + \frac{1}{2}$

**b**  $A = 5 + \sqrt{4-3B}$   
 $\sqrt{4-3B} = A-5$

kwadrateren geeft  $4-3B = (A-5)^2$   
 $4-3B = A^2 - 10A + 25$   
 $-3B = A^2 - 10A + 21$   
 $B = -\frac{1}{3}A^2 + 3\frac{1}{3}A - 7$

**c**  $2x\sqrt{y} - 5 = 0$

$$2x\sqrt{y} = 5$$

kwadrateren geeft  $4x^2y = 25$

$$y = \frac{25}{4x^2}$$

**d**  $R\sqrt{q} - \sqrt{R} = 6$

$$R\sqrt{q} = 6 + \sqrt{R}$$

kwadrateren geeft  $R^2q = (6 + \sqrt{R})^2$

$$R^2q = 36 + 12\sqrt{R} + R$$

$$q = \frac{36 + 12\sqrt{R} + R}{R^2}$$

### Bladzijde 18

**27 a**  $y = 2 - \frac{1}{2}\sqrt{5 - 4x}$

$$\frac{1}{2}\sqrt{5 - 4x} = 2 - y$$

$$\sqrt{5 - 4x} = 4 - 2y$$

kwadrateren geeft  $5 - 4x = (4 - 2y)^2$

$$5 - 4x = 16 - 16y + 4y^2$$

$$-4x = 4y^2 - 16y + 11$$

$$x = -y^2 + 4y - 2\frac{3}{4}$$

**b**  $y = 2 \cdot \sqrt[3]{5x - 1}$

$$2 \cdot \sqrt[3]{5x - 1} = y$$

tot de derde macht geeft  $8(5x - 1) = y^3$

$$5x - 1 = \frac{1}{8}y^3$$

$$5x = \frac{1}{8}y^3 + 1$$

$$x = \frac{1}{40}y^3 + \frac{1}{5}$$

**c**  $y = \frac{2}{3x - 1} + 4$

$$y(3x - 1) = 2 + 4(3x - 1)$$

$$3xy - y = 2 + 12x - 4$$

$$3xy - 12x = y - 2$$

$$x(3y - 12) = y - 2$$

$$x = \frac{y - 2}{3y - 12}$$

**28 a** Voor  $f$  geldt  $y = 2\sqrt{x + 3} - 3$ , dus voor  $f^{\text{inv}}$  geldt  $x = 2\sqrt{y + 3} - 3$ .

$$x = 2\sqrt{y + 3} - 3$$

$$2\sqrt{y + 3} = x + 3$$

kwadrateren geeft  $4(y + 3) = (x + 3)^2$

$$y + 3 = \frac{1}{4}(x^2 + 6x + 9)$$

$$y + 3 = \frac{1}{4}x^2 + 1\frac{1}{2}x + 2\frac{1}{4}$$

$$y = \frac{1}{4}x^2 + 1\frac{1}{2}x - \frac{3}{4}$$

Dus  $f^{\text{inv}}(x) = \frac{1}{4}x^2 + 1\frac{1}{2}x - \frac{3}{4}$ .

$B_f = [-3, \rightarrow)$  geeft  $D_{f^{\text{inv}}} = [-3, \rightarrow)$ , dus  $p = -3$ .

$$\begin{aligned}
 \text{b } x + 3 &= 0 \\
 x &= -3 \\
 f(-3) &= -3, \text{ dus randpunt } (-3, -3). \\
 f(x) = 0 &\text{ geeft } 2\sqrt{x+3} - 3 = 0 \\
 2\sqrt{x+3} &= 3 \\
 4(x+3) &= 9 \\
 x+3 &= 2\frac{1}{4} \\
 x &= -\frac{3}{4}, \text{ dus } A(-\frac{3}{4}, 0).
 \end{aligned}$$

$$k: y = ax + b$$

$$a = \frac{0 - (-3)}{-\frac{3}{4} - (-3)} = 1\frac{1}{3}$$

$$\left. \begin{aligned} y &= 1\frac{1}{3}x + b \\ \text{door } A(-\frac{3}{4}, 0) \end{aligned} \right\} \begin{aligned} 1\frac{1}{3} \cdot -\frac{3}{4} + b &= 0 \\ -1 + b &= 0 \\ b &= 1 \end{aligned}$$

$$\text{Dus } k: y = 1\frac{1}{3}x + 1.$$

$$\begin{aligned}
 \text{De grafiek van } f^{\text{inv}} \text{ en } k \text{ snijden geeft } \frac{1}{4}x^2 + 1\frac{1}{2}x - \frac{3}{4} &= 1\frac{1}{3}x + 1 \\
 3x^2 + 18x - 9 &= 16x + 12 \\
 3x^2 + 2x - 21 &= 0 \\
 D = 2^2 - 4 \cdot 3 \cdot (-21) &= 256, \text{ dus } \sqrt{D} = 16 \\
 x = \frac{-2 \pm 16}{6} &= 2\frac{1}{3} \vee x = \frac{-2 - 16}{6} = -3
 \end{aligned}$$

$$x = 2\frac{1}{3} \text{ geeft } y = 1\frac{1}{3} \cdot 2\frac{1}{3} + 1 = 4\frac{1}{9}, \text{ dus } B(2\frac{1}{3}, 4\frac{1}{9}).$$

$$29 \text{ a } T = -27,4 \text{ geeft } v = 331 \cdot \sqrt{1 - \frac{27,4}{273}} = 313,9 \dots$$

$$T = 38,6 \text{ geeft } v = 331 \cdot \sqrt{1 + \frac{38,6}{273}} = 353,6 \dots$$

Het verschil is  $353,6 \dots - 313,9 \dots \approx 40 \text{ m/s}$ .

$$\text{b } v = 331 \cdot \sqrt{1 + \frac{T}{273}}$$

$$331 \cdot \sqrt{1 + \frac{T}{273}} = v$$

$$\text{kwadrateren geeft } 109561 \left( 1 + \frac{T}{273} \right) = v^2$$

$$1 + \frac{T}{273} = \frac{1}{109561} v^2$$

$$\frac{T}{273} = \frac{1}{109561} v^2 - 1$$

$$T = \frac{273}{109561} v^2 - 273$$

$$\text{c } v = 340 \text{ geeft } T = \frac{273}{109561} \cdot 340^2 - 273 \approx 15$$

Bij een temperatuur van  $15^\circ\text{C}$  is de geluidssnelheid  $340 \text{ m/s}$ .

$$\text{d } T_0 = 25 \text{ en } h = 2,5 \text{ geeft } T = 25 - 6,5 \cdot 2,5 = 8,75$$

$$T = 8,75 \text{ geeft } v = 331 \cdot \sqrt{1 + \frac{8,75}{273}} = 336,2 \dots \text{ m/s} \approx 1211 \text{ km/u}$$

De geluidssnelheid is  $1211 \text{ km/u}$ .

$$\text{e } T = -1 \text{ en } h = 2 \text{ geeft } -1 = T_0 - 6,5 \cdot 2$$

$$T_0 = 12$$

$$\text{Dus } T = 12 - 6,5h.$$

$$\text{Dit geeft } v = 331 \cdot \sqrt{1 + \frac{12 - 6,5h}{273}} = 331 \cdot \sqrt{1 + \frac{12}{273} - \frac{6,5}{273}h} \approx 331 \cdot \sqrt{-0,0238h + 1,0440}$$

$$\text{Dus } v = 331 \cdot \sqrt{-0,0238h + 1,0440}.$$

**30** Voor  $f$  geldt  $y = a + \sqrt{bx + c}$ , dus voor  $f^{\text{inv}}$  geldt  $x = a + \sqrt{by + c}$ .

$$x = a + \sqrt{by + c}$$

$$\sqrt{by + c} = x - a$$

kwadrateren geeft  $by + c = (x - a)^2$

$$by + c = x^2 - 2ax + a^2$$

$$by = x^2 - 2ax + a^2 - c$$

$$y = \frac{1}{b}x^2 - \frac{2a}{b}x + \frac{a^2 - c}{b}$$

$$f^{\text{inv}}(x) = \frac{1}{b}x^2 - \frac{2a}{b}x + \frac{a^2 - c}{b} \left\{ \frac{1}{b} = \frac{1}{2}, \text{ dus } b = 2. \right.$$

$$g(x) = \frac{1}{2}x^2 - 3x$$

$$-\frac{2a}{b} = -3 \text{ oftewel } 2a = 3 \cdot 2 = 6, \text{ dus } a = 3.$$

$$\frac{a^2 - c}{b} = 0 \text{ oftewel } a^2 - c = 0, \text{ dus } c = a^2 = 9.$$

Dus  $a = 3$ ,  $b = 2$  en  $c = 9$ .

### Bladzijde 19

**C2**

**a**  $y = \sqrt{x}$

↓  $\times -1$  t.o.v. de  $x$ -as

$$y = -\sqrt{x}$$

↓ translatie  $(-2, 0)$

$$f(x) = -\sqrt{x+2}$$

$$y = \sqrt{x}$$

↓  $\times 4$  t.o.v. de  $x$ -as

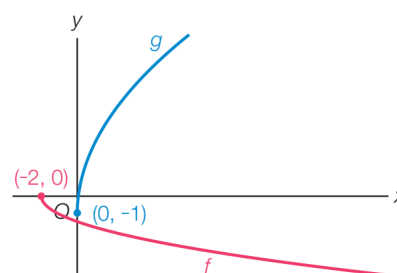
$$y = 4\sqrt{x}$$

↓ translatie  $(0, -1)$

$$g(x) = 4\sqrt{x} - 1$$

**b** Zie de schets hiernaast.

**c**  $D_f = [-2, \rightarrow)$ ,  $B_f = \langle \leftarrow, 0]$ ,  $D_g = [0, \rightarrow)$  en  $B_g = [-1, \rightarrow)$



**C3**

**a**  $2x + 5 = 0$

$$2x = -5$$

$$x = -2\frac{1}{2}$$

$$f(-2\frac{1}{2}) = 1, \text{ dus randpunt } (-2\frac{1}{2}, 1).$$

$$f(x) = 3 \text{ geeft } 1 + \sqrt{2x+5} = 3$$

$$\sqrt{2x+5} = 2$$

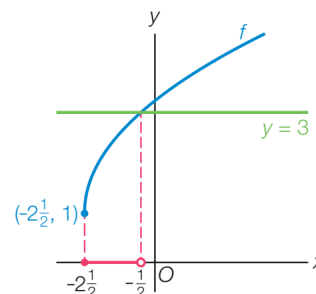
$$2x + 5 = 4$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Zie de schets hiernaast.

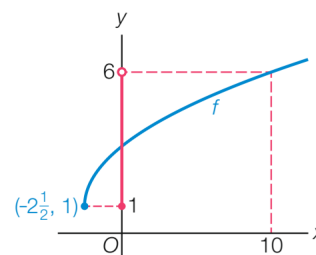
$$f(x) < 3 \text{ geeft } -2\frac{1}{2} \leq x < -\frac{1}{2}$$



**b**  $f(10) = 6$

Zie de schets hiernaast.

$$x < 10 \text{ geeft } 1 \leq f(x) < 6$$



**C4**

$$N = 4 - \frac{1}{2}\sqrt{3t-5}$$

$$\frac{1}{2}\sqrt{3t-5} = 4 - N$$

$$\sqrt{3t-5} = 8 - 2N$$

kwadrateren geeft  $3t - 5 = (8 - 2N)^2$

$$3t - 5 = 64 - 32N + 4N^2$$

$$3t = 4N^2 - 32N + 69$$

$$t = \frac{1}{3}N^2 - 10\frac{2}{3}N + 23$$

### 5.3 Machten met negatieve en gebroken exponenten

#### Bladzijde 20

V1

a  $2x^3 \cdot 3x^2 = 6x^5$

b  $(x^2y)^3 = x^6y^3$

c  $-2x^4y^3 \cdot -3xy = 6x^5y^4$

d  $(-4xy^4)^2 = 16x^2y^8$

e  $(3x)^2 \cdot (2x^2)^3 = 9x^2 \cdot 8x^6 = 72x^8$

f  $(3x^3)^2 + (-2x^2)^3 = 9x^6 + -8x^6 = x^6$

V2

a  $\frac{12x^6}{4x^2} = 3x^4$

b  $\frac{5x^{10}}{15x^5} = \frac{1}{3}x^5$

c  $\frac{24x^4y^2}{6xy} = 4x^3y$

d  $\frac{-15x^6y}{5x^2y} = -3x^4$

e  $\frac{(2xy)^3}{(3xy)^2} = \frac{8x^3y^3}{9x^2y^2} = \frac{8}{9}xy$

f  $\left(\frac{9x^4}{x}\right)^2 = (9x^3)^2 = 81x^6$

#### Bladzijde 21

31

a  $\frac{a^5}{a^5} = a^0$

b  $\left. \begin{array}{l} \frac{a^5}{a^5} = a^0 \\ \frac{a^5}{a^5} = 1 \end{array} \right\} a^0 = 1$

c  $\frac{1}{a^5} = \frac{a^0}{a^5} = a^{-5}$

#### Bladzijde 22

32

a  $\left(\frac{3}{4}\right)^{-1} = \frac{1}{\left(\frac{3}{4}\right)^1} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

b  $\left(\frac{1}{5}\right)^{-2} = 5^2 = 25$   
 $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8} = 3\frac{3}{8}$   
 $\left(1\frac{1}{4}\right)^{-1} = \left(\frac{5}{4}\right)^{-1} = \frac{4}{5}$

33

a  $\frac{1}{x^2} = x^{-2}$

b  $x^4 \cdot \frac{1}{x^6} = x^4 \cdot x^{-6} = x^{-2}$

c  $\frac{x^4}{\left(\frac{1}{x^4}\right)} = \frac{x^4}{x^{-4}} = x^8$

d  $\frac{x^8}{x^0} = \frac{x^8}{1} = x^8$

e  $(x^3)^{-2} = x^{-6}$

f  $\frac{\left(\frac{1}{x^5}\right)}{x} = \frac{x^{-5}}{x} = x^{-6}$

g  $\frac{x}{x^{12}} = x^{-11}$

h  $\frac{1}{x^8} \cdot (x^3)^{-1} = x^{-8} \cdot x^{-3} = x^{-11}$

i  $\frac{\left(\frac{1}{x}\right)}{x^{-3}} = \frac{x^{-1}}{x^{-3}} = x^2$

34

a  $\frac{1}{2}x^{-3} = \frac{1}{2x^3}$

b  $\left(\frac{1}{2}x\right)^{-3} = \left(\frac{1}{2}\right)^{-3} \cdot x^{-3} = 2^3 \cdot \frac{1}{x^3} = \frac{8}{x^3}$

c  $3x^{-4}y^3 = \frac{3y^3}{x^4}$

d  $(3x)^{-4}y^3 = \frac{y^3}{(3x)^4} = \frac{y^3}{81x^4}$

e  $\frac{3}{5}x^{-1} = \frac{3}{5x}$

f  $\left(\frac{3}{5}x\right)^{-1} = \left(\frac{3}{5}\right)^{-1} \cdot x^{-1} = \frac{5}{3} \cdot \frac{1}{x} = \frac{5}{3x}$

g  $-4 \cdot 3x^{-2} = -\frac{12}{x^2}$

h  $-4 \cdot (3x)^{-2} = -\frac{4}{(3x)^2} = -\frac{4}{9x^2}$

i  $(-4 \cdot 3x)^{-2} = \frac{1}{(-12x)^2} = \frac{1}{144x^2}$

## Bladzijde 23

**35** Uit  $(\sqrt[3]{x})^3 = x$  en  $(x^{\frac{1}{3}})^3 = x$  volgt  $x^{\frac{1}{3}} = \sqrt[3]{x}$ .

## Bladzijde 24

**36 a**  $x \cdot \sqrt[3]{x} = x \cdot x^{\frac{1}{3}} = x^{1\frac{1}{3}}$

**b**  $\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$

**c**  $x^2 \cdot \sqrt{x} = x^2 \cdot x^{\frac{1}{2}} = x^{2\frac{1}{2}}$

**d**  $\sqrt[3]{\frac{1}{x^2}} = \sqrt[3]{x^{-2}} = x^{-\frac{2}{3}}$

**e**  $\sqrt[3]{x^{12}} = x^{\frac{12}{3}} = x^4$

**f**  $\frac{1}{x^4} \cdot \sqrt[3]{x} = x^{-4} \cdot x^{\frac{1}{3}} = x^{-3\frac{2}{3}}$

**37 a**  $8\sqrt{2} = 2^3 \cdot 2^{\frac{1}{2}} = 2^{3\frac{1}{2}}$

**b**  $\frac{1}{3}\sqrt{3} = 3^{-1} \cdot 3^{\frac{1}{2}} = 3^{-\frac{1}{2}}$

**c**  $\frac{1}{100}\sqrt{10} = \frac{1}{10^2} \cdot 10^{\frac{1}{2}} = 10^{-2} \cdot 10^{\frac{1}{2}} = 10^{-1\frac{1}{2}}$

**d**  $\frac{1}{2} \cdot \sqrt[3]{2} = 2^{-1} \cdot 2^{\frac{1}{3}} = 2^{-\frac{2}{3}}$

**e**  $\frac{1}{27} \cdot \sqrt[3]{\frac{1}{9}} = \frac{1}{3^3} \cdot \sqrt[3]{\frac{1}{3^2}} = 3^{-3} \cdot \sqrt[3]{3^{-2}} = 3^{-3} \cdot 3^{-\frac{2}{3}} = 3^{-3\frac{2}{3}}$

**f**  $\frac{100\sqrt{10}}{\sqrt[3]{10}} = \frac{10^2 \cdot 10^{\frac{1}{2}}}{10^{\frac{1}{3}}} = \frac{10^{2\frac{1}{2}}}{10^{\frac{1}{3}}} = 10^{2\frac{1}{6}}$

**38 a**  $5x^{\frac{3}{3}} = 5x^3 \cdot x^{\frac{1}{3}} = 5x^3 \cdot \sqrt[3]{x}$

**b**  $\frac{1}{2}x^{-\frac{1}{4}}y = \frac{y}{2x^{\frac{1}{4}}} = \frac{y}{2 \cdot \sqrt[4]{x}}$

**c**  $3x^{-\frac{2}{3}} = \frac{3}{x^{\frac{2}{3}}} = \frac{3}{\sqrt[3]{x^2}}$

**d**  $\frac{2}{3}x^{-3}y^{\frac{1}{2}} = \frac{2y^{\frac{1}{2}}}{3x^3} = \frac{2y \cdot y^{\frac{1}{2}}}{3x^3} = \frac{2y\sqrt{y}}{3x^3}$

**e**  $\frac{1}{5}x^{-\frac{1}{2}}y^{\frac{1}{3}} = \frac{y^{\frac{1}{3}}}{5x^{\frac{1}{2}}} = \frac{\sqrt[3]{y}}{5\sqrt{x}}$

**f**  $(5x)^{-\frac{1}{2}} = \frac{1}{(5x)^{\frac{1}{2}}} = \frac{1}{\sqrt{5x}}$

**39 a**  $y = \frac{50x^{1,9}}{10x^{1,1}} = 5x^{0,8}$

**b**  $y = \sqrt[6]{x} \cdot \sqrt[3]{x} = x^{\frac{1}{6}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{2}}$

**c**  $y = \frac{3}{x} \cdot \sqrt[4]{x^3} = 3x^{-1} \cdot x^{\frac{3}{4}} = 3x^{-\frac{1}{4}}$

**d**  $y = \frac{x}{\sqrt[5]{x}} = \frac{x}{x^{\frac{1}{5}}} = x^{\frac{4}{5}}$

**e**  $y = \frac{5}{x\sqrt{x}} = \frac{5}{x \cdot x^{\frac{1}{2}}} = \frac{5}{x^{1\frac{1}{2}}} = 5x^{-1\frac{1}{2}}$

**f**  $y = 4x\sqrt{x^3} = 4x \cdot x^{\frac{3}{2}} = 4x^{2\frac{1}{2}}$

**40 a**  $\frac{5}{6}x^{-\frac{1}{2}}y^{-\frac{2}{3}} = \frac{5}{6x^{\frac{1}{2}}y^{\frac{2}{3}}} = \frac{5}{6\sqrt{x} \cdot \sqrt[3]{y^2}}$

**b**  $(2x^{-\frac{2}{7}})^3 = 8x^{-\frac{6}{7}} = \frac{8}{x^{\frac{6}{7}}} = \frac{8}{\sqrt[7]{x^6}}$

**c**  $\frac{6y^{-2}}{3x^{-1\frac{1}{4}}} = \frac{2x^{\frac{1}{4}}}{y^2} = \frac{2x \cdot x^{\frac{1}{4}}}{y^2} = \frac{2x \cdot \sqrt[4]{x}}{y^2}$

**41**  $x^{\frac{2}{3}} = 10$

$(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$

$x = 10^1 \cdot 10^{\frac{1}{2}}$

$x = 10\sqrt{10}$

## Bladzijde 25

**42 a**  $x^{\frac{2}{3}} = 9$

$x = 9^{\frac{3}{2}}$

$x = 9\sqrt{9} = 27$

**b**  $8x^{-1\frac{1}{2}} = 1$

$x^{-\frac{3}{2}} = \frac{1}{8}$

$x = (\frac{1}{8})^{-\frac{2}{3}}$

$x = (2^{-3})^{-\frac{2}{3}}$

$x = 2^2 = 4$



$$\begin{aligned} \text{c} \quad 5 - 2x^{-3} &= 4 \\ 2x^{-3} &= 1 \\ x^{-3} &= \frac{1}{2} \\ x &= \left(\frac{1}{2}\right)^{-\frac{1}{3}} \\ x &= (2^{-1})^{-\frac{1}{3}} \\ x &= 2^{\frac{1}{3}} = \sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} \text{d} \quad \sqrt{(2x)^3} &= \frac{1}{8} \\ (2x)^{\frac{3}{2}} &= \frac{1}{8} \\ 2x &= \left(\frac{1}{8}\right)^{\frac{2}{3}} \\ 2x &= (2^{-3})^{\frac{2}{3}} \\ 2x &= 2^{-2} \\ 2x &= \frac{1}{4} \\ x &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} 43 \quad \text{a} \quad \frac{1}{3}x^{1\frac{1}{2}} &= 2\frac{2}{3} - \frac{1}{3}x^{1\frac{1}{2}} \\ \frac{2}{3}x^{1\frac{1}{2}} &= 2\frac{2}{3} \\ x^{\frac{3}{2}} &= 4 \\ x &= 4^{\frac{2}{3}} \\ x &= \sqrt[3]{4^2} = \sqrt[3]{16} \\ \text{b} \quad 3 \cdot \sqrt[4]{(2x)^{-1}} &= 6 \\ \sqrt[4]{(2x)^{-1}} &= 2 \\ (2x)^{-\frac{1}{4}} &= 2 \\ 2x &= 2^{-4} \\ 2x &= \frac{1}{16} \\ x &= \frac{1}{32} \end{aligned}$$

$$\begin{aligned} \text{c} \quad (x+9)^{-1\frac{1}{2}} &= \frac{8}{27} \\ x+9 &= \left(\frac{8}{27}\right)^{-\frac{2}{3}} \\ x+9 &= \left(\left(\frac{2}{3}\right)^3\right)^{-\frac{2}{3}} \\ x+9 &= \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4} \\ x &= -6\frac{3}{4} \\ \text{d} \quad \frac{1}{3}(x^2-3)^{2\frac{1}{2}} &= 81 \\ (x^2-3)^{\frac{5}{2}} &= 243 \\ x^2-3 &= 243^{\frac{2}{5}} \\ x^2-3 &= (3^5)^{\frac{2}{5}} \\ x^2-3 &= 3^2 \\ x^2 &= 12 \\ x &= 2\sqrt{3} \vee x = -2\sqrt{3} \end{aligned}$$

$$\begin{aligned} 44 \quad \text{a} \quad y &= ax^{1,3} \\ x &= 2 \text{ en } y = 17 \end{aligned} \left\{ \begin{aligned} a \cdot 2^{1,3} &= 17 \\ a &= \frac{17}{2^{1,3}} \\ a &\approx 6,90 \end{aligned} \right.$$

$$\begin{aligned} \text{b} \quad y &= \frac{b}{x^{3,1}} \\ x &= 5 \text{ en } y = 0,32 \end{aligned} \left\{ \begin{aligned} \frac{b}{5^{3,1}} &= 0,32 \\ b &= 0,32 \cdot 5^{3,1} \\ b &\approx 47 \end{aligned} \right.$$

#### Bladzijde 26

$$\begin{aligned} 45 \quad y &= 21x \cdot \sqrt[3]{x} \text{ geeft } 21x^{1\frac{1}{3}} = y \\ x^{\frac{4}{3}} &= \frac{1}{21}y \\ x &= \left(\frac{1}{21}y\right)^{\frac{3}{4}} = \left(\frac{1}{21}\right)^{\frac{3}{4}} \cdot y^{\frac{3}{4}} \approx 0,102y^{0,75} \\ \text{Dus } b &\approx 0,102. \end{aligned}$$

#### Bladzijde 27

$$\begin{aligned} 46 \quad \text{a} \quad y &= ax^{2,3} \\ x &= 3 \text{ en } y = 37 \end{aligned} \left\{ \begin{aligned} a \cdot 3^{2,3} &= 37 \\ a &= \frac{37}{3^{2,3}} \\ a &\approx 2,96 \end{aligned} \right.$$

Dus  $y = 2,96x^{2,3}$ .

$$\begin{aligned} \text{b} \quad y &= \frac{a}{x^{1,65}} \\ x &= 8 \text{ en } y = 3 \end{aligned} \left\{ \begin{aligned} \frac{a}{8^{1,65}} &= 3 \\ a &= 3 \cdot 8^{1,65} \\ a &\approx 93 \end{aligned} \right.$$

Dus  $y = \frac{93}{x^{1,65}}$ .

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$$\text{a } \left. \begin{array}{l} y = ax^{-\frac{2}{5}} \\ x = 32 \text{ en } y = 5 \end{array} \right\} \begin{array}{l} a \cdot 32^{-\frac{2}{5}} = 5 \\ \frac{1}{4}a = 5 \\ a = 20 \end{array}$$

$$\text{Dus } y = 20x^{-\frac{2}{5}}.$$

$$y = 20x^{-\frac{2}{5}} \text{ geeft } 20x^{-\frac{2}{5}} = y$$

$$x^{-\frac{2}{5}} = \frac{1}{20}y$$

$$x = \left(\frac{1}{20}y\right)^{-\frac{5}{2}} = \left(\frac{1}{20}\right)^{-\frac{5}{2}} \cdot y^{-\frac{5}{2}} \approx 1789y^{-2,5}$$

$$\text{Dus } x = 1789y^{-2,5}.$$

$$\text{b } \left. \begin{array}{l} y = \frac{a}{x \cdot \sqrt[3]{x}} \\ x = 64 \text{ en } y = 3,5 \end{array} \right\} \begin{array}{l} \frac{a}{64 \cdot \sqrt[3]{64}} = 3,5 \\ \frac{a}{256} = 3,5 \\ a = 896 \end{array}$$

$$\text{Dus } y = \frac{896}{x \cdot \sqrt[3]{x}}.$$

$$y = \frac{896}{x \cdot \sqrt[3]{x}} \text{ geeft } y = \frac{896}{x^{1\frac{1}{3}}}$$

$$x^{1\frac{1}{3}}y = 896$$

$$x^{\frac{4}{3}} = \frac{896}{y}$$

$$x^{\frac{4}{3}} = 896y^{-1}$$

$$x = (896y^{-1})^{\frac{3}{4}} = 896^{\frac{3}{4}} \cdot y^{-\frac{3}{4}} \approx 164y^{-0,75}$$

$$\text{Dus } x = 164y^{-0,75}.$$

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$$\text{a } \left. \begin{array}{l} y = a\sqrt{2x-5} \\ x = 7 \text{ en } y = 1 \end{array} \right\} \begin{array}{l} a\sqrt{2 \cdot 7 - 5} = 1 \\ 3a = 1 \\ a = \frac{1}{3} \end{array}$$

$$\text{Dus } y = \frac{1}{3}\sqrt{2x-5}.$$

$$\text{b } \left. \begin{array}{l} y = \frac{a}{3x+4} \\ x = 2 \text{ en } y = 8 \end{array} \right\} \begin{array}{l} \frac{a}{3 \cdot 2 + 4} = 8 \\ \frac{a}{10} = 8 \\ a = 80 \end{array}$$

$$\text{Dus } y = \frac{80}{3x+4}.$$

$$\text{c } \left. \begin{array}{l} y = a(x+4)^3 \\ x = -7 \text{ en } y = 81 \end{array} \right\} \begin{array}{l} a(-7+4)^3 = 81 \\ -27a = 81 \\ a = -3 \end{array}$$

$$\text{Dus } y = -3(x+4)^3.$$

d  $y = \frac{1}{3}\sqrt{2x-5}$

$$\sqrt{2x-5} = 3y$$

kwadrateren geeft  $2x-5 = (3y)^2$

$$2x-5 = 9y^2$$

$$2x = 9y^2 + 5$$

$$x = 4\frac{1}{2}y^2 + 2\frac{1}{2}$$

$$y = \frac{80}{3x+4} \text{ geeft } y(3x+4) = 80$$

$$3x+4 = \frac{80}{y}$$

$$3x = \frac{80}{y} - 4$$

$$x = \frac{80}{3y} - 1\frac{1}{3}$$

$$y = -3(x+4)^3 \text{ geeft } -3(x+4)^3 = y$$

$$(x+4)^3 = -\frac{1}{3}y$$

$$x+4 = \sqrt[3]{-\frac{1}{3}y}$$

$$x = \sqrt[3]{-\frac{1}{3}y} - 4$$

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$$\left. \begin{aligned} D &= a(1 - 0,0226h)^{5,26} \\ h &= 1,5 \text{ en } D = 846 \end{aligned} \right\} \begin{aligned} a(1 - 0,0226 \cdot 1,5)^{5,26} &= 846 \\ 0,834\dots a &= 846 \\ a &= 1014,271\dots \end{aligned}$$

Dus  $D = 1014,271\dots \cdot (1 - 0,0226h)^{5,26}$ .

$$D = 500 \text{ geeft } 1014,271\dots \cdot (1 - 0,0226h)^{5,26} = 500$$

$$(1 - 0,0226h)^{5,26} = 0,492\dots$$

$$1 - 0,0226h = 0,492\dots^{\frac{1}{5,26}}$$

$$1 - 0,0226h = 0,874\dots$$

$$-0,0226h = -0,125\dots$$

$$h = 5,567\dots$$

Dus op een hoogte van 5600 m boven zeeniveau is de luchtdruk 0,5 bar.

50  $T^2 = a \cdot r^3$

$$T_A^2 = a \cdot r_A^3, \text{ dus } a = \frac{T_A^2}{r_A^3}.$$

$$T_V^2 = a \cdot r_V^3 \text{ met } a = \frac{T_A^2}{r_A^3} \text{ en } r_V = \frac{3}{4}r_A \text{ geeft } T_V^2 = \frac{T_A^2}{r_A^3} \cdot \left(\frac{3}{4}r_A\right)^3 = \frac{T_A^2}{r_A^3} \cdot \frac{27}{64}r_A^3 = \frac{27}{64}T_A^2$$

$$T_V^2 = \frac{27}{64} \cdot 365^2 = 56204,2\dots$$

$$T_V \approx 237$$

Dus een Venusjaar duurt 237 aardse dagen.

### Bladzijde 28

C5 a  $\sqrt[4]{x^3} \cdot \sqrt{x} = x^{\frac{3}{4}} \cdot x^{\frac{1}{2}} = x^{1\frac{1}{4}}$

b  $\sqrt[3]{x} \cdot \frac{1}{\sqrt{x^3}} = x^{\frac{1}{3}} \cdot \frac{1}{x^{\frac{3}{2}}} = x^{\frac{1}{3}} \cdot x^{-1\frac{1}{2}} = x^{-1\frac{1}{6}}$

c  $\frac{x\sqrt{x}}{x^2 \cdot \sqrt[4]{x}} = \frac{x \cdot x^{\frac{1}{2}}}{x^2 \cdot x^{\frac{1}{4}}} = \frac{x^{1\frac{1}{2}}}{x^{2\frac{1}{4}}} = x^{-\frac{3}{4}}$

C6 a  $x^{3\frac{2}{5}} = x^3 \cdot x^{\frac{2}{5}} = x^3 \cdot \sqrt[5]{x^2}$

b  $\frac{1}{4}x^{1\frac{1}{2}}y^{-\frac{1}{3}} = \frac{x^{1\frac{1}{2}}}{4y^{\frac{1}{3}}} = \frac{x \cdot x^{\frac{1}{2}}}{4y^{\frac{1}{3}}} = \frac{x\sqrt{x}}{4 \cdot \sqrt[3]{y}}$

c  $\frac{\frac{3}{7}x^{-2}y^{1\frac{1}{4}}}{7x^2} = \frac{3y^{1\frac{1}{4}}}{7x^2} = \frac{3y \cdot y^{\frac{1}{4}}}{7x^2} = \frac{3y \cdot \sqrt[4]{y}}{7x^2}$

**C7**  $6x^{-\frac{1}{2}} + 5 = 53$   
 $6x^{-\frac{1}{2}} = 48$   
 $x^{-\frac{1}{2}} = 8$   
 $x = 8^{-2}$   
 $x = (2^3)^{-2}$   
 $x = 2^{-2}$   
 $x = \frac{1}{4}$

**C8 a**  $y = ax^{2,8}$   
 $x = 3$  en  $y = 117$   $\left\{ \begin{array}{l} a \cdot 3^{2,8} = 117 \\ a = \frac{117}{3^{2,8}} \\ a \approx 5,4 \end{array} \right.$

Dus  $y = 5,4x^{2,8}$ .

**b**  $y = \frac{a}{x\sqrt{x}}$   
 $x = 25$  en  $y = 4$   $\left\{ \begin{array}{l} \frac{a}{25\sqrt{25}} = 4 \\ \frac{a}{125} = 4 \\ a = 500 \end{array} \right.$

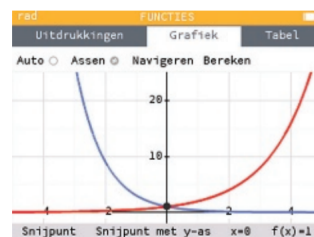
Dus  $y = \frac{500}{x\sqrt{x}}$ .

**C9**  $y = \frac{3,7}{x^{1,36}}$  geeft  $x^{1,36}y = 3,7$   
 $x^{1,36} = \frac{3,7}{y}$   
 $x^{1,36} = 3,7y^{-1}$   
 $x = (3,7y^{-1})^{\frac{1}{1,36}} = 3,7^{\frac{1}{1,36}} \cdot y^{-\frac{1}{1,36}} \approx 2,62y^{-0,74}$   
Dus  $x = 2,62y^{-0,74}$ .

## 5.4 Exponentiële functies

### Bladzijde 29

- 51 a** Zie de plot hiernaast.  
**b** De grafiek van  $f$  is stijgend en de grafiek van  $g$  is dalend.  
**c**  $f(-10) = 2^{-10} \approx 9,77 \cdot 10^{-4}$   
 $f(-20) = 2^{-20} \approx 9,54 \cdot 10^{-7}$   
 $f(-100) = 2^{-100} \approx 7,89 \cdot 10^{-31}$   
**d** Voor elke  $x$  is  $2^x > 0$ , dus er is geen origineel te vinden waarvan het beeld 0 is.  
**e**  $B_f = \langle 0, \rightarrow \rangle$  en  $B_g = \langle 0, \rightarrow \rangle$



### Bladzijde 31

- 52 a** Het gaat hier om de vermenigvuldiging met  $\frac{1}{4}$  ten opzichte van de  $y$ -as.  
In dit geval geldt dat  $a = 4$ .  
**b** Het gaat hier om de vermenigvuldiging met  $-1$  ten opzichte van de  $y$ -as.  
 $k(x) = 2^{-x} = (2^{-1})^x = (\frac{1}{2})^x$   
**c**  $l(x) = (\frac{1}{8})^x = (2^{-3})^x = 2^{-3x}$   
Het gaat hier dus om de vermenigvuldiging met  $-\frac{1}{3}$  ten opzichte van de  $y$ -as.

- 53**  $y = 0,7^x$   
 $\downarrow \times -1$  t.o.v. de  $x$ -as  
 $y = -0,7^x$   
 $\downarrow \times 5$  t.o.v. de  $y$ -as  
 $y = -0,7^{0,2x}$   
 $\downarrow$  translatie  $(3, 150)$   
 $y = 150 - 0,7^{0,2(x-3)} = 150 - 0,7^{0,2x-0,6}$   
Dus je krijgt de grafiek van  $y = 150 - 0,7^{0,2x-0,6}$ .

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- a**  $y = 0,4^x$   
 $\downarrow \times -6$  t.o.v. de  $x$ -as  
 $y = -6 \cdot 0,4^x$   
 $\downarrow$  translatie  $(0, 5)$   
 $f(x) = 5 - 6 \cdot 0,4^x$   
 $B_f = \langle \leftarrow, 5 \rangle$  en de asymptoot is de lijn  $y = 5$ .

- b**  $y = 2^x$   
 $\downarrow \times 3$  t.o.v. de  $x$ -as  
 $y = 3 \cdot 2^x$   
 $\downarrow$  translatie  $(0, 4)$   
 $y = 3 \cdot 2^x + 4$   
 $\downarrow \times \frac{1}{3}$  t.o.v. de  $y$ -as  
 $g(x) = 3 \cdot 2^{3x} + 4$   
 $B_g = \langle 4, \rightarrow \rangle$  en de asymptoot is de lijn  $y = 4$ .

- c**  $y = 0,8^x$   
 $\downarrow$  translatie  $(4, -10)$   
 $y = 0,8^{x-4} - 10$   
 $\downarrow \times 2$  t.o.v. de  $y$ -as  
 $h(x) = 0,8^{0,5x-4} - 10$   
 $B_h = \langle -10, \rightarrow \rangle$  en de asymptoot is de lijn  $y = -10$ .

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- a**  $y = 0,8^x$   
 $\downarrow \times 2$  t.o.v. de  $x$ -as  
 $y = 2 \cdot 0,8^x$   
 $\downarrow$  translatie  $(0, -7)$   
 $y = 2 \cdot 0,8^x - 7$   
 $\downarrow \times \frac{1}{2}$  t.o.v. de  $y$ -as  
 $f(x) = 2 \cdot 0,8^{2x} - 7$   
 $B_f = \langle -7, \rightarrow \rangle$  en de asymptoot is de lijn  $y = -7$ .

- b**  $y = 0,3^x$   
 $\downarrow \times -1$  t.o.v. de  $x$ -as  
 $y = -0,3^x$   
 $\downarrow$  translatie  $(-2, 1000)$   
 $y = 1000 - 0,3^{x+2}$   
 $\downarrow \times 10$  t.o.v. de  $y$ -as  
 $g(x) = 1000 - 0,3^{0,1x+2}$   
 $B_g = \langle \leftarrow, 1000 \rangle$  en de asymptoot is de lijn  $y = 1000$ .

- c**  $h(x) = 1000(1 - 1,3^x) = 1000 - 1000 \cdot 1,3^x$   
 $y = 1,3^x$   
 $\downarrow \times -1000$  t.o.v. de  $x$ -as  
 $y = -1000 \cdot 1,3^x$   
 $\downarrow$  translatie  $(0, 1000)$   
 $h(x) = 1000 - 1000 \cdot 1,3^x$   
 $B_h = \langle \leftarrow, 1000 \rangle$  en de asymptoot is de lijn  $y = 1000$ .

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- a**  $y = 0,7^x$   
 $\downarrow \times -1$  t.o.v. de  $x$ -as  
 $y = -0,7^x$   
 $\downarrow$  translatie  $(-1, 4)$   
 $y = 4 - 0,7^{x+1}$   
 $\downarrow \times -5$  t.o.v. de  $y$ -as  
 $f(x) = 4 - 0,7^{-0,2x+1}$

- b**  $y = \sqrt{x}$   
 $\downarrow \times 2$  t.o.v. de  $x$ -as  
 $y = 2\sqrt{x}$   
 $\downarrow$  translatie  $(5, -1)$   
 $y = -1 + 2\sqrt{x-5}$   
 $\downarrow \times \frac{1}{3}$  t.o.v. de  $y$ -as  
 $h(x) = -1 + 2\sqrt{3x-5}$

**57 a**  $y = 3^x$   
 $\downarrow \times -1$  t.o.v. de  $x$ -as  
 $y = -3^x$   
 $\downarrow$  translatie  $(0, -1)$   
 $y = -3^x - 1$

**b**  $y = 3^x$   
 $\downarrow$  translatie  $(2, 5)$   
 $y = 3^{x-2} + 5$   
 $\downarrow \times 6$  t.o.v. de  $y$ -as  
 $y = 3^{\frac{1}{6}x-2} + 5$

**c**  $y = 3^x$   
 $\downarrow$  translatie  $(4, -5)$   
 $y = 3^{x-4} - 5$   
 $\downarrow \times 3$  t.o.v. de  $x$ -as  
 $y = 3(3^{x-4} - 5) = 3 \cdot 3^{x-4} - 15 (= 3^{x-3} - 15)$

**d**  $y = 3^x$   
 $\downarrow \times 3$  t.o.v. de  $x$ -as  
 $y = 3 \cdot 3^x$   
 $\downarrow$  translatie  $(4, -5)$   
 $y = 3 \cdot 3^{x-4} - 5 (= 3^{x-3} - 5)$

**58 a**  $y = 2^x$   
 $\downarrow$  translatie  $(2, -3)$   
 $f(x) = 2^{x-2} - 3$

**b**  $y = 0,5^x$   
 $\downarrow \times 4$  t.o.v. de  $x$ -as  
 $y = 4 \cdot 0,5^x$   
 $\downarrow$  translatie  $(3, -1)$   
 $g(x) = 4 \cdot 0,5^{x-3} - 1$

**c**  $f(2) = -2$  en de asymptoot van de grafiek van  $f$  is de lijn  $y = -3$ .  
Dus voor  $x \leq 2$  is  $-3 < f(x) \leq -2$ .

**d**  $g(2) = 7$  en de asymptoot van de grafiek van  $g$  is de lijn  $y = -1$ .  
Dus voor  $x \geq 2$  is  $-1 < g(x) \leq 7$ .

**e** Voor  $-3 < p \leq -1$ .

**f**  $f(1) = 2^{-1} - 3 = \frac{1}{2} - 3 = -2\frac{1}{2}$   
 $g(1) = 4 \cdot 0,5^{-2} - 1 = 4 \cdot 4 - 1 = 15$   
Dus  $PQ = 15 - -2\frac{1}{2} = 17\frac{1}{2}$ .

**59 a**  $8 \cdot 4^x = 2^3 \cdot (2^2)^x = 2^3 \cdot 2^{2x} = 2^{2x+3}$

**b** Bij de herleiding van  $(2^2)^x$  tot  $2^{2x}$  gebruik je de regel  $(a^p)^q = a^{pq}$ .  
Bij de herleiding van  $2^3 \cdot 2^{2x}$  tot  $2^{2x+3}$  gebruik je de regel  $a^p \cdot a^q = a^{p+q}$ .

### Bladzijde 33

**60 a**  $y = 15 \cdot 2^{3x+2} = 15 \cdot 2^{3x} \cdot 2^2 = 15 \cdot (2^3)^x \cdot 4 = 60 \cdot 8^x$   
Dus  $y = 60 \cdot 8^x$ .

**b**  $y = 260 \cdot 4^{1\frac{1}{2}x-1} = 260 \cdot 4^{1\frac{1}{2}x} \cdot 4^{-1} = 260 \cdot (4^{1\frac{1}{2}})^x \cdot \frac{1}{4} = 65 \cdot 8^x$   
Dus  $y = 65 \cdot 8^x$ .

**c**  $y = 8 \cdot 4^{-2x-1} = 8 \cdot 4^{-2x} \cdot 4^{-1} = 8 \cdot (4^{-2})^x \cdot \frac{1}{4} = 2 \cdot (\frac{1}{16})^x$   
Dus  $y = 2 \cdot (\frac{1}{16})^x$ .

**61 a**  $y = 2^x \cdot 2^{2x-3} = 2^x \cdot 2^{2x} \cdot 2^{-3} = 2^{3x} \cdot \frac{1}{8} = (2^3)^x \cdot \frac{1}{8} = \frac{1}{8} \cdot 8^x$   
Dus  $y = \frac{1}{8} \cdot 8^x$ .

**b**  $y = 63 \cdot 3^{\frac{1}{2}x-2} = 63 \cdot 3^{\frac{1}{2}x} \cdot 3^{-2} = 63 \cdot (3^{\frac{1}{2}})^x \cdot \frac{1}{9} = 7 \cdot (\sqrt{3})^x$   
Dus  $y = 7 \cdot (\sqrt{3})^x$ .

**c**  $y = 50000 \cdot 100^{-x-2\frac{1}{2}} = 50000 \cdot 100^{-x} \cdot 100^{-2\frac{1}{2}} = 50000 \cdot (100^{-1})^x \cdot \frac{1}{100000} = \frac{1}{2} \cdot (\frac{1}{100})^x$   
Dus  $y = \frac{1}{2} \cdot (\frac{1}{100})^x$ .

**62 a**  $y = 50 \cdot 2^{3x-1} = 50 \cdot 2^{3x} \cdot 2^{-1} = 50 \cdot (2^3)^x \cdot \frac{1}{2} = 25 \cdot 8^x$   
Dus  $y = 25 \cdot 8^x$ .

**b**  $y = 10x \cdot \sqrt[3]{x^2} = 10x \cdot x^{\frac{2}{3}} = 10x^{1\frac{2}{3}}$   
Dus  $y = 10x^{1\frac{2}{3}}$ .

**c**  $y = 162 \cdot 9^{-\frac{1}{2}x-2} = 162 \cdot 9^{-\frac{1}{2}x} \cdot 9^{-2} = 162 \cdot (9^{-\frac{1}{2}})^x \cdot \frac{1}{81} = 2 \cdot (\frac{1}{3})^x$   
Dus  $y = 2 \cdot (\frac{1}{3})^x$ .

**d**  $y = \frac{3\sqrt{x}}{x \cdot \sqrt[3]{x}} = \frac{3x^{\frac{1}{2}}}{x \cdot x^{\frac{1}{3}}} = \frac{3x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = 3x^{\frac{5}{6}}$   
Dus  $y = 3x^{\frac{5}{6}}$ .

$$\text{e } y = \frac{1}{4x^2 \cdot \sqrt{x}} = \frac{1}{4x^2 \cdot x^{\frac{1}{2}}} = \frac{1}{4x^{2\frac{1}{2}}} = \frac{1}{4}x^{-2\frac{1}{2}}$$

$$\text{Dus } y = \frac{1}{4}x^{-2\frac{1}{2}}.$$

$$\text{f } y = \frac{64}{4^{\frac{1}{2}x+2}} = 64 \cdot 4^{-(\frac{1}{2}x+2)} = 64 \cdot 4^{-\frac{1}{2}x-2} = 64 \cdot 4^{-\frac{1}{2}x} \cdot 4^{-2} = 64 \cdot (4^{-\frac{1}{2}})^x \cdot \frac{1}{16} = 4 \cdot (\frac{1}{2})^x$$

$$\text{Dus } y = 4 \cdot (\frac{1}{2})^x.$$

$$\text{63 a } g(x) = \frac{1}{3} \cdot 3^{x-3} - 2 = 3^{-1} \cdot 3^{x-3} - 2 = 3^{x-4} - 2$$

$$f(x) = 3^x$$

↓ translatie (4, -2)

$$g(x) = 3^{x-4} - 2$$

Dus  $p = 4$  en  $q = -2$ .

$$\text{b } h(x) = 2^{x-3}$$

↓ translatie (-5, 0)

$$k(x) = 2^{x+5-3} = 2^{x-3} \cdot 2^5 = 32 \cdot 2^{x-3}$$

Door de grafiek van  $h$  met 32 te vermenigvuldigen ten opzichte van de  $x$ -as krijg je ook de grafiek van  $k$ .

Dus  $a = 32$ .

### Bladzijde 34

$$\text{64 } f(x) = 5 - 2^{\frac{1}{2}x+4}$$

↓ translatie (12, -8)

$$h(x) = 5 - 2^{\frac{1}{2}(x-12)+4} - 8 = -3 - 2^{\frac{1}{2}x-2} = -3 - 2^{\frac{1}{2}x} \cdot 2^{-2} = -3 - (2^{\frac{1}{2}})^x \cdot \frac{1}{4} = -3 - \frac{1}{4} \cdot (\sqrt{2})^x$$

$$\text{Dus } a = -3, b = -\frac{1}{4} \text{ en } g = \sqrt{2}.$$

$$\text{65 a } h = 10 \text{ geeft } d = 1,225 \cdot 0,88^{10} = 0,341 \dots$$

$$900 \text{ km/u} = 250 \text{ m/s, dus } v = 250.$$

$$\text{Na een uur vliegen is } G = 0,03 \cdot 0,341 \dots \cdot 250^2 \cdot 541 = 346\,067,9 \dots$$

$$\text{Dus het startgewicht is } 346\,067,9 \dots + 0,8 \cdot 12\,000 \approx 355\,668 \text{ kg.}$$

$$\text{b Na 10 uur is } G = 355\,667,9 \dots - 10 \cdot 12\,000 \cdot 0,8 = 259\,667,9 \dots$$

$$\text{Dit geeft } 0,03d \cdot 250^2 \cdot 541 = 259\,667,9 \dots$$

$$d = \frac{259\,667,9 \dots}{0,03 \cdot 250^2 \cdot 541} = 0,255 \dots$$

$$d = 0,255 \dots \text{ geeft } 1,225 \cdot 0,88^h = 0,255 \dots$$

$$\text{Los op } 1,225 \cdot 0,88^h = 0,255 \dots$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft  $h \approx 12,2$ .

Dus de vlieghoogte tien uur na de start is 12,2 km.

$$\text{c } G = 355\,667,9 \dots - 12\,000 \cdot 0,8t = 355\,668 - 9600t \text{ met } t \text{ in uren}$$

$$\text{Dit geeft } 355\,667,9 \dots - 9600t = 0,03 \cdot 1,225 \cdot 0,88^h \cdot 250^2 \cdot 541$$

$$355\,667,9 \dots - 9600t = 1\,242\,609,375 \cdot 0,88^h$$

$$0,88^h = -0,00772 \dots \cdot t + 0,286 \dots$$

$$\text{Dus } g = 0,88, a \approx -0,0077 \text{ en } b \approx 0,29.$$

$$\text{66 } 4\sqrt{2} = 2^2 \cdot 2^{\frac{1}{2}} = 2^{2\frac{1}{2}}$$

$$2^{x-1} = 4\sqrt{2}$$

$$2^{x-1} = 2^{2\frac{1}{2}}$$

$$x-1 = 2\frac{1}{2}$$

$$x = 3\frac{1}{2}$$

**67**  $2 \cdot 9^{\frac{1}{2}x-3} = 6$

$$9^{\frac{1}{2}x-3} = 3$$

$$9^{\frac{1}{2}x-3} = 9^{\frac{1}{2}}$$

$$\frac{1}{2}x - 3 = \frac{1}{2}$$

$$\frac{1}{2}x = 3\frac{1}{2}$$

$$x = 7$$

\*

**68 a**  $2^{x-3} = \frac{1}{8}$   
 $2^{x-3} = 2^{-3}$

$$x - 3 = -3$$

$$x = 0$$

**b**  $3^{2x} - 2 = 25$

$$3^{2x} = 27$$

$$3^{2x} = 3^3$$

$$2x = 3$$

$$x = 1\frac{1}{2}$$

**c**  $5 \cdot 2^x = 80$

$$2^x = 16$$

$$2^x = 2^4$$

$$x = 4$$

**d**  $3^{2x+1} = 27\sqrt{3}$

$$3^{2x+1} = 3^3 \cdot 3^{\frac{1}{2}}$$

$$3^{2x+1} = 3^{3\frac{1}{2}}$$

$$2x + 1 = 3\frac{1}{2}$$

$$2x = 2\frac{1}{2}$$

$$x = 1\frac{1}{4}$$

**e**  $10 \cdot 3^{2x-1} = 270$

$$3^{2x-1} = 27$$

$$3^{2x-1} = 3^3$$

$$2x - 1 = 3$$

$$2x = 4$$

$$x = 2$$

**f**  $3 \cdot 8^{2-x} = 48$

$$8^{2-x} = 16$$

$$(2^3)^{2-x} = 2^4$$

$$2^{6-3x} = 2^4$$

$$6 - 3x = 4$$

$$-3x = -2$$

$$x = \frac{2}{3}$$

**69 a**  $3 \cdot 2^x + 4 = 28$

$$3 \cdot 2^x = 24$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

**b**  $5^{2x-6} = \frac{1}{25}$

$$5^{2x-6} = 5^{-2}$$

$$2x - 6 = -2$$

$$2x = 4$$

$$x = 2$$

**c**  $7^{3x+1} = 49^{2x}$

$$7^{3x+1} = (7^2)^{2x}$$

$$7^{3x+1} = 7^{4x}$$

$$3x + 1 = 4x$$

$$-x = -1$$

$$x = 1$$

**d**  $32^{x-2} = \frac{1}{16}$

$$(2^5)^{x-2} = 2^{-4}$$

$$2^{5x-10} = 2^{-4}$$

$$5x - 10 = -4$$

$$5x = 6$$

$$x = 1\frac{1}{5}$$

**e**  $5 \cdot 4^{x-1} = 2\frac{1}{2}$

$$4^{x-1} = \frac{1}{2}$$

$$(2^2)^{x-1} = 2^{-1}$$

$$2^{2x-2} = 2^{-1}$$

$$2x - 2 = -1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

**f**  $8 \cdot 2^x = 4^{x+1}$

$$2^3 \cdot 2^x = (2^2)^{x+1}$$

$$2^{3+x} = 2^{2x+2}$$

$$3 + x = 2x + 2$$

$$-x = -1$$

$$x = 1$$

**70 a**  $4 \cdot 2^x = 2^x + 3$

$$3 \cdot 2^x = 3$$

$$2^x = 1$$

$$x = 0$$

**b**  $\frac{1}{5}(2x+1) + 2 = \frac{1}{2}x + 3$

$$\frac{2}{5}x + \frac{1}{5} + 2 = \frac{1}{2}x + 3$$

$$4x + 2 + 20 = 5x + 30$$

$$-x = 8$$

$$x = -8$$

**c**  $3x^3 + 17x^2 = 5x^2 + 36x$

$$3x^3 + 12x^2 - 36x = 0$$

$$3x(x^2 + 4x - 12) = 0$$

$$3x(x-2)(x+6) = 0$$

$$x = 0 \vee x = 2 \vee x = -6$$

**d**  $2\sqrt{x+5} = x+2$

kwadrateren geeft

$$4(x+5) = (x+2)^2$$

$$4x + 20 = x^2 + 4x + 4$$

$$x^2 = 16$$

$$x = 4 \vee x = -4$$

$x = -4$  voldoet niet, dus  $x = 4$ .

**e**  $5^{x+6} = (\frac{1}{5})^x$

$$5^{x+6} = (5^{-1})^x$$

$$5^{x+6} = 5^{-x}$$

$$x + 6 = -x$$

$$2x = -6$$

$$x = -3$$

**f**  $\frac{3x+2}{2x+5} = \frac{3x+2}{3x^2}$

$$3x + 2 = 0 \vee 2x + 5 = 3x^2$$

$$3x = -2 \quad \vee -3x^2 + 2x + 5 = 0$$

$$x = -\frac{2}{3} \quad \vee D = 2^2 - 4 \cdot -3 \cdot 5 = 64, \text{ dus } \sqrt{D} = 8$$

$$x = -\frac{2}{3} \quad \vee x = \frac{-2+8}{-6} = -1 \vee x = \frac{-2-8}{-6} = 1\frac{2}{3}$$



**71**  $g(x) = 0$  geeft  $\frac{1}{4} \cdot 2^{x-3} - 1 = 0$

$$\frac{1}{4} \cdot 2^{x-3} = 1$$

$$2^{x-3} = 4$$

$$2^{x-3} = 2^2$$

$$x - 3 = 2$$

$$x = 5, \text{ dus } A(5, 0).$$

$$f(5) = 3, \text{ dus } B(5, 3).$$

$$g(x) = 3 \text{ geeft } \frac{1}{4} \cdot 2^{x-3} - 1 = 3$$

$$\frac{1}{4} \cdot 2^{x-3} = 4$$

$$2^{x-3} = 16$$

$$2^{x-3} = 2^4$$

$$x - 3 = 4$$

$$x = 7, \text{ dus } C(7, 3).$$

$$AB = 3 - 0 = 3 \text{ en } BC = 7 - 5 = 2$$

$$\text{Dus } AC = \sqrt{3^2 + 2^2} = \sqrt{13}.$$

### Bladzijde 36

**72**  $g(x) = \frac{1}{3} \cdot 3^{x-3} - 2$

↓  $\times -6$  t.o.v. de  $x$ -as

$$y = -6\left(\frac{1}{3} \cdot 3^{x-3} - 2\right) = -2 \cdot 3^{x-3} + 12$$

↓ translatie  $(0, -7)$

$$h(x) = -2 \cdot 3^{x-3} + 12 - 7 = -2 \cdot 3^{x-3} + 5$$

$$g(x) = h(x) \text{ geeft } \frac{1}{3} \cdot 3^{x-3} - 2 = -2 \cdot 3^{x-3} + 5$$

$$2\frac{1}{3} \cdot 3^{x-3} = 7$$

$$3^{x-3} = 3$$

$$x - 3 = 1$$

$$x = 4$$

$$g(4) = -1, \text{ dus } S(4, -1).$$

**73 a**  $2^{x+1} + 2^x = 48$  geeft  $2^x \cdot 2^1 + 2^x = 48$

$$\text{Hieruit volgt } 2 \cdot 2^x + 2^x = 48.$$

**b**  $2 \cdot 2^x + 2^x = 48$

$$2 \cdot 2^x + 1 \cdot 2^x = 48$$

$$3 \cdot 2^x = 48$$

$$2^x = 16$$

$$2^x = 2^4$$

$$x = 4$$

**74 a**  $3^{x+2} + 3^x = 810$

$$3^x \cdot 3^2 + 3^x = 810$$

$$9 \cdot 3^x + 3^x = 810$$

$$10 \cdot 3^x = 810$$

$$3^x = 81$$

$$3^x = 3^4$$

$$x = 4$$

**b**  $2^{x-1} + 2^{x+1} = 10$

$$2^x \cdot 2^{-1} + 2^x \cdot 2 = 10$$

$$\frac{1}{2} \cdot 2^x + 2 \cdot 2^x = 10$$

$$2\frac{1}{2} \cdot 2^x = 10$$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

**c**  $2^{x+3} - 2^x = \frac{7}{8}$

$$2^x \cdot 2^3 - 2^x = \frac{7}{8}$$

$$8 \cdot 2^x - 2^x = \frac{7}{8}$$

$$7 \cdot 2^x = \frac{7}{8}$$

$$2^x = \frac{1}{8}$$

$$2^x = 2^{-3}$$

$$x = -3$$

**d**  $3^{x+2} = 24 + 3^x$

$$3^{x+2} - 3^x = 24$$

$$3^x \cdot 3^2 - 3^x = 24$$

$$9 \cdot 3^x - 3^x = 24$$

$$8 \cdot 3^x = 24$$

$$3^x = 3$$

$$x = 1$$

**e**  $3^x - 3^{x-1} = 2\sqrt{3}$

$$3^x - 3^x \cdot 3^{-1} = 2\sqrt{3}$$

$$3^x - \frac{1}{3} \cdot 3^x = 2\sqrt{3}$$

$$\frac{2}{3} \cdot 3^x = 2\sqrt{3}$$

$$3^x = 3\sqrt{3}$$

$$3^x = 3 \cdot 3^{\frac{1}{2}}$$

$$3^x = 3^{1\frac{1}{2}}$$

$$x = 1\frac{1}{2}$$

**f**  $5^{x-1} + 5^{x-2} = 6\sqrt{5}$

$$5^x \cdot 5^{-1} + 5^x \cdot 5^{-2} = 6\sqrt{5}$$

$$\frac{1}{5} \cdot 5^x + \frac{1}{25} \cdot 5^x = 6\sqrt{5}$$

$$\frac{6}{25} \cdot 5^x = 6\sqrt{5}$$

$$5^x = 25\sqrt{5}$$

$$5^x = 5^2 \cdot 5^{\frac{1}{2}}$$

$$5^x = 5^{2\frac{1}{2}}$$

$$x = 2\frac{1}{2}$$

**75 a**  $3^{x+1} = 9^{x+2}$   
 $3^{x+1} = (3^2)^{x+2}$   
 $3^{x+1} = 3^{2x+4}$   
 $x+1 = 2x+4$   
 $-x = 3$   
 $x = -3$

**b**  $3^{x+1} - 3^{x-1} = 8\sqrt{3}$   
 $3^x \cdot 3 - 3^x \cdot 3^{-1} = 8\sqrt{3}$   
 $3 \cdot 3^x - \frac{1}{3} \cdot 3^x = 8\sqrt{3}$   
 $2\frac{2}{3} \cdot 3^x = 8\sqrt{3}$   
 $3^x = 3\sqrt{3}$   
 $3^x = 3 \cdot 3^{\frac{1}{2}}$   
 $3^x = 3^{1\frac{1}{2}}$   
 $x = 1\frac{1}{2}$

**c**  $3^{x^2} = (\frac{1}{3})^{x-6}$   
 $3^{x^2} = (3^{-1})^{x-6}$   
 $3^{x^2} = 3^{-x+6}$   
 $x^2 = -x+6$   
 $x^2 + x - 6 = 0$   
 $(x-2)(x+3) = 0$   
 $x = 2 \vee x = -3$

**d**  $5^{x^2+5} = 125^{x+1}$   
 $5^{x^2+5} = (5^3)^{x+1}$   
 $5^{x^2+5} = 5^{3x+3}$   
 $x^2 + 5 = 3x + 3$   
 $x^2 - 3x + 2 = 0$   
 $(x-1)(x-2) = 0$   
 $x = 1 \vee x = 2$

**e**  $2^{x+2} - (\frac{1}{2})^{-x+1} = 28$   
 $2^{x+2} - (2^{-1})^{-x+1} = 28$   
 $2^{x+2} - 2^{x-1} = 28$   
 $2^x \cdot 2^2 - 2^x \cdot 2^{-1} = 28$   
 $4 \cdot 2^x - \frac{1}{2} \cdot 2^x = 28$   
 $3\frac{1}{2} \cdot 2^x = 28$   
 $2^x = 8$   
 $2^x = 2^3$   
 $x = 3$

**f**  $4^{x^2+1} = 8^{x^2-1}$   
 $(2^2)^{x^2+1} = (2^3)^{x^2-1}$   
 $2^{2x^2+2} = 2^{3x^2-3}$   
 $2x^2 + 2 = 3x^2 - 3$   
 $x^2 = 5$   
 $x = \sqrt{5} \vee x = -\sqrt{5}$

**76 a**  $3^{2x} - 12 \cdot 3^x + 27 = 0$   
 $(3^x)^2 - 12 \cdot 3^x + 27 = 0$   
 Stel  $3^x = u$ .  
 $u^2 - 12u + 27 = 0$   
 $(u-3)(u-9) = 0$   
 $u = 3 \vee u = 9$   
 $3^x = 3 \vee 3^x = 9$   
 $3^x = 3 \vee 3^x = 3^2$   
 $x = 1 \vee x = 2$

**b**  $2^{2x+1} + 2^{x+2} = 6$   
 $2^{2x} \cdot 2 + 2^x \cdot 2^2 = 6$   
 $2 \cdot (2^x)^2 + 4 \cdot 2^x - 6 = 0$   
 $(2^x)^2 + 2 \cdot 2^x - 3 = 0$   
 Stel  $2^x = u$ .  
 $u^2 + 2u - 3 = 0$   
 $(u-1)(u+3) = 0$   
 $u = 1 \vee u = -3$   
 $2^x = 1 \vee 2^x = -3$   
 $x = 0$

**c**  $4^{x+1\frac{1}{2}} - 2^{x+4} = 384$   
 $(2^2)^{x+1\frac{1}{2}} - 2^{x+4} = 384$   
 $2^{2x+3} - 2^{x+4} = 384$   
 $2^{2x} \cdot 2^3 - 2^x \cdot 2^4 = 384$   
 $8 \cdot (2^x)^2 - 16 \cdot 2^x - 384 = 0$   
 $(2^x)^2 - 2 \cdot 2^x - 48 = 0$   
 Stel  $2^x = u$ .  
 $u^2 - 2u - 48 = 0$   
 $(u+6)(u-8) = 0$   
 $u = -6 \vee u = 8$   
 $2^x = -6 \vee 2^x = 8$   
 $2^x = 2^3$   
 $x = 3$

### Bladzijde 37

**77 a**  $f(x) = g(x)$  geeft  $3^{x+1} - 4 = 6 - 3^{x-1}$   
 $3^{x+1} + 3^{x-1} = 10$   
 $3^x \cdot 3 + 3^x \cdot 3^{-1} = 10$   
 $3 \cdot 3^x + \frac{1}{3} \cdot 3^x = 10$   
 $3\frac{1}{3} \cdot 3^x = 10$   
 $3^x = 3$   
 $x = 1$

Uit de figuur in het leerboek volgt  $f(x) \leq g(x)$  geeft  $x \leq 1$ .

**b**  $f(2\frac{1}{2}) = 3^{3\frac{1}{2}} - 4 = 3^3 \cdot 3^{\frac{1}{2}} - 4 = 27\sqrt{3} - 4$   
 $g(2\frac{1}{2}) = 6 - 3^{1\frac{1}{2}} = 6 - 3 \cdot 3^{\frac{1}{2}} = 6 - 3\sqrt{3}$   
 Dus  $AB = 27\sqrt{3} - 4 - (6 - 3\sqrt{3}) = 30\sqrt{3} - 10$ .

**c**  $f(x) - g(x) = 80$  geeft  $3^{x+1} - 4 - (6 - 3^{x-1}) = 80$   
 $3^{x+1} + 3^{x-1} = 90$   
 $3^x \cdot 3 + 3^x \cdot 3^{-1} = 90$   
 $3 \cdot 3^x + \frac{1}{3} \cdot 3^x = 90$   
 $3\frac{1}{3} \cdot 3^x = 90$   
 $3^x = 27$   
 $3^x = 3^3$   
 $x = 3$

**d**  $g(x) - f(x) = 6 - 3^{x-1} - (3^{x+1} - 4)$   
 $= 10 - 3^{x-1} - 3^{x+1}$   
 $= 10 - 3^x \cdot 3^{-1} - 3^x \cdot 3$   
 $= 10 - \frac{1}{3} \cdot 3^x - 3 \cdot 3^x$   
 $= 10 - 3\frac{1}{3} \cdot 3^x$

Het bereik van  $g(x) - f(x)$  is  $\langle \leftarrow, 10 \rangle$ .

Dus voor  $p \geq 10$  heeft  $g(x) - f(x) = p$  geen oplossing.

**C10**

**a**  $y = 0,5^x$

 $\downarrow \times -3$  t.o.v. de  $x$ -as

$y = -3 \cdot 0,5^x$

 $\downarrow$  translatie  $(0, 1)$ 

$f(x) = 1 - 3 \cdot 0,5^x$

 $B_f = \langle \leftarrow, 1 \rangle$  en de asymptoot is de lijn  $y = 1$ .

**b**  $y = 3^x$

 $\downarrow \times 4$  t.o.v. de  $x$ -as

$y = 4 \cdot 3^x$

 $\downarrow$  translatie  $(-1, 0)$ 

$y = 4 \cdot 3^{x+1}$

 $\downarrow \times 5$  t.o.v. de  $y$ -as

$g(x) = 4 \cdot 3^{0,2x+1}$

 $B_g = \langle 0, \rightarrow \rangle$  en de asymptoot is de lijn  $y = 0$ .**C11**

**a**  $y = 5 \cdot 3^{2x+1} = 5 \cdot 3^{2x} \cdot 3 = 15 \cdot (3^2)^x = 15 \cdot 9^x$

Dus  $y = 15 \cdot 9^x$ .

**b**  $y = (\frac{1}{2})^x \cdot 2^{3x+4} = (2^{-1})^x \cdot 2^{3x} \cdot 2^4 = 2^{-x} \cdot 2^{3x} \cdot 16 = 16 \cdot 2^{2x} = 16 \cdot (2^2)^x = 16 \cdot 4^x$

Dus  $y = 16 \cdot 4^x$ .

**c**  $y = \frac{1}{5^{2x-1}} = 5^{-(2x-1)} = 5^{-2x+1} = 5^{-2x} \cdot 5 = 5 \cdot (5^{-2})^x = 5 \cdot (\frac{1}{25})^x$

Dus  $y = 5 \cdot (\frac{1}{25})^x$ .**C12**

**a**  $3^{x+2} = 9\sqrt{3}$

$3^{x+2} = 3^2 \cdot 3^{\frac{1}{2}}$

$3^{x+2} = 3^{2\frac{1}{2}}$

$x+2 = 2\frac{1}{2}$

$x = \frac{1}{2}$

**b**  $5 \cdot 2^{1-3x} = 20$

$2^{1-3x} = 4$

$2^{1-3x} = 2^2$

$1-3x = 2$

$-3x = 1$

$x = -\frac{1}{3}$

**c**  $8 \cdot 4^x = 2^{x-5}$

$2^3 \cdot (2^2)^x = 2^{x-5}$

$2^3 \cdot 2^{2x} = 2^{x-5}$

$2^{2x+3} = 2^{x-5}$

$2x+3 = x-5$

$x = -8$

**d**  $3^{x+1} - 3^x = 162$

$3^x \cdot 3 - 3^x = 162$

$2 \cdot 3^x = 162$

$3^x = 81$

$3^x = 3^4$

$x = 4$

**e**  $2^{x+1} + 2^{x-1} = \frac{5}{16}$

$2^x \cdot 2 + 2^x \cdot 2^{-1} = \frac{5}{16}$

$2 \cdot 2^x + \frac{1}{2} \cdot 2^x = \frac{5}{16}$

$2\frac{1}{2} \cdot 2^x = \frac{5}{16}$

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$

$x = -3$

**f**  $5^{2x-1} - (\frac{1}{25})^{-x+1} = 100$

$5^{2x-1} - (5^{-2})^{-x+1} = 100$

$5^{2x-1} - 5^{2x-2} = 100$

$5^{2x} \cdot 5^{-1} - 5^{2x} \cdot 5^{-2} = 100$

$\frac{1}{5} \cdot 5^{2x} - \frac{1}{25} \cdot 5^{2x} = 100$

$\frac{4}{25} \cdot 5^{2x} = 100$

$5^{2x} = 625$

$5^{2x} = 5^4$

$2x = 4$

$x = 2$

## 5.5 Logaritmen

### Bladzijde 38

**78**

**a**  $81 \times 2187 = 3^4 \times 3^7 = 3^{11} = 177\,147$

**b**  $27 \times 729 = 3^3 \times 3^6 = 3^9 = 19\,683$

$9 \times 6561 = 3^2 \times 3^8 = 3^{10} = 59\,049$

**c, d \***

**e**  $291 \times 1234 \approx 3^{5,16408} \times 3^{6,47910} = 3^{11,64318}$

In de tabel zou de logaritme  $5,16408 + 6,47910 = 11,64318$  moeten staan om de uitkomst te kunnen geven. $3^{11,64318} \approx 359\,094$  en  $291 \times 1234 = 359\,094$ , dus de gevonden waarde klopt.Bij de logaritme  $11,64318$  hoort dus, afgerond op gehele, het getal  $359\,094$ .

### Bladzijde 39

**79**

**a**  ${}^5\log(125) = {}^5\log(5^3) = 3$

**b**  ${}^{10}\log(0,1) = {}^{10}\log(\frac{1}{10}) = {}^{10}\log(10^{-1}) = -1$

**c**  ${}^2\log(4) = {}^2\log(2^2) = 2$

**d**  ${}^7\log(49) = {}^7\log(7^2) = 2$

**e**  ${}^2\log(\sqrt{2}) = {}^2\log(2^{\frac{1}{2}}) = \frac{1}{2}$

**f**  ${}^2\log(0,5) = {}^2\log(\frac{1}{2}) = {}^2\log(2^{-1}) = -1$

**g**  ${}^4\log(0,25) = {}^4\log(\frac{1}{4}) = {}^4\log(4^{-1}) = -1$

**h**  ${}^4\log(4) = {}^4\log(4^1) = 1$

**i**  ${}^4\log(1) = {}^4\log(4^0) = 0$

### Bladzijde 40

- 80** **a**  ${}^2\log(64\sqrt{2}) = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{6\frac{1}{2}}) = 6\frac{1}{2}$   
**b**  ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$   
**c**  ${}^3\log(3^{21,5}) = 21,5$   
**d**  ${}^5\log(\frac{1}{125}) = {}^5\log(5^{-3}) = -3$   
**e**  ${}^{\frac{1}{3}}\log(\frac{1}{27}) = {}^{\frac{1}{3}}\log((\frac{1}{3})^3) = 3$   
**f**  ${}^{\frac{1}{2}}\log(\frac{1}{4}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) = 2$   
**g**  ${}^2\log(\frac{1}{32} \cdot \sqrt[3]{2}) = {}^2\log(2^{-5} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-4\frac{2}{3}}) = -4\frac{2}{3}$   
**h**  ${}^5\log(1) = {}^5\log(5^0) = 0$   
**i**  ${}^3\log(81 \cdot \sqrt[5]{27}) = {}^3\log(3^4 \cdot \sqrt[5]{3^3}) = {}^3\log(3^4 \cdot 3^{\frac{3}{5}}) = {}^3\log(3^{4\frac{3}{5}}) = 4\frac{3}{5}$

- 81** **a**  ${}^3\log(9) = {}^3\log(3^2) = 2$ , dus  ${}^3\log(x) = 2$  geeft  $x = 9$ .  
**b**  ${}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$ , dus  ${}^5\log(x) = -2$  geeft  $x = \frac{1}{25}$ .

- 82** **a**  $x = {}^5\log(0,2) = {}^5\log(\frac{1}{5}) = {}^5\log(5^{-1}) = -1$   
**b**  ${}^9\log(x) = \frac{1}{2}$   
 $x = 9^{\frac{1}{2}} = \sqrt{9} = 3$   
**c**  ${}^x\log(1000) = 3$   
 $x^3 = 1000$   
 $x = 10$

- |                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                      |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>83</b> <b>a</b> <math>{}^3\log(x+2) = 2</math><br/> <math>x+2 = 3^2</math><br/> <math>x+2 = 9</math><br/> <math>x = 7</math></p> <p><b>b</b> <math>1 + {}^{\frac{1}{2}}\log(x) = 4</math><br/> <math>{}^{\frac{1}{2}}\log(x) = 3</math><br/> <math>x = (\frac{1}{2})^3</math><br/> <math>x = \frac{1}{8}</math></p> | <p><b>c</b> <math>{}^3\log(2x+1) = 4</math><br/> <math>2x+1 = 3^4</math><br/> <math>2x+1 = 81</math><br/> <math>2x = 80</math><br/> <math>x = 40</math></p> <p><b>d</b> <math>5 + {}^4\log(x) = 3</math><br/> <math>{}^4\log(x) = -2</math><br/> <math>x = 4^{-2}</math><br/> <math>x = \frac{1}{16}</math></p> | <p><b>e</b> <math>{}^{\frac{1}{2}}\log(x-1) = 3</math><br/> <math>x-1 = (\frac{1}{2})^3</math><br/> <math>x-1 = \frac{1}{8}</math><br/> <math>x = 1\frac{1}{8}</math></p> <p><b>f</b> <math>{}^2\log(x^2-4) = 5</math><br/> <math>x^2-4 = 2^5</math><br/> <math>x^2-4 = 32</math><br/> <math>x^2 = 36</math><br/> <math>x = 6 \vee x = -6</math></p> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

- |                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>84</b> <b>a</b> <math>4 \cdot {}^3\log(x) = 2</math><br/> <math>{}^3\log(x) = \frac{1}{2}</math><br/> <math>x = 3^{\frac{1}{2}}</math><br/> <math>x = \sqrt{3}</math></p> <p><b>b</b> <math>{}^3\log(4x-1) = -2</math><br/> <math>4x-1 = 3^{-2}</math><br/> <math>4x-1 = \frac{1}{9}</math><br/> <math>4x = 1\frac{1}{9}</math><br/> <math>x = \frac{5}{18}</math></p> | <p><b>c</b> <math>3 + {}^2\log(x) = -1</math><br/> <math>{}^2\log(x) = -4</math><br/> <math>x = 2^{-4}</math><br/> <math>x = \frac{1}{16}</math></p> <p><b>d</b> <math>{}^5\log(3x+2) = 1</math><br/> <math>3x+2 = 5</math><br/> <math>3x = 3</math><br/> <math>x = 1</math></p> | <p><b>e</b> <math>{}^3\log(0,4x-5) = 2</math><br/> <math>0,4x-5 = 3^2</math><br/> <math>0,4x-5 = 9</math><br/> <math>0,4x = 14</math><br/> <math>x = 35</math></p> <p><b>f</b> <math>4 + 2 \cdot {}^2\log(x) = 7</math><br/> <math>2 \cdot {}^2\log(x) = 3</math><br/> <math>{}^2\log(x) = 1\frac{1}{2}</math><br/> <math>x = 2^{1\frac{1}{2}}</math><br/> <math>x = 2 \cdot 2^{\frac{1}{2}}</math><br/> <math>x = 2\sqrt{2}</math></p> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

### Bladzijde 41

- 85**  ${}^2\log({}^3\log(x)) = -1$   
 ${}^3\log(x) = 2^{-1}$   
 ${}^3\log(x) = \frac{1}{2}$   
 $x = 3^{\frac{1}{2}}$   
 $x = \sqrt{3}$

- 86** Voor  $f$  geldt  $y = 2^x$ , dus voor  $f^{\text{inv}}$  geldt  $x = 2^y$ .  
 $x = 2^y$  geeft  $y = {}^2\log(x)$   
Dus  $f^{\text{inv}}(x) = {}^2\log(x)$  en er geldt inderdaad dat  $g(x) = f^{\text{inv}}(x)$ .

87

$${}^3\log(2x+6) - 1 = 2$$

$${}^3\log(2x+6) = 3$$

$$2x+6 = 3^3$$

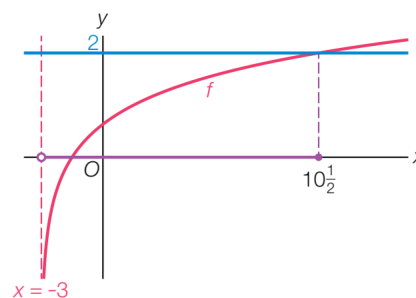
$$2x+6 = 27$$

$$2x = 21$$

$$x = 10\frac{1}{2}$$

Zie de schets hiernaast.

$${}^3\log(2x+6) - 1 \leq 2 \text{ geeft } -3 < x \leq 10\frac{1}{2}$$



88

a  $5x - 8 > 0$

$$5x > 8$$

$$x > 1\frac{3}{5}$$

$$\text{Dus } D_f = \langle 1\frac{3}{5}, \rightarrow \rangle.$$

De asymptoot van de grafiek van  $f$  is de lijn  $x = 1\frac{3}{5}$ .

b  $f(x) = 0$  geeft  $-3 + {}^2\log(5x-8) = 0$

$${}^2\log(5x-8) = 3$$

$$5x-8 = 2^3$$

$$5x-8 = 8$$

$$5x = 16$$

$$x = 3\frac{1}{5}$$

Uit de figuur in het leerboek volgt  $f(x) \leq 0$  geeft  $1\frac{3}{5} < x \leq 3\frac{1}{5}$ .

c  $f(8) = -3 + {}^2\log(32) = -3 + {}^2\log(2^5) = -3 + 5 = 2$

Uit de figuur in het leerboek volgt  $x \leq 8$  geeft  $f(x) \leq 2$ .

89

a  $2x - 5 = 0$

$$2x = 5$$

$$x = 2\frac{1}{2}$$

Dus de asymptoot van de grafiek van  $f$  is de lijn  $x = 2\frac{1}{2}$ .

$$f(x) = 7 \text{ geeft } {}^3\log(2x-5) + 4 = 7$$

$${}^3\log(2x-5) = 3$$

$$2x-5 = 3^3$$

$$2x-5 = 27$$

$$2x = 32$$

$$x = 16$$

Uit de figuur in het leerboek volgt  $f(x) < 7$  geeft  $2\frac{1}{2} < x < 16$ .

b  $f(7) = {}^3\log(9) + 4 = {}^3\log(3^2) + 4 = 2 + 4 = 6$

Uit de figuur in het leerboek volgt  $x \leq 7$  geeft  $f(x) \leq 6$ .

90

a  $x - 1 = 0$

$$x = 1$$

Dus de asymptoot van de grafiek van  $f$  is de lijn  $x = 1$ .

$$f(x) = 1\frac{1}{2} \text{ geeft } {}^4\log(x-1) = 1\frac{1}{2}$$

$$x-1 = 4^{1\frac{1}{2}}$$

$$x-1 = 4\sqrt{4}$$

$$x-1 = 8$$

$$x = 9$$

Uit de figuur in het leerboek volgt  $f(x) \leq 1\frac{1}{2}$  geeft  $1 < x \leq 9$ .

b  $x - 3 = 0$

$$x = 3$$

Dus de asymptoot van de grafiek van  $g$  is de lijn  $x = 3$ .

$$g(x) = 1 \text{ geeft } {}^{\frac{1}{4}}\log(x-3) + 2 = 1$$

$${}^{\frac{1}{4}}\log(x-3) = -1$$

$$x-3 = (\frac{1}{4})^{-1}$$

$$x-3 = 4$$

$$x = 7$$

Uit de figuur in het leerboek volgt  $g(x) \geq 1$  geeft  $3 < x \leq 7$ .

c  $f(5) = {}^4\log(4) = 1$

$$g(5) = {}^{\frac{1}{4}}\log(2) + 2 = {}^{\frac{1}{4}}\log(\sqrt{4}) + 2 = {}^{\frac{1}{4}}\log(4^{\frac{1}{2}}) + 2 = {}^{\frac{1}{4}}\log((\frac{1}{4})^{-\frac{1}{2}}) + 2 = -\frac{1}{2} + 2 = 1\frac{1}{2}$$

$$\text{Dus } AB = 1\frac{1}{2} - 1 = \frac{1}{2}.$$

**91** Voor  $f$  geldt  $y = -3 + {}^2\log(2x - 1)$ , dus voor  $f^{\text{inv}}$  geldt  $x = -3 + {}^2\log(2y - 1)$ .

$$x = -3 + {}^2\log(2y - 1)$$

$${}^2\log(2y - 1) = x + 3$$

$$2y - 1 = 2^{x+3}$$

$$2y = 2^{x+3} + 1$$

$$y = \frac{1}{2} \cdot 2^{x+3} + \frac{1}{2}$$

$$y = \frac{1}{2} \cdot 2^x \cdot 2^3 + \frac{1}{2}$$

$$y = \frac{1}{2} + 4 \cdot 2^x$$

$$\text{Dus } f^{\text{inv}}(x) = \frac{1}{2} + 4 \cdot 2^x.$$

Hieruit volgt  $a = \frac{1}{2}$  en  $b = 4$ .

#### Bladzijde 43

**C13** **a**  ${}^2\log(16\sqrt{2}) = {}^2\log(2^4 \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{4\frac{1}{2}}) = 4\frac{1}{2}$

**b**  ${}^3\log(\frac{1}{27}\sqrt{3}) = {}^3\log(3^{-3} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-2\frac{1}{2}}) = -2\frac{1}{2}$

**c**  ${}^5\log(\frac{1}{5}) = {}^5\log(5^{-1}) = -1$

**d**  $\frac{1}{3}\log(\frac{1}{9}) = \frac{1}{3}\log((\frac{1}{3})^2) = 2$

**e**  ${}^2\log(\frac{1}{8} \cdot \sqrt[5]{2}) = {}^2\log(2^{-3} \cdot 2^{\frac{1}{5}}) = {}^2\log(2^{-2\frac{4}{5}}) = -2\frac{4}{5}$

**f**  ${}^{10}\log(1) = {}^{10}\log(10^0) = 0$

**C14** **a**  $2 + {}^4\log(x) = 1$

$${}^4\log(x) = -1$$

$$x = 4^{-1}$$

$$x = \frac{1}{4}$$

**b**  ${}^3\log(2x - 5) = 2$

$$2x - 5 = 3^2$$

$$2x - 5 = 9$$

$$2x = 14$$

$$x = 7$$

**c**  $\frac{1}{2}\log(x - 3) + 1 = 0$

$$\frac{1}{2}\log(x - 3) = -1$$

$$x - 3 = (\frac{1}{2})^{-1}$$

$$x - 3 = 2$$

$$x = 5$$

**C15** **a**  $x + 1 > 0$

$$x > -1$$

Dus  $D_f = \langle -1, \rightarrow \rangle$ .

De asymptoot van de grafiek van  $f$  is de lijn  $x = -1$ .

**b**  $f(x) = 0$  geeft  ${}^2\log(x + 1) + 2 = 0$

$${}^2\log(x + 1) = -2$$

$$x + 1 = 2^{-2}$$

$$x + 1 = \frac{1}{4}$$

$$x = -\frac{3}{4}$$

Uit de figuur in het leerboek volgt  $f(x) \leq 0$  geeft  $-1 < x \leq -\frac{3}{4}$ .

**c**  $f(0) = {}^2\log(1) + 2 = {}^2\log(2^0) + 2 = 0 + 2 = 2$

Uit de figuur in het leerboek volgt  $x \leq 0$  geeft  $f(x) \leq 2$ .

### Kennis- en vaardigheidentest

#### Bladzijde 44

**1**  $f(x) = 0,3(x + 2)^4 - 6$

↓  $\times -4$  t.o.v. de  $x$ -as

$$y = -4(0,3(x + 2)^4 - 6) = -1,2(x + 2)^4 + 24$$

↓ translatie  $(3, -15)$

$$g(x) = -1,2(x - 3 + 2)^4 + 24 - 15 = -1,2(x - 1)^4 + 9$$

max. is  $g(1) = 9$

**2** **a**  $y = \sqrt{x}$

↓ translatie  $(2, 3)$

$$f(x) = \sqrt{x - 2} + 3$$

$$y = \sqrt{x}$$

↓  $\times -1$  t.o.v. de  $x$ -as

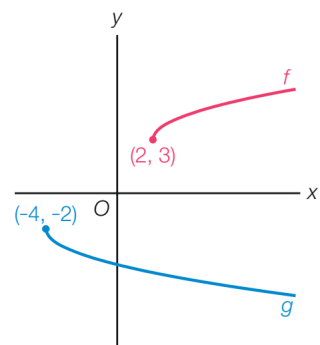
$$y = -\sqrt{x}$$

↓ translatie  $(-4, -2)$

$$g(x) = -2 - \sqrt{x + 4}$$

**b** Zie de schets hiernaast.

**c**  $D_f = [2, \rightarrow)$ ,  $B_f = [3, \rightarrow)$ ,  $D_g = [-4, \rightarrow)$  en  $B_g = \langle \leftarrow, -2 \rangle$



**3 a**  $3 - 4x = 0$

$$4x = 3$$

$$x = \frac{3}{4}$$

$$f\left(\frac{3}{4}\right) = 3, \text{ dus randpunt } \left(\frac{3}{4}, 3\right).$$

$$f(x) = 2 \text{ geeft } 3 - \sqrt{3 - 4x} = 2$$

$$\sqrt{3 - 4x} = 1$$

$$3 - 4x = 1$$

$$4x = 2$$

$$x = \frac{1}{2}$$

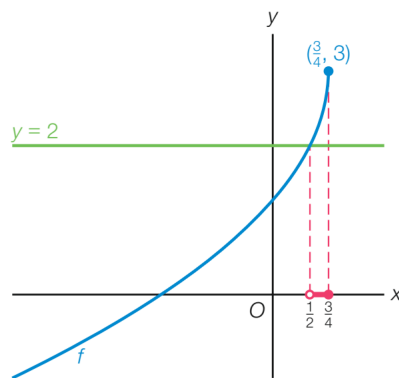
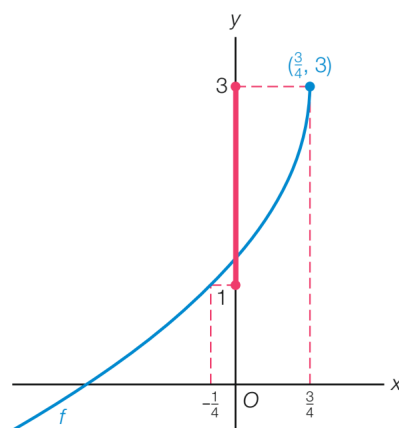
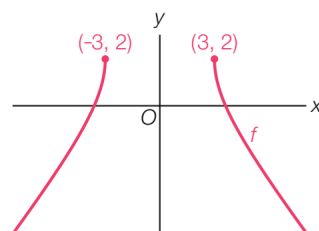
Zie de schets hiernaast.

$$f(x) > 2 \text{ geeft } \frac{1}{2} < x \leq \frac{3}{4}$$

**b**  $f\left(-\frac{1}{4}\right) = 1$

Zie de schets hiernaast.

$$x \geq -\frac{1}{4} \text{ geeft } 1 \leq f(x) \leq 3$$



**4**  $x^2 - 9 = 0$

$$x^2 = 9$$

$$x = 3 \vee x = -3$$

$$f(3) = 2 \text{ en } f(-3) = 2$$

Dus randpunten  $(3, 2)$  en  $(-3, 2)$ .

Zie de schets hiernaast.

$$B_f = \langle \leftarrow, 2 \rangle$$

**5 a**  $N = 2\sqrt{-5t + 1}$

$$2\sqrt{-5t + 1} = N$$

$$\text{kwadrateren geeft } 4(-5t + 1) = N^2$$

$$-5t + 1 = \frac{1}{4}N^2$$

$$-5t = \frac{1}{4}N^2 - 1$$

$$t = -\frac{1}{20}N^2 + \frac{1}{5}$$

**b**  $2x\sqrt{y} - \sqrt{x} = 1$

$$2x\sqrt{y} = \sqrt{x} + 1$$

$$\text{kwadrateren geeft } 4x^2y = (\sqrt{x} + 1)^2$$

$$4x^2y = x + 2\sqrt{x} + 1$$

$$y = \frac{x + 2\sqrt{x} + 1}{4x^2}$$

**6 a**  $x^{1\frac{2}{3}} = x \cdot x^{\frac{2}{3}} = x \cdot \sqrt[3]{x^2}$

**b**  $\frac{2}{3}xy^{-1\frac{1}{2}} = \frac{2x}{3y^{1\frac{1}{2}}} = \frac{2x}{3y \cdot y^{\frac{1}{2}}} = \frac{2x}{3y\sqrt{y}}$

**c**  $(5x^{-\frac{1}{4}})^3 = 125x^{-\frac{3}{4}} = \frac{125}{x^{\frac{3}{4}}} = \frac{125}{\sqrt[4]{x^3}}$

**d**  $\left(\frac{2}{3}x\right)^{-1} = \left(\frac{2}{3}\right)^{-1} \cdot x^{-1} = \frac{3}{2} \cdot \frac{1}{x} = \frac{3}{2x}$

**e**  $(4x^{-\frac{1}{3}})^{-2} = 4^{-2} \cdot x^{\frac{2}{3}} = \frac{1}{16} \cdot \sqrt[3]{x^2}$

**f**  $\frac{3}{5}x^{-2}y^{\frac{3}{4}} = \frac{3 \cdot \sqrt[4]{y^3}}{5x^2}$

**7 a**  $y = x \cdot \sqrt[4]{x^3} = x \cdot x^{\frac{3}{4}} = x^{1\frac{3}{4}}$

$$\text{Dus } y = x^{1\frac{3}{4}}.$$

**b**  $y = \frac{6}{2x^2 \cdot \sqrt{x}} = \frac{3}{x^2 \cdot x^{\frac{1}{2}}} = \frac{3}{x^{2\frac{1}{2}}} = 3x^{-2\frac{1}{2}}$

$$\text{Dus } y = 3x^{-2\frac{1}{2}}.$$

$$\text{c } y = \frac{10}{x} \cdot \sqrt[5]{x^3} = 10 \cdot x^{-1} \cdot x^{\frac{3}{5}} = 10x^{-\frac{2}{5}}$$

Dus  $y = 10x^{-\frac{2}{5}}$ .

$$\text{8 a } 3x^{-\frac{1}{2}} = 96$$

$$x^{-\frac{1}{2}} = 32$$

$$x = 32^{-\frac{2}{5}} = (2^5)^{-\frac{2}{5}} = 2^{-2} = \frac{1}{4}$$

$$\text{b } \sqrt[3]{(6x)^{-1}} = \frac{1}{4}$$

$$(6x)^{-\frac{1}{3}} = \frac{1}{4}$$

$$6x = \left(\frac{1}{4}\right)^{-3}$$

$$6x = (4^{-1})^{-3}$$

$$6x = 4^3$$

$$6x = 64$$

$$x = 10\frac{2}{3}$$

$$\text{c } (2x + 3)^{-\frac{2}{3}} = \frac{1}{9}$$

$$2x + 3 = \left(\frac{1}{9}\right)^{-\frac{3}{2}}$$

$$2x + 3 = (3^{-2})^{-\frac{3}{2}}$$

$$2x + 3 = 3^3$$

$$2x + 3 = 27$$

$$2x = 24$$

$$x = 12$$

$$\text{9 a } \left. \begin{array}{l} y = \frac{a}{x^{\frac{3}{4}}} \\ x = 16 \text{ en } y = 3 \end{array} \right\} \frac{a}{16^{\frac{3}{4}}} = 3$$

$$\frac{a}{8} = 3$$

$$a = 24$$

$$\text{Dus } y = \frac{24}{x^{\frac{3}{4}}}$$

$$\text{b } y = \frac{24}{x^{\frac{3}{4}}} \text{ geeft } x^{\frac{3}{4}} = \frac{24}{y}$$

$$x^{\frac{3}{4}} = 24y^{-1}$$

$$x = (24y^{-1})^{\frac{4}{3}} = 24^{\frac{4}{3}} \cdot y^{-\frac{4}{3}} \approx 69,2y^{-1\frac{1}{3}}$$

$$\text{Dus } x = 69,2y^{-1\frac{1}{3}}$$

#### Bladzijde 45

$$\text{10 a } y = 2^x$$

↓  $\times 0,3$  t.o.v. de  $x$ -as

$$y = 0,3 \cdot 2^x$$

↓ translatie (1, -5)

$$f(x) = 0,3 \cdot 2^{x-1} - 5$$

$B_f = \langle -5, \rightarrow \rangle$  en de asymptoot is de lijn  $y = -5$ .

$$\text{b } y = 0,7^x$$

↓ translatie (0, 2)

$$y = 2 + 0,7^x$$

↓  $\times \frac{1}{3}$  t.o.v. de  $y$ -as

$$g(x) = 2 + 0,7^{3x}$$

$B_g = \langle 2, \rightarrow \rangle$  en de asymptoot is de lijn  $y = 2$ .

$$\text{c } y = 3^x$$

↓  $\times -5$  t.o.v. de  $x$ -as

$$y = -5 \cdot 3^x$$

↓ translatie (0, 4)

$$y = 4 - 5 \cdot 3^x$$

↓  $\times 4$  t.o.v. de  $y$ -as

$$h(x) = 4 - 5 \cdot 3^{0,25x}$$

$B_h = \langle 4, \leftarrow \rangle$  en de asymptoot is de lijn  $y = 4$ .

$$\text{11 a } y = 108 \cdot 3^{4x-3} = 108 \cdot 3^{4x} \cdot 3^{-3} = 108 \cdot (3^4)^x \cdot \frac{1}{27} = 4 \cdot 81^x$$

$$\text{Dus } y = 4 \cdot 81^x.$$

$$\text{b } y = \frac{250}{5^{2x+3}} = 250 \cdot 5^{-2x-3} = 250 \cdot 5^{-2x} \cdot 5^{-3} = 250 \cdot (5^{-2})^x \cdot \frac{1}{125} = 2 \cdot \left(\frac{1}{25}\right)^x$$

$$\text{Dus } y = 2 \cdot \left(\frac{1}{25}\right)^x.$$

$$\text{c } y = 32 \cdot 4^{2\frac{1}{2}x-2} = 32 \cdot 4^{2\frac{1}{2}x} \cdot 4^{-2} = 32 \cdot (4^{\frac{1}{2}})^x \cdot \frac{1}{16} = 2 \cdot 32^x$$

$$\text{Dus } y = 2 \cdot 32^x.$$



**12 a**  $5^{x-1} = 125\sqrt{5}$

$$5^{x-1} = 5^3 \cdot 5^{\frac{1}{2}}$$

$$5^{x-1} = 5^{3\frac{1}{2}}$$

$$x-1 = 3\frac{1}{2}$$

$$x = 4\frac{1}{2}$$

**b**  $4^{2x-1} - 3 = 61$

$$4^{2x-1} = 64$$

$$4^{2x-1} = 4^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$x = 2$$

**c**  $3^{2x-5} = (\frac{1}{27})^x$

$$3^{2x-5} = (3^{-3})^x$$

$$3^{2x-5} = 3^{-3x}$$

$$2x-5 = -3x$$

$$5x = 5$$

$$x = 1$$

**d**  $9^{x-1} = 27^{x+1}$

$$(3^2)^{x-1} = (3^3)^{x+1}$$

$$3^{2x-2} = 3^{3x+3}$$

$$2x-2 = 3x+3$$

$$-x = 5$$

$$x = -5$$

**e**  $2^{x+2} + 2^{x-1} = 36$

$$2^x \cdot 2^2 + 2^x \cdot 2^{-1} = 36$$

$$4 \cdot 2^x + \frac{1}{2} \cdot 2^x = 36$$

$$4\frac{1}{2} \cdot 2^x = 36$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

**f**  $2^{x^2} = (\frac{1}{8})^x$

$$2^{x^2} = (2^{-3})^x$$

$$2^{x^2} = 2^{-3x}$$

$$x^2 = -3x$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = 0 \vee x = -3$$

**13 a**  $f(x) = g(x)$  geeft  $2^{x+2} - 3 = 6 - 2^{x-1}$

$$2^{x+2} + 2^{x-1} = 9$$

$$2^x \cdot 2^2 + 2^x \cdot 2^{-1} = 9$$

$$4 \cdot 2^x + \frac{1}{2} \cdot 2^x = 9$$

$$4\frac{1}{2} \cdot 2^x = 9$$

$$2^x = 2$$

$$x = 1$$

Uit de figuur in het leerboek volgt  $f(x) \geq g(x)$  geeft  $x \geq 1$ .

**b** De asymptoot van de grafiek van  $g$  is de lijn  $y = 6$ .

$$g(4) = -2$$

Uit de figuur in het leerboek volgt  $x \leq 4$  geeft  $-2 \leq g(x) < 6$ .

**c**  $f(x) + g(x) = 2^{x+2} - 3 + 6 - 2^{x-1} = 2^x \cdot 2^2 + 3 - 2^x \cdot 2^{-1} = 4 \cdot 2^x + 3 - \frac{1}{2} \cdot 2^x = 3\frac{1}{2} \cdot 2^x + 3$

Het bereik van  $f(x) + g(x)$  is  $\langle 3, \rightarrow \rangle$ .

Dus de vergelijking  $f(x) + g(x) = p$  heeft geen oplossing voor  $p \leq 3$ .

**14 a**  ${}^2\log(\frac{1}{4}\sqrt{8}) = {}^2\log(2^{-2} \cdot (2^3)^{\frac{1}{2}}) = \log(2^{-2} \cdot 2^{\frac{3}{2}}) = {}^2\log(2^{-\frac{1}{2}}) = -\frac{1}{2}$

**b**  ${}^3\log(27\sqrt{3}) = {}^3\log(3^3 \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{3\frac{1}{2}}) = 3\frac{1}{2}$

**c**  ${}^5\log(\frac{1}{125} \cdot \sqrt[3]{5}) = {}^5\log(5^{-3} \cdot 5^{\frac{1}{3}}) = {}^5\log(5^{-2\frac{2}{3}}) = -2\frac{2}{3}$

**15 a**  ${}^4\log(2x-3) = 2$

$$2x-3 = 4^2$$

$$2x-3 = 16$$

$$2x = 19$$

$$x = 9\frac{1}{2}$$

**b**  ${}^{\frac{1}{2}}\log(x-3) = -4$

$$x-3 = (\frac{1}{2})^{-4}$$

$$x-3 = 2^4$$

$$x-3 = 16$$

$$x = 19$$

**c**  $5 + 3 \cdot {}^2\log(x) = 20$

$$3 \cdot {}^2\log(x) = 15$$

$${}^2\log(x) = 5$$

$$x = 2^5$$

$$x = 32$$

**16 a**  $2x+3 = 0$

$$2x = -3$$

$$x = -1\frac{1}{2}$$

Dus de asymptoot van de grafiek van  $f$  is de lijn  $x = -1\frac{1}{2}$ .

$$f(x) = 4 \text{ geeft } 2 + {}^3\log(2x+3) = 4$$

$${}^3\log(2x+3) = 2$$

$$2x+3 = 3^2$$

$$2x+3 = 9$$

$$2x = 6$$

$$x = 3$$

Uit de figuur in het leerboek volgt  $f(x) \leq 4$  geeft  $-1\frac{1}{2} < x \leq 3$ .

**b**  $f(-1\frac{1}{3}) = 2 + {}^3\log(\frac{1}{3}) = 2 + {}^3\log(3^{-1}) = 2 - 1 = 1$

Uit de figuur in het leerboek volgt  $x < -1\frac{1}{3}$  geeft  $f(x) < 1$ .

## Gemengde opgaven

### Bladzijde 46

1

$$a \quad y = \frac{6x^4}{(2\sqrt{x})^3} = \frac{6x^4}{2^3 \cdot (\sqrt{x})^3} = \frac{6x^4}{8 \cdot (x^{\frac{1}{2}})^3} = \frac{6x^4}{8x^{\frac{3}{2}}} = \frac{3}{4}x^{2\frac{1}{2}}$$

$$\text{Dus } y = \frac{3}{4}x^{2\frac{1}{2}}.$$

$$b \quad y = (\frac{1}{4})^{2-\frac{1}{2}x} = (\frac{1}{4})^2 \cdot (\frac{1}{4})^{-\frac{1}{2}x} = \frac{1}{16} \cdot ((\frac{1}{4})^{-\frac{1}{2}})^x = \frac{1}{16} \cdot (4^{\frac{1}{2}})^x = \frac{1}{16} \cdot 2^x$$

$$\text{Dus } y = \frac{1}{16} \cdot 2^x.$$

$$c \quad y = 4x^{-1\frac{2}{3}} \cdot (3x^4)^2 = 4x^{-1\frac{2}{3}} \cdot 9x^8 = 36x^{6\frac{1}{3}}$$

$$36x^{6\frac{1}{3}} = y \text{ geeft } x^{\frac{19}{3}} = \frac{1}{36}y$$

$$x = (\frac{1}{36}y)^{\frac{3}{19}} = (\frac{1}{36})^{\frac{3}{19}} \cdot y^{\frac{3}{19}} \approx 0,57y^{0,16}$$

$$\text{Dus } x = 0,57y^{0,16}.$$

$$d \quad A = 6 + 2\sqrt{3B + 2}$$

$$2\sqrt{3B + 2} = A - 6$$

$$\text{kwadrateren geeft } 4(3B + 2) = (A - 6)^2$$

$$12B + 8 = A^2 - 12A + 36$$

$$12B = A^2 - 12A + 28$$

$$B = \frac{1}{12}A^2 - A + 2\frac{1}{3}$$

$$\text{Dus } B = \frac{1}{12}A^2 - A + 2\frac{1}{3}.$$

2

$$a \quad \frac{1}{2}(2x^2 - 4)^{\frac{1}{2}} = 4$$

$$(2x^2 - 4)^{\frac{1}{2}} = 8$$

$$2x^2 - 4 = 8^{\frac{2}{3}}$$

$$2x^2 - 4 = (2^3)^{\frac{2}{3}}$$

$$2x^2 - 4 = 2^2$$

$$2x^2 - 4 = 4$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = 2 \vee x = -2$$

$$b \quad 2^{x+4} - 2^{x+3} = 16\sqrt{2}$$

$$2^x \cdot 2^4 - 2^x \cdot 2^3 = 16\sqrt{2}$$

$$16 \cdot 2^x - 8 \cdot 2^x = 16\sqrt{2}$$

$$8 \cdot 2^x = 16\sqrt{2}$$

$$2^x = 2\sqrt{2}$$

$$2^x = 2 \cdot 2^{\frac{1}{2}}$$

$$2^x = 2^{1\frac{1}{2}}$$

$$x = 1\frac{1}{2}$$

$$c \quad (2x - 5)^{-\frac{3}{4}} = \frac{1}{27}$$

$$2x - 5 = (\frac{1}{27})^{-\frac{4}{3}}$$

$$2x - 5 = (3^{-3})^{-\frac{4}{3}}$$

$$2x - 5 = 3^4$$

$$2x - 5 = 81$$

$$2x = 86$$

$$x = 43$$

$$d \quad (\frac{1}{2}x^2 + 9) \cdot \sqrt[3]{\frac{1}{2}x^2 + 9} = 81$$

$$(\frac{1}{2}x^2 + 9)(\frac{1}{2}x^2 + 9)^{\frac{1}{3}} = 81$$

$$(\frac{1}{2}x^2 + 9)^{\frac{4}{3}} = 81$$

$$\frac{1}{2}x^2 + 9 = 81^{\frac{3}{4}}$$

$$\frac{1}{2}x^2 + 9 = (3^4)^{\frac{3}{4}}$$

$$\frac{1}{2}x^2 + 9 = 3^3$$

$$\frac{1}{2}x^2 + 9 = 27$$

$$\frac{1}{2}x^2 = 18$$

$$x^2 = 36$$

$$x = 6 \vee x = -6$$

$$e \quad (\sqrt{3})^{x+2} = 9\sqrt{3}$$

$$(3^{\frac{1}{2}})^{x+2} = 3^2 \cdot 3^{\frac{1}{2}}$$

$$3^{\frac{1}{2}x+1} = 3^{2\frac{1}{2}}$$

$$\frac{1}{2}x + 1 = 2\frac{1}{2}$$

$$\frac{1}{2}x = 1\frac{1}{2}$$

$$x = 3$$

$$f \quad {}^5\log(x^2 \cdot \sqrt{x} - 7) = 2$$

$$x^2 \cdot \sqrt{x} - 7 = 5^2$$

$$x^2 \cdot x^{\frac{1}{2}} - 7 = 25$$

$$x^{2\frac{1}{2}} = 32$$

$$x = 32^{\frac{2}{5}}$$

$$x = (2^5)^{\frac{2}{5}}$$

$$x = 2^2$$

$$x = 4$$

3

$$a \quad f(1) = {}^2\log(8) = {}^2\log(2^3) = 3$$

$$g(1) = 4 - 3\sqrt{8} = 4 - 3 \cdot 2\sqrt{2} = 4 - 6\sqrt{2}$$

$$\text{Dus } AB = 3 - (4 - 6\sqrt{2}) = 6\sqrt{2} - 1.$$

$$b \quad 10 - 2x = 0 \text{ oftewel } 2x = 10, \text{ dus } x = 5.$$

$$g(5) = 4, \text{ dus randpunt } (5, 4).$$

$$f(5) = {}^2\log(16) = {}^2\log(2^4) = 4$$

Dus de grafiek van  $f$  gaat door het randpunt van de grafiek van  $g$ .

$$c \quad 2x + 6 = 0 \text{ oftewel } 2x = -6, \text{ dus } x = -3.$$

$$\text{Dus } p = -3.$$

$$g(-3) = -8$$

$$x > -3 \text{ geeft } -8 < g(x) \leq 4$$

**d**  $f(x) = -2$  geeft  ${}^2\log(2x + 6) = -2$

$$2x + 6 = 2^{-2}$$

$$2x + 6 = \frac{1}{4}$$

$$2x = -5\frac{3}{4}$$

$$x = -2\frac{7}{8}$$

$$f(x) > -2 \text{ geeft } x > -2\frac{7}{8}$$

$$g(x) = -2 \text{ geeft } 4 - 3\sqrt{10 - 2x} = -2$$

$$3\sqrt{10 - 2x} = 6$$

$$\sqrt{10 - 2x} = 2$$

$$10 - 2x = 4$$

$$2x = 6$$

$$x = 3$$

$$g(x) < -2 \text{ geeft } x < 3$$

$$f(x) > -2 \text{ en } g(x) < -2 \text{ geeft } -2\frac{7}{8} < x < 3$$

**4 a**  $g(x) = 7 - 9^{0,5x-1} = 7 - (3^2)^{0,5x-1} = 7 - 3^{x-2}$

$$f(x) = 2 \cdot 3^{x-1}$$

↓  $\times -\frac{1}{2}$  t.o.v. de  $x$ -as

$$y = -3^{x-1}$$

↓ translatie  $(1, 7)$

$$g(x) = 7 - 3^{x-1-1} = 7 - 3^{x-2}$$

**b** De asymptoot van de grafiek van  $f$  is de lijn  $y = 0$ .

De asymptoot van de grafiek van  $g$  is de lijn  $y = 7$ .

Voor  $p \geq 7$  heeft de vergelijking  $f(x) = p$  één oplossing en de vergelijking

$g(x) = p$  geen oplossing.

**c**  $f(x) = g(x)$  geeft  $2 \cdot 3^{x-1} = 7 - 9^{0,5x-1}$

$$2 \cdot 3^{x-1} = 7 - 3^{x-2}$$

$$2 \cdot 3^{x-1} + 3^{x-2} = 7$$

$$2 \cdot 3^x \cdot 3^{-1} + 3^x \cdot 3^{-2} = 7$$

$$\frac{2}{3} \cdot 3^x + \frac{1}{9} \cdot 3^x = 7$$

$$\frac{7}{9} \cdot 3^x = 7$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x = 2$$

$$f(2) = 6, \text{ dus } S(2, 6).$$

**d** De asymptoten van de grafieken van  $f$  en  $g$  zijn de lijnen  $y = 0$  en  $y = 7$ , dus

$AB > 14$  is alleen mogelijk als de lijn  $x = q$  rechts van  $S$  ligt.

$$AB = f(q) - g(q)$$

$$= 2 \cdot 3^{q-1} - (7 - 9^{0,5q-1})$$

$$= 2 \cdot 3^{q-1} - 7 + 3^{q-2}$$

$$= \frac{7}{9} \cdot 3^q - 7$$

$$AB = 14 \text{ geeft } \frac{7}{9} \cdot 3^q - 7 = 14$$

$$\frac{7}{9} \cdot 3^q = 21$$

$$3^q = 27$$

$$3^q = 3^3$$

$$q = 3$$

Uit de figuur in het leerboek volgt  $AB > 14$  geeft  $q > 3$ .

**5 a**  $T = aR^{1,5}$  geeft  $a = \frac{T}{R^{1,5}}$

$$R = 5,28 \text{ en } T = 4,5 \text{ geeft } a = \frac{4,5}{5,28^{1,5}} \approx 0,37$$

$$R = 3,78 \text{ en } T = 2,7 \text{ geeft } a = \frac{2,7}{3,78^{1,5}} \approx 0,37$$

$$R = 2,95 \text{ en } T = 1,9 \text{ geeft } a = \frac{1,9}{2,95^{1,5}} \approx 0,37$$

Dus  $a \approx 0,37$ .

**b** Straal is  $3,56 \cdot 10^6 = 35,6 \cdot 10^5$ , dus  $R = 35,6$ .

$$R = 35,6 \text{ geeft } T = 0,37 \cdot 35,6^{1,5} \approx 78,6$$

Dus de omlooptijd is ongeveer 78,6 dagen.

$$\begin{aligned} \left. \begin{aligned} T_{\text{Hyperion}} &= 0,37R_{\text{Hyperion}}^{1,5} \\ T_{\text{Teleso}} &= 0,37R_{\text{Teleso}}^{1,5} \\ R_{\text{Hyperion}} &= 5,0 \cdot R_{\text{Teleso}} \end{aligned} \right\} \begin{aligned} \frac{T_{\text{Hyperion}}}{T_{\text{Teleso}}} &= \frac{0,37R_{\text{Hyperion}}^{1,5}}{0,37R_{\text{Teleso}}^{1,5}} \\ \frac{T_{\text{Hyperion}}}{T_{\text{Teleso}}} &= \left( \frac{R_{\text{Hyperion}}}{R_{\text{Teleso}}} \right)^{1,5} \\ \frac{T_{\text{Hyperion}}}{T_{\text{Teleso}}} &= \left( \frac{5,0 \cdot R_{\text{Teleso}}}{R_{\text{Teleso}}} \right)^{1,5} \\ \frac{T_{\text{Hyperion}}}{T_{\text{Teleso}}} &= 5,0^{1,5} \approx 11,2 \end{aligned}$$

Dus de omlooptijd van Hyperion is ongeveer 11,2 keer zo groot.

**d**  $T = 0,37R^{1,5}$  geeft  $0,37R^{1,5} = T$

$$R^{1,5} = \frac{1}{0,37} T$$

$$R = \left( \frac{1}{0,37} T \right)^{\frac{1}{1,5}} = \left( \frac{1}{0,37} \right)^{\frac{1}{1,5}} \cdot T^{\frac{1}{1,5}} \approx 1,94 T^{0,67}$$

Dus  $R = 1,94 T^{0,67}$ .

**e** Omlooptijd 15 uur geeft  $T = \frac{15}{24}$ .

$$R = 1,94 \cdot \left( \frac{15}{24} \right)^{0,67} \approx 1,42$$

De straal is ongeveer  $1,42 \cdot 10^5$  km.

**6** Voor  $f$  geldt  $y = 1 + \sqrt[3]{2x + 8}$ , dus voor  $f^{\text{inv}}$  geldt  $x = 1 + \sqrt[3]{2y + 8}$ .

$$x = 1 + \sqrt[3]{2y + 8}$$

$$\sqrt[3]{2y + 8} = x - 1$$

tot de derde macht geeft  $2y + 8 = (x - 1)^3$

$$2y = (x - 1)^3 - 8$$

$$y = \frac{1}{2}(x - 1)^3 - 4$$

Dus  $f^{\text{inv}}(x) = \frac{1}{2}(x - 1)^3 - 4$ .

Het punt van symmetrie van de grafiek van  $f^{\text{inv}}$  is  $A(1, -4)$ .

$$f^{\text{inv}}(x) = 0 \text{ geeft } \frac{1}{2}(x - 1)^3 - 4 = 0$$

$$\frac{1}{2}(x - 1)^3 = 4$$

$$(x - 1)^3 = 8$$

$$x - 1 = 2$$

$$x = 3$$

Dus  $B(3, 0)$ .

Het punt  $A(1, -4)$  is het randpunt van de grafiek van  $g(x) = p + \sqrt{qx + r}$ .

Dus  $p = -4$  en  $q \cdot 1 + r = q + r = 0$  geeft  $r = -q$ .

$$g(3) = 0 \text{ geeft } -4 + \sqrt{3q - q} = 0$$

$$\sqrt{2q} = 4$$

$$2q = 16$$

$$q = 8$$

Dus  $p = -4$ ,  $q = 8$  en  $r = -8$ .

**a**  $f_{-8}(x) = {}^2\log(x^2 - 8x + 12)$

$$x^2 - 8x + 12 = 0$$

$$(x - 2)(x - 6) = 0$$

$$x = 2 \vee x = 6$$

Uit de figuur in het leerboek volgt  $x < 2$  of  $x > 6$ .

Dus het domein van  $f_{-8}$  bestaat uit de intervallen  $\langle -, 2 \rangle$  en  $\langle 6, \rightarrow \rangle$ .

**b**  $f_{-8}(x) = 3$  geeft  ${}^2\log(x^2 - 8x + 12) = 3$

$$x^2 - 8x + 12 = 2^3$$

$$x^2 - 8x + 12 = 8$$

$$x^2 - 8x + 4 = 0$$

$$D = (-8)^2 - 4 \cdot 1 \cdot 4 = 48, \text{ dus } \sqrt{D} = 4\sqrt{3}$$

$$x = \frac{8 + 4\sqrt{3}}{2} = 4 + 2\sqrt{3} \vee x = \frac{8 - 4\sqrt{3}}{2} = 4 - 2\sqrt{3}$$

De asymptoten van de grafiek van  $f_{-8}$  zijn de lijnen  $x = 2$  en  $x = 6$ .

Uit de figuur in het leerboek volgt dat

$$f_{-8}(x) \leq 3 \text{ geeft } 4 - 2\sqrt{3} \leq x < 2 \vee 6 < x \leq 4 + 2\sqrt{3}.$$

**c**  $f_{-8}(-2) = {}^2\log(32) = {}^2\log(2^5) = 5$

$$f_{-8}(10) = {}^2\log(32) = 5$$

Uit de figuur in het leerboek volgt dat  $-2 \leq x \leq 10$  geeft  $f_{-8}(x) \leq 5$ .

**d** De grafiek van  $f_p$  heeft twee asymptoten als de vergelijking  $x^2 + px + 12 = 0$  twee oplossingen heeft.

Er moet gelden  $D > 0$ .

$$D = p^2 - 4 \cdot 1 \cdot 12 = p^2 - 48 \left\{ \begin{array}{l} p^2 - 48 > 0 \\ p^2 > 48 \end{array} \right.$$

$$p < -4\sqrt{3} \vee p > 4\sqrt{3}$$

Dus de grafiek van  $f_p$  heeft twee asymptoten voor  $p < -4\sqrt{3} \vee p > 4\sqrt{3}$ .

**e**  $x^2 + px + 12 = 0$

$$D = p^2 - 48$$

$$x = \frac{-p + \sqrt{p^2 - 48}}{2} = -\frac{1}{2}p + \frac{1}{2}\sqrt{p^2 - 48} \vee x = \frac{-p - \sqrt{p^2 - 48}}{2} = -\frac{1}{2}p - \frac{1}{2}\sqrt{p^2 - 48}$$

$$\text{De afstand tussen de verticale asymptoten is } -\frac{1}{2}p + \frac{1}{2}\sqrt{p^2 - 48} - (-\frac{1}{2}p - \frac{1}{2}\sqrt{p^2 - 48}) = \sqrt{p^2 - 48}.$$

$$\sqrt{p^2 - 48} = 1$$

$$p^2 - 48 = 1$$

$$p^2 = 49$$

$$p = 7 \vee p = -7$$

$$p = 7 \text{ geeft } x = -3\frac{1}{2} + \frac{1}{2}\sqrt{49 - 48} = -3 \text{ en } x = -3\frac{1}{2} - \frac{1}{2}\sqrt{49 - 48} = -4$$

Dus  $p = 7$  geeft als asymptoten de lijnen  $x = -3$  en  $x = -4$ .

$$p = -7 \text{ geeft } x = 3\frac{1}{2} + \frac{1}{2}\sqrt{49 - 48} = 4 \text{ en } x = 3\frac{1}{2} - \frac{1}{2}\sqrt{49 - 48} = 3$$

Dus  $p = -7$  geeft als asymptoten de lijnen  $x = 3$  en  $x = 4$ .

# 6 Differentiaalrekening

## 6.1 Toppen en buigpunten

### Bladzijde 51

- V1**
- a**  $f(x) = \frac{1}{2}x^4 - 3x^2 + 6x - 1$  geeft  $f'(x) = 2x^3 - 6x + 6$
- b**  $g(x) = (5x^2 - 2)^2 = 25x^4 - 20x^2 + 4$  geeft  $g'(x) = 100x^3 - 40x$
- c**  $h(x) = \frac{x-3}{x^2+4}$  geeft  $h'(x) = \frac{(x^2+4) \cdot 1 - (x-3) \cdot 2x}{(x^2+4)^2} = \frac{x^2+4-2x^2+6x}{(x^2+4)^2} = \frac{-x^2+6x+4}{(x^2+4)^2}$
- d**  $k(p) = (2p^2 - 3p + 1)(5p - 4)$  geeft  $k'(p) = (4p - 3)(5p - 4) + (2p^2 - 3p + 1) \cdot 5$   
 $= 20p^2 - 16p - 15p + 12 + 10p^2 - 15p + 5$   
 $= 30p^2 - 46p + 17$
- e**  $l(q) = 10 - 5q^2 + 9q^3 - a^4$  geeft  $l'(q) = -10q + 27q^2$
- f**  $m(x) = 2x^2 - \frac{6}{x+1}$  geeft  $m'(x) = 4x - \frac{(x+1) \cdot 0 - 6 \cdot 1}{(x+1)^2} = 4x + \frac{6}{(x+1)^2}$

- V2**
- a**  $f(x) = x^3 - x^2 - 4x + 2$  geeft  $f'(x) = 3x^2 - 2x - 4$   
 $k: y = ax + b$   
 $a = f'(0) = -4$   
 $y = -4x + b$   
 $f(0) = 2$ , dus  $A(0, 2)$  }  $b = 2$   
Dus  $k: y = -4x + 2$ .
- b**  $f'(x) = 1$  geeft  $3x^2 - 2x - 4 = 1$   
 $3x^2 - 2x - 5 = 0$   
 $D = (-2)^2 - 4 \cdot 3 \cdot -5 = 64$ , dus  $\sqrt{D} = 8$   
 $x = \frac{2+8}{6} = 1\frac{2}{3}$  of  $x = \frac{2-8}{6} = -1$   
 $f(-1) = 4$  en  $f(1\frac{2}{3}) = -2\frac{22}{27}$ , dus  $B(-1, 4)$  en  $C(1\frac{2}{3}, -2\frac{22}{27})$ .

### Bladzijde 52

- V3**
- a**  $f(x) = \frac{2x-1}{x+1}$  geeft  $f'(x) = \frac{(x+1) \cdot 2 - (2x-1) \cdot 1}{(x+1)^2} = \frac{2x+2-2x+1}{(x+1)^2} = \frac{3}{(x+1)^2}$   
 $k: y = ax + b$   
 $a = f'(2) = \frac{3}{(2+1)^2} = \frac{1}{3}$   
 $y = \frac{1}{3}x + b$   
 $f(2) = 1$ , dus  $A(2, 1)$  }  $\frac{1}{3} \cdot 2 + b = 1$   
 $\frac{2}{3} + b = 1$   
 $b = \frac{1}{3}$   
Dus  $k: y = \frac{1}{3}x + \frac{1}{3}$ .
- b**  $f'(x) = \frac{3}{4}$  geeft  $\frac{3}{(x+1)^2} = \frac{3}{4}$   
 $(x+1)^2 = 4$   
 $x+1 = 2$  of  $x+1 = -2$   
 $x = 1$  of  $x = -3$   
 $f(-3) = 3\frac{1}{2}$  en  $f(1) = \frac{1}{2}$ , dus  $B(-3, 3\frac{1}{2})$  en  $C(1, \frac{1}{2})$ .
- $l: y = \frac{3}{4}x + b$  }  $\frac{3}{4} \cdot -3 + b = 3\frac{1}{2}$   
door  $B(-3, 3\frac{1}{2})$  }  $-2\frac{1}{4} + b = 3\frac{1}{2}$   
 $b = 5\frac{3}{4}$   
Dus  $l: y = \frac{3}{4}x + 5\frac{3}{4}$ .
- $m: y = \frac{3}{4}x + b$  }  $\frac{3}{4} \cdot 1 + b = \frac{1}{2}$   
door  $C(1, \frac{1}{2})$  }  $b = -\frac{1}{4}$   
Dus  $m: y = \frac{3}{4}x - \frac{1}{4}$ .

**1**  $f(x) = x^3 - 1\frac{1}{2}x^2 - 6x + 3$  geeft  $f'(x) = 3x^2 - 3x - 6$   
 $f'(x) = 0$  geeft  $3x^2 - 3x - 6 = 0$   
 $x^2 - x - 2 = 0$   
 $(x+1)(x-2) = 0$   
 $x = -1 \vee x = 2$

$f(-1) = 6\frac{1}{2}$ , dus  $A(-1, 6\frac{1}{2})$ .

$f(2) = -7$ , dus  $B(2, -7)$ .

De formule van de raaklijn in  $A$  is  $y = 6\frac{1}{2}$ .

De formule van de raaklijn in  $B$  is  $y = -7$ .

### Bladzijde 53

**2 a**  $f(x) = \frac{1}{3}x^3 + 3\frac{1}{2}x^2 + 10x + 5$  geeft  $f'(x) = x^2 + 7x + 10$   
 $f'(x) = 0$  geeft  $x^2 + 7x + 10 = 0$   
 $(x+2)(x+5) = 0$   
 $x = -2 \vee x = -5$

Je krijgt max. is  $f(-5) = \frac{5}{6}$  en min. is  $f(-2) = -3\frac{2}{3}$ .

**b**  $g(x) = -\frac{1}{6}x^3 - \frac{1}{8}x^2 + 3\frac{3}{4}x + 3$  geeft  $g'(x) = -\frac{1}{2}x^2 - \frac{1}{4}x + 3\frac{3}{4}$   
 $g'(x) = 0$  geeft  $-\frac{1}{2}x^2 - \frac{1}{4}x + 3\frac{3}{4} = 0$   
 $2x^2 + x - 15 = 0$   
 $D = 1^2 - 4 \cdot 2 \cdot -15 = 121$ , dus  $\sqrt{D} = 11$   
 $x = \frac{-1 \pm 11}{4} = 2\frac{1}{2} \vee x = -3$

Je krijgt min. is  $g(-3) = -4\frac{7}{8}$  en max. is  $g(2\frac{1}{2}) = 8\frac{95}{96}$ .

### Bladzijde 54

**3 a**  $f(x) = \frac{5x}{x^2 + 4}$  geeft  $f'(x) = \frac{(x^2 + 4) \cdot 5 - 5x \cdot 2x}{(x^2 + 4)^2} = \frac{5x^2 + 20 - 10x^2}{(x^2 + 4)^2} = \frac{-5x^2 + 20}{(x^2 + 4)^2}$   
 $f'(x) = 0$  geeft  $-5x^2 + 20 = 0$   
 $-5x^2 = -20$   
 $x^2 = 4$   
 $x = 2 \vee x = -2$

Je krijgt min. is  $f(-2) = -1\frac{1}{4}$  en max. is  $f(2) = 1\frac{1}{4}$ .

$f(x) = 0$  geeft  $5x = 0$

$x = 0$

De grafiek snijdt de  $x$ -as alleen in  $(0, 0)$ , dus  $B_f = [-1\frac{1}{4}, 1\frac{1}{4}]$ .

**b**  $k: y = ax + b$

$a = f'(6) = \frac{-5 \cdot 6^2 + 20}{(6^2 + 4)^2} = -\frac{1}{10}$

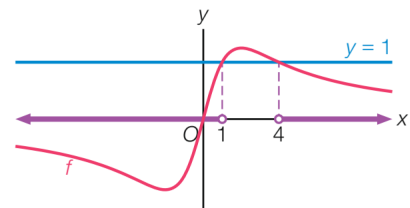
$y = -\frac{1}{10}x + b$   
 $f(6) = \frac{3}{4}$ , dus  $A(6, \frac{3}{4})$   $\left. \begin{array}{l} -\frac{1}{10} \cdot 6 + b = \frac{3}{4} \\ -\frac{3}{5} + b = \frac{3}{4} \\ b = 1\frac{7}{20} \end{array} \right\}$

Dus  $k: y = -\frac{1}{10}x + 1\frac{7}{20}$ .

**c**  $f(x) = 1$  geeft  $\frac{5x}{x^2 + 4} = 1$   
 $x^2 + 4 = 5x$   
 $x^2 - 5x + 4 = 0$   
 $(x-1)(x-4) = 0$   
 $x = 1 \vee x = 4$

Zie de schets hiernaast.

$f(x) < 1$  geeft  $x < 1 \vee x > 4$



- 4 a**  $f(x) = 3x^4 - 4x^3 - 12x^2 + 40$  geeft  $f'(x) = 12x^3 - 12x^2 - 24x$   
 $f'(x) = 0$  geeft  $12x^3 - 12x^2 - 24x = 0$   
 $12x(x^2 - x - 2) = 0$   
 $12x(x+1)(x-2) = 0$   
 $x = 0 \vee x = -1 \vee x = 2$   
 $f(-1) = 35$ ,  $f(0) = 40$  en  $f(2) = 8$ , dus  $A(-1, 35)$ ,  $B(0, 40)$  en  $C(2, 8)$ .
- b**  $B_f = [8, \rightarrow)$
- c** Alle horizontale lijnen die liggen tussen  $y = y_A$  en  $y = y_B$  snijden de grafiek van  $f$  vier keer.  
Dus de vergelijking  $f(x) = p$  heeft vier oplossingen voor  $35 < p < 40$ .
- d**  $k: y = ax + b$   
 $a = \frac{8-40}{2-0} = -16$   
 $y = -16x + b$   
door  $B(0, 40)$  }  $k: y = -16x + 40$   
 $k$  snijden met de grafiek van  $f$  geeft  $3x^4 - 4x^3 - 12x^2 + 40 = -16x + 40$ .  
Invoeren van de formules van  $f$  en  $k$  en gebruiken van de optie snijpunt geeft  $x = -2$  en  $y = 72$  en  $x = 1,333\dots$  en  $y = 18,666\dots$   
Dus  $D(-2, 72)$  en  $E(1,33; 18,67)$ .
- e**  $f'(x) = -24$  geeft  $12x^3 - 12x^2 - 24x = -24$   
Invoeren van de formule van  $f'$  en de formule  $y = -24$  en gebruiken van de optie snijpunt geeft voor  $0 < x < 2$  dat  $x = 1$  en  $x = 1,414\dots$   
 $f(1) = 27$  en  $f(1,414\dots) = 16,686\dots$   
Dus de punten zijn  $(1, 27)$  en  $(1,41; 16,69)$ .

- 5 a**  $f_2(x) = (x^2 - 2)(x^2 - 16) = x^4 - 18x^2 + 32$  geeft  $f_2'(x) = 4x^3 - 36x$   
 $f_2'(x) = 0$  geeft  $4x^3 - 36x = 0$   
 $4x(x^2 - 9) = 0$   
 $4x = 0 \vee x^2 = 9$   
 $x = 0 \vee x = 3 \vee x = -3$

Zie de figuur hiernaast.

Je krijgt min. is  $f_2(-3) = -49$ , max. is  $f_2(0) = 32$

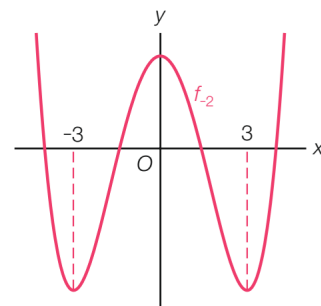
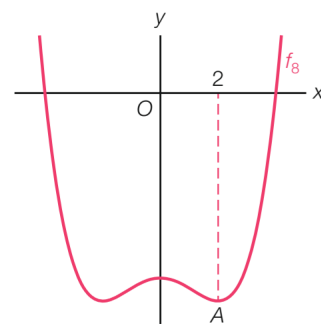
en min. is  $f_2(3) = -49$ .

Dus  $B_{f_2} = [-49, \rightarrow)$ .

- b**  $f_p(x) = (x^2 + p)(x^2 - 16) = x^4 - 16x^2 + px^2 - 16p$  geeft  
 $f_p'(x) = 4x^3 - 32x + 2px$   
 $f_p'(2) = 0$  geeft  $32 - 64 + 4p = 0$   
 $4p = 32$   
 $p = 8$

Zie de figuur hiernaast.

Dus voor  $p = 8$ .



- 6**  $f_p(x) = x^4 - 16x^2 + px^2 - 16p$  geeft  $f_p'(x) = 4x^3 - 32x + 2px$   
Voor  $x = x_T$  geldt  $4x^3 - 32x + 2px = 0$   
 $p = 14x$  }  $4x^3 - 32x + 2 \cdot 14x \cdot x = 0$   
 $4x^3 + 28x^2 - 32x = 0$   
 $4x(x^2 + 7x - 8) = 0$   
 $4x(x-1)(x+8) = 0$   
 $x = 0 \vee x = 1 \vee x = -8$

Alleen voor  $x = -8$  geldt dat  $p < 0$ , namelijk  $p = -112$ .



$$f_{-112}(x) = x^4 - 16x^2 - 112x^2 - 16 \cdot -112 = x^4 - 128x^2 + 1792 \text{ geeft}$$

$$f_{-112}'(x) = 4x^3 - 256x$$

$$f_{-112}'(x) = 0 \text{ geeft } 4x^3 - 256x = 0$$

$$4x(x^2 - 64) = 0$$

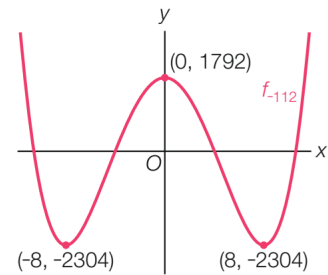
$$4x = 0 \vee x^2 = 64$$

$$x = 0 \vee x = 8 \vee x = -8$$

$$f_{-112}(-8) = -2304, f_{-112}(0) = 1792 \text{ en } f_{-112}(8) = -2304.$$

Zie de figuur hiernaast.

$$B_{f_{-112}} = [-2304, \rightarrow)$$



**7 a**  $f(x) = \frac{1}{4}x^4 - x^2 - 4x + 3$  geeft  $f'(x) = x^3 - 2x - 4$

$$f'(2) = 2^3 - 2 \cdot 2 - 4 = 0$$

- b** Uit de figuur in het leerboek blijkt dat er een laagste punt is met een horizontale raaklijn en omdat  $f'(2) = 0$  is er dus een extreme waarde voor  $x = 2$ .

#### Bladzijde 55

**8 a**  $f'(-1) = 4 \cdot (-1)^3 + 6 \cdot (-1)^2 - 2 = 0$

De functie heeft geen extreme waarde voor  $x = -1$ , want de grafiek heeft geen top voor  $x = -1$ .

**b**  $f(-1) = 4$ , dus horizontale raaklijn  $y = 4$ .

$$f(\frac{1}{2}) = 2\frac{5}{16}, \text{ dus horizontale raaklijn } y = 2\frac{5}{16}.$$

#### Bladzijde 56

**9 a**  $f(x) = \frac{1}{2}x^5 - \frac{1}{2}x^3 - 7x$  geeft  $f'(x) = 2\frac{1}{2}x^4 - 1\frac{1}{2}x^2 - 7$

$$f'(\sqrt{2}) = 2\frac{1}{2} \cdot (\sqrt{2})^4 - 1\frac{1}{2} \cdot (\sqrt{2})^2 - 7 = 0$$

$f'(\sqrt{2}) = 0$  en in de figuur in het leerboek is te zien dat de grafiek een top heeft voor  $x = \sqrt{2}$ .

Dus  $f$  heeft een extreme waarde voor  $x = \sqrt{2}$ .

**b**  $g(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 1\frac{1}{2}x^2 - 3x$  geeft  $g'(x) = x^3 + x^2 - 3x - 3$

$$g'(\sqrt{3}) = (\sqrt{3})^3 + (\sqrt{3})^2 - 3 \cdot \sqrt{3} - 3 = 0$$

$g'(\sqrt{3}) = 0$  en in de figuur in het leerboek is te zien dat de grafiek een top heeft voor  $x = \sqrt{3}$ .

Dus  $g$  heeft een extreme waarde voor  $x = \sqrt{3}$ .

**10 a**  $f(x) = (x^2 + 1)(x^2 - 4) = x^4 - 4x^2 + x^2 - 4 = x^4 - 3x^2 - 4$  geeft  $f'(x) = 4x^3 - 6x$

$$f'(1) = 4 \cdot 1^3 - 6 \cdot 1 = -2$$

$f'(1) \neq 0$ , dus  $f$  heeft geen extreme waarde voor  $x = 1$ .

**b**  $f'(\sqrt{1\frac{1}{2}}) = 4 \cdot (\sqrt{1\frac{1}{2}})^3 - 6 \cdot \sqrt{1\frac{1}{2}} = 0$

$f'(\sqrt{1\frac{1}{2}}) = 0$  en in de figuur in het leerboek is te zien dat de grafiek een top heeft voor  $x = \sqrt{1\frac{1}{2}}$ .

Dus  $f$  heeft een extreme waarde voor  $x = \sqrt{1\frac{1}{2}}$ .

Dus  $f$  heeft een extreme waarde voor  $x = \sqrt{1\frac{1}{2}}$ .

**c**  $f(0) = -4$ ,  $f(\sqrt{1\frac{1}{2}}) = -6\frac{1}{4}$  en  $f(-\sqrt{1\frac{1}{2}}) = -6\frac{1}{4}$

Alle horizontale lijnen die liggen tussen  $y = -6\frac{1}{4}$  en  $y = -4$  snijden de grafiek van  $f$  vier keer.

Dus de vergelijking  $f(x) = p$  heeft vier oplossingen voor  $-6\frac{1}{4} < p < -4$ .

**d**  $k: y = ax + b$

$$a = f'(-1) = 4 \cdot (-1)^3 - 6 \cdot -1 = 2$$

$$y = 2x + b$$

$$f(-1) = -6, \text{ dus } A(-1, -6) \left\{ \begin{array}{l} 2 \cdot -1 + b = -6 \\ -2 + b = -6 \end{array} \right.$$

$$b = -4$$

Dus  $k: y = 2x - 4$ .

$$y = 0 \text{ geeft } 2x - 4 = 0$$

$$2x = 4$$

$$x = 2$$

Dus  $k$  snijdt de  $x$ -as in  $(2, 0)$ .

$$f(2) = (2^2 + 1)(2^2 - 4) = 0$$

Dus de grafiek van  $f$  snijdt de positieve  $x$ -as in  $(2, 0)$ .

Dus  $k$  snijdt de grafiek van  $f$  in het snijpunt van de grafiek van  $f$  met de positieve  $x$ -as.

**e**  $y = ax + b$

$$a = f'(1\frac{1}{2}) = 4 \cdot (1\frac{1}{2})^3 - 6 \cdot 1\frac{1}{2} = 4\frac{1}{2}$$

$$y = 4\frac{1}{2}x + b$$

$$f(1\frac{1}{2}) = -5\frac{11}{16}, \text{ dus } B(1\frac{1}{2}, -5\frac{11}{16}) \left\{ \begin{array}{l} 4\frac{1}{2} \cdot 1\frac{1}{2} + b = -5\frac{11}{16} \\ 6\frac{3}{4} + b = -5\frac{11}{16} \end{array} \right.$$

$$b = -12\frac{7}{16}$$

Dus de lijn met vergelijking  $y = 4\frac{1}{2}x - 12\frac{7}{16}$  raakt de grafiek van  $f$  in het punt  $B$  met  $x_B = 1\frac{1}{2}$ .

**11 a**  $f(x) = \frac{1}{2}x^4 - 4x^3 + 9x^2 - 8x + 6$  geeft  $f'(x) = 2x^3 - 12x^2 + 18x - 8$

$f'(1) = 2 \cdot 1^3 - 12 \cdot 1^2 + 18 \cdot 1 - 8 = 0$ , dus de grafiek van  $f$  heeft een horizontale raaklijn in  $A$ .

$f(1) = 3\frac{1}{2}$ , dus de formule van deze raaklijn is  $y = 3\frac{1}{2}$ .

**b**  $f'(4) = 2 \cdot 4^3 - 12 \cdot 4^2 + 18 \cdot 4 - 8 = 0$

$f'(4) = 0$  en in de figuur in het leerboek is te zien dat de grafiek een top heeft voor  $x = 4$ .

Dus  $f$  heeft een extreme waarde voor  $x = 4$ .

$$f(4) = -10, \text{ dus } B_f = [-10, \rightarrow).$$

**c**  $k: y = ax + b$

$$a = f'(2) = 2 \cdot 2^3 - 12 \cdot 2^2 + 18 \cdot 2 - 8 = -4$$

$$y = -4x + b$$

$$f(2) = 2, \text{ dus } C(2, 2) \left\{ \begin{array}{l} -4 \cdot 2 + b = 2 \\ -8 + b = 2 \end{array} \right.$$

$$b = 10$$

Dus  $k: y = -4x + 10$ .

$$x = 4 \text{ geeft } y = -4 \cdot 4 + 10 = -6, \text{ dus } D(4, -6).$$

$$y_B = -10, \text{ dus } y_D - y_B = -6 - (-10) = 4.$$

### Bladzijde 57

**12 a**  $f(x) = \frac{1}{x^2 - x + 1}$  geeft  $f'(x) = \frac{(x^2 - x + 1) \cdot 0 - 1 \cdot (2x - 1)}{(x^2 - x + 1)^2} = \frac{-2x + 1}{(x^2 - x + 1)^2}$

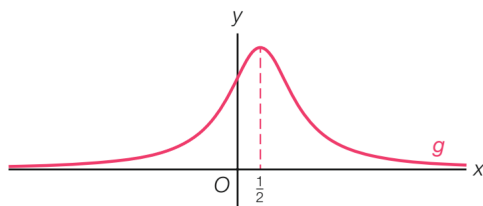
$$f'(x) = 0 \text{ geeft } -2x + 1 = 0$$

$$-2x = -1$$

$$x = \frac{1}{2}$$

Je krijgt max. is  $f(\frac{1}{2}) = 1\frac{1}{3}$ .

$$\begin{aligned} \text{b } g(x) &= \frac{x+1}{x^3+1} \text{ geeft } g'(x) = \frac{(x^3+1) \cdot 1 - (x+1) \cdot 3x^2}{(x^3+1)^2} = \frac{x^3+1-3x^3-3x^2}{(x^3+1)^2} = \frac{-2x^3-3x^2+1}{(x^3+1)^2} \\ g'(\tfrac{1}{2}) &= \frac{-2 \cdot (\tfrac{1}{2})^3 - 3 \cdot (\tfrac{1}{2})^2 + 1}{((\tfrac{1}{2})^3+1)^2} = \frac{-2 \cdot \tfrac{1}{8} - 3 \cdot \tfrac{1}{4} + 1}{(\tfrac{1}{8}+1)^2} = \frac{-\tfrac{1}{4} - \tfrac{3}{4} + 1}{(\tfrac{9}{8})^2} = \frac{0}{\tfrac{81}{64}} = 0 \end{aligned}$$



$g'(\frac{1}{2}) = 0$  en in de schets is te zien dat de grafiek een top heeft voor  $x = \frac{1}{2}$ .  
Dus  $g$  heeft een extreme waarde voor  $x = \frac{1}{2}$ .

$$\text{c } f(x) \cdot \frac{x+1}{x+1} = \frac{1}{x^2-x+1} \cdot \frac{x+1}{x+1} = \frac{x+1}{(x^2-x+1)(x+1)} = \frac{x+1}{x^3+x^2-x^2-x+x+1} = \frac{x+1}{x^3+1}$$

Voor  $x \neq -1$  geldt dat  $f(x) \cdot \frac{x+1}{x+1} = g(x)$ .

$$\text{d } f(-1) = \frac{1}{(-1)^2 - (-1) + 1} = \frac{1}{3} \text{ en } g(-1) = \frac{-1+1}{(-1)^3+1} = \frac{0}{0}$$

Dus de coördinaten van dat punt zijn  $(-1, \frac{1}{3})$ .

$$\begin{aligned} \text{e } g(x) &= 1 \text{ geeft } \frac{x+1}{x^3+1} = 1 \\ x^3+1 &= x+1 \\ x^3-x &= 0 \\ x(x^2-1) &= 0 \\ x=0 \vee x^2 &= 1 \\ x=0 \vee x &= 1 \vee x=-1 \end{aligned}$$

$x = -1$  voldoet niet, dus  $x = 0 \vee x = 1$ .

Uit de figuur in het leerboek volgt dat  $g(x) < 1$  geeft  $x < -1 \vee -1 < x < 0 \vee x > 1$ .

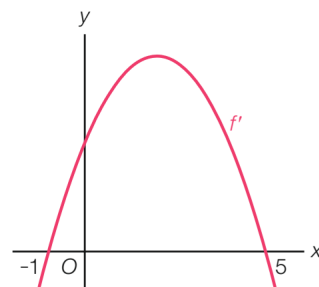
$$\text{f } g(-10) = \frac{1}{111} \text{ en } g(0) = 1, \text{ dus voor } -10 < x < 0 \text{ geldt dat } \frac{1}{111} < g(x) < \frac{1}{3} \vee \frac{1}{3} < g(x) < 1.$$

**13 a** Toenemend stijgend op  $\langle -1, 2 \rangle$ .

Afnemend stijgend op  $\langle 2, 5 \rangle$ .

$$\begin{aligned} \text{b } f(x) &= -x^3 + 6x^2 + 15x - 5 \text{ geeft } f'(x) = -3x^2 + 12x + 15 \\ f'(x) &= 0 \text{ geeft } -3x^2 + 12x + 15 = 0 \\ x^2 - 4x - 5 &= 0 \\ (x+1)(x-5) &= 0 \\ x &= -1 \vee x = 5 \end{aligned}$$

Zie de figuur hiernaast.



$$\text{c } x_p = \frac{-1+5}{2} = 2$$

**d** Bij  $x = x_p$  gaat de grafiek over van toenemend stijgend naar afnemend stijgend, want voor  $-1 < x < x_p$  is de helling positief en neemt de helling toe en voor  $x_p < x < 5$  is de helling positief en neemt de helling af.

#### Bladzijde 59

$$\begin{aligned} \text{14 a } f(x) &= x^4 - 12x^3 + 30x^2 + 48x + 5 \\ f'(x) &= 4x^3 - 36x^2 + 60x + 48 \\ f''(x) &= 12x^2 - 72x + 60 \\ f''(x) &= 0 \text{ geeft } 12x^2 - 72x + 60 = 0 \\ x^2 - 6x + 5 &= 0 \\ (x-1)(x-5) &= 0 \\ x &= 1 \vee x = 5 \end{aligned}$$

$$f(1) = 72 \text{ en } f(5) = 120$$

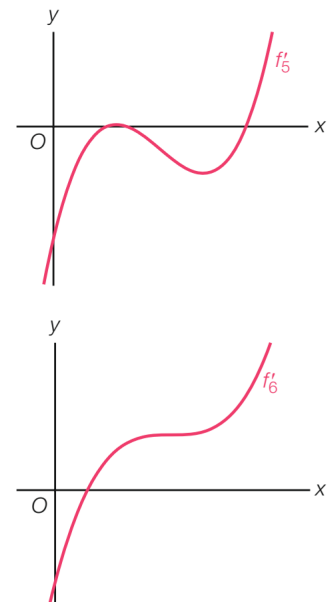
Uit de figuur volgt dat  $(1, 72)$  en  $(5, 120)$  de buigpunten zijn.

$$\begin{aligned}
 \text{b } g(x) &= \frac{1}{3}x^3 - 3x^2 + 6x + 4 \\
 g'(x) &= x^2 - 6x + 6 \\
 g''(x) &= 2x - 6 \\
 g''(x) = 0 &\text{ geeft } 2x - 6 = 0 \\
 2x &= 6 \\
 x &= 3 \\
 k: y &= ax + b \\
 a = g'(3) &= 3^2 - 6 \cdot 3 + 6 = -3 \\
 y &= -3x + b \\
 g(3) = 4, \text{ dus buigpunt } (3, 4) &\left. \begin{array}{l} -3 \cdot 3 + b = 4 \\ -9 + b = 4 \\ b = 13 \end{array} \right\}
 \end{aligned}$$

Dus  $k: y = -3x + 13$ .

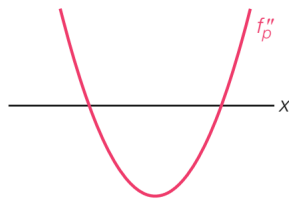
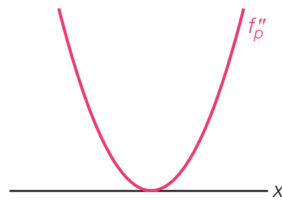
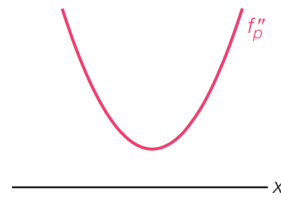
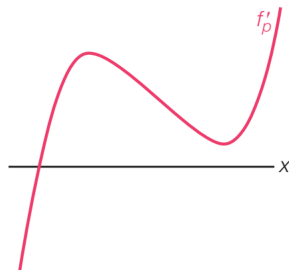
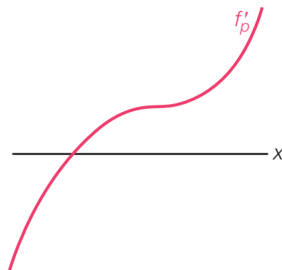
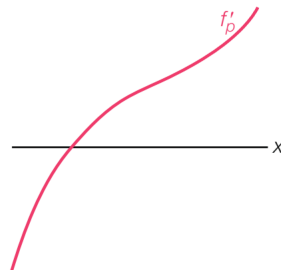
$$\begin{aligned}
 \text{15 a } f(x) &= -\frac{1}{12}x^4 - \frac{1}{3}x^3 \\
 f'(x) &= -\frac{1}{3}x^3 - x^2 \\
 f''(x) &= -x^2 - 2x \\
 f''(x) = 0 &\text{ geeft } -x^2 - 2x = 0 \\
 -x(x+2) &= 0 \\
 x &= 0 \vee x = -2 \\
 f'(-2) &= -1\frac{1}{3} \\
 f'(0) &= 0, \text{ dus de grafiek van } f \text{ heeft een horizontale buigraaklijn.} \\
 \text{b } f(0) = 0 \text{ en } f(-2) &= 1\frac{1}{3} \\
 \text{Uit de schets volgt dat } (-2, 1\frac{1}{3}) \text{ en } (0, 0) &\text{ de buigpunten zijn.} \\
 y = -x^2 + bx &\left. \begin{array}{l} -(-2)^2 + b \cdot -2 = 1\frac{1}{3} \\ -4 - 2b = 1\frac{1}{3} \\ -2b = 5\frac{1}{3} \\ b = -2\frac{2}{3} \end{array} \right\}
 \end{aligned}$$

$$\begin{aligned}
 \text{16 a } f_5(x) &= \frac{1}{4}x^4 - 2x^3 + 5x^2 - 5x - 5 \\
 f_5'(x) &= x^3 - 6x^2 + 10x - 5 \\
 f_5''(x) &= 3x^2 - 12x + 10 \\
 f_5''(x) = 0 &\text{ geeft } 3x^2 - 12x + 10 = 0 \\
 D &= (-12)^2 - 4 \cdot 3 \cdot 10 = 24 > 0, \text{ dus twee oplossingen} \\
 \text{Dus } f_5' &\text{ heeft twee extremen.} \\
 \text{Zie de figuur hiernaast.} \\
 \text{De grafiek van } f_5 &\text{ heeft twee buigpunten, omdat } f_5' \text{ twee extremen heeft.} \\
 \text{b } f_6(x) &= \frac{1}{4}x^4 - 2x^3 + 6x^2 - 5x - 5 \\
 f_6'(x) &= x^3 - 6x^2 + 12x - 5 \\
 f_6''(x) &= 3x^2 - 12x + 12 \\
 f_6''(x) = 0 &\text{ geeft } 3x^2 - 12x + 12 = 0 \\
 D &= (-12)^2 - 4 \cdot 3 \cdot 12 = 0, \text{ dus één oplossing.} \\
 \text{Dus de grafiek van } f_6' &\text{ heeft één horizontale raaklijn.} \\
 \text{Zie de figuur hiernaast.} \\
 \text{De grafiek van } f_6 &\text{ heeft geen buigpunten omdat } f_6' \text{ geen extremen heeft.} \\
 \text{c } f_p(x) &= \frac{1}{4}x^4 - 2x^3 + px^2 - 5x - 5 \\
 f_p'(x) &= x^3 - 6x^2 + 2px - 5 \\
 f_p''(x) &= 3x^2 - 12x + 2p \\
 \text{Geen buigpunten als } f_p''(x) = 0 &\text{ geen of één oplossing heeft, dus } D \leq 0 \\
 (-12)^2 - 4 \cdot 3 \cdot 2p &\leq 0 \\
 144 - 24p &\leq 0 \\
 -24p &\leq -144 \\
 p &\geq 6
 \end{aligned}$$



d  $f_p''(x) = 3x^2 - 12x + 2p$

De grafiek van  $f_p''$  is een dalparabool met twee, één of nul snijpunten met de  $x$ -as.



Heeft de grafiek van  $f_p''$  twee snijpunten met de  $x$ -as, dan is de grafiek van  $f_p'$  stijgend-dalend-stijgend, dus  $f_p'$  heeft twee extremen, dus de grafiek van  $f_p$  heeft twee buigpunten.

Heeft de grafiek van  $f_p''$  één raakpunt met de  $x$ -as, dan is de grafiek van  $f_p'$  stijgend-stijgend, dus  $f_p'$  heeft geen extremen, dus de grafiek van  $f_p$  heeft geen buigpunten.

Heeft de grafiek van  $f_p''$  geen snijpunten met de  $x$ -as, dan is de grafiek van  $f_p'$  stijgend, dus  $f_p'$  heeft geen extremen, dus de grafiek van  $f_p$  heeft geen buigpunten.

Dus het is niet mogelijk dat de grafiek van  $f_p$  één buigpunt heeft.

#### Bladzijde 60

17

a  $f(x) = \frac{3}{4}x^5 - 5x^3$

$$f'(x) = \frac{15}{4}x^4 - 15x^2$$

$$f'(x) = 0 \text{ geeft } \frac{15}{4}x^4 - 15x^2 = 0$$

$$\frac{15}{4}x^2(x^2 - 4) = 0$$

$$\frac{15}{4}x^2 = 0 \vee x^2 = 4$$

$$x = 0 \vee x = 2 \vee x = -2$$

$$f(0) = 0, f(2) = -16 \text{ en } f(-2) = 16$$

$$\left. \begin{array}{l} g_p(x) = -x^3 + px \\ \text{door } (2, -16) \end{array} \right\} \begin{array}{l} -2^3 + 2p = -16 \\ -8 + 2p = -16 \end{array}$$

$$2p = -8$$

$$p = -4$$

$g_{-4}(-2) = -(-2)^3 - 4 \cdot -2 = 16$ , dus voor  $p = -4$  gaat de grafiek van  $g_p$  door de toppen van de grafiek van  $f$ .

b  $f''(x) = 15x^3 - 30x$

$$f''(x) = 0 \text{ geeft } 15x^3 - 30x = 0$$

$$15x(x^2 - 2) = 0$$

$$15x = 0 \vee x^2 = 2$$

$$x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2}$$

$$f(\sqrt{2}) = \frac{3}{4} \cdot 4\sqrt{2} - 5 \cdot 2\sqrt{2} = -7\sqrt{2} \text{ en } f(-\sqrt{2}) = \frac{3}{4} \cdot -4\sqrt{2} - 5 \cdot -2\sqrt{2} = 7\sqrt{2}$$

$$\left. \begin{array}{l} g_p(x) = -x^3 + px \\ \text{door } B(\sqrt{2}, -7\sqrt{2}) \end{array} \right\} \begin{array}{l} -(\sqrt{2})^3 + p\sqrt{2} = -7\sqrt{2} \\ -2\sqrt{2} + p\sqrt{2} = -7\sqrt{2} \end{array}$$

$$-2 + p = -7$$

$$p = -5$$

$g_{-5}(-\sqrt{2}) = -(-\sqrt{2})^3 - 5 \cdot -\sqrt{2} = 7\sqrt{2}$ , dus de grafiek van  $g_{-5}$  gaat ook door  $A$ .

**c**  $k: y = ax + b$   
 $a = f'(\sqrt{2}) = \frac{15}{4} \cdot (\sqrt{2})^4 - 15 \cdot (\sqrt{2})^2 = -15$   

$$\left. \begin{array}{l} y = -15x + b \\ \text{door } B(\sqrt{2}, -7\sqrt{2}) \end{array} \right\} \begin{array}{l} -15\sqrt{2} + b = -7\sqrt{2} \\ b = 8\sqrt{2} \end{array}$$
  
Dus  $k: y = -15x + 8\sqrt{2}$ .  
 $y = 0$  geeft  $-15x + 8\sqrt{2} = 0$   
 $-15x = -8\sqrt{2}$   
 $x = \frac{8}{15}\sqrt{2}$ , dus  $C(\frac{8}{15}\sqrt{2}, 0)$ .  
 $x = 0$  geeft  $y = 8\sqrt{2}$ , dus  $D(0, 8\sqrt{2})$ .  
Hieruit volgt dat  $O(\triangle OCD) = \frac{1}{2} \cdot \frac{8}{15}\sqrt{2} \cdot 8\sqrt{2} = 4\frac{4}{15}$ .

**18 a**  $f_{-11}(x) = x^4 - x^3 - 11x^2 + 50$   
 $f_{-11}'(x) = 4x^3 - 3x^2 - 22x$   
 $f_{-11}'(x) = 0$  geeft  $4x^3 - 3x^2 - 22x = 0$   
 $x(4x^2 - 3x - 22) = 0$   
 $x = 0 \vee 4x^2 - 3x - 22 = 0$   
 $D = (-3)^2 - 4 \cdot 4 \cdot -22 = 361$ , dus  $\sqrt{D} = 19$   
 $x = \frac{3 + 19}{8} = 2\frac{3}{4} \vee x = \frac{3 - 19}{8} = -2$   
 $f_{-11}(0) = 50$  en  $f_{-11}(-2) = 30$   
Dus voor  $30 < a < 50$  heeft de vergelijking  $f_{-11}(x) = a$  vier oplossingen.

**b**  $f_{-18}(x) = x^4 - x^3 - 18x^2 + 50$   
 $f_{-18}'(x) = 4x^3 - 3x^2 - 36x$   
 $f_{-18}''(x) = 12x^2 - 6x - 36$   
 $f_{-18}''(x) = 0$  geeft  $12x^2 - 6x - 36 = 0$   
 $2x^2 - x - 6 = 0$   
 $D = (-1)^2 - 4 \cdot 2 \cdot -6 = 49$ , dus  $\sqrt{D} = 7$   
 $x = \frac{1 + 7}{4} = 2 \vee x = \frac{1 - 7}{4} = -1\frac{1}{2}$

$f_{-18}(2) = -14$  en  $f_{-18}(-1\frac{1}{2}) = 17\frac{15}{16}$

$k: y = ax + b$

$a = \frac{-14 - 17\frac{15}{16}}{2 - (-1\frac{1}{2})} = -9\frac{1}{8}$

$$\left. \begin{array}{l} y = -9\frac{1}{8}x + b \\ \text{door } (2, -14) \end{array} \right\} \begin{array}{l} -9\frac{1}{8} \cdot 2 + b = -14 \\ -18\frac{1}{4} + b = -14 \\ b = 4\frac{1}{4} \end{array}$$

Dus  $y_A = 4\frac{1}{4}$ .

**c**  $f_p(x) = x^4 - x^3 + px^2 + 50$   
 $f_p'(x) = 4x^3 - 3x^2 + 2px$   
 $f_p''(x) = 12x^2 - 6x + 2p$   
 $f_p''(-\frac{1}{2}) = 0$  geeft  $12 \cdot (-\frac{1}{2})^2 - 6 \cdot -\frac{1}{2} + 2p = 0$   
 $3 + 3 + 2p = 0$   
 $2p = -6$   
 $p = -3$   
 $f_{-3}''(x) = 0$  geeft  $12x^2 - 6x + 2 \cdot -3 = 0$   
 $12x^2 - 6x - 6 = 0$   
 $2x^2 - x - 1 = 0$   
 $D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$ , dus  $\sqrt{D} = 3$   
 $x = \frac{1 + 3}{4} = 1 \vee x = \frac{1 - 3}{4} = -\frac{1}{2}$

$f_{-3}(1) = 47$ , dus het andere buigpunt is  $(1, 47)$ .

- d** De grafiek van  $f_p$  heeft twee buigpunten als voor  $12x^2 - 6x + 2p = 0$  geldt  $D > 0$ .  
 $D = (-6)^2 - 4 \cdot 12 \cdot 2p = 36 - 96p$   
 $D > 0$  geeft  $36 - 96p > 0$   
 $-96p > -36$   
 $p < \frac{3}{8}$   
Dus voor  $p < \frac{3}{8}$  heeft de grafiek van  $f_p$  twee buigpunten.

- 19 a**  $f(x) = ax^3 + bx^2 + cx + d$  geeft  $f'(x) = 3ax^2 + 2bx + c$   
Gegeven is dat  $f$  een derdegraadsfunctie is, dus  $a \neq 0$ .  
Hieruit volgt dat de grafiek van  $f'$  een parabool is, dus  $f'$  heeft één extreem.  
Dus de grafiek van elke derdegraadsfunctie heeft één buigpunt.

- b** Omdat de grafiek van  $f$  twee toppen heeft, heeft de grafiek van  $f'$  twee snijpunten met de  $x$ -as.

$$\left. \begin{array}{l} f'(x) = 0 \text{ geeft } 3ax^2 + 2bx + c = 0 \\ \text{twee oplossingen, dus } D > 0 \\ D = (2b)^2 - 4 \cdot 3a \cdot c = 4b^2 - 12ac \end{array} \right\} \begin{array}{l} 4b^2 - 12ac > 0 \\ 4b^2 > 12ac \\ b^2 > 3ac \end{array}$$

- c**  $f'(x) = 3ax^2 + 2bx + c$  geeft  $f''(x) = 6ax + 2b$   
 $f''(x) = 0$  geeft  $6ax + 2b = 0$

$$6ax = -2b$$

$$x = -\frac{b}{3a}$$

$$\text{Dus } x_R = -\frac{b}{3a}.$$

$$f'(x) = 0 \text{ geeft } 3ax^2 + 2bx + c = 0$$

$$D = 4b^2 - 12ac$$

$$x = \frac{-2b + \sqrt{D}}{6a} \vee x = \frac{-2b - \sqrt{D}}{6a}$$

$$x_P + x_Q = \frac{-2b + \sqrt{D}}{6a} + \frac{-2b - \sqrt{D}}{6a} = -\frac{4b}{6a} = -\frac{2b}{3a} = 2 \cdot -\frac{b}{3a} = 2x_R$$

$$\text{Dus } 2x_R = x_P + x_Q \text{ oftewel } x_R = \frac{x_P + x_Q}{2}.$$

#### Bladzijde 61

- 20 a**  $h = \frac{1}{3}ax^3 - \frac{1}{2}abx^2$  geeft  $h' = ax^2 - abx$   
 $x = 0$  geeft  $h' = a \cdot 0^2 - ab \cdot 0 = 0$ , dus de oprit is aan het begin horizontaal.  
 $x = b$  geeft  $h' = a \cdot b^2 - ab \cdot b = ab^2 - ab^2 = 0$ , dus de oprit is aan het eind horizontaal.  
 $h' = ax^2 - abx$  geeft  $h'' = 2ax - ab$   
 $x = \frac{1}{2}b$  geeft  $h'' = 2a \cdot \frac{1}{2}b - ab = ab - ab = 0$   
Omdat geldt dat  $a < 0$ , is de grafiek van  $h' = ax^2 - abx$  een bergparabool, dus de helling is maximaal voor  $x = \frac{1}{2}b$ .

Dus de formule is met de drie eisen in overeenstemming.

- b**  $x = b$  en  $h = 6$  geeft  $\frac{1}{3}ab^3 - \frac{1}{2}ab \cdot b^2 = 6$

$$\frac{1}{3}ab^3 - \frac{1}{2}ab^3 = 6$$

$$-\frac{1}{6}ab^3 = 6$$

$$a = \frac{-36}{b^3}$$

$$a = \frac{-36}{b^3} \text{ invullen in } h = \frac{1}{3}ax^3 - \frac{1}{2}abx^2 \text{ geeft } h = \frac{1}{3} \cdot \frac{-36}{b^3} \cdot x^3 - \frac{1}{2} \cdot \frac{-36}{b^3} \cdot bx^2 = \frac{-12}{b^3}x^3 + \frac{18}{b^2}x^2$$

$$\text{c } h = \frac{-12}{b^3}x^3 + \frac{18}{b^2}x^2 \text{ geeft } h' = \frac{-36}{b^3}x^2 + \frac{36}{b^2}x$$

$$b = 240 \text{ en } x = \frac{1}{2}b = 120 \text{ geeft } h' = \frac{-36}{240^3} \cdot 120^2 + \frac{36}{240^2} \cdot 120 = 0,0375$$

Dus het maximale hellingspercentage is 3,75%.

$$\text{d } x = \frac{1}{2}b \text{ geeft } h' = \frac{-36}{b^3} \cdot (\frac{1}{2}b)^2 + \frac{36}{b^2} \cdot \frac{1}{2}b = \frac{-36}{b^3} \cdot \frac{1}{4}b^2 + \frac{18}{b} = \frac{-9}{b} + \frac{18}{b} = \frac{9}{b}$$

Voor  $x = \frac{1}{2}b$  moet gelden dat de helling  $\frac{3}{100}$  is, dus  $\frac{9}{b} = \frac{3}{100}$  en dit geeft  $3b = 900$  oftewel  $b = 300$ .

Dus bij een maximale helling van 3% is de minimale lengte van de oprit 300 meter.

$$\text{e } h = 8 \text{ en } x = b = 400 \text{ invullen in } h = \frac{1}{3}ax^3 - \frac{1}{2}abx^2 \text{ geeft } \frac{1}{3}a \cdot 400^3 - \frac{1}{2}a \cdot 400 \cdot 400^2 = 8$$

$$\frac{1}{3}a \cdot 400^3 - \frac{1}{2}a \cdot 400^3 = 8$$

$$-\frac{1}{6}a \cdot 400^3 = 8$$

$$a = \frac{-48}{400^3} = -\frac{3}{4000000}$$

$$\text{Dus } h = \frac{1}{3} \cdot -\frac{3}{4000000}x^3 - \frac{1}{2} \cdot -\frac{3}{4000000} \cdot 400x^2 = -\frac{1}{4000000}x^3 + \frac{3}{20000}x^2 \text{ en dit geeft } h' = -\frac{3}{4000000}x^2 + \frac{3}{10000}x.$$

$$x = 200 \text{ geeft } h' = -\frac{3}{4000000} \cdot 200^2 + \frac{3}{10000} \cdot 200 = 0,03 \text{ oftewel een helling van } 3\%.$$

Dus de maximaal acceptabele helling van 3% wordt niet overschreden.

### Bladzijde 62

$$\text{C1 } f(x) = \frac{2}{3}x^3 - 3x^2 + 4 \text{ geeft } f'(x) = 2x^2 - 6x$$

$$f'(x) = 0 \text{ geeft } 2x^2 - 6x = 0$$

$$2x(x - 3) = 0$$

$$x = 0 \vee x = 3$$

Je krijgt max. is  $f(0) = 4$  en min. is  $f(3) = -5$ .

$$\text{C2 } f(x) = x^4 + x^3 - 3x^2 - x - 3 \text{ geeft } f'(x) = 4x^3 + 3x^2 - 6x - 1$$

$$f'(1) = 4 \cdot 1^3 + 3 \cdot 1^2 - 6 \cdot 1 - 1 = 0$$

$f'(1) = 0$  en in de figuur is te zien dat de grafiek een top heeft voor  $x = 1$ .

Dus  $f$  heeft een extreme waarde voor  $x = 1$ .

$$\text{C3 } f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 - 6x^2 + 100$$

$$f'(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x$$

$$f''(x) = x^2 - x - 12$$

$$f''(x) = 0 \text{ geeft } x^2 - x - 12 = 0$$

$$(x + 3)(x - 4) = 0$$

$$x = -3 \vee x = 4$$

$$f(-3) = \frac{1}{12} \cdot (-3)^4 - \frac{1}{6} \cdot (-3)^3 - 6 \cdot (-3)^2 + 100 = 57\frac{1}{4}$$

$$f(4) = \frac{1}{12} \cdot 4^4 - \frac{1}{6} \cdot 4^3 - 6 \cdot 4^2 + 100 = 14\frac{2}{3}$$

Dus de buigpunten zijn  $(-3, 57\frac{1}{4})$  en  $(4, 14\frac{2}{3})$ .

## 6.2 De afgeleide van machtsfuncties

### Bladzijde 63

$$\text{21 a } f(x) = \frac{1}{x^2} \text{ geeft } f'(x) = \frac{x^2 \cdot 0 - 1 \cdot 2x}{x^4} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

$$\text{b } f(x) = x^{-2} = \frac{1}{x^2} \text{ geeft } f'(x) = -\frac{2}{x^3} = -2x^{-3}$$

$$\text{c } g(x) = x^{-5} = \frac{1}{x^5} \text{ geeft } g'(x) = \frac{x^5 \cdot 0 - 1 \cdot 5x^4}{x^{10}} = \frac{-5x^4}{x^{10}} = -5x^{-6}$$



**22** Voor  $n = 0$ :

$$\left. \begin{array}{l} f(x) = x^0 = 1 \text{ geeft } f'(x) = 0 \\ f'(x) = 0 \cdot x^{0-1} = 0 \end{array} \right\} f(x) = x^n \text{ geeft } f'(x) = nx^{n-1} \text{ klopt voor } n = 0.$$

Voor  $n = 1$ :

$$\left. \begin{array}{l} f(x) = x^1 = x \text{ geeft } f'(x) = 1 \\ f'(x) = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1 = 1 \end{array} \right\} f(x) = x^n \text{ geeft } f'(x) = nx^{n-1} \text{ klopt voor } n = 1.$$

**23 a** De functies  $k$  en  $l$  kun je differentiëren door uit te delen.

$$\text{b } g(x) = \frac{x^3 + 1}{3x^2} \text{ geeft } g'(x) = \frac{3x^2 \cdot 3x^2 - (x^3 + 1) \cdot 6x}{(3x^2)^2} = \frac{9x^4 - 6x^4 - 6x}{9x^4} = \frac{3x^4 - 6x}{9x^4} = \frac{3x(x^3 - 2)}{9x^4} = \frac{x^3 - 2}{3x^3}$$

\*

$$\text{24 a } f(x) = \frac{1}{x^6} = x^{-6} \text{ geeft } f'(x) = -6x^{-7} = \frac{-6}{x^7}$$

$$\text{b } g(x) = 5 - \frac{3}{x^2} = 5 - 3x^{-2} \text{ geeft } g'(x) = 6x^{-3} = \frac{6}{x^3}$$

$$\text{c } h(x) = ax^4 - \frac{b}{x^4} = ax^4 - bx^{-4} \text{ geeft } h'(x) = 4ax^3 + 4bx^{-5} = 4ax^3 + \frac{4b}{x^5}$$

$$\text{25 a } f(x) = \frac{2x-1}{3x^2} = \frac{2x}{3x^2} - \frac{1}{3x^2} = \frac{2}{3}x^{-1} - \frac{1}{3}x^{-2} \text{ geeft } f'(x) = -\frac{2}{3}x^{-2} + \frac{2}{3}x^{-3} = -\frac{2}{3x^2} + \frac{2}{3x^3} = \frac{-2x}{3x^3} + \frac{2}{3x^3} = \frac{2-2x}{3x^3}$$

$$\text{b } g(x) = \frac{3x^2}{2x-1} \text{ geeft } g'(x) = \frac{(2x-1) \cdot 6x - 3x^2 \cdot 2}{(2x-1)^2} = \frac{12x^2 - 6x - 6x^2}{(2x-1)^2} = \frac{6x^2 - 6x}{(2x-1)^2}$$

$$\text{c } h(x) = \frac{3x^6-3}{x^3} = \frac{3x^6}{x^3} - \frac{3}{x^3} = 3x^3 - 3x^{-3} \text{ geeft } h'(x) = 9x^2 + 9x^{-4} = 9x^2 + \frac{9}{x^4} = \frac{9x^6}{x^4} + \frac{9}{x^4} = \frac{9x^6+9}{x^4}$$

$$\text{26 a } f(x) = 5x^2 - \frac{5}{x^2} = 5x^2 - 5x^{-2} \text{ geeft } f'(x) = 10x + 10x^{-3} = 10x + \frac{10}{x^3}$$

$$\text{b } g(x) = \frac{5}{2x^2} - \frac{2x^2}{5} = \frac{5}{2}x^{-2} - \frac{2}{5}x^2 \text{ geeft } g'(x) = -5x^{-3} - \frac{4}{5}x = -\frac{5}{x^3} - \frac{4x}{5}$$

$$\text{c } h(x) = 6 - \frac{x^2-1}{x} = 6 - \left(\frac{x^2}{x} - \frac{1}{x}\right) = 6 - x + x^{-1} \text{ geeft } h'(x) = -1 - x^{-2} = -1 - \frac{1}{x^2} \left(= \frac{-x^2-1}{x^2}\right)$$

$$\text{27 a } f(x) = 0 \text{ geeft } 3 + \frac{3}{x} = 0$$

$$\frac{3}{x} = -3$$

$$x = -1$$

Dus  $A(-1, 0)$ .

$$f(x) = 3 + \frac{3}{x} = 3 + 3x^{-1} \text{ geeft } f'(x) = -3x^{-2} = -\frac{3}{x^2}$$

$$k: y = ax + b$$

$$a = f'(-1) = -\frac{3}{(-1)^2} = -3$$

$$\left. \begin{array}{l} y = -3x + b \\ \text{door } A(-1, 0) \end{array} \right\} \begin{array}{l} -3 \cdot -1 + b = 0 \\ 3 + b = 0 \\ b = -3 \end{array}$$

Dus  $k: y = -3x - 3$  en  $y_B = -3$ .

$$\text{b } f'(x) = -\frac{3}{4} \text{ geeft } -\frac{3}{x^2} = -\frac{3}{4}$$

$$x^2 = 4$$

$$x = 2 \vee x = -2$$

 $f(2) = 4\frac{1}{2}$  en  $f(-2) = 1\frac{1}{2}$ , dus de raakpunten zijn  $(2, 4\frac{1}{2})$  en  $(-2, 1\frac{1}{2})$ .

**28 a**  $f(x) = \frac{x^2 + 4}{x} = x + 4x^{-1}$  geeft  $f'(x) = 1 - 4x^{-2} = 1 - \frac{4}{x^2}$

$k: y = ax + b$

$a = f'(3) = 1 - \frac{4}{3^2} = \frac{5}{9}$

$$\left. \begin{array}{l} y = \frac{5}{9}x + b \\ f(3) = 4\frac{1}{3}, \text{ dus } A(3, 4\frac{1}{3}) \end{array} \right\} \begin{array}{l} \frac{5}{9} \cdot 3 + b = 4\frac{1}{3} \\ \frac{5}{3} + b = 4\frac{1}{3} \\ b = 2\frac{2}{3} \end{array}$$

Dus  $k: y = \frac{5}{9}x + 2\frac{2}{3}$ .

$y = 0$  geeft  $\frac{5}{9}x + 2\frac{2}{3} = 0$

$\frac{5}{9}x = -2\frac{2}{3}$

$x = -4\frac{4}{5}$

Dus  $x_B = -4\frac{4}{5}$ .

**b**  $f'(x) = -3$  geeft  $1 - \frac{4}{x^2} = -3$

$-\frac{4}{x^2} = -4$

$x^2 = 1$

$x = 1 \vee x = -1$

$f(1) = 5$  en  $f(-1) = -5$ , dus de raakpunten zijn  $(1, 5)$  en  $(-1, -5)$ .

**c**  $f'(x) = 0$  geeft  $1 - \frac{4}{x^2} = 0$

$\frac{4}{x^2} = 1$

$x^2 = 4$

$x = 2 \vee x = -2$

Je krijgt max. is  $f(-2) = -4$  en min. is  $f(2) = 4$ .

**d**  $f'(x) = 2$  geeft  $1 - \frac{4}{x^2} = 2$

$-\frac{4}{x^2} = 1$

$x^2 = -4$

geen oplossing

Dus de grafiek van  $f$  heeft geen raaklijn met richtingscoëfficiënt 2.

**29 a**  $f(x) = \frac{x^2}{3} + \frac{3}{x^2} = \frac{1}{3}x^2 + 3x^{-2}$  geeft  $f'(x) = \frac{2}{3}x - 6x^{-3} = \frac{2x}{3} - \frac{6}{x^3}$

$f'(x) = 0$  geeft  $\frac{2x}{3} - \frac{6}{x^3} = 0$

$\frac{2x}{3} = \frac{6}{x^3}$

$2x^4 = 18$

$x^4 = 9$

$x = \sqrt[4]{9} \vee x = -\sqrt[4]{9}$

Uit de figuur in het leerboek volgt min. is  $f(\sqrt[4]{9}) = 2$  en min. is  $f(-\sqrt[4]{9}) = 2$ .

Dus de vergelijking  $\frac{x^2}{3} + \frac{3}{x^2} = p$  heeft vier oplossingen voor  $p > 2$ .

$$\text{b } f(x) = \frac{x^2}{a} + \frac{a}{x^2} = \frac{1}{a}x^2 + ax^{-2} \text{ geeft } f'(x) = \frac{2}{a}x - 2ax^{-3} = \frac{2x}{a} - \frac{2a}{x^3}$$

$$f'(x) = 0 \text{ geeft } \frac{2x}{a} - \frac{2a}{x^3} = 0$$

$$\frac{2x}{a} = \frac{2a}{x^3}$$

$$2x^4 = 2a^2$$

$$x^4 = a^2$$

$$x^2 = a \vee x^2 = -a$$

$$x = \sqrt{a} \vee x = -\sqrt{a}$$

$$x_{\text{top}} = \sqrt{a} \text{ geeft } y_{\text{top}} = f(\sqrt{a}) = \frac{a}{a} + \frac{a}{a} = 1 + 1 = 2$$

$$x_{\text{top}} = -\sqrt{a} \text{ geeft } y_{\text{top}} = f(-\sqrt{a}) = \frac{a}{a} + \frac{a}{a} = 1 + 1 = 2$$

Dus voor elke waarde van  $a$  liggen de toppen van de grafiek van  $f$  op de lijn  $y = 2$ .

$$\text{30 a } x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x$$

$$[x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}]' = [x]'$$

$$[x^{\frac{1}{2}}]' \cdot x^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$$

$$2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$$

$$\text{b } 2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$$

$$x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = \frac{1}{2}$$

$$[x^{\frac{1}{2}}]' = \frac{\frac{1}{2}}{x^{\frac{1}{2}}}$$

$$[x^{\frac{1}{2}}]' = \frac{1}{2}x^{-\frac{1}{2}}$$

### Bladzijde 67

$$\text{31 a } f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}} \text{ geeft } f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2\sqrt{x}}$$

$$\text{b } g(x) = x \cdot \sqrt[3]{x} = x^{\frac{4}{3}} \text{ geeft } g'(x) = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3} \cdot \sqrt[3]{x}$$

$$\text{c } h(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}} \text{ geeft } h'(x) = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}} = -\frac{1}{2x\sqrt{x}}$$

$$\text{d } k(x) = (3 - \sqrt{x})^2 = 9 - 6\sqrt{x} + x = 9 - 6x^{\frac{1}{2}} + x \text{ geeft } k'(x) = -3x^{-\frac{1}{2}} + 1 = -\frac{3}{\sqrt{x}} + 1$$

$$\text{32 a } f(x) = x^2 \cdot \sqrt[4]{x} = x^2 \cdot x^{\frac{1}{4}} \text{ geeft } f'(x) = 2x \cdot x^{\frac{1}{4}} = 2x^{\frac{5}{4}} = 2\sqrt[4]{x} \cdot x$$

$$\text{b } g(x) = \frac{4x+1}{\sqrt{x}} = \frac{4x}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}} = 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \text{ geeft } g'(x) = 2x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{2}{x^{\frac{1}{2}}} - \frac{1}{2x^{\frac{3}{2}}} = \frac{2}{\sqrt{x}} - \frac{1}{2x\sqrt{x}} = \frac{4x}{2x\sqrt{x}} - \frac{1}{2x\sqrt{x}} = \frac{4x-1}{2x\sqrt{x}}$$

$$\text{c } h(x) = (x+1)(1+\sqrt{x}) = x + x\sqrt{x} + 1 + \sqrt{x} = x + x^{\frac{3}{2}} + 1 + x^{\frac{1}{2}} \text{ geeft}$$

$$h'(x) = 1 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$$

$$\text{d } k(x) = \frac{x\sqrt{x}+1}{2\sqrt{x}} = \frac{x}{2} + \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2}x + \frac{1}{2}x^{-\frac{1}{2}} \text{ geeft } k'(x) = \frac{1}{2} - \frac{1}{4}x^{-\frac{3}{2}} = \frac{1}{2} - \frac{1}{4x^{\frac{3}{2}}} = \frac{1}{2} - \frac{1}{4x\sqrt{x}}$$

$$= \frac{2x\sqrt{x}}{4x\sqrt{x}} - \frac{1}{4x\sqrt{x}} = \frac{2x\sqrt{x}-1}{4x\sqrt{x}}$$

$$\text{33 a } f(x) = \frac{1}{x\sqrt{x}} = \frac{1}{x^{\frac{3}{2}}} = x^{-\frac{3}{2}} \text{ geeft } f'(x) = -\frac{3}{2}x^{-\frac{5}{2}} = -\frac{3}{2} \cdot \frac{1}{x^2 \cdot \sqrt{x}}$$

$$\text{b } g(x) = \sqrt{x} - \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} - x^{-\frac{1}{2}} \text{ geeft } g'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{2x^{\frac{3}{2}}} = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}}$$

$$\begin{aligned} \text{c } h(x) &= \frac{x^2-2}{x\sqrt{x}} = \frac{x^2}{x^{1\frac{1}{2}}} - \frac{2}{x^{1\frac{1}{2}}} = x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} \text{ geeft } h'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 3x^{-2\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} + \frac{3}{x^{2\frac{1}{2}}} = \frac{1}{2\sqrt{x}} + \frac{3}{x^2 \cdot \sqrt{x}} \\ &= \frac{x^2}{2x^2 \cdot \sqrt{x}} + \frac{6}{2x^2 \cdot \sqrt{x}} = \frac{x^2+6}{2x^2 \cdot \sqrt{x}} \\ \text{d } k(x) &= x^2(x\sqrt{x}-3) = x^3 \cdot \sqrt{x} - 3x^2 = x^{3\frac{1}{2}} - 3x^2 \text{ geeft } k'(x) = 3\frac{1}{2}x^{2\frac{1}{2}} - 6x = 3\frac{1}{2}x^2 \cdot \sqrt{x} - 6x \end{aligned}$$

**34 a**  $f(x) = (x\sqrt{x}-3)^2 = x^3 - 6x\sqrt{x} + 9 = x^3 - 6x^{1\frac{1}{2}} + 9$  geeft  $f'(x) = 3x^2 - 9x^{\frac{1}{2}} = 3x^2 - 9\sqrt{x}$

**b**  $g(x) = \frac{2x-3}{x\sqrt{x}} = \frac{2x}{x^{1\frac{1}{2}}} - \frac{3}{x^{1\frac{1}{2}}} = 2x^{-\frac{1}{2}} - 3x^{-1\frac{1}{2}}$  geeft  
 $g'(x) = -x^{-\frac{1}{2}} + 4\frac{1}{2}x^{-2\frac{1}{2}} = -\frac{1}{x^{\frac{1}{2}}} + \frac{9}{2x^{2\frac{1}{2}}} = -\frac{1}{x\sqrt{x}} + \frac{9}{2x^2 \cdot \sqrt{x}} = -\frac{2x}{2x^2 \cdot \sqrt{x}} + \frac{9}{2x^2 \cdot \sqrt{x}} = \frac{9-2x}{2x^2 \cdot \sqrt{x}}$

**c**  $h(x) = (x - \sqrt[3]{x})^2 = (x - x^{\frac{1}{3}})^2 = x^2 - 2x^{1\frac{1}{3}} + x^{\frac{2}{3}}$  geeft

$$h'(x) = 2x - 2\frac{2}{3}x^{\frac{1}{3}} + \frac{2}{3}x^{-\frac{1}{3}} = 2x - 2\frac{2}{3} \cdot \sqrt[3]{x} + \frac{2}{3 \cdot \sqrt[3]{x}}$$

**d**  $k(x) = \frac{x^2+4}{\sqrt[4]{x}} = \frac{x^2}{x^{\frac{1}{4}}} + \frac{4}{x^{\frac{1}{4}}} = x^{1\frac{3}{4}} + 4x^{-\frac{1}{4}}$  geeft  $k'(x) = 1\frac{3}{4}x^{\frac{3}{4}} - x^{-1\frac{1}{4}} = \frac{7x^{\frac{3}{4}}}{4} - \frac{1}{x^{1\frac{1}{4}}} = \frac{7x^2}{4x^{1\frac{1}{4}}} - \frac{4}{4x^{1\frac{1}{4}}} = \frac{7x^2-4}{4x \cdot \sqrt[4]{x}}$

**35**  $f(x) = 3 \cdot \sqrt[3]{x^2} = 3x^{\frac{2}{3}}$  geeft  $f'(x) = 2x^{-\frac{1}{3}} = \frac{2}{x^{\frac{1}{3}}} = \frac{2}{\sqrt[3]{x}}$

$k: y = ax + b$

$a = f'(\frac{1}{8}) = \frac{2}{\sqrt[3]{\frac{1}{8}}} = 4$

$\left. \begin{aligned} y &= 4x + b \\ f(\frac{1}{8}) &= \frac{3}{4}, \text{ dus } A(\frac{1}{8}, \frac{3}{4}) \end{aligned} \right\} \begin{aligned} 4 \cdot \frac{1}{8} + b &= \frac{3}{4} \\ \frac{1}{2} + b &= \frac{3}{4} \\ b &= \frac{1}{4} \end{aligned}$

Dus  $k: y = 4x + \frac{1}{4}$ .

$k$  snijden met  $l$  geeft  $4x + \frac{1}{4} = x + 4$

$$3x = 3\frac{3}{4}$$

$$x = 1\frac{1}{4}$$

$x = 1\frac{1}{4}$  geeft  $y = 1\frac{1}{4} + 4 = 5\frac{1}{4}$ , dus  $C(1\frac{1}{4}, 5\frac{1}{4})$ .

$l: y = ax + b$

$a = f'(8) = \frac{2}{\sqrt[3]{8}} = 1$

$\left. \begin{aligned} y &= x + b \\ f(8) &= 12, \text{ dus } B(8, 12) \end{aligned} \right\} \begin{aligned} 8 + b &= 12 \\ b &= 4 \end{aligned}$

Dus  $l: y = x + 4$ .

### Bladzijde 68

**36 a**  $f(x) = x\sqrt{x} - 3x = x^{1\frac{1}{2}} - 3x$  geeft  $f'(x) = 1\frac{1}{2}x^{\frac{1}{2}} - 3 = 1\frac{1}{2}\sqrt{x} - 3$

$f'(x) = 0$  geeft  $1\frac{1}{2}\sqrt{x} - 3 = 0$

$$1\frac{1}{2}\sqrt{x} = 3$$

$$\sqrt{x} = 2$$

$$x = 4$$

Je krijgt min. is  $f(4) = -4$ .

Dus  $B_f = [-4, \rightarrow)$ .

**b**  $f(x) = 0$  geeft  $x\sqrt{x} - 3x = 0$

$$x(\sqrt{x} - 3) = 0$$

$$x = 0 \vee \sqrt{x} = 3$$

$$x = 0 \vee x = 9$$

Dus  $A(9, 0)$ .

$l: y = ax + b$

$a = rc_l = rc_k = f'(0) = 1\frac{1}{2}\sqrt{0} - 3 = -3$

$\left. \begin{aligned} y &= -3x + b \\ \text{door } A(9, 0) \end{aligned} \right\} \begin{aligned} -3 \cdot 9 + b &= 0 \\ b &= 27 \end{aligned}$

$l: y = -3x + 27$ , dus  $y_B = 27$ .

**c**  $f'(x) = 3$  geeft  $1\frac{1}{2}\sqrt{x} - 3 = 3$

$$1\frac{1}{2}\sqrt{x} = 6$$

$$\sqrt{x} = 4$$

$$x = 16$$

$$\left. \begin{array}{l} m: y = 3x + b \\ f(16) = 16, \text{ dus } C(16, 16) \end{array} \right\} \begin{array}{l} 3 \cdot 16 + b = 16 \\ 48 + b = 16 \\ b = -32 \end{array}$$

Dus  $m: y = 3x - 32$ .

$y = 0$  geeft  $3x - 32 = 0$

$$3x = 32$$

$$x = 10\frac{2}{3}$$

Dus  $x_D = 10\frac{2}{3}$ .

**37 a**  $f(x) = 8\frac{1}{4}$  geeft  $\frac{x^3 + 2}{x\sqrt{x}} = 8\frac{1}{4}$

$$x^3 + 2 = 8\frac{1}{4}x\sqrt{x}$$

$$x^3 - 8\frac{1}{4}x\sqrt{x} + 2 = 0$$

Stel  $x\sqrt{x} = u$ .

$$u^2 - 8\frac{1}{4}u + 2 = 0$$

$$4u^2 - 33u + 8 = 0$$

$$D = (-33)^2 - 4 \cdot 4 \cdot 8 = 961, \text{ dus } \sqrt{D} = 31$$

$$u = \frac{33 + 31}{8} = 8 \vee u = \frac{33 - 31}{8} = \frac{1}{4}$$

$$x\sqrt{x} = 8 \vee x\sqrt{x} = \frac{1}{4}$$

$$x^{1\frac{1}{2}} = 2^3 \vee x^{1\frac{1}{2}} = 2^{-2}$$

$$x = (2^3)^{\frac{2}{3}} \vee x = (2^{-2})^{\frac{2}{3}}$$

$$x = 2^2 \vee x = 2^{-1\frac{1}{3}}$$

Dus  $a = -1\frac{1}{3}$  en  $b = 2$ .

**b**  $f(x) = \frac{x^3 + 2}{x\sqrt{x}} = \frac{x^3}{x^{1\frac{1}{2}}} + \frac{2}{x^{1\frac{1}{2}}} = x^{1\frac{1}{2}} + 2x^{-1\frac{1}{2}}$  geeft

$$f'(x) = 1\frac{1}{2}x^{\frac{1}{2}} - 3x^{-2\frac{1}{2}} = 1\frac{1}{2}\sqrt{x} - \frac{3}{x^{2\frac{1}{2}}} = 1\frac{1}{2}\sqrt{x} - \frac{3}{x^2\sqrt{x}} = \frac{1\frac{1}{2}x^3}{x^2 \cdot \sqrt{x}} - \frac{3}{x^2 \cdot \sqrt{x}} = \frac{1\frac{1}{2}x^3 - 3}{x^2 \cdot \sqrt{x}}$$

$f'(x) = 0$  geeft  $1\frac{1}{2}x^3 - 3 = 0$

$$1\frac{1}{2}x^3 = 3$$

$$x^3 = 2$$

$$x = 2^{\frac{1}{3}}$$

$$f(2^{\frac{1}{3}}) = \frac{(2^{\frac{1}{3}})^3 + 2}{(2^{\frac{1}{3}})^{1\frac{1}{2}}} = \frac{2 + 2}{2^{\frac{1}{2}}} = \frac{2^2}{2^{\frac{1}{2}}} = 2^{1\frac{1}{2}}$$

Dus  $p = \frac{1}{3}$  en  $q = 1\frac{1}{2}$ .

**c**  $k: y = ax + b$

$$a = f'(1) = \frac{\frac{1}{2} \cdot 1^3 - 3}{1^2 \cdot \sqrt{1}} = -1\frac{1}{2}$$

$$\left. \begin{array}{l} y = -1\frac{1}{2}x + b \\ f(1) = 3, \text{ dus } C(1, 3) \end{array} \right\} -1\frac{1}{2} \cdot 1 + b = 3$$

$$b = 4\frac{1}{2}$$

Dus  $k: y = -1\frac{1}{2}x + 4\frac{1}{2}$ .

$y = 0$  geeft  $-1\frac{1}{2}x + 4\frac{1}{2} = 0$

$$-1\frac{1}{2}x = -4\frac{1}{2}$$

$$x = 3, \text{ dus } D(3, 0).$$

Dus  $O(\triangle OCD) = \frac{1}{2} \cdot 3 \cdot 3 = 4\frac{1}{2}$ .

**38 a**  $s(t) = 10t\sqrt{t} = 10t^{\frac{3}{2}}$  geeft  $s'(t) = 15t^{\frac{1}{2}} = 15\sqrt{t}$

$$s'(8) = 15\sqrt{8} = 15 \cdot 2\sqrt{2} = 30\sqrt{2}$$

Dus de snelheid na acht seconden is  $30\sqrt{2}$  m/s.

**b**  $108 \text{ km/u} = \frac{108}{3,6} \text{ m/s} = 30 \text{ m/s}$

$$s'(t) = 30 \text{ geeft } 15\sqrt{t} = 30$$

$$\sqrt{t} = 2$$

$$t = 4$$

Dus na vier seconden is de snelheid 108 km/u.

**c**  $s(9) = 10 \cdot 9 \cdot \sqrt{9} = 270$  en  $s'(9) = 15 \cdot \sqrt{9} = 45$

Dus in de eerste negen seconden legt de trein 270 meter af en in de volgende 51 seconden  $51 \cdot 45 = 2295$  meter.

Dus in de eerste minuut legt de trein  $270 + 2295 = 2565$  meter af.

**39**  $f_p(x) = (2x - p\sqrt{x})^2 = 4x^2 - 4px\sqrt{x} + p^2x = 4x^2 - 4p \cdot x^{\frac{3}{2}} + p^2x$  geeft

$$f'_p(x) = 8x - 6p \cdot x^{\frac{1}{2}} + p^2 = 8x - 6p\sqrt{x} + p^2$$

$$f'_p(2\frac{1}{4}) = 0 \text{ geeft } 8 \cdot 2\frac{1}{4} - 6p\sqrt{2\frac{1}{4}} + p^2 = 0$$

$$18 - 6p \cdot 1\frac{1}{2} + p^2 = 0$$

$$p^2 - 9p + 18 = 0$$

$$(p - 3)(p - 6) = 0$$

$$p = 3 \vee p = 6$$

$p = 3$  geeft  $f_3(x) = (2x - 3\sqrt{x})^2$

Zie de figuur hiernaast.

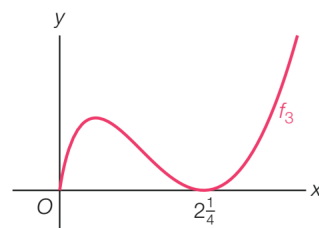
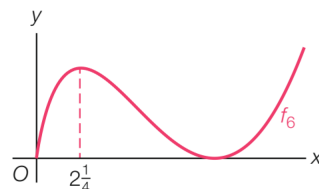
$f_3$  heeft geen maximum voor  $x = 2\frac{1}{4}$ .

$p = 6$  geeft  $f_6(x) = (2x - 6\sqrt{x})^2$

Zie de figuur hiernaast.

$f_6$  heeft een maximum voor  $x = 2\frac{1}{4}$ .

Je krijgt max. is  $f_6(2\frac{1}{4}) = 20\frac{1}{4}$ .



### Bladzijde 69

C4

- a**  $f(x) = \frac{2}{x^4} = 2x^{-4}$  geeft  $f'(x) = -8x^5 = -\frac{8}{x^5}$
- b**  $g(x) = \frac{x^2 + 1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + x^{-1}$  geeft  $g'(x) = 1 - x^{-2} = \frac{x^2}{x^2} - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$
- c**  $h(x) = \frac{1}{2}x + \frac{9}{2x} = \frac{1}{2}x + \frac{9}{2}x^{-1}$  geeft  $h'(x) = \frac{1}{2} - \frac{9}{2}x^{-2} = \frac{1}{2} - \frac{9}{2x^2}$   
 $h'(x) = -4$  geeft  $\frac{1}{2} - \frac{9}{2x^2} = -4$   
 $-\frac{9}{2x^2} = -4\frac{1}{2}$   
 $-\frac{9}{2x^2} = -\frac{9}{2}$   
 $x^2 = 1$   
 $x = 1 \vee x = -1$   
 $h(1) = 5$  en  $h(-1) = -5$ , dus de raakpunten zijn  $(-1, -5)$  en  $(1, 5)$ .

C5

- a**  $f(x) = x^3 \cdot \sqrt{x} = x^{3\frac{1}{2}}$  geeft  $f'(x) = 3\frac{1}{2}x^{2\frac{1}{2}} = 3\frac{1}{2}x^2 \cdot \sqrt{x}$
- b**  $g(x) = \frac{x^2 + 1}{x\sqrt{x}} = \frac{x^2}{x^{1\frac{1}{2}}} + \frac{1}{x^{1\frac{1}{2}}} = x^{\frac{1}{2}} + x^{-1\frac{1}{2}}$  geeft  
 $g'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-2\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} - \frac{3}{2x^{2\frac{1}{2}}} = \frac{1}{2\sqrt{x}} - \frac{3}{2x^2 \cdot \sqrt{x}} = \frac{x^2}{2x^2 \cdot \sqrt{x}} - \frac{3}{2x^2 \cdot \sqrt{x}} = \frac{x^2 - 3}{2x^2 \cdot \sqrt{x}}$
- c**  $h(x) = 6x - 2x\sqrt{x} = 6x - 2x^{1\frac{1}{2}}$  geeft  $h'(x) = 6 - 3x^{\frac{1}{2}} = 6 - 3\sqrt{x}$   
 $h'(x) = 0$  geeft  $6 - 3\sqrt{x} = 0$   
 $3\sqrt{x} = 6$   
 $\sqrt{x} = 2$   
 $x = 4$   
 Je krijgt max. is  $h(4) = 8$ .

## 6.3 De kettingregel

### Bladzijde 70

40

- a**  $v(3) = 3^2 - 5 \cdot 3 = -6$   
 $u(v(3)) = u(-6) = (-6)^4 = 1296$
- b**  $v(4) = 4^2 - 5 \cdot 4 = -4$   
 $u(v(4)) = u(-4) = (-4)^4 = 256$

41

- a**  $f(x) = u(v(x))$  met  $u(v) = v^6$  en  $v(x) = 3 - x^5$ .
- b**  $f(x) = u(v(x))$  met  $u(v) = \sqrt{v}$  en  $v(x) = x^2 + 1$ .
- c**  $f(x) = u(v(x))$  met  $u(v) = \frac{2}{v^3}$  en  $v(x) = x + 8$ .

### Bladzijde 71

42

- a**  $f(x) = -2(2x + 1)^4$  geeft  $f'(x) = -8(2x + 1)^3 \cdot 2 = -16(2x + 1)^3$
- b**  $g(x) = \frac{1}{(3x - 2)^2} = (3x - 2)^{-2}$  geeft  $g'(x) = -2(3x - 2)^{-3} \cdot 3 = \frac{-6}{(3x - 2)^3}$
- c**  $h(x) = \sqrt{2x^2 + 4x}$  geeft  $h'(x) = \frac{1}{2\sqrt{2x^2 + 4x}} \cdot (4x + 4) = \frac{2x + 2}{\sqrt{2x^2 + 4x}}$
- d**  $j(x) = \frac{1}{\sqrt{4x - 1}} = (4x - 1)^{-\frac{1}{2}}$  geeft  $j'(x) = -\frac{1}{2}(4x - 1)^{-\frac{1}{2}} \cdot 4 = \frac{-2}{(4x - 1)\sqrt{4x - 1}}$
- e**  $k(x) = (x^2 + 3)\sqrt{x^2 + 3} = (x^2 + 3)^{\frac{1}{2}}$  geeft  $k'(x) = \frac{1}{2}(x^2 + 3)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2 + 3}}$
- f**  $l(x) = \frac{1}{\sqrt{x^2 + 2x + 3}} = (x^2 + 2x + 3)^{-\frac{1}{2}}$  geeft  
 $l'(x) = -\frac{1}{2}(x^2 + 2x + 3)^{-\frac{1}{2}} \cdot (2x + 2) = \frac{-x - 1}{(x^2 + 2x + 3)\sqrt{x^2 + 2x + 3}}$

- 43** **a**  $f(x) = 4(x^3 + 7x - 2)^2$  geeft  $f'(x) = 8(x^3 + 7x - 2)(3x^2 + 7)$   
**b**  $g(x) = -\frac{6}{(x^2 + 3x)^3} = -6(x^2 + 3x)^{-3}$  geeft  $g'(x) = 18(x^2 + 3x)^{-4} \cdot (2x + 3) = \frac{36x + 54}{(x^2 + 3x)^4}$   
**c**  $h(x) = \sqrt[3]{x^3 + 3x} = (x^3 + 3x)^{\frac{1}{3}}$  geeft  $h'(x) = \frac{1}{3}(x^3 + 3x)^{-\frac{2}{3}} \cdot (3x^2 + 3) = \frac{x^2 + 1}{\sqrt[3]{(x^3 + 3x)^2}}$   
**d**  $j(x) = \frac{1}{(4-x)\sqrt{4-x}} = (4-x)^{-1\frac{1}{2}}$  geeft  $j'(x) = -1\frac{1}{2}(4-x)^{-2\frac{1}{2}} \cdot -1 = \frac{3}{2(4-x)^2 \cdot \sqrt{4-x}}$   
**e**  $k(x) = 5\sqrt{2x^4 + x^2} + 4x^2$  geeft  $k'(x) = 5 \cdot \frac{1}{2\sqrt{2x^4 + x^2}} \cdot (8x^3 + 2x) + 8x = \frac{20x^3 + 5x}{\sqrt{2x^4 + x^2}} + 8x$   
**f**  $l(x) = \frac{x^2 + 4}{\sqrt{x^2 + 4}} = \sqrt{x^2 + 4}$  geeft  $l'(x) = \frac{1}{2\sqrt{x^2 + 4}} \cdot 2x = \frac{x}{\sqrt{x^2 + 4}}$

- 44** **a**  $f(x) = (x^2 - 3x)^3$  geeft  $f'(x) = 3(x^2 - 3x)^2 \cdot (2x - 3)$   
 $f'(x) = 0$  geeft  $3(x^2 - 3x)^2 \cdot (2x - 3) = 0$   
 $x^2 - 3x = 0 \vee 2x - 3 = 0$   
 $x(x - 3) = 0 \vee x = 1\frac{1}{2}$   
 $x = 0 \vee x = 3 \vee x = 1\frac{1}{2}$   
 $f(0) = 0, f(1\frac{1}{2}) = -11\frac{25}{64}$  en  $f(3) = 0$   
Dus de coördinaten van de punten zijn  $(0, 0)$ ,  $(1\frac{1}{2}, -11\frac{25}{64})$  en  $(3, 0)$ .  
**b**  $k: y = ax + b$   
 $a = f'(1) = 3(1^2 - 3 \cdot 1)^2 \cdot (2 \cdot 1 - 3) = -12$   
 $y = -12x + b$   
 $f(1) = -8$ , dus  $A(1, -8)$   $\left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} -12 + b = -8 \\ b = 4 \end{array}$   
Dus  $k: y = -12x + 4$ .  
 $y = 0$  geeft  $-12x + 4 = 0$   
 $-12x = -4$   
 $x = \frac{1}{3}$   
Dus  $x_B = \frac{1}{3}$ .

#### Bladzijde 72

- 45** **a**  $f(x) = (\frac{1}{4}x - 1)^4 - x + 2$  geeft  $f'(x) = 4(\frac{1}{4}x - 1)^3 \cdot \frac{1}{4} - 1 = (\frac{1}{4}x - 1)^3 - 1$   
 $f'(x) = 0$  geeft  $(\frac{1}{4}x - 1)^3 - 1 = 0$   
 $(\frac{1}{4}x - 1)^3 = 1$   
 $\frac{1}{4}x - 1 = 1$   
 $\frac{1}{4}x = 2$   
 $x = 8$   
Je krijgt min. is  $f(8) = -5$ , dus  $B_f = [-5, \rightarrow)$ .  
**b** Uit vraag a volgt  $A(8, -5)$ .  
 $f(0) = 3$ , dus  $B(0, 3)$ .  
 $k: y = ax + b$   
 $a = \frac{-5 - 3}{8 - 0} = -1$   
 $y = -x + b$   $\left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} k: y = -x + 3 \\ \text{door } B(0, 3) \end{array}$   
 $y = 0$  geeft  $-x + 3 = 0$   
 $x = 3$   
Dus  $x_C = 3$ .



c  $f'(x) = -1$  geeft  $(\frac{1}{4}x - 1)^3 - 1 = -1$

$$(\frac{1}{4}x - 1)^3 = 0$$

$$\frac{1}{4}x - 1 = 0$$

$$\frac{1}{4}x = 1$$

$$x = 4$$

$$\left. \begin{array}{l} l: y = -x + b \\ f(4) = -2, \text{ dus raakpunt } (4, -2) \end{array} \right\} \begin{array}{l} -4 + b = -2 \\ b = 2 \end{array}$$

Dus  $l: y = -x + 2$ .

$f'(x) = 7$  geeft  $(\frac{1}{4}x - 1)^3 - 1 = 7$

$$(\frac{1}{4}x - 1)^3 = 8$$

$$\frac{1}{4}x - 1 = 2$$

$$\frac{1}{4}x = 3$$

$$x = 12$$

$$\left. \begin{array}{l} p: y = 7x + b \\ f(12) = 6, \text{ dus raakpunt } (12, 6) \end{array} \right\} \begin{array}{l} 7 \cdot 12 + b = 6 \\ 84 + b = 6 \\ b = -78 \end{array}$$

Dus  $p: y = 7x - 78$ .

$l$  snijden met  $p$  geeft  $-x + 2 = 7x - 78$

$$-8x = -80$$

$$x = 10$$

$x = 10$  geeft  $y = -10 + 2 = -8$ , dus  $S(10, -8)$ .

46 a  $f(3) = \frac{1}{4}(2 \cdot 3 - 5)^3 + 2 = 2\frac{1}{4}$

$$g(3) = -\frac{1}{4}(3 \cdot 3 - 10)^4 + 2\frac{1}{2} = 2\frac{1}{4}$$

$f(3) = g(3) = 2\frac{1}{4}$ , dus  $A$  ligt zowel op de grafiek van  $f$  als op die van  $g$ .

b  $f(x) = \frac{1}{4}(2x - 5)^3 + 2$  geeft  $f'(x) = \frac{3}{4}(2x - 5)^2 \cdot 2 = 1\frac{1}{2}(2x - 5)^2$

$$f'(3) = 1\frac{1}{2}(2 \cdot 3 - 5)^2 = 1\frac{1}{2}$$

$$g(x) = -\frac{1}{4}(3x - 10)^4 + 2\frac{1}{2} \text{ geeft } g'(x) = -(3x - 10)^3 \cdot 3 = -3(3x - 10)^3$$

$$g'(3) = -3(3 \cdot 3 - 10)^3 = 3$$

$f'(3) \neq g'(3)$ , dus de grafieken raken elkaar niet in  $A$ .

c  $f'(x) = 13\frac{1}{2}$  geeft  $1\frac{1}{2}(2x - 5)^2 = 13\frac{1}{2}$

$$(2x - 5)^2 = 9$$

$$2x - 5 = 3 \vee 2x - 5 = -3$$

$$2x = 8 \vee 2x = 2$$

$$x = 4 \vee x = 1$$

$f(4) = 8\frac{3}{4}$  en  $f(1) = -4\frac{3}{4}$ , dus de punten zijn  $(4, 8\frac{3}{4})$  en  $(1, -4\frac{3}{4})$ .

De afstand tussen de punten is  $\sqrt{(4 - 1)^2 + (8\frac{3}{4} - (-4\frac{3}{4}))^2} = \sqrt{191\frac{1}{4}} = \sqrt{\frac{765}{4}} = \frac{1}{2}\sqrt{765} = \frac{1}{2} \cdot 3\sqrt{85} = 1\frac{1}{2}\sqrt{85}$ .

### Bladzijde 73

47 a  $f(x) = \sqrt{x^2 + 9} - x^2 + 5x$  geeft  $f'(x) = \frac{1}{2\sqrt{x^2 + 9}} \cdot 2x - 2x + 5 = \frac{x}{\sqrt{x^2 + 9}} - 2x + 5$

$$f'(2\frac{1}{2}) = \frac{2\frac{1}{2}}{\sqrt{(2\frac{1}{2})^2 + 9}} - 2 \cdot 2\frac{1}{2} + 5 = 0,64...$$

$$f'(3) = \frac{3}{\sqrt{3^2 + 9}} - 2 \cdot 3 + 5 = -0,29...$$

Voor  $x = 2\frac{1}{2}$  is de grafiek van  $f$  stijgend en voor  $x = 3$  is de grafiek van  $f$  dalend.

In de figuur in het leerboek is te zien dat de grafiek van  $f$  bij de overgang van stijgend naar dalend een extreme waarde heeft.

**b**  $y = 0$  geeft  $5x - 2 = 0$

$$5x = 2$$

$$x = \frac{2}{5}$$

Dus  $A(\frac{2}{5}, 0)$ .

$$f'(x) = 5 \text{ geeft } \frac{x}{\sqrt{x^2 + 9}} - 2x + 5 = 5$$

$$\frac{x}{\sqrt{x^2 + 9}} = 2x$$

kwadrateren geeft

$$\frac{x^2}{x^2 + 9} = 4x^2$$

$$4x^2(x^2 + 9) = x^2$$

$$4x^4 + 36x^2 = x^2$$

$$4x^4 + 35x^2 = 0$$

$$x^2(4x^2 + 35) = 0$$

$$x^2 = 0 \vee 4x^2 = -35$$

$$x = 0 \quad \text{geen opl.}$$

$$\left. \begin{array}{l} l: y = 5x + b \\ f(0) = 3, \text{ dus } (0, 3) \end{array} \right\} l: y = 5x + 3$$

$$y = 0 \text{ geeft } 5x + 3 = 0$$

$$5x = -3$$

$$x = -\frac{3}{5}, \text{ dus } B(-\frac{3}{5}, 0).$$

Dus de afstand tussen  $A$  en  $B$  is  $\frac{2}{5} - (-\frac{3}{5}) = 1$ .

**c**  $f(0) = 3$ , dus  $C(0, 3)$ .

$$m: y = ax + b$$

$$a = f'(4) = \frac{4}{\sqrt{4^2 + 9}} - 2 \cdot 4 + 5 = -2\frac{1}{5}$$

$$\left. \begin{array}{l} y = -2\frac{1}{5}x + b \\ f(4) = 9, \text{ dus } D(4, 9) \end{array} \right\} \begin{array}{l} -2\frac{1}{5} \cdot 4 + b = 9 \\ -8\frac{4}{5} + b = 9 \\ b = 17\frac{4}{5} \end{array}$$

$$m: y = -2\frac{1}{5}x + 17\frac{4}{5}, \text{ dus } E(0, 17\frac{4}{5}).$$

Dus de afstand tussen  $C$  en  $E$  is  $17\frac{4}{5} - 3 = 14\frac{4}{5}$ .

**48 a** De tijd nodig van  $A$  naar  $B$  via  $C$  is  $\frac{2}{80} + \frac{10}{100} = \frac{1}{8}$  uur, dus 450 seconden.

De afstand van  $A$  naar  $B$  is  $\sqrt{2^2 + 10^2} = \sqrt{104}$  km.

De tijd nodig van  $A$  direct naar  $B$  is  $\frac{\sqrt{104}}{80}$  uur, dus  $\approx 458,9 \dots$  seconden.

Het verschil is  $458,9 \dots - 450 \approx 9$  seconden.

**b**  $t = \frac{AD}{80} + \frac{BD}{100} = \frac{\sqrt{x^2 + 2^2}}{80} + \frac{10 - x}{100} = \frac{1}{80} \sqrt{x^2 + 4} + \frac{10}{100} - \frac{x}{100} = \frac{1}{80} \sqrt{x^2 + 4} + \frac{1}{10} - \frac{1}{100}x$

$$\text{Dus } t = \frac{1}{80} \sqrt{x^2 + 4} + \frac{1}{10} - \frac{1}{100}x.$$

$$\begin{aligned}
 \text{c } t &= \frac{1}{80} \sqrt{x^2 + 4} + \frac{1}{10} - \frac{1}{100}x \text{ geeft } t' = \frac{1}{80} \cdot \frac{1}{2\sqrt{x^2 + 4}} \cdot 2x - \frac{1}{100} = \frac{x}{80\sqrt{x^2 + 4}} - \frac{1}{100} \\
 t' &= 0 \text{ geeft } \frac{x}{80\sqrt{x^2 + 4}} - \frac{1}{100} = 0 \\
 \frac{x}{80\sqrt{x^2 + 4}} &= \frac{1}{100} \\
 100x &= 80\sqrt{x^2 + 4} \\
 x &= \frac{4}{5}\sqrt{x^2 + 4} \\
 \text{kwadrateren geeft} \\
 x^2 &= \frac{16}{25}(x^2 + 4) \\
 x^2 &= \frac{16}{25}x^2 + 2\frac{14}{25} \\
 \frac{9}{25}x^2 &= 2\frac{14}{25} \\
 x^2 &= \frac{64}{9} \\
 x &= 2\frac{2}{3}
 \end{aligned}$$

Dus  $t$  is minimaal voor  $x = 2\frac{2}{3}$ .

Dit geeft  $t_{\min} = \frac{1}{80}\sqrt{(2\frac{2}{3})^2 + 4} + \frac{1}{10} - \frac{1}{100} \cdot 2\frac{2}{3} = 0,115$ , wat overeenkomt met 414 seconden.

De snelste route is  $450 - 414 = 36$  seconden sneller dan via  $C$  naar  $B$ .

**49 a**  $f(x) = ax + b + \sqrt{ax + b}$  geeft  $f'(x) = a + \frac{1}{2\sqrt{ax + b}} \cdot a = a + \frac{a}{2\sqrt{ax + b}}$

$$f'(x) = 0 \text{ geeft } a + \frac{a}{2\sqrt{ax + b}} = 0$$

$$a\left(1 + \frac{1}{2\sqrt{ax + b}}\right) = 0$$

$$a = 0 \vee \frac{1}{2\sqrt{ax + b}} = -1$$

$$a = 0 \vee 2\sqrt{ax + b} = -1$$

$$a = 0 \vee \sqrt{ax + b} = -\frac{1}{2}$$

$a = 0$  geeft  $f(x) = b + \sqrt{b}$  en dit is voor elke  $b$  een constante functie, dus  $f$  heeft geen extreme waarde in dit geval.

$\sqrt{ax + b} = -\frac{1}{2}$  heeft geen oplossing, dus ook in dit geval heeft  $f$  geen extreme waarde.

*Alternatieve uitwerking*

Voor  $a > 0$  is zowel de grafiek van  $y = ax + b$  als de grafiek van  $y = \sqrt{ax + b}$  stijgend.

Dus de grafiek van  $f$  is stijgend en dus heeft  $f$  geen extreme waarde.

Voor  $a < 0$  is zowel de grafiek van  $y = ax + b$  als de grafiek van  $y = \sqrt{ax + b}$  dalend.

Dus de grafiek van  $f$  is dalend en dus heeft  $f$  geen extreme waarde.

**b**  $g(x) = (ax - 2)^4 + \frac{1}{2}ax$  geeft  $g'(x) = 4(ax - 2)^3 \cdot a + \frac{1}{2}a = 4a(ax - 2)^3 + \frac{1}{2}a$

$$g'(x) = 0 \text{ geeft } 4a(3a - 2)^3 + \frac{1}{2}a = 0$$

$$\frac{1}{2}a(8(3a - 2)^3 + 1) = 0$$

$$\frac{1}{2}a = 0 \vee 8(3a - 2)^3 = -1$$

$$a = 0 \vee (3a - 2)^3 = -\frac{1}{8}$$

$$a = 0 \vee 3a - 2 = -\frac{1}{2}$$

$$a = 0 \vee 3a = 1\frac{1}{2}$$

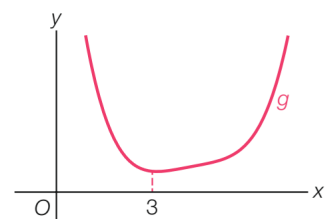
$$a = 0 \vee a = \frac{1}{2}$$

$a = 0$  geeft  $g(x) = (-2)^4 + 0 = 16$  en deze functie heeft geen extreme waarde voor  $x = 3$ .

$$a = \frac{1}{2} \text{ geeft } g(x) = (\frac{1}{2}x - 2)^4 + \frac{1}{4}x$$

In de schets hiernaast is te zien dat de grafiek een top heeft voor  $x = 3$ .

Dus voor  $a = \frac{1}{2}$  heeft  $g$  een extreme waarde voor  $x = 3$ .



### Bladzijde 74

**50**  $f(x) = x\sqrt{2x+1}$  is het product van de factoren  $x$  en  $\sqrt{2x+1}$ .

Dus volgens de productregel is  $f'(x) = [x]' \cdot \sqrt{2x+1} + x \cdot [\sqrt{2x+1}]'$ .

De afgeleide van  $\sqrt{2x+1}$  bereken je met de kettingregel.

### Bladzijde 75

**51**  $f'(x) = \sqrt{2x+1} + \frac{x}{2\sqrt{2x+1}} = \frac{2x+1}{2\sqrt{2x+1}} + \frac{x}{2\sqrt{2x+1}} = \frac{3x+1}{2\sqrt{2x+1}}$

**52 a**  $f(x) = x(3x+1)^3$  geeft  $f'(x) = 1 \cdot (3x+1)^3 + x \cdot 3(3x+1)^2 \cdot 3 = (3x+1)^3 + 9x(3x+1)^2$

**b**  $g(x) = x\sqrt{3x+4}$  geeft  $g'(x) = 1 \cdot \sqrt{3x+4} + x \cdot \frac{1}{2\sqrt{3x+4}} \cdot 3 = \sqrt{3x+4} + \frac{3x}{2\sqrt{3x+4}}$

$$= \frac{2(3x+4)}{2\sqrt{3x+4}} + \frac{3x}{2\sqrt{3x+4}} = \frac{6x+8+3x}{2\sqrt{3x+4}} = \frac{9x+8}{2\sqrt{3x+4}}$$

**c**  $h(x) = \frac{\sqrt{x^2+1}}{2x+1}$  geeft  $h'(x) = \frac{(2x+1) \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x - \sqrt{x^2+1} \cdot 2}{(2x+1)^2} = \frac{\frac{2x^2+x}{\sqrt{x^2+1}} - 2\sqrt{x^2+1}}{(2x+1)^2}$

$$= \frac{2x^2+x-2(x^2+1)}{(2x+1)^2 \cdot \sqrt{x^2+1}} = \frac{2x^2+x-2x^2-2}{(2x+1)^2 \cdot \sqrt{x^2+1}} = \frac{x-2}{(2x+1)^2 \cdot \sqrt{x^2+1}}$$

**53**  $\sqrt{3x^3+4x^2} = \sqrt{x^2(3x+4)} = \sqrt{x^2} \cdot \sqrt{3x+4} = |x| \cdot \sqrt{3x+4}$

En voor  $x < 0$  is  $|x| \cdot \sqrt{3x+4} \neq x\sqrt{3x+4}$ , dus daarom mag de functie  $g$  niet zo door Harm herschreven worden.

**54 a**  $f(x) = \frac{1}{4}x\sqrt{3x+12}$  geeft  $f'(x) = \frac{1}{4} \cdot \sqrt{3x+12} + \frac{1}{4}x \cdot \frac{1}{2\sqrt{3x+12}} \cdot 3 = \frac{1}{4}\sqrt{3x+12} + \frac{3x}{8\sqrt{3x+12}}$

$$= \frac{2(3x+12)}{8\sqrt{3x+12}} + \frac{3x}{8\sqrt{3x+12}} = \frac{6x+24+3x}{8\sqrt{3x+12}} = \frac{9x+24}{8\sqrt{3x+12}}$$

**b**  $f'(x) = 0$  geeft  $9x+24=0$   
 $9x = -24$   
 $x = -\frac{24}{9} = -\frac{8}{3}$

Je krijgt min. is  $f(-\frac{8}{3}) = -1\frac{1}{3}$ .

**c**  $l: y = ax + b$

$$a = f'(8) = \frac{9 \cdot 8 + 24}{8\sqrt{3 \cdot 8 + 12}} = 2$$

$$\left. \begin{array}{l} y = 2x + b \\ f(8) = 12, \text{ dus } A(8, 12) \end{array} \right\} \begin{array}{l} 2 \cdot 8 + b = 12 \\ 16 + b = 12 \\ b = -4 \end{array}$$

Dus  $l: y = 2x - 4$  en  $C(0, -4)$ .

$$3x + 12 = 0$$

$$3x = -12$$

$x = -4$ , dus randpunt  $(-4, 0)$ .

$k: y = ax + b$  met  $a = rc_k = rc_l = 2$ .

$$\left. \begin{array}{l} y = 2x + b \\ \text{door } (-4, 0) \end{array} \right\} \begin{array}{l} 2 \cdot -4 + b = 0 \\ -8 + b = 0 \\ b = 8 \end{array}$$

Dus  $k: y = 2x + 8$  en  $B(0, 8)$ .

Uit  $B(0, 8)$  en  $C(0, -4)$  volgt  $BC = 8 - (-4) = 12$ .

**d**  $f'(x) = \frac{5}{8}$  geeft  $\frac{9x+24}{8\sqrt{3x+12}} = \frac{5}{8}$   
 $9x+24 = 5\sqrt{3x+12}$   
kwadrateren geeft  
 $81x^2 + 432x + 576 = 25(3x+12)$   
 $81x^2 + 432x + 576 = 75x + 300$   
 $81x^2 + 357x + 276 = 0$   
 $D = 357^2 - 4 \cdot 81 \cdot 276 = 38025$ , dus  $\sqrt{D} = 195$   
 $x = \frac{-357 \pm 195}{162} = -1 \vee x = \frac{-357 - 195}{162} = -3\frac{11}{27}$   
 $x = -3\frac{11}{27}$  voldoet niet, dus  $x = -1$ .  
 $f(-1) = -\frac{3}{4}$ , dus  $D(-1, -\frac{3}{4})$ .

#### Bladzijde 76

**55 a**  $f(x) = (x+2)\sqrt{8-2x}$  geeft  $f'(x) = 1 \cdot \sqrt{8-2x} + (x+2) \cdot \frac{1}{2\sqrt{8-2x}} \cdot -2 = \sqrt{8-2x} - \frac{x+2}{\sqrt{8-2x}}$   
 $= \frac{8-2x}{\sqrt{8-2x}} - \frac{x+2}{\sqrt{8-2x}} = \frac{8-2x-x-2}{\sqrt{8-2x}} = \frac{-3x+6}{\sqrt{8-2x}}$

**b**  $8-2x=0$   
 $2x=8$

$x=4$ , dus randpunt  $(4, 0)$  en uit de figuur in het leerboek volgt dat  $D_f = \langle \leftarrow, 4 \rangle$ .

$f'(x) = 0$  geeft  $-3x+6=0$   
 $-3x=-6$   
 $x=2$

Je krijgt max. is  $f(2) = 8$  en uit de figuur in het leerboek volgt dat  $B_f = \langle \leftarrow, 8 \rangle$ .

**c**  $k: y = ax + b$

$a = f'(-4) = \frac{-3 \cdot -4 + 6}{\sqrt{8-2 \cdot -4}} = 4\frac{1}{2}$

$y = 4\frac{1}{2}x + b$   
 $f(-4) = -8$ , dus  $A(-4, -8)$   $\left. \begin{array}{l} 4\frac{1}{2} \cdot -4 + b = -8 \\ -18 + b = -8 \end{array} \right\}$   
 $b = 10$

Dus  $k: y = 4\frac{1}{2}x + 10$ .

$l: y = ax + b$

$a = f'(-\frac{1}{2}) = \frac{-3 \cdot -\frac{1}{2} + 6}{\sqrt{8-2 \cdot -\frac{1}{2}}} = 2\frac{1}{2}$

$y = 2\frac{1}{2}x + b$   
 $f(-\frac{1}{2}) = 4\frac{1}{2}$ , dus  $B(-\frac{1}{2}, 4\frac{1}{2})$   $\left. \begin{array}{l} 2\frac{1}{2} \cdot -\frac{1}{2} + b = 4\frac{1}{2} \\ -1\frac{1}{4} + b = 4\frac{1}{2} \end{array} \right\}$   
 $b = 5\frac{3}{4}$

Dus  $l: y = 2\frac{1}{2}x + 5\frac{3}{4}$ .

$k$  en  $l$  snijden geeft  $4\frac{1}{2}x + 10 = 2\frac{1}{2}x + 5\frac{3}{4}$

$2x = -4\frac{1}{4}$

$x = -2\frac{1}{8}$

$x = -2\frac{1}{8}$  geeft  $y = 4\frac{1}{2} \cdot -2\frac{1}{8} + 10 = \frac{7}{16}$

Dus  $k$  en  $l$  snijden elkaar in een punt boven de  $x$ -as.

**d**  $f'(x) = -4\frac{1}{2}$  geeft  $\frac{-3x+6}{\sqrt{8-2x}} = \frac{-9}{2}$   
 $2(-3x+6) = -9\sqrt{8-2x}$   
 $-6x+12 = -9\sqrt{8-2x}$   
kwadrateren geeft  
 $36x^2 - 144x + 144 = 81(8-2x)$   
 $36x^2 - 144x + 144 = 648 - 162x$   
 $36x^2 + 18x - 504 = 0$   
 $2x^2 + x - 28 = 0$   
 $D = 1^2 - 4 \cdot 2 \cdot -28 = 225$ , dus  $\sqrt{D} = 15$   
 $x = \frac{-1 \pm 15}{4} = 3\frac{1}{2} \vee x = \frac{-1 - 15}{4} = -4$   
 $x = -4$  voldoet niet, dus  $x = 3\frac{1}{2}$ .

$$m: y = -4\frac{1}{2}x + b \quad \left. \begin{array}{l} f(3\frac{1}{2}) = 5\frac{1}{2}, \text{ dus door } (3\frac{1}{2}, 5\frac{1}{2}) \end{array} \right\} \begin{array}{l} -4\frac{1}{2} \cdot 3\frac{1}{2} + b = 5\frac{1}{2} \\ -15\frac{3}{4} + b = 5\frac{1}{2} \\ b = 21\frac{1}{4} \end{array}$$

$m: y = -4\frac{1}{2}x + 21\frac{1}{4}$ , dus  $y_C = 21\frac{1}{4}$ .

**56 a**  $f(x) = \frac{4x+4}{\sqrt{x^2+4}}$  geeft  $f'(x) = \frac{\sqrt{x^2+4} \cdot 4 - (4x+4) \cdot \frac{1}{2\sqrt{x^2+4}} \cdot 2x}{x^2+4} = \frac{4\sqrt{x^2+4} - \frac{x(4x+4)}{\sqrt{x^2+4}}}{x^2+4}$   
 $= \frac{4(x^2+4) - x(4x+4)}{(x^2+4)\sqrt{x^2+4}} = \frac{4x^2+16-4x^2-4x}{(x^2+4)\sqrt{x^2+4}} = \frac{16-4x}{(x^2+4)\sqrt{x^2+4}}$

$f'(x) = 0$  geeft  $16 - 4x = 0$   
 $4x = 16$   
 $x = 4$

$f(4) = \frac{16+4}{\sqrt{4^2+4}} = \frac{20}{\sqrt{20}} = \sqrt{20} = 2\sqrt{5}$

De coördinaten van de top zijn  $(4, 2\sqrt{5})$ .

**b**  $f(x) = 0$  geeft  $4x+4 = 0$   
 $4x = -4$   
 $x = -1$

Dus  $A(-1, 0)$ .

$f(0) = \frac{0+4}{\sqrt{0^2+4}} = 2$ , dus  $B(0, 2)$ .

$k: y = ax + b$

$a = f'(-1) = \frac{16+4}{((-1)^2+4)\sqrt{(-1)^2+4}} = \frac{20}{5\sqrt{5}} = \frac{4}{\sqrt{5}} = \frac{4}{5}\sqrt{5}$

$y = \frac{4}{5}\sqrt{5} \cdot x + b$   
door  $A(-1, 0)$   $\left. \begin{array}{l} \frac{4}{5}\sqrt{5} \cdot (-1) + b = 0 \\ b = \frac{4}{5}\sqrt{5} \end{array} \right\}$

Dus  $k: y = \frac{4}{5}\sqrt{5} \cdot x + \frac{4}{5}\sqrt{5}$ .

Het snijpunt van  $k$  met de  $y$ -as is  $(0, \frac{4}{5}\sqrt{5})$ .

$k$  gaat niet door  $B$ , dus bewering I is niet waar.

$l: y = ax + b$

$a = f'(0) = \frac{16-0}{(0^2+4)\sqrt{0^2+4}} = 2$

$y = 2x + b$   
door  $B(0, 2)$   $\left. \begin{array}{l} b = 2 \end{array} \right\}$

Dus  $l: y = 2x + 2$ .

$$y = 0 \text{ geeft } 2x + 2 = 0$$

$$2x = -2$$

$$x = -1$$

Dus het snijpunt van  $l$  met de  $x$ -as is  $(-1, 0)$ .

$l$  gaat door  $A$ , dus bewering II is waar.

**57 a**  $9 - 2x = 0$

$$2x = 9$$

$x = 4\frac{1}{2}$ , dus randpunt  $(4\frac{1}{2}, -3)$  en uit de figuur in het leerboek volgt dat  $D_f = \langle \leftarrow, 4\frac{1}{2} \rangle$ .

$$\begin{aligned} f(x) = 2x\sqrt{9-2x} - 3 \text{ geeft } f'(x) &= 2 \cdot \sqrt{9-2x} + 2x \cdot \frac{1}{2\sqrt{9-2x}} \cdot -2 = 2\sqrt{9-2x} - \frac{2x}{\sqrt{9-2x}} \\ &= \frac{2(9-2x)}{\sqrt{9-2x}} - \frac{2x}{\sqrt{9-2x}} = \frac{18-4x-2x}{\sqrt{9-2x}} = \frac{18-6x}{\sqrt{9-2x}} \end{aligned}$$

$$f'(x) = 0 \text{ geeft } 18 - 6x = 0$$

$$6x = 18$$

$$x = 3$$

Je krijgt max. is  $f(3) = 6\sqrt{3} - 3$ , dus  $B_f = \langle \leftarrow, 6\sqrt{3} - 3 \rangle$ .

**b**  $k: y = ax + b$

$$l: y = ax + b$$

$$a = f'(0) = \frac{18 - 6 \cdot 0}{\sqrt{9 - 2 \cdot 0}} = 6$$

$$a = f'(4) = \frac{18 - 6 \cdot 4}{\sqrt{9 - 2 \cdot 4}} = -6$$

$$\left. \begin{aligned} y &= 6x + b \\ f(0) &= -3, \text{ dus } A(0, -3) \end{aligned} \right\} b = -3$$

$$\left. \begin{aligned} y &= -6x + b \\ f(4) &= 5, \text{ dus } B(4, 5) \end{aligned} \right\} \begin{aligned} -6 \cdot 4 + b &= 5 \\ -24 + b &= 5 \\ b &= 29 \end{aligned}$$

Dus  $k: y = 6x - 3$ .

Dus  $l: y = -6x + 29$ .

$k$  en  $l$  snijden geeft  $6x - 3 = -6x + 29$

$$12x = 32$$

$$x = 2\frac{2}{3}$$

$x = 2\frac{2}{3}$  geeft  $y = 6 \cdot 2\frac{2}{3} - 3 = 13$

**c**  $f'(x) = 1\frac{1}{2}$  geeft  $\frac{18-6x}{\sqrt{9-2x}} = \frac{3}{2}$

$$36 - 12x = 3\sqrt{9-2x}$$

$$12 - 4x = \sqrt{9-2x}$$

kwadrateren geeft

$$144 - 96x + 16x^2 = 9 - 2x$$

$$16x^2 - 94x + 135 = 0$$

$$D = (-94)^2 - 4 \cdot 16 \cdot 135 = 196, \text{ dus } \sqrt{D} = 14$$

$$x = \frac{94 \pm 14}{32} = 3\frac{3}{8} \vee x = \frac{94 - 14}{32} = 2\frac{1}{2}$$

$x = 3\frac{3}{8}$  voldoet niet, dus  $x = 2\frac{1}{2}$ .

$$\left. \begin{aligned} m: y &= 1\frac{1}{2}x + b \\ f(2\frac{1}{2}) &= 7, \text{ dus door } (2\frac{1}{2}, 7) \end{aligned} \right\} \begin{aligned} 1\frac{1}{2} \cdot 2\frac{1}{2} + b &= 7 \\ 3\frac{3}{4} + b &= 7 \\ b &= 3\frac{1}{4} \end{aligned}$$

Dus  $m: y = 1\frac{1}{2}x + 3\frac{1}{4}$ .

$y = 0$  geeft  $1\frac{1}{2}x + 3\frac{1}{4} = 0$

$$1\frac{1}{2}x = -3\frac{1}{4}$$

$$x = -2\frac{1}{6}$$

Dus  $x_S = -2\frac{1}{6}$ .

$$f_p(x) = \frac{px}{\sqrt{x^2+9}} \text{ geeft}$$

$$f_p'(x) = \frac{\sqrt{x^2+9} \cdot p - px \cdot \frac{1}{2\sqrt{x^2+9}} \cdot 2x}{x^2+9} = \frac{p\sqrt{x^2+9} - \frac{px^2}{\sqrt{x^2+9}}}{x^2+9} = \frac{p(x^2+9) - px^2}{(x^2+9)\sqrt{x^2+9}} = \frac{9p}{(x^2+9)\sqrt{x^2+9}}$$

$$f_p'(4) = 1 \text{ geeft } \frac{9p}{(4^2+9)\sqrt{4^2+9}} = 1$$

$$\frac{9p}{125} = 1$$

$$p = 13\frac{8}{9}$$

$$k: y = x + b$$

$$f_{13\frac{8}{9}}(4) = \frac{13\frac{8}{9} \cdot 4}{\sqrt{4^2+9}} = 11\frac{1}{9}, \text{ dus door } (4, 11\frac{1}{9}) \left. \begin{array}{l} 4 + b = 11\frac{1}{9} \\ b = 7\frac{1}{9} \end{array} \right\}$$

$$k: y = x + 7\frac{1}{9}, \text{ dus } y_B = 7\frac{1}{9}.$$

### Bladzijde 77

**C6** a  $f(x) = 5(x^2 - 4)^3$  geeft  $f'(x) = 15(x^2 - 4)^2 \cdot 2x = 30x(x^2 - 4)^2$

b  $g(x) = \sqrt{x^2 + x}$  geeft  $g'(x) = \frac{1}{2\sqrt{x^2 + x}} \cdot (2x + 1) = \frac{2x + 1}{2\sqrt{x^2 + x}}$

c  $h(x) = \frac{1}{(x^2 + 1)\sqrt{x^2 + 1}} = (x^2 + 1)^{-1\frac{1}{2}}$  geeft  $h'(x) = -1\frac{1}{2}(x^2 + 1)^{-2\frac{1}{2}} \cdot 2x = \frac{-3x}{(x^2 + 1)^2 \cdot \sqrt{x^2 + 1}}$

**C7** a  $f(x) = x\sqrt{x^2 + x}$  geeft  $f'(x) = 1 \cdot \sqrt{x^2 + x} + x \cdot \frac{1}{2\sqrt{x^2 + x}} \cdot (2x + 1) = \sqrt{x^2 + x} + \frac{2x^2 + x}{2\sqrt{x^2 + x}}$

$$= \frac{2(x^2 + x)}{2\sqrt{x^2 + x}} + \frac{2x^2 + x}{2\sqrt{x^2 + x}} = \frac{4x^2 + 3x}{2\sqrt{x^2 + x}}$$

b  $g(x) = \frac{\sqrt{x^2 + x}}{x + 1}$  geeft  $g'(x) = \frac{(x + 1) \cdot \frac{1}{2\sqrt{x^2 + x}} \cdot (2x + 1) - \sqrt{x^2 + x} \cdot 1}{(x + 1)^2} = \frac{\frac{2x^2 + x + 2x + 1}{2\sqrt{x^2 + x}} - \sqrt{x^2 + x}}{(x + 1)^2}$

$$= \frac{2x^2 + 3x + 1 - 2(x^2 + x)}{2(x + 1)^2 \cdot \sqrt{x^2 + x}} = \frac{x + 1}{2(x + 1)^2 \cdot \sqrt{x^2 + x}} = \frac{1}{2(x + 1)\sqrt{x^2 + x}}$$

## 6.4 Raaklijnen, toppen, rakende en loodrecht snijdende grafieken

### Bladzijde 78

**59** a Raaklijn  $k$  heeft richtingscoëfficiënt  $2\frac{1}{2}$ , dus  $f_p'(x) = 2\frac{1}{2}$ .

Raken in het punt  $A$  met  $x_A = 3$ , dus  $f_p'(3) = 2\frac{1}{2}$ .

$$f_p(x) = \frac{1}{4}x^2 + px + 2 \text{ geeft } f_p'(x) = \frac{1}{2}x + p$$

$$f_p'(3) = 2\frac{1}{2} \text{ geeft } \frac{1}{2} \cdot 3 + p = 2\frac{1}{2}$$

$$1\frac{1}{2} + p = 2\frac{1}{2}$$

$$p = 1$$

b  $f_p'(x) = 0$  geeft  $\frac{1}{2}x + p = 0$

$$p = -\frac{1}{2}x$$

Substitutie van  $p = -\frac{1}{2}x$  in  $y = \frac{1}{4}x^2 + px + 2$  geeft  $y = \frac{1}{4}x^2 - \frac{1}{2}x \cdot x + 2$

$$y = \frac{1}{4}x^2 - \frac{1}{2}x^2 + 2$$

$$y = -\frac{1}{4}x^2 + 2$$

Dus de gevraagde formule is  $y = -\frac{1}{4}x^2 + 2$ .



60

**a**  $f_p(x) = \frac{1}{3}x^3 + px^2 + 3x + 5$  geeft  $f_p'(x) = x^2 + 2px + 3$   
 $f_p'(x_A) = \text{rc}_k$  oftewel  $f_p'(4) = 3$  geeft  $4^2 + 2p \cdot 4 + 3 = 3$   
 $16 + 8p = 0$   
 $8p = -16$   
 $p = -2$

$$f_{-2}(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 5$$

$$\left. \begin{array}{l} y = 3x + q \\ f_{-2}(4) = 6\frac{1}{3}, \text{ dus } A(4, 6\frac{1}{3}) \end{array} \right\} 3 \cdot 4 + q = 6\frac{1}{3} \text{ geeft } q = -5\frac{2}{3}$$

Dus  $p = -2$  en  $q = -5\frac{2}{3}$ .

**b**  $f_p'(x) = 0$  geeft  $x^2 + 2px + 3 = 0$

$$2px = -x^2 - 3$$

$$\left. \begin{array}{l} p = \frac{-x^2 - 3}{2x} \text{ voor } x \neq 0 \\ y = \frac{1}{3}x^3 + px^2 + 3x + 5 \end{array} \right\} \begin{array}{l} y = \frac{1}{3}x^3 + \frac{-x^2 - 3}{2x} \cdot x^2 + 3x + 5 \\ y = \frac{1}{3}x^3 + \frac{1}{2}x(-x^2 - 3) + 3x + 5 \\ y = \frac{1}{3}x^3 - \frac{1}{2}x^3 - \frac{1}{2}x + 3x + 5 \\ y = -\frac{1}{6}x^3 + 1\frac{1}{2}x + 5 \end{array}$$

Dus de gevraagde formule is  $y = -\frac{1}{6}x^3 + 1\frac{1}{2}x + 5$ .

61

**a**  $f_p(x) = (x^2 + p) \cdot \sqrt{x} = x^{2\frac{1}{2}} + px^{\frac{1}{2}}$  geeft  $f_p'(x) = 2\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}px^{-\frac{1}{2}} = \frac{5x\sqrt{x}}{2} + \frac{p}{2\sqrt{x}} = \frac{5x^2}{2\sqrt{x}} + \frac{p}{2\sqrt{x}} = \frac{5x^2 + p}{2\sqrt{x}}$

$$f_p'(x_A) = \text{rc}_k \text{ oftewel } f_p'(4) = 18 \text{ geeft } \frac{5 \cdot 4^2 + p}{2\sqrt{4}} = 18$$

$$\frac{80 + p}{4} = 18$$

$$80 + p = 72$$

$$p = -8$$

$$f_{-8}(x) = (x^2 - 8) \cdot \sqrt{x}$$

$$\left. \begin{array}{l} y = 18x + q \\ f_{-8}(4) = 16, \text{ dus } A(4, 16) \end{array} \right\} 18 \cdot 4 + q = 16 \text{ geeft } q = -56$$

Dus  $p = -8$  en  $q = -56$ .

**b**  $f_p'(x) = 0$  geeft  $5x^2 + p = 0$

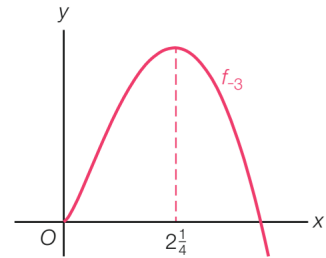
$$\left. \begin{array}{l} p = -5x^2 \\ y = (x^2 + p) \cdot \sqrt{x} \end{array} \right\} \begin{array}{l} y = (x^2 - 5x^2) \cdot \sqrt{x} \\ y = -4x^2 \cdot \sqrt{x} \end{array}$$

Dus de gevraagde formule is  $y = -4x^2 \cdot \sqrt{x}$ .

**62 a**  $f_p(x) = 6x\sqrt{x} + px^2 = 6x^{\frac{1}{2}} + px^2$  geeft  $f'_p(x) = 9x^{\frac{1}{2}} + 2px = 9\sqrt{x} + 2px$   
 $f'_p(2\frac{1}{4}) = 0$  geeft  $9 \cdot \sqrt{2\frac{1}{4}} + 2p \cdot 2\frac{1}{4} = 0$   
 $9 \cdot 1\frac{1}{2} + 4\frac{1}{2}p = 0$   
 $4\frac{1}{2}p = -13\frac{1}{2}$   
 $p = -3$

Zie de schets hiernaast.

Dus  $f_p$  heeft een maximum voor  $x = 2\frac{1}{4}$  voor  $p = -3$ .



**b**  $f'_p(x_A) = rc_k$  oftewel  $f'_p(1) = 5$  geeft  $9 \cdot \sqrt{1} + 2p \cdot 1 = 5$   
 $9 + 2p = 5$   
 $2p = -4$   
 $p = -2$

$$f_2(x) = 6x\sqrt{x} - 2x^2$$

$$\left. \begin{array}{l} y = 5x + q \\ f_2(1) = 4, \text{ dus } A(1, 4) \end{array} \right\} 5 \cdot 1 + q = 4 \text{ geeft } q = -1$$

Dus  $p = -2$  en  $q = -1$ .

**c**  $f'_p(x) = 0$  geeft  $9\sqrt{x} + 2px = 0$   
 $2px = -9\sqrt{x}$   
 $p = -\frac{9}{2\sqrt{x}}$  voor  $x \neq 0$   
 $y = 6x\sqrt{x} + px^2 \quad \left\{ \begin{array}{l} y = 6x\sqrt{x} - \frac{9}{2\sqrt{x}} \cdot x^2 \\ y = 6x\sqrt{x} - 4\frac{1}{2}x\sqrt{x} \\ y = 1\frac{1}{2}x\sqrt{x} \end{array} \right.$

Dus de gevraagde formule is  $y = 1\frac{1}{2}x\sqrt{x}$ .

**63 a**  $f_p(x) = \frac{4x+p}{x^2+1}$  geeft  $f'_p(x) = \frac{(x^2+1) \cdot 4 - (4x+p) \cdot 2x}{(x^2+1)^2} = \frac{4x^2+4-8x^2-2px}{(x^2+1)^2} = \frac{-4x^2-2px+4}{(x^2+1)^2}$

$$a = f'_p(0) = \frac{-4 \cdot 0^2 - 2p \cdot 0 + 4}{(0^2+1)^2} = 4$$

$$\left. \begin{array}{l} k: y = 4x + 4 \text{ snijdt de } y\text{-as in } (0, 4) \\ f_p(0) = p, \text{ dus } (0, p) \end{array} \right\} p = 4$$

Dus  $a = 4$  en  $p = 4$ .

**b**  $f'_p(x_A) = rc_l$  oftewel  $f'_p(-1) = -1$  geeft  $\frac{-4 \cdot (-1)^2 - 2p \cdot -1 + 4}{((-1)^2+1)^2} = -1$   
 $\frac{-4 + 2p + 4}{(1+1)^2} = -1$   
 $\frac{2p}{4} = -1$   
 $2p = -4$   
 $p = -2$

$$f_2(x) = \frac{4x-2}{x^2+1}$$

$$\left. \begin{array}{l} l: y = -x + b \\ f_2(-1) = -3, \text{ dus } A(-1, -3) \end{array} \right\} \begin{array}{l} -1 + b = -3 \\ 1 + b = -3 \\ b = -4 \end{array}$$

Dus  $l: y = -x - 4$ .

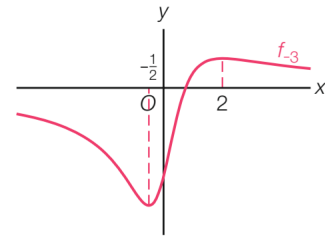
$$\begin{aligned} \text{c } f'_p(2) = 0 \text{ geeft } \frac{-4 \cdot 2^2 - 2p \cdot 2 + 4}{(2^2 + 1)^2} &= 0 \\ -12 - 4p &= 0 \\ -4p &= 12 \\ p &= -3 \end{aligned}$$

$$f_{-3}(x) = \frac{4x-3}{x^2+1} \text{ en } f'_{-3}(x) = \frac{-4x^2+6x+4}{(x^2+1)^2}$$

$$\begin{aligned} f'_{-3}(x) = 0 \text{ geeft } -4x^2 + 6x + 4 &= 0 \\ D = 6^2 - 4 \cdot (-4) \cdot 4 &= 100, \text{ dus } \sqrt{D} = 10 \\ x = \frac{-6 \pm 10}{-8} = -\frac{1}{2} \vee x = \frac{-6 \pm 10}{-8} &= 2 \end{aligned}$$

Zie de schets hiernaast.

Je krijgt min. is  $f_{-3}(-\frac{1}{2}) = -4$  en max. is  $f_{-3}(2) = 1$ .



- 64 a**  $f_p(x) = \frac{1}{3}x^3 + px^2 - 3x - p$  geeft  $f'_p(x) = x^2 + 2px - 3$   
 $f_p$  heeft twee extremen als  $f'_p(x) = 0$  twee oplossingen heeft.  
 $D = (2p)^2 - 4 \cdot 1 \cdot (-3) = 4p^2 + 12$   
Omdat  $p^2 > 0$  geldt dat  $D > 0$  voor elke  $p$ .  
Dus  $f_p$  heeft voor elke  $p$  twee extreme waarden.

$$\begin{aligned} \text{b } f'_p(3) = 0 \text{ geeft } 3^2 + 2p \cdot 3 - 3 &= 0 \\ 9 + 6p - 3 &= 0 \\ 6p &= -6 \\ p &= -1 \end{aligned}$$

$$\begin{aligned} f_{-1}(x) &= \frac{1}{3}x^3 - x^2 - 3x + 1 \text{ en } f'_{-1}(x) = x^2 - 2x - 3 \\ f'_{-1}(x) = 0 \text{ geeft } x^2 - 2x - 3 &= 0 \\ (x+1)(x-3) &= 0 \\ x &= -1 \vee x = 3 \end{aligned}$$

Zie de schets hiernaast.

Je krijgt max. is  $f_{-1}(-1) = 2\frac{2}{3}$  en min. is  $f_{-1}(3) = -8$ .

$$\begin{aligned} \text{c } f'_p(x_B) = r_{cl} \text{ oftewel } f'_p(-2) = -1 \text{ geeft } (-2)^2 + 2p \cdot (-2) - 3 &= -1 \\ 4 - 4p - 3 &= -1 \\ -4p &= -2 \\ p &= \frac{1}{2} \end{aligned}$$

$$f_{\frac{1}{2}}(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 3x - \frac{1}{2}$$

$$\left. \begin{aligned} l: y &= -x + q \\ f'_{\frac{1}{2}}(-2) &= 4\frac{5}{6}, \text{ dus } A(-2, 4\frac{5}{6}) \end{aligned} \right\} -2 + q = 4\frac{5}{6} \text{ geeft } q = 2\frac{5}{6}$$

Dus  $p = \frac{1}{2}$  en  $q = 2\frac{5}{6}$ .

$$\text{d } f'_p(x) = 0 \text{ geeft } x^2 + 2px - 3 = 0$$

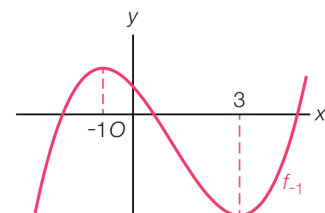
$$2px = 3 - x^2$$

$$\left. \begin{aligned} p &= \frac{3-x^2}{2x} \text{ voor } x \neq 0 \\ y &= \frac{1}{3}x^3 + px^2 - 3x - p \end{aligned} \right\} \begin{aligned} y &= \frac{1}{3}x^3 + \frac{3-x^2}{2x} \cdot x^2 - 3x - \frac{3-x^2}{2x} \\ y &= \frac{1}{3}x^3 + \frac{1}{2}x(3-x^2) - 3x - \frac{3}{2x} + \frac{1}{2}x \end{aligned}$$

$$y = \frac{1}{3}x^3 + 1\frac{1}{2}x - \frac{1}{2}x^3 - 3x - \frac{3}{2x} + \frac{1}{2}x$$

$$y = -\frac{1}{6}x^3 - x - \frac{3}{2x}$$

Dus de gevraagde formule is  $y = -\frac{1}{6}x^3 - x - \frac{3}{2x}$ .



$$\begin{aligned}
 65 \quad f_{p,q}(x) &= px\sqrt{x^2+9} + qx^2 \text{ geeft } f_{p,q}'(x) = p \cdot \sqrt{x^2+9} + px \cdot \frac{1}{2\sqrt{x^2+9}} \cdot 2x + 2qx \\
 &= p\sqrt{x^2+9} + \frac{px^2}{\sqrt{x^2+9}} + 2qx \\
 f_{p,q}'(x_A) &= r_{c_k} \text{ oftewel } f_{p,q}'(4) = 1 \text{ geeft } p\sqrt{4^2+9} + \frac{p \cdot 4^2}{\sqrt{4^2+9}} + 2q \cdot 4 = 1 \\
 &5p + 3\frac{1}{5}p + 8q = 1 \\
 &8\frac{1}{5}p + 8q = 1
 \end{aligned}$$

$$\begin{aligned}
 f_{p,q}(4) &= 20 \text{ geeft } p \cdot 4\sqrt{4^2+9} + q \cdot 4^2 = 20 \\
 20p + 16q &= 20 \\
 10p + 8q &= 10
 \end{aligned}$$

$$\begin{cases} 10p + 8q = 10 \\ 8\frac{1}{5}p + 8q = 1 \end{cases} \quad -$$

$$\frac{1\frac{4}{5}p}{p} = 9$$

$$\begin{aligned}
 p &= 5 \\
 p = 5 \text{ geeft } 10 \cdot 5 + 8q &= 10 \\
 50 + 8q &= 10 \\
 8q &= -40 \\
 q &= -5
 \end{aligned}$$

Dus  $p = 5$  en  $q = -5$ .

$$66 \quad \mathbf{a} \quad f(1) = \frac{1}{2} \cdot 1^2 + 1 + 1\frac{1}{2} = 3 \text{ en } g(1) = -1^2 + 4 \cdot 1 = 3$$

Dus  $A$  ligt zowel op de grafiek van  $f$  als op de grafiek van  $g$ .

$$\mathbf{b} \quad f(x) = \frac{1}{2}x^2 + x + 1\frac{1}{2} \text{ geeft } f'(x) = x + 1$$

$$k: y = ax + b$$

$$a = f'(1) = 1 + 1 = 2$$

$$\left. \begin{aligned} y &= 2x + b \\ \text{door } A(1, 3) \end{aligned} \right\} \begin{aligned} 2 + b &= 3 \\ b &= 1 \end{aligned}$$

$$\text{Dus } k: y = 2x + 1.$$

$$g(x) = -x^2 + 4x \text{ geeft } g'(x) = -2x + 4$$

$$l: y = ax + b$$

$$a = g'(1) = -2 + 4 = 2$$

$$\left. \begin{aligned} y &= 2x + b \\ \text{door } A(1, 3) \end{aligned} \right\} \begin{aligned} 2 + b &= 3 \\ b &= 1 \end{aligned}$$

$$\text{Dus } l: y = 2x + 1.$$

$\mathbf{c}$   $A$  is het punt waarin de grafieken van  $f$  en  $g$  elkaar raken.

$$67 \quad k: y = ax + b$$

Bij  $p = -9$  hoort  $x = 3$ .

$$a = f'(3) = 3^2 - 4 \cdot 3 + 1 = -2$$

$$\left. \begin{aligned} k: y &= -2x + b \\ f(3) &= -6, \text{ dus raakpunt } (3, -6) \end{aligned} \right\} \begin{aligned} -2 \cdot 3 + b &= -6 \\ -6 + b &= -6 \\ b &= 0 \end{aligned}$$

$$\text{Dus } k: y = -2x.$$

**68 a**  $f(x) = x^3 + 4x^2 + 2x + 1$  geeft  $f'(x) = 3x^2 + 8x + 2$

$g(x) = x^2 + 11x + 28$  geeft  $g'(x) = 2x + 11$

$f'(x) = g'(x)$  geeft  $3x^2 + 8x + 2 = 2x + 11$

$$3x^2 + 6x - 9 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1 \vee x = -3$$

$f(1) = 8$  en  $g(1) = 40$ , dus voor  $x = 1$  raken de grafieken van  $f$  en  $g$  elkaar niet.

$f(-3) = 4$  en  $g(-3) = 4$ , dus voor  $x = -3$  raken de grafieken elkaar.

**b**  $k: y = ax + b$

Uit vraag a volgt dat het raakpunt  $(-3, 4)$  is.

$$g'(-3) = 2 \cdot -3 + 11 = 5$$

$$\left. \begin{array}{l} y = 5x + b \\ \text{door } (-3, 4) \end{array} \right\} \begin{array}{l} 5 \cdot -3 + b = 4 \\ b = 19 \end{array}$$

Dus  $k: y = 5x + 19$ .

**69**  $f(x) = \sqrt{2x}$  geeft  $f'(x) = \frac{1}{2\sqrt{2x}} \cdot 2 = \frac{1}{\sqrt{2x}}$

$g_p(x) = x^2 + p$  geeft  $g_p'(x) = 2x$

$f'(x) = g_p'(x)$  geeft  $\frac{1}{\sqrt{2x}} = 2x$

$$2x\sqrt{2x} = 1$$

kwadrateren geeft

$$4x^2 \cdot 2x = 1$$

$$8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$f(\frac{1}{2}) = g_p(\frac{1}{2})$  geeft  $1 = \frac{1}{4} + p$ , dus  $p = \frac{3}{4}$ .

**70**  $f(x) = \frac{6x}{\sqrt{x^2+5}}$  geeft  $f'(x) = \frac{\sqrt{x^2+5} \cdot 6 - 6x \cdot \frac{1}{2\sqrt{x^2+5}} \cdot 2x}{x^2+5} = \frac{6\sqrt{x^2+5} - \frac{6x^2}{\sqrt{x^2+5}}}{x^2+5}$

$g(x) = \frac{1}{18}x^2 + \frac{8}{9}x + 2$  geeft  $g'(x) = \frac{1}{9}x + \frac{8}{9}$

$$6\sqrt{2^2+5} - \frac{6 \cdot 2^2}{\sqrt{2^2+5}}$$

$f'(2) = \frac{6\sqrt{2^2+5} - \frac{6 \cdot 2^2}{\sqrt{2^2+5}}}{2^2+5} = 1\frac{1}{9}$  en  $g'(2) = \frac{1}{9} \cdot 2 + \frac{8}{9} = 1\frac{1}{9}$

$f(2) = 4$  en  $g(2) = 4$

Er geldt dat  $f(2) = g(2)$  en  $f'(2) = g'(2)$ , dus voor  $x = 2$  raken de grafieken van  $f$  en  $g$  elkaar.

$k: y = 1\frac{1}{9}x + b$   $\left\{ \begin{array}{l} 1\frac{1}{9} \cdot 2 + b = 4 \\ 2\frac{2}{9} + b = 4 \\ b = 1\frac{7}{9} \end{array} \right.$

$k: y = 1\frac{1}{9}x + 1\frac{7}{9}$

$$9y = 10x + 16$$

$$10x - 9y = -16$$

71

a  $f_p(x) = -\frac{2}{3}x^3 - x^2 + px$  geeft  $f'_p(x) = -2x^2 - 2x + p$   
 $g(x) = x^2 + 2x$  geeft  $g'(x) = 2x + 2$   
 $f'_p(x) = g'(x)$  geeft  $-2x^2 - 2x + p = 2x + 2$   
 $p = 2x^2 + 4x + 2$

b  $f_p(x) = g(x)$  met  $p = 2x^2 + 4x + 2$  geeft  $-\frac{2}{3}x^3 - x^2 + (2x^2 + 4x + 2)x = x^2 + 2x$   
 $-\frac{2}{3}x^3 - x^2 + 2x^3 + 4x^2 + 2x = x^2 + 2x$   
 $1\frac{1}{3}x^3 + 3x^2 + 2x = x^2 + 2x$   
 $1\frac{1}{3}x^3 + 2x^2 = 0$

c  $1\frac{1}{3}x^3 + 2x^2 = 0$

$$x^2(1\frac{1}{3}x + 2) = 0$$

$$x^2 = 0 \vee 1\frac{1}{3}x = -2$$

$$x = 0 \vee x = -1\frac{1}{2}$$

$$x = 0 \text{ geeft } p = 2 \text{ en } x = -1\frac{1}{2} \text{ geeft } p = \frac{1}{2}.$$

Dus de grafieken raken elkaar voor  $p = 2$  en voor  $p = \frac{1}{2}$ .

72

a  $f(x) = x^3 - 2x^2$  geeft  $f'(x) = 3x^2 - 4x$   
 $g(x) = \frac{1}{3}x^3 - 2\frac{2}{3}$  geeft  $g'(x) = x^2$   
 $f'(x) = g'(x)$  geeft  $3x^2 - 4x = x^2$   
 $2x^2 - 4x = 0$   
 $2x(x - 2) = 0$   
 $x = 0 \vee x = 2$

$$f(0) = 0 \text{ en } g(0) = -2\frac{2}{3}, \text{ dus voor } x = 0 \text{ raken de grafieken van } f \text{ en } g \text{ elkaar niet.}$$

$$f(2) = 0 \text{ en } g(2) = 0$$

$$f(2) = g(2) \text{ en } f'(2) = g'(2), \text{ dus voor } x = 2 \text{ raken de grafieken van } f \text{ en } g \text{ elkaar.}$$

Het raakpunt is  $(2, 0)$ .

b  $h(x) = -x^2 + 8x - 12$  geeft  $h'(x) = -2x + 8$   
 $k_p(x) = x^2 + px$  geeft  $k'_p(x) = 2x + p$   
 $h'(x) = k'_p(x)$  geeft  $-2x + 8 = 2x + p$   
 $p = -4x + 8$

$$h(x) = k_p(x) \text{ met } p = -4x + 8 \text{ geeft } -x^2 + 8x - 12 = x^2 + (-4x + 8)x$$

$$-x^2 + 8x - 12 = x^2 - 4x^2 + 8x$$

$$2x^2 = 12$$

$$x^2 = 6$$

$$x = \sqrt{6} \vee x = -\sqrt{6}$$

$$x = \sqrt{6} \text{ geeft } p = -4\sqrt{6} + 8 \text{ en } x = -\sqrt{6} \text{ geeft } p = 4\sqrt{6} + 8.$$

$$\text{Dus } p = -4\sqrt{6} + 8 \vee p = 4\sqrt{6} + 8.$$

73

$$f(x) = \frac{10x}{\sqrt{x^2 + 16}} \text{ geeft } f'(x) = \frac{\sqrt{x^2 + 16} \cdot 10 - 10x \cdot \frac{1}{2\sqrt{x^2 + 16}} \cdot 2x}{x^2 + 16} = \frac{10\sqrt{x^2 + 16} - \frac{10x^2}{\sqrt{x^2 + 16}}}{x^2 + 16}$$

$$g(x) = ax^2 + bx + 3 \text{ geeft } g'(x) = 2ax + b$$

$$f(3) = g(3) \text{ geeft } \frac{10 \cdot 3}{\sqrt{3^2 + 16}} = a \cdot 3^2 + b \cdot 3 + 3$$

$$6 = 9a + 3b + 3$$

$$3 = 9a + 3b$$

$$1 = 3a + b$$

$$f'(3) = g'(3) \text{ geeft } \frac{10\sqrt{3^2 + 16} - \frac{10 \cdot 3^2}{\sqrt{3^2 + 16}}}{3^2 + 16} = 2a \cdot 3 + b$$

$$1\frac{7}{25} = 6a + b$$

$$\begin{cases} 6a + b = 1\frac{7}{25} \\ 3a + b = 1 \end{cases}$$

$$\frac{3a}{3a} = \frac{\frac{7}{25}}{\frac{7}{25}} -$$

$$a = \frac{7}{75}$$

$$a = \frac{7}{75} \text{ geeft } b = 1 - 3 \cdot \frac{7}{75} = \frac{18}{25}$$

74

a lijn  $k$ 

|     |    |   |
|-----|----|---|
| $x$ | 0  | 3 |
| $y$ | -2 | 4 |

lijn  $l$ 

|     |   |   |
|-----|---|---|
| $x$ | 0 | 6 |
| $y$ | 3 | 0 |

Zie de figuur hiernaast.

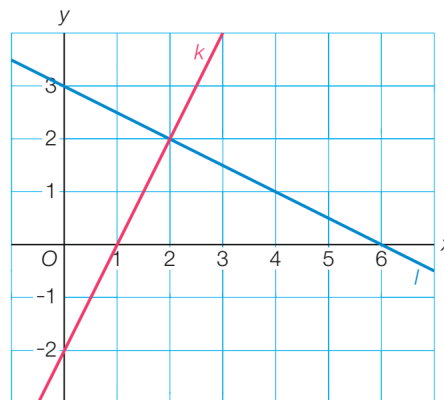
b ja

c  $m: y = ax + b$  $m \perp k$  oftewel  $m \parallel l$ , dus  $a = rc_m = rc_l = -\frac{1}{2}$ .

$$\left. \begin{array}{l} y = -\frac{1}{2}x + b \\ \text{door } A(4, 0) \end{array} \right\} \begin{array}{l} -\frac{1}{2} \cdot 4 + b = 0 \\ -2 + b = 0 \\ b = 2 \end{array}$$

Dus  $m: y = -\frac{1}{2}x + 2$ .d  $n: y = ax + b$  $rc_p = \frac{1}{4}$ , dus bij  $p$  ga je 4 naar rechts en 1 omhoog.Dus bij  $n$  ga je 4 omlaag en 1 naar rechts, dus  $a = rc_n = -4$ .

$$\left. \begin{array}{l} y = -4x + b \\ \text{door } B(1, 3) \end{array} \right\} \begin{array}{l} -4 \cdot 1 + b = 3 \\ -4 + b = 3 \\ b = 7 \end{array}$$

Dus  $n: y = -4x + 7$ .

## Bladzijde 85

75

$$2\sqrt{x} = \frac{p}{x} \text{ geeft } p = 2x\sqrt{x}$$

$$\left. \begin{array}{l} \frac{1}{\sqrt{x}} \cdot \frac{-p}{x^2} = -1 \text{ geeft } \frac{p}{x^2 \cdot \sqrt{x}} = 1 \text{ oftewel } p = x^2 \cdot \sqrt{x} \\ 2x\sqrt{x} = x^2 \cdot \sqrt{x} \\ 2x\sqrt{x} - x^2 \cdot \sqrt{x} = 0 \\ x\sqrt{x}(2-x) = 0 \\ x = 0 \vee x = 2 \end{array} \right\}$$

 $x = 0$  voldoet niet, dus  $x = 2$ .

**a**  $f(x) = \sqrt{x}$  geeft  $f'(x) = \frac{1}{2\sqrt{x}}$

$g(x) = -\frac{1}{2}x^2 + 10$  geeft  $g'(x) = -x$

$f(x) = g(x) \wedge f'(x) \cdot g'(x) = -1$

$\sqrt{x} = -\frac{1}{2}x^2 + 10 \wedge \frac{1}{2\sqrt{x}} \cdot -x = -1$

$$\frac{x}{2\sqrt{x}} = 1$$

$$\frac{1}{2}\sqrt{x} = 1$$

$$\sqrt{x} = 2$$

$$x = 4$$

$x = 4$  substitueren in  $\sqrt{x} = -\frac{1}{2}x^2 + 10$  geeft  $\sqrt{4} = -\frac{1}{2} \cdot 4^2 + 10$

$$2 = 2$$

Dus de grafieken van  $f$  en  $g$  snijden elkaar loodrecht.

**b**  $h_p(x) = p\sqrt{x}$  geeft  $h_p'(x) = p \cdot \frac{1}{2\sqrt{x}} = \frac{p}{2\sqrt{x}}$

$k(x) = -\frac{1}{6}x^2 + 8\frac{2}{3}$  geeft  $k'(x) = -\frac{1}{3}x$

$h_p(x) = k(x) \wedge h_p'(x) \cdot k'(x) = -1$

$p\sqrt{x} = -\frac{1}{6}x^2 + 8\frac{2}{3} \wedge \frac{p}{2\sqrt{x}} \cdot -\frac{1}{3}x = -1$

$$-\frac{1}{6}p\sqrt{x} = -1$$

$$p\sqrt{x} = 6$$

$$\left. \begin{array}{l} p\sqrt{x} = -\frac{1}{6}x^2 + 8\frac{2}{3} \\ p\sqrt{x} = 6 \end{array} \right\} \begin{array}{l} 6 = -\frac{1}{6}x^2 + 8\frac{2}{3} \\ \frac{1}{6}x^2 = 2\frac{2}{3} \end{array}$$

$$x^2 = 16$$

$$x = 4 \vee x = -4$$

$x = -4$  voldoet niet en  $x = 4$  geeft  $p\sqrt{4} = 6$

$$2p = 6$$

$$p = 3$$

$h_3(4) = 3\sqrt{4} = 6$ , dus de grafieken snijden elkaar loodrecht in het punt  $(4, 6)$ .

### Bladzijde 86

**a**  $f(x) = x^2 - 4x$  geeft  $f'(x) = 2x - 4$

$f'(5) = 2 \cdot 5 - 4 = 6$

$rc_k \cdot 6 = -1$  geeft  $rc_k = -\frac{1}{6}$

$k: y = -\frac{1}{6}x + b \left\{ \begin{array}{l} -\frac{1}{6} \cdot 5 + b = 5 \\ f(5) = 5, \text{ dus } A(5, 5) \end{array} \right.$

$$-\frac{5}{6} + b = 5$$

$$b = 5\frac{5}{6}$$

Dus  $k: y = -\frac{1}{6}x + 5\frac{5}{6}$ .

$y = 0$  geeft  $-\frac{1}{6}x + 5\frac{5}{6} = 0$

$$-\frac{1}{6}x = -5\frac{5}{6}$$

$$x = 35$$

Dus  $x_B = 35$ .



$$\text{b } g(x) = \frac{2x-1}{x+2} \text{ geeft } g'(x) = \frac{(x+2) \cdot 2 - (2x-1) \cdot 1}{(x+2)^2} = \frac{2x+4-2x+1}{(x+2)^2} = \frac{5}{(x+2)^2}$$

$$g(x) = -5x + p \wedge g'(x) \cdot \text{rc}_l = -1$$

$$\frac{2x-1}{x+2} = -5x + p \wedge \frac{5}{(x+2)^2} \cdot -5 = -1$$

$$p = 5x + \frac{2x-1}{x+2} \wedge \frac{25}{(x+2)^2} = 1$$

$$(x+2)^2 = 25$$

$$x+2 = 5 \vee x+2 = -5$$

$$x = 3 \vee x = -7$$

$$\left. \begin{array}{l} x = 3 \\ p = 5x + \frac{2x-1}{x+2} \end{array} \right\} p = 5 \cdot 3 + \frac{2 \cdot 3 - 1}{3 + 2} = 16$$

$$\left. \begin{array}{l} x = -7 \\ p = 5x + \frac{2x-1}{x+2} \end{array} \right\} p = 5 \cdot -7 + \frac{2 \cdot -7 - 1}{-7 + 2} = -32$$

$$\text{Dus } p = 16 \vee p = -32.$$

$$\text{78 a } f(x) = x^2 - x \text{ geeft } f'(x) = 2x - 1$$

$$g_p(x) = \frac{p}{x} = px^{-1} \text{ geeft } g_p'(x) = -px^{-2} = \frac{-p}{x^2}$$

$$f'(x) = g_p'(x) \text{ geeft } 2x - 1 = \frac{-p}{x^2}$$

$$x^2(2x - 1) = -p$$

$$p = -2x^3 + x^2$$

$$f(x) = g_p(x) \text{ met } p = -2x^3 + x^2 \text{ geeft } x^2 - x = \frac{-2x^3 + x^2}{x}$$

$$x^2 - x = -2x^2 + x$$

$$3x^2 - 2x = 0$$

$$x(3x - 2) = 0$$

$$x = 0 \vee 3x = 2$$

$$x = 0 \vee x = \frac{2}{3}$$

$$x = 0 \text{ voldoet niet, dus } x = \frac{2}{3}.$$

$$x = \frac{2}{3} \text{ geeft } p = -2 \cdot \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 = -\frac{4}{27}$$

$$f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 - \frac{2}{3} = -\frac{2}{9}, \text{ dus } A\left(\frac{2}{3}, -\frac{2}{9}\right).$$

$$\text{b } f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$$

$$x^2 - x = \frac{p}{x} \wedge (2x - 1) \cdot \frac{-p}{x^2} = -1$$

$$p = x^3 - x^2 \wedge p(2x - 1) = x^2$$

$$\left. \begin{array}{l} p = x^3 - x^2 \\ p(2x - 1) = x^2 \end{array} \right\} (x^3 - x^2)(2x - 1) = x^2$$

$$x^2(x - 1)(2x - 1) = x^2$$

$$x^2 = 0 \vee (x - 1)(2x - 1) = 1$$

$$2x^2 - 3x + 1 = 1$$

$$2x^2 - 3x = 0$$

$$x(2x - 3) = 0$$

$$x = 0 \vee x = 1\frac{1}{2}$$

$$x = 0 \text{ voldoet niet, dus } x = 1\frac{1}{2}.$$

$$x = 1\frac{1}{2} \text{ geeft } p = \left(1\frac{1}{2}\right)^3 - \left(1\frac{1}{2}\right)^2 = 1\frac{1}{8}$$

$$f\left(1\frac{1}{2}\right) = \frac{3}{4}, \text{ dus } B\left(1\frac{1}{2}, \frac{3}{4}\right).$$

79

$$f_{p,q}(x) = px\sqrt{x^2+3} + qx \text{ geeft } f_{p,q}'(x) = p \cdot \sqrt{x^2+3} + px \cdot \frac{1}{2\sqrt{x^2+3}} \cdot 2x + q = p\sqrt{x^2+3} + \frac{px^2}{\sqrt{x^2+3}} + q$$

$$g(x) = -x^2 + 1\frac{4}{5}x + 2\frac{1}{5} \text{ geeft } g'(x) = -2x + 1\frac{4}{5}$$

$$f_{p,q}(1) = g(1) \quad \wedge \quad f_{p,q}'(1) \cdot g'(1) = -1$$

$$p \cdot 1\sqrt{1^2+3} + q \cdot 1 = -1^2 + 1\frac{4}{5} \cdot 1 + 2\frac{1}{5} \wedge \left( p\sqrt{1^2+3} + \frac{p \cdot 1^2}{\sqrt{1^2+3}} + q \right) \cdot (-2 \cdot 1 + 1\frac{4}{5}) = -1$$

$$2p + q = 3 \quad \wedge \quad (2p + \frac{1}{2}p + q) \cdot -\frac{1}{5} = -1$$

$$(2\frac{1}{2}p + q) \cdot -\frac{1}{5} = -1$$

$$2\frac{1}{2}p + q = 5$$

$$\begin{cases} 2\frac{1}{2}p + q = 5 \\ 2p + q = 3 \end{cases}$$

$$\frac{\frac{1}{2}p}{\frac{1}{2}p} = \frac{2}{2}$$

$$p = 4$$

$$p = 4 \text{ geeft } 2 \cdot 4 + q = 3$$

$$8 + q = 3$$

$$q = -5$$

Dus  $p = 4$  en  $q = -5$ .

### Bladzijde 87

**C8**  $f_p(x) = \sqrt{2x^2+p}$  geeft  $f_p'(x) = \frac{1}{2\sqrt{2x^2+p}} \cdot 4x = \frac{2x}{\sqrt{2x^2+p}}$

$$f_p'(x_A) = \text{rc}_k \text{ oftewel } f_p'(1) = \frac{2}{3} \text{ geeft } \frac{2 \cdot 1}{\sqrt{2 \cdot 1^2 + p}} = \frac{2}{3}$$

$$\sqrt{2+p} = 3$$

$$2+p = 9$$

$$p = 7$$

$$f_7(x) = \sqrt{2x^2+7}$$

$$y = \frac{2}{3}x + q \quad \left. \vphantom{y = \frac{2}{3}x + q} \right\} \frac{2}{3} \cdot 1 + q = 3 \text{ geeft } q = 2\frac{1}{3}$$

Dus  $p = 7$  en  $q = 2\frac{1}{3}$ .

**C9**  $f_p(x) = -\frac{1}{3}x^3 + 1\frac{1}{2}x^2 + px - 5$  geeft  $f_p'(x) = -x^2 + 3x + p$

$$f_p'(x) = 0 \text{ geeft } -x^2 + 3x + p = 0$$

$$\left. \begin{aligned} p &= x^2 - 3x \\ y &= -\frac{1}{3}x^3 + 1\frac{1}{2}x^2 + px - 5 \end{aligned} \right\} \begin{aligned} y &= -\frac{1}{3}x^3 + 1\frac{1}{2}x^2 + (x^2 - 3x)x - 5 \\ y &= -\frac{1}{3}x^3 + 1\frac{1}{2}x^2 + x^3 - 3x^2 - 5 \\ y &= \frac{2}{3}x^3 - 1\frac{1}{2}x^2 - 5 \end{aligned}$$

Dus de gevraagde formule is  $y = \frac{2}{3}x^3 - 1\frac{1}{2}x^2 - 5$ .

**C10** a  $f(x) = \frac{1}{3}x^3 + 2x$  geeft  $f'(x) = x^2 + 2$

$$g_{-1}(x) = -2x^2 - x \text{ geeft } g_{-1}'(x) = -4x - 1$$

$$f'(x) = g_{-1}'(x) \text{ geeft } x^2 + 2 = -4x - 1$$

$$x^2 + 4x + 3 = 0$$

$$(x+1)(x+3) = 0$$

$$x = -1 \vee x = -3$$

$f(-1) = -2\frac{1}{3}$  en  $g_{-1}(-1) = -1$ , dus voor  $x = -1$  raken de grafieken elkaar niet.

$f(-3) = -15$  en  $g_{-1}(-3) = -15$ , dus voor  $x = -3$  raken de grafieken elkaar.

Het raakpunt is  $(-3, -15)$ .

$$\begin{aligned}
\text{b } g_p(x) &= -2x^2 + p \text{ geeft } g_p'(x) = -4x + p \\
f'(x) &= g_p'(x) \text{ geeft } x^2 + 2 = -4x + p \\
p &= x^2 + 4x + 2 \\
f(x) &= g_p(x) \text{ met } p = x^2 + 4x + 2 \text{ geeft } \frac{1}{3}x^3 + 2x = -2x^2 + (x^2 + 4x + 2)x \\
\frac{1}{3}x^3 + 2x &= -2x^2 + x^3 + 4x^2 + 2x \\
-\frac{2}{3}x^3 - 2x^2 &= 0 \\
-x^2(\frac{2}{3}x + 2) &= 0 \\
-x^2 = 0 \vee \frac{2}{3}x &= -2 \\
x = 0 \vee x &= -3 \\
x = 0 \text{ geeft } p &= 0^2 + 4 \cdot 0 + 2 = 2
\end{aligned}$$

**C11**  $f(x) = -x^2 + 5$  geeft  $f'(x) = -2x$

$$\begin{aligned}
g_p(x) &= p\sqrt{x} \text{ geeft } g_p'(x) = p \cdot \frac{1}{2\sqrt{x}} = \frac{p}{2\sqrt{x}} \\
f(x) &= g_p(x) \wedge f'(x) \cdot g_p'(x) = -1 \\
-x^2 + 5 &= p\sqrt{x} \wedge -2x \cdot \frac{p}{2\sqrt{x}} = -1 \\
p\sqrt{x} &= 1 \\
\left. \begin{aligned} -x^2 + 5 &= p\sqrt{x} \\ p\sqrt{x} &= 1 \end{aligned} \right\} \begin{aligned} -x^2 + 5 &= 1 \\ -x^2 &= -4 \\ x^2 &= 4 \\ x &= 2 \vee x = -2 \end{aligned} \\
x = -2 &\text{ voldoet niet, dus } x = 2. \\
x = 2 \text{ geeft } p\sqrt{2} &= 1 \\
p &= \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{2}\sqrt{2} \\
f(2) &= 1, \text{ dus de grafieken snijden elkaar loodrecht in het punt } (2, 1).
\end{aligned}$$

## Kennis- en vaardighedentest

### Bladzijde 88

**1**  $f(x) = \frac{6x}{x^2 + 9}$  geeft  $f'(x) = \frac{(x^2 + 9) \cdot 6 - 6x \cdot 2x}{(x^2 + 9)^2} = \frac{6x^2 + 54 - 12x^2}{(x^2 + 9)^2} = \frac{-6x^2 + 54}{(x^2 + 9)^2}$

$$\begin{aligned}
f'(x) = 0 \text{ geeft } -6x^2 + 54 &= 0 \\
-6x^2 &= -54 \\
x^2 &= 9 \\
x &= 3 \vee x = -3 \\
\text{Je krijgt min. is } f(-3) &= -1 \text{ en max. is } f(3) = 1. \\
f(x) = 0 \text{ geeft } 6x &= 0 \\
x &= 0 \\
\text{Dus de grafiek snijdt de } x\text{-as alleen in } (0, 0), \text{ dus } B_f &= [-1, 1].
\end{aligned}$$

**2**  $f(x) = 9x^4 + 4x^3 - 45x^2 - 30x + 5$  geeft  $f'(x) = 36x^3 + 12x^2 - 90x - 30$

$$\begin{aligned}
f'(\sqrt{2\frac{1}{2}}) &= 36 \cdot 2\frac{1}{2}\sqrt{2\frac{1}{2}} + 12 \cdot 2\frac{1}{2} - 90\sqrt{2\frac{1}{2}} - 30 = 0 \\
f'(\sqrt{2\frac{1}{2}}) &= 0 \text{ en in de figuur in het leerboek is te zien dat de grafiek een top heeft voor } x = \sqrt{2\frac{1}{2}}. \\
\text{Dus } f \text{ heeft een extreme waarde voor } x &= \sqrt{2\frac{1}{2}}.
\end{aligned}$$

**3**  $f(x) = x^5 - 10x^3$   
 $f'(x) = 5x^4 - 30x^2$   
 $f''(x) = 20x^3 - 60x$   
 $f''(x) = 0$  geeft  $20x^3 - 60x = 0$   
 $20x(x^2 - 3) = 0$   
 $20x = 0 \vee x^2 = 3$   
 $x = 0 \vee x = \sqrt{3} \vee x = -\sqrt{3}$

Dus  $x_B = \sqrt{3}$ .

$k: y = ax + b$

$a = f'(\sqrt{3}) = 5 \cdot (\sqrt{3})^4 - 30 \cdot (\sqrt{3})^2 = -45$

$y = -45x + b$   
 $f(\sqrt{3}) = -21\sqrt{3}$ , dus  $B(\sqrt{3}, -21\sqrt{3})$   $\left. \begin{array}{l} -45 \cdot \sqrt{3} + b = -21\sqrt{3} \\ b = 24\sqrt{3} \end{array} \right\}$

Dus  $k: y = -45x + 24\sqrt{3}$ .

**4 a**  $f(x) = \frac{5}{x^6} = 5x^{-6}$  geeft  $f'(x) = -30x^{-7} = -\frac{30}{x^7}$

**b**  $g(x) = \frac{x^6 - 2x^4}{x^5} = \frac{x^6}{x^5} - \frac{2x^4}{x^5} = x - 2x^{-1}$  geeft  $g'(x) = 1 + 2x^{-2} = 1 + \frac{2}{x^2} = \frac{x^2 + 2}{x^2}$

**c**  $h(x) = 4 - \frac{3x - 2}{x^3} = 4 - \frac{3x}{x^3} + \frac{2}{x^3} = 4 - 3x^{-2} + 2x^{-3}$  geeft  $h'(x) = 6x^{-3} - 6x^{-4} = \frac{6}{x^3} - \frac{6}{x^4} = \frac{6x - 6}{x^4}$

**5 a**  $f(x) = 2x + \frac{8}{x} + 1 = 2x + 8x^{-1} + 1$  geeft  $f'(x) = 2 - 8x^{-2} = 2 - \frac{8}{x^2}$

$f'(x) = 0$  geeft  $2 - \frac{8}{x^2} = 0$

$\frac{8}{x^2} = 2$

$x^2 = 4$

$x = 2 \vee x = -2$

Je krijgt max. is  $f(-2) = -7$  en min. is  $f(2) = 9$ .

**b**  $f'(x) = -6$  geeft  $2 - \frac{8}{x^2} = -6$

$\frac{8}{x^2} = 8$

$x^2 = 1$

$x = 1 \vee x = -1$

$f(1) = 11$  en  $f(-1) = -9$ , dus de raakpunten zijn  $(1, 11)$  en  $(-1, -9)$ .

**c**  $f'(x) = 3$  geeft  $2 - \frac{8}{x^2} = 3$

$\frac{8}{x^2} = -1$

$x^2 = -8$

geen oplossing

De grafiek van  $f$  heeft dus geen raaklijn met richtingscoëfficiënt 3.

**6 a**  $f(x) = \frac{8x^6 - x^4}{x\sqrt{x}} = \frac{8x^6}{x^{1\frac{1}{2}}} - \frac{x^4}{x^{1\frac{1}{2}}} = 8x^{4\frac{1}{2}} - x^{2\frac{1}{2}}$  geeft  $f'(x) = 36x^{3\frac{1}{2}} - 2\frac{1}{2}x^{1\frac{1}{2}} = 36x^3 \cdot \sqrt{x} - 2\frac{1}{2}x\sqrt{x}$

**b**  $g(x) = \frac{3}{x^2 \cdot \sqrt[3]{x}} = \frac{3}{x^{2\frac{1}{3}}} = 3x^{-2\frac{1}{3}}$  geeft  $g'(x) = -7x^{-3\frac{1}{3}} = -7 \cdot \frac{1}{x^{3\frac{1}{3}}} = -\frac{7}{x^3 \cdot \sqrt[3]{x}}$

**c**  $h(x) = (x^2 + 2\sqrt{x})^2 = x^4 + 4x^2 \cdot \sqrt{x} + 4x = x^4 + 4x^{2\frac{1}{2}} + 4x$  geeft

$h'(x) = 4x^3 + 10x^{1\frac{1}{2}} + 4 = 4x^3 + 10x\sqrt{x} + 4$

**7**  $f(x) = \frac{1}{4}x^2 \cdot \sqrt{x} - 5x = \frac{1}{4}x^{2\frac{1}{2}} - 5x$  geeft  $f'(x) = \frac{5}{8}x^{\frac{1}{2}} - 5 = \frac{5}{8}x\sqrt{x} - 5$   
 $f'(x) = 0$  geeft  $\frac{5}{8}x\sqrt{x} - 5 = 0$   
 $\frac{5}{8}x\sqrt{x} = 5$   
 $x\sqrt{x} = 8$   
kwadrateren geeft  
 $x^3 = 64$   
 $x = 4$

Je krijgt min. is  $f(4) = -12$ , dus  $B_f = [-12, \rightarrow)$ .

### Bladzijde 89

**8 a**  $f(x) = \frac{5}{(2x^2 - x)^4} = 5(2x^2 - x)^{-4}$  geeft  $f'(x) = -20(2x^2 - x)^{-5} \cdot (4x - 1) = \frac{-80x + 20}{(2x^2 - x)^5}$

**b**  $g(x) = (x^2 + 3x) \cdot \sqrt[3]{x^2 + 3x} = (x^2 + 3x)^{\frac{4}{3}}$  geeft  $g'(x) = \frac{4}{3}(x^2 + 3x)^{\frac{1}{3}} \cdot (2x + 3) = (2\frac{2}{3}x + 4) \cdot \sqrt[3]{x^2 + 3x}$

**9 a**  $f(x) = \frac{1}{3}(x^2 - 2)^3 - x^2$  geeft  $f'(x) = (x^2 - 2)^2 \cdot 2x - 2x = 2x(x^2 - 2)^2 - 2x$   
 $f'(x) = 0$  geeft  $2x(x^2 - 2)^2 - 2x = 0$

$$2x((x^2 - 2)^2 - 1) = 0$$

$$2x = 0 \vee (x^2 - 2)^2 = 1$$

$$x = 0 \vee x^2 - 2 = 1 \vee x^2 - 2 = -1$$

$$x = 0 \vee x^2 = 3 \vee x^2 = 1$$

$$x = 0 \vee x = \sqrt{3} \vee x = -\sqrt{3} \vee x = 1 \vee x = -1$$

**b**  $k: y = ax + b$

$$a = f'(2) = 2 \cdot 2 \cdot (2^2 - 2)^2 - 2 \cdot 2 = 12$$

$$y = 12x + b$$

$$f(2) = -1\frac{1}{3}, \text{ dus } A(2, -1\frac{1}{3}) \left\{ \begin{array}{l} 12 \cdot 2 + b = -1\frac{1}{3} \\ 24 + b = -1\frac{1}{3} \\ b = -25\frac{1}{3} \end{array} \right.$$

Dus  $k: y = 12x - 25\frac{1}{3}$ .

$$y = 0 \text{ geeft } 12x - 25\frac{1}{3} = 0$$

$$12x = 25\frac{1}{3}$$

$$x = 2\frac{1}{9}$$

Dus  $B(2\frac{1}{9}, 0)$ .

**10 a**  $f(x) = x^2 \cdot \sqrt{2x - 3}$  geeft  $f'(x) = 2x \cdot \sqrt{2x - 3} + x^2 \cdot \frac{1}{2\sqrt{2x - 3}} \cdot 2 = \frac{2x(2x - 3)}{\sqrt{2x - 3}} + \frac{x^2}{\sqrt{2x - 3}} = \frac{5x^2 - 6x}{\sqrt{2x - 3}}$

**b**  $g(x) = \frac{2x}{\sqrt{x^3 - 3}}$  geeft  $g'(x) = \frac{\sqrt{x^3 - 3} \cdot 2 - 2x \cdot \frac{1}{2\sqrt{x^3 - 3}} \cdot 3x^2}{x^3 - 3} = \frac{2\sqrt{x^3 - 3} - \frac{3x^3}{\sqrt{x^3 - 3}}}{x^3 - 3}$   

$$= \frac{2(x^3 - 3) - 3x^3}{(x^3 - 3)\sqrt{x^3 - 3}} = \frac{-x^3 - 6}{(x^3 - 3)\sqrt{x^3 - 3}}$$

**11 a**  $50 - x^2 = 0$

$$x^2 = 50$$

$$x = 5\sqrt{2} \vee x = -5\sqrt{2}$$

Je krijgt  $D_f = [-5\sqrt{2}, 5\sqrt{2}]$ .

$$f(x) = x\sqrt{50 - x^2} \text{ geeft}$$

$$f'(x) = 1 \cdot \sqrt{50 - x^2} + x \cdot \frac{1}{2\sqrt{50 - x^2}} \cdot -2x = \sqrt{50 - x^2} - \frac{x^2}{\sqrt{50 - x^2}} = \frac{50 - x^2}{\sqrt{50 - x^2}} - \frac{x^2}{\sqrt{50 - x^2}} = \frac{50 - 2x^2}{\sqrt{50 - x^2}}$$

$$f'(x) = 0 \text{ geeft } 50 - 2x^2 = 0$$

$$2x^2 = 50$$

$$x^2 = 25$$

$$x = 5 \vee x = -5$$

Je krijgt min. is  $f(-5) = -25$  en max. is  $f(5) = 25$ .

Dus  $B_f = [-25, 25]$ .

**b**  $k: y = ax + b$

$$a = f'(1) = \frac{50 - 2 \cdot 1^2}{\sqrt{50 - 1^2}} = 6\frac{6}{7}$$

$$\left. \begin{array}{l} y = 6\frac{6}{7}x + b \\ f(1) = 7, \text{ dus } A(1, 7) \end{array} \right\} \begin{array}{l} 6\frac{6}{7} \cdot 1 + b = 7 \\ b = \frac{1}{7} \end{array}$$

$$\text{Dus } k: y = 6\frac{6}{7}x + \frac{1}{7}.$$

**12 a**  $f_p(x) = \frac{2}{3}x^3 + px^2 - 4x + 2$  geeft  $f'_p(x) = 2x^2 + 2px - 4$

$$f'_p(1) = 0 \text{ geeft } 2 \cdot 1^2 + 2p \cdot 1 - 4 = 0$$

$$2 + 2p - 4 = 0$$

$$2p = 2$$

$$p = 1$$

$$f_1(x) = \frac{2}{3}x^3 + x^2 - 4x + 2 \text{ en } f'_1(x) = 2x^2 + 2x - 4$$

$$f'_1(x) = 0 \text{ geeft } 2x^2 + 2x - 4 = 0$$

$$x^2 + x - 2 = 0$$

$$(x - 1)(x + 2) = 0$$

$$x = 1 \vee x = -2$$

Zie de schets hiernaast.

Je krijgt max. is  $f_1(-2) = 8\frac{2}{3}$  en min. is  $f_1(1) = -\frac{1}{3}$ .

**b**  $f'_p(x_A) = r_{cl}$  oftewel  $f'_p(-2) = 8$  geeft  $2 \cdot (-2)^2 + 2p \cdot -2 - 4 = 8$

$$8 - 4p = 12$$

$$-4p = 4$$

$$p = -1$$

$$f_{-1}(x) = \frac{2}{3}x^3 - x^2 - 4x + 2$$

$$k: y = 8x + q$$

$$\left. \begin{array}{l} f_{-1}(-2) = \frac{2}{3}, \text{ dus } A(-2, \frac{2}{3}) \\ y = 8x + q \end{array} \right\} \begin{array}{l} 8 \cdot -2 + q = \frac{2}{3} \\ q = 16\frac{2}{3} \end{array}$$

Dus  $p = -1$  en  $q = 16\frac{2}{3}$ .

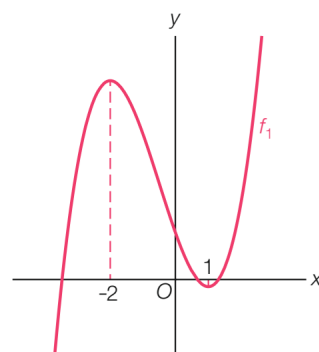
**c**  $f'_p(x) = 0$  geeft  $2x^2 + 2px - 4 = 0$

$$2px = 4 - 2x^2$$

$$\left. \begin{array}{l} p = \frac{2 - x^2}{x} \text{ voor } x \neq 0 \\ y = \frac{2}{3}x^3 + px^2 - 4x + 2 \end{array} \right\} \begin{array}{l} y = \frac{2}{3}x^3 + \frac{2 - x^2}{x} \cdot x^2 - 4x + 2 \\ y = \frac{2}{3}x^3 + 2x - x^3 - 4x + 2 \end{array}$$

$$y = -\frac{1}{3}x^3 - 2x + 2$$

Dus de gevraagde formule is  $y = -\frac{1}{3}x^3 - 2x + 2$ .



**13**  $f_p(x) = \frac{1}{3}x^3 + p$  geeft  $f'_p(x) = x^2$   
 $g(x) = 2x^2 - 3x$  geeft  $g'(x) = 4x - 3$   
 $f'_p(x) = g'(x)$  geeft  $x^2 = 4x - 3$   
 $x^2 - 4x + 3 = 0$   
 $(x - 1)(x - 3) = 0$   
 $x = 1 \vee x = 3$

$f_p(1) = g(1)$  geeft  $\frac{1}{3} + p = -1$  oftewel  $p = -1\frac{1}{3}$ .

$f_p(3) = g(3)$  geeft  $9 + p = 9$  oftewel  $p = 0$ .

Dus de grafieken van  $f_p$  en  $g$  raken elkaar voor  $p = 0$  en voor  $p = -1\frac{1}{3}$ .

**14**  $f(x) = \frac{1}{4}x^2 + 3$  geeft  $f'(x) = \frac{1}{2}x$

$g_p(x) = \frac{p}{x^2} + 4 = px^{-2} + 4$  geeft  $g'_p(x) = -2px^{-3} = -\frac{2p}{x^3}$

$f(x) = g_p(x) \quad \wedge \quad f'(x) \cdot g'_p(x) = -1$

$\frac{1}{4}x^2 + 3 = \frac{p}{x^2} + 4 \quad \wedge \quad \frac{1}{2}x \cdot -\frac{2p}{x^3} = -1$

$\frac{p}{x^2} = \frac{1}{4}x^2 - 1 \quad \wedge \quad \frac{p}{x^2} = 1$

$p = \frac{1}{4}x^4 - x^2 \quad \wedge \quad p = x^2$

$$\left. \begin{array}{l} p = \frac{1}{4}x^4 - x^2 \\ p = x^2 \end{array} \right\} \begin{array}{l} \frac{1}{4}x^4 - x^2 = x^2 \\ \frac{1}{4}x^4 - 2x^2 = 0 \end{array}$$

$\frac{1}{4}x^4(x^2 - 8) = 0$

$x^4 = 0 \vee x^2 = 8$

$x = 0 \vee x = 2\sqrt{2} \vee x = -2\sqrt{2}$

$x = 0$  voldoet niet,  $x^2 = 8$  geeft  $p = 8$ .

$f(2\sqrt{2}) = 5$  en  $f(-2\sqrt{2}) = 5$

Dus voor  $p = 8$  snijden de grafieken van  $f$  en  $g_p$  elkaar loodrecht in de punten  $A(-2\sqrt{2}, 5)$  en  $B(2\sqrt{2}, 5)$ .

## Gemengde opgaven

### Bladzijde 90

**1 a**  $f(x) = \frac{x^2 - 5\sqrt{x}}{2x^3 \cdot \sqrt{x}} = \frac{x^2}{2x^{3\frac{1}{2}}} - \frac{5x^{\frac{1}{2}}}{2x^{3\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{2}}} - \frac{5}{2x^3} = \frac{1}{2}x^{-\frac{1}{2}} - 2\frac{1}{2}x^{-3}$  geeft

$f'(x) = -\frac{3}{4}x^{-\frac{3}{2}} + 7\frac{1}{2}x^{-4} = -\frac{3}{4x^{\frac{3}{2}}} + \frac{15}{2x^4} = -\frac{3}{4x^2 \cdot \sqrt{x}} + \frac{15}{2x^4} = -\frac{3x\sqrt{x}}{4x^4} + \frac{30}{4x^4} = \frac{30 - 3x\sqrt{x}}{4x^4}$

**b**  $g(x) = \frac{4}{3(x^2 - 3x)^6} = \frac{4}{3}(x^2 - 3x)^{-6}$  geeft  $g'(x) = -8(x^2 - 3x)^{-7} \cdot (2x - 3) = \frac{24 - 16x}{(x^2 - 3x)^7}$

**c**  $h(x) = \frac{3x}{2(4 - x^2)^4}$  geeft  $h'(x) = \frac{2(4 - x^2)^4 \cdot 3 - 3x \cdot 8(4 - x^2)^3 \cdot -2x}{4(4 - x^2)^8} = \frac{6(4 - x^2)^4 + 48x^2(4 - x^2)^3}{4(4 - x^2)^8}$   
 $= \frac{6(4 - x^2) + 48x^2}{4(4 - x^2)^5} = \frac{24 + 42x^2}{4(4 - x^2)^5} = \frac{12 + 21x^2}{2(4 - x^2)^5}$

**d**  $k(x) = (x - \frac{1}{2}x^3) \cdot \sqrt[4]{x^3 - 2x} = -\frac{1}{2}(x^3 - 2x) \cdot (x^3 - 2x)^{\frac{1}{4}} = -\frac{1}{2}(x^3 - 2x)^{\frac{5}{4}}$  geeft  
 $k'(x) = -\frac{5}{8}(x^3 - 2x)^{\frac{1}{4}} \cdot (3x^2 - 2) = -\frac{5}{8}(3x^2 - 2) \cdot \sqrt[4]{x^3 - 2x}$

2

a  $f(x) = \frac{3x^2 - 9}{x^3} = 3x^{-1} - 9x^{-3}$  geeft  $f'(x) = -3x^{-2} + 27x^{-4} = -\frac{3}{x^2} + \frac{27}{x^4} = \frac{-3x^2 + 27}{x^4}$

$$f'(x) = 0 \text{ geeft } -3x^2 + 27 = 0$$

$$-3x^2 = -27$$

$$x^2 = 9$$

$$x = 3 \vee x = -3$$

Uit de figuur in het leerboek volgt min. is  $f(-3) = -\frac{2}{3}$  en max. is  $f(3) = \frac{2}{3}$ .

b  $f(x) = 24$  geeft  $\frac{-3x^2 + 27}{x^4} = 24$

$$-3x^2 + 27 = 24x^4$$

$$24x^4 + 3x^2 - 27 = 0$$

$$8x^4 + x^2 - 9 = 0$$

$$\text{Stel } x^2 = u.$$

$$8u^2 + u - 9 = 0$$

$$D = 1^2 - 4 \cdot 8 \cdot -9 = 289, \text{ dus } \sqrt{D} = 17$$

$$u = \frac{-1 + 17}{16} = 1 \vee u = \frac{-1 - 17}{16} = -1\frac{1}{8}$$

$$x^2 = 1 \quad \vee \quad x^2 = -1\frac{1}{8}$$

$$x = 1 \vee x = -1 \quad \text{geen opl.}$$

$f(1) = -6$  en  $f(-1) = 6$ , dus de raakpunten zijn  $(1, -6)$  en  $(-1, 6)$ .

c  $g_p(x) = px^2$  geeft  $g_p'(x) = 2px$

$$f(x) = g_p(x) \wedge f'(x) = g_p'(x)$$

$$\frac{3x^2 - 9}{x^3} = px^2 \wedge \frac{-3x^2 + 27}{x^4} = 2px$$

$$p = \frac{3x^2 - 9}{x^5} \wedge p = \frac{-3x^2 + 27}{2x^5}$$

$$\left. \begin{aligned} p &= \frac{3x^2 - 9}{x^5} \\ p &= \frac{-3x^2 + 27}{2x^5} \end{aligned} \right\} \begin{aligned} \frac{3x^2 - 9}{x^5} &= \frac{-3x^2 + 27}{2x^5} \\ \frac{2(3x^2 - 9)}{2x^5} &= \frac{-3x^2 + 27}{2x^5} \end{aligned}$$

$$6x^2 - 18 = -3x^2 + 27$$

$$9x^2 = 45$$

$$x^2 = 5$$

$$x = \sqrt{5} \vee x = -\sqrt{5}$$

$$x = \sqrt{5} \text{ geeft } p = \frac{3 \cdot 5 - 9}{(\sqrt{5})^5} = \frac{6}{25\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6}{125}\sqrt{5}$$

$$x = -\sqrt{5} \text{ geeft } p = \frac{3 \cdot 5 - 9}{(-\sqrt{5})^5} = -\frac{6}{125}\sqrt{5}$$

Dus voor  $p = \frac{6}{125}\sqrt{5}$  en voor  $p = -\frac{6}{125}\sqrt{5}$ .

3

a  $f_p(x) = x^4 + px^3 + 3x^2 + 4x + 1$

$$f_p'(x) = 4x^3 + 3px^2 + 6x + 4$$

$$f_p''(x) = 12x^2 + 6px + 6$$

$$f_p''(x) = 0 \text{ geeft } 12x^2 + 6px + 6 = 0$$

$$2x^2 + px + 1 = 0$$

$$D = p^2 - 4 \cdot 2 \cdot 1 = p^2 - 8$$

De grafiek van  $f_p$  heeft twee buigpunten als  $D > 0$ .

$$\text{Dus } p^2 - 8 > 0$$

$$p^2 > 8$$

$$p < -2\sqrt{2} \vee p > 2\sqrt{2}$$



$$\text{b } f_p''(-\frac{1}{2}) = 0 \text{ geeft } 12 \cdot (-\frac{1}{2})^2 + 6p \cdot -\frac{1}{2} + 6 = 0$$

$$3 - 3p + 6 = 0$$

$$-3p = -9$$

$$p = 3$$

$$f_3''(x) = 0 \text{ geeft } 12x^2 + 18x + 6 = 0$$

$$2x^2 + 3x + 1 = 0$$

$$x^2 + 1\frac{1}{2}x + \frac{1}{2} = 0$$

$$(x+1)(x+\frac{1}{2}) = 0$$

$$x = -1 \vee x = -\frac{1}{2}$$

$$f_3(x) = x^4 + 3x^3 + 3x^2 + 4x + 1 \text{ en } f_3'(x) = 4x^3 + 9x^2 + 6x + 4$$

$$k: y = ax + b$$

$$a = f_3'(-1) = 4 \cdot (-1)^3 + 9 \cdot (-1)^2 + 6 \cdot -1 + 4 = 3$$

$$y = 3x + b$$

$$f_3(-1) = -2, \text{ dus } A(-1, -2) \left\{ \begin{array}{l} 3 \cdot -1 + b = -2 \\ -3 + b = -2 \end{array} \right.$$

$$b = 1$$

$$\text{Dus } k: y = 3x + 1.$$

$$\text{c } f_p'(x) = rc_k \text{ geeft } 4x^3 + 3px^2 + 6x + 4 = 4$$

$$4x^3 + 3px^2 + 6x = 0$$

$$x(4x^2 + 3px + 6) = 0$$

$$x = 0 \vee 4x^2 + 3px + 6 = 0$$

$$x(4x^2 + 3px + 6) = 0 \text{ heeft drie oplossingen als } 4x^2 + 3px + 6 = 0 \text{ twee oplossingen heeft, dus } D > 0.$$

$$D = (3p)^2 - 4 \cdot 4 \cdot 6 = 9p^2 - 96 \left\{ \begin{array}{l} 9p^2 - 96 > 0 \\ 9p^2 > 96 \end{array} \right.$$

$$D > 0$$

$$p^2 > 10\frac{2}{3}$$

$$p < -\sqrt{10\frac{2}{3}} \vee p > \sqrt{10\frac{2}{3}}$$

$$\text{4 a } f(x) = 15 \text{ geeft } \frac{3x^2}{\sqrt{x^2-4}} = 15$$

$$3x^2 = 15\sqrt{x^2-4}$$

$$\text{kwadrateren geeft}$$

$$9x^4 = 225(x^2-4)$$

$$9x^4 = 225x^2 - 900$$

$$9x^4 - 225x^2 + 900 = 0$$

$$\text{Stel } x^2 = u.$$

$$9u^2 - 225u + 900 = 0$$

$$u^2 - 25u + 100 = 0$$

$$(u-5)(u-20) = 0$$

$$u = 5 \vee u = 20$$

$$x^2 = 5 \vee x^2 = 20$$

$$x = \sqrt{5} \vee x = -\sqrt{5} \vee x = 2\sqrt{5} \vee x = -2\sqrt{5}$$

$$\text{Uit de figuur in het leerboek volgt dat } f(x) \leq 15 \text{ geeft } -2\sqrt{5} \leq x \leq -\sqrt{5} \vee \sqrt{5} \leq x \leq 2\sqrt{5}.$$

$$\text{b } f(x) = \frac{3x^2}{\sqrt{x^2-4}} \text{ geeft } f'(x) = \frac{\sqrt{x^2-4} \cdot 6x - 3x^2 \cdot \frac{1}{2\sqrt{x^2-4}} \cdot 2x}{x^2-4} = \frac{6x\sqrt{x^2-4} - \frac{3x^3}{\sqrt{x^2-4}}}{x^2-4}$$

$$= \frac{6x(x^2-4) - 3x^3}{(x^2-4)\sqrt{x^2-4}} = \frac{3x^3 - 24x}{(x^2-4)\sqrt{x^2-4}}$$

$$\text{c } f'(x) = 0 \text{ geeft } 3x^3 - 24x = 0$$

$$3x(x^2-8) = 0$$

$$3x = 0 \vee x^2 = 8$$

$$x = 0 \vee x = \sqrt{8} \vee x = -\sqrt{8}$$

$$x = 0 \text{ voldoet niet, dus } x = \sqrt{8} \vee x = -\sqrt{8}$$

$$f(\sqrt{8}) = 12 \text{ en } f(-\sqrt{8}) = 12$$

$$\text{Uit de figuur in het leerboek volgt dat voor } p < 12 \text{ de vergelijking } f(x) = p \text{ geen oplossing heeft.}$$

**5**  $f(x) = \sqrt{9-x^2} + \frac{1}{2}x^2 - 2$  geeft  $f'(x) = \frac{1}{2\sqrt{9-x^2}} \cdot -2x + x = \frac{-x}{\sqrt{9-x^2}} + x$

$$f'(x) = 0 \text{ geeft } \frac{-x}{\sqrt{9-x^2}} + x = 0$$

$$\frac{x}{\sqrt{9-x^2}} = x$$

$$x = 0 \vee \sqrt{9-x^2} = 1$$

$$x = 0 \vee 9 - x^2 = 1$$

$$x = 0 \vee x^2 = 8$$

$$x = 0 \vee x = 2\sqrt{2} \vee x = -2\sqrt{2}$$

Je krijgt max. is  $f(-2\sqrt{2}) = f(2\sqrt{2}) = 3$  en min. is  $f(0) = 1$ .

Uit de figuur in het leerboek volgt dat  $B_f = [1, 3]$ .

**6 a**  $f(x) = x^2\sqrt{6x-x^2}$  geeft  $f'(x) = 2x \cdot \sqrt{6x-x^2} + x^2 \cdot \frac{1}{2\sqrt{6x-x^2}} \cdot (6-2x)$

$$= 2x\sqrt{6x-x^2} + \frac{3x^2-x^3}{\sqrt{6x-x^2}} = \frac{2x(6x-x^2)}{\sqrt{6x-x^2}} + \frac{3x^2-x^3}{\sqrt{6x-x^2}}$$

$$= \frac{15x^2-3x^3}{\sqrt{6x-x^2}}$$

**b** Er moet gelden dat  $6x - x^2 \geq 0$ .

$$6x - x^2 = 0$$

$$x(6-x) = 0$$

$$x = 0 \vee x = 6$$

Uit de figuur in het leerboek volgt dat  $0 \leq x \leq 6$ , dus  $D_f = [0, 6]$ .

$$f'(x) = 0 \text{ geeft } 15x^2 - 3x^3 = 0$$

$$3x^2(5-x) = 0$$

$$x = 0 \vee x = 5$$

$$f(0) = 0, f(6) = 0 \text{ en } f(5) = 25\sqrt{5}, \text{ dus } B_f = [0, 25\sqrt{5}].$$

**c**  $k: y = ax + b$

$$a = f'(3) = \frac{15 \cdot 3^2 - 3 \cdot 3^3}{\sqrt{6 \cdot 3 - 3^2}} = 18$$

$$\left. \begin{array}{l} y = 18x + b \\ f(3) = 27, \text{ dus } A(3, 27) \end{array} \right\} \begin{array}{l} 18 \cdot 3 + b = 27 \\ 54 + b = 27 \\ b = -27 \end{array}$$

Dus  $k: y = 18x - 27$  en  $B(0, -27)$ .

$l: y = ax + b$  met

$$rc_l \cdot rc_k = -1 \text{ geeft } a \cdot 18 = -1 \text{ oftewel } a = -\frac{1}{18}.$$

$$\left. \begin{array}{l} y = -\frac{1}{18}x + b \\ \text{door } A(3, 27) \end{array} \right\} \begin{array}{l} -\frac{1}{18} \cdot 3 + b = 27 \\ -\frac{1}{6} + b = 27 \end{array}$$

$$b = 27\frac{1}{6}$$

Dus  $l: y = -\frac{1}{18}x + 27\frac{1}{6}$  en  $C(0, 27\frac{1}{6})$ .

$$\text{Uit } B(0, -27) \text{ en } C(0, 27\frac{1}{6}) \text{ volgt } BC = 27\frac{1}{6} - (-27) = 54\frac{1}{6}.$$

**7 a**  $f_{p,0}(x) = \frac{px}{\sqrt{x^2+1}}$  geeft

$$f'_{p,0}(x) = \frac{\sqrt{x^2+1} \cdot p - px \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1} = \frac{p\sqrt{x^2+1} - \frac{px^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{p(x^2+1) - px^2}{(x^2+1)\sqrt{x^2+1}} = \frac{p}{(x^2+1)\sqrt{x^2+1}}$$

$$f'_{p,0}(x) = 0 \text{ geeft } p = 0$$

Voor  $p = 0$  is de grafiek van  $f_{p,0}$  de lijn  $y = 0$ , dus dan heeft  $f_{p,0}$  geen extreme waarde.

$$\begin{aligned} \text{b } f_{p,q}(x) &= \frac{px+q}{\sqrt{x^2+1}} \text{ geeft } f_{p,q}'(x) = \frac{\sqrt{x^2+1} \cdot p - (px+q) \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1} = \frac{p\sqrt{x^2+1} - \frac{px^2+qx}{\sqrt{x^2+1}}}{x^2+1} \\ &= \frac{p(x^2+1) - px^2 - qx}{(x^2+1)\sqrt{x^2+1}} = \frac{p - qx}{(x^2+1)\sqrt{x^2+1}} \end{aligned}$$

$$f_{p,q}'(\sqrt{3}) = 0 \text{ geeft } p - q\sqrt{3} = 0$$

$$p = q\sqrt{3}$$

$$y_{\text{top}} = f_{q\sqrt{3},q}(\sqrt{3}) = \frac{q\sqrt{3} \cdot \sqrt{3} + q}{\sqrt{3+1}} = \frac{4q}{2} = 2q, \text{ dus de top is } (\sqrt{3}, 2q).$$

$$\left. \begin{aligned} y &= x^2 - q \\ \text{door } (\sqrt{3}, 2q) \end{aligned} \right\} \begin{aligned} 3 - q &= 2q \\ -3q &= -3 \\ q &= 1 \end{aligned}$$

$$q = 1 \text{ geeft } p = \sqrt{3}$$

$$\text{8 a } f_p(x) = x\sqrt{x} + p\sqrt{x} = x^{\frac{1}{2}} + p\sqrt{x} \text{ geeft } f_p'(x) = \frac{1}{2}x^{-\frac{1}{2}} + p \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2}\sqrt{x} + \frac{p}{2\sqrt{x}}$$

$$f_p'(4) = 5 \text{ geeft } \frac{1}{2} \cdot 2 + \frac{p}{2 \cdot 2} = 5$$

$$3 + \frac{p}{4} = 5$$

$$\frac{1}{4}p = 2$$

$$p = 8$$

$$f_8(x) = x\sqrt{x} + 8\sqrt{x}$$

$$\left. \begin{aligned} k: y &= 5x + q \\ f_8(4) &= 24, \text{ dus } A(4, 24) \end{aligned} \right\} 5 \cdot 4 + q = 24 \text{ geeft } q = 4$$

Dus  $q = 4$ .

$$\text{b } f_p'(x) = 0 \text{ geeft } \frac{1}{2}\sqrt{x} + \frac{p}{2\sqrt{x}} = 0$$

$$\frac{p}{2\sqrt{x}} = -\frac{1}{2}\sqrt{x}$$

$$\left. \begin{aligned} p &= -3x \\ y &= x\sqrt{x} + p\sqrt{x} \end{aligned} \right\} y = x\sqrt{x} - 3x\sqrt{x} = -2x\sqrt{x}$$

Dus de gevraagde kromme is  $y = -2x\sqrt{x}$ .

$$\text{c } f_{-14}(x) = x\sqrt{x} - 14\sqrt{x} \text{ en } f_{-14}'(x) = \frac{1}{2}\sqrt{x} - \frac{14}{2\sqrt{x}} = \frac{1}{2}\sqrt{x} - \frac{7}{\sqrt{x}}$$

$$g_r(x) = \frac{2}{3}x\sqrt{x} + r = \frac{2}{3}x^{\frac{3}{2}} + r \text{ geeft } g_r'(x) = x^{\frac{1}{2}} = \sqrt{x}$$

$$f_{-14}(x) = g_r(x) \wedge f_{-14}'(x) \cdot g_r'(x) = -1$$

$$f_{-14}'(x) \cdot g_r'(x) = -1 \text{ geeft } \left( \frac{1}{2}\sqrt{x} - \frac{7}{\sqrt{x}} \right) \cdot \sqrt{x} = -1$$

$$\frac{1}{2}x - 7 = -1$$

$$\frac{1}{2}x = 6$$

$$x = 4$$

$$f_{-14}(4) = g_r(4) \text{ geeft } 4\sqrt{4} - 14\sqrt{4} = \frac{2}{3} \cdot 4\sqrt{4} + r$$

$$-20 = 5\frac{1}{3} + r$$

$$r = -25\frac{1}{3}$$

$$f_{-14}(4) = -20, \text{ dus } B(4, -20).$$

# 7 Meetkunde met coördinaten

## 7.1 Lijnen en hoeken

### Bladzijde 94

- 1 a**  $k: 2x + 3y = 12$        $l: 4x + 6y = 15$   
 $k: 3y = -2x + 12$        $l: 6y = -4x + 15$   
 $k: y = -\frac{2}{3}x + 4$        $l: y = -\frac{2}{3}x + 3\frac{1}{2}$
- b**  $rc_k = rc_l = -\frac{2}{3}$ , dus  $k$  en  $l$  zijn evenwijdig.  
 $k$  snijdt de  $y$ -as in het punt  $(0, 4)$  en  $l$  snijdt de  $y$ -as in het punt  $(0, 3\frac{1}{2})$ ,  
dus  $k$  en  $l$  vallen niet samen.
- c**  $k$  en  $l$  hebben geen punt gemeenschappelijk, dus het stelsel  $\begin{cases} 2x + 3y = 12 \\ 4x + 6y = 15 \end{cases}$   
heeft geen oplossing.

### Bladzijde 95

- 2 a** Voor  $p = \frac{1}{3}$  en  $q \neq -10$  hebben de lijnen geen punt gemeenschappelijk.
- b** Voor  $p \neq \frac{1}{3}$  en  $q$  elk getal van  $\mathbb{R}$  hebben de lijnen een snijpunt.
- c**  $(2, -3)$  op  $k_p$  geeft  $2p - 3(p + 1) = 5$   
 $2p - 3p - 3 = 5$   
 $-p = 8$   
 $p = -8$   
 $p = -8$  geeft  $l_{-8,q}: -9x - 11y = q$   
 $(2, -3)$  op  $l_{-8,q}$  geeft  $q = -9 \cdot 2 - 11 \cdot -3 = 15$   
Dus voor  $p = -8$  en  $q = 15$  snijden de lijnen elkaar in het punt  $(2, -3)$ .

- 3 a** Samenvallen, dus  $\frac{3}{p-1} = \frac{p}{p+4} = \frac{5}{q}$ .  
 $\frac{3}{p-1} = \frac{p}{p+4}$  geeft  $p(p-1) = 3(p+4)$   
 $p^2 - p = 3p + 12$   
 $p^2 - 4p - 12 = 0$   
 $(p+2)(p-6) = 0$   
 $p = -2 \vee p = 6$   
 $p = -2$  geeft  $\frac{3}{-2-1} = \frac{5}{q}$  oftewel  $\frac{3}{-3} = \frac{5}{q}$ , dus  $q = -5$ .  
 $p = 6$  geeft  $\frac{3}{6-1} = \frac{5}{q}$  oftewel  $\frac{3}{5} = \frac{5}{q}$ , dus  $q = \frac{5 \cdot 5}{3} = 8\frac{1}{3}$ .  
Dus voor  $p = -2$  en  $q = -5$  en voor  $p = 6$  en  $q = 8\frac{1}{3}$  vallen de lijnen samen.
- b** Voor  $p = -2$  en  $q \neq -5$  en voor  $p = 6$  en  $q \neq 8\frac{1}{3}$  hebben de lijnen geen punt gemeenschappelijk.

- 4** Samenvallen, dus  $\frac{p}{q+3} = \frac{q}{p-1} = \frac{4}{1}$ .

Uit  $\frac{p}{q+3} = \frac{4}{1}$  volgt  $p = 4(q+3)$  oftewel  $p = 4q + 12$ .

Uit  $\frac{q}{p-1} = \frac{4}{1}$  volgt  $q = 4(p-1)$  oftewel  $q = 4p - 4$ .

$$\begin{aligned} p = 4q + 12 \text{ en } q = 4p - 4 \text{ geeft } q &= 4(4q + 12) - 4 \\ q &= 16q + 48 - 4 \\ -15q &= 44 \\ q &= -2\frac{14}{15} \end{aligned}$$

$$q = -2\frac{14}{15} \text{ geeft } p = 4 \cdot -2\frac{14}{15} + 12 = \frac{4}{15}$$

Dus voor  $p = \frac{4}{15}$  en  $q = -2\frac{14}{15}$  vallen de lijnen samen.

**5 a**  $y = 0$  geeft  $2x = 18$

$$x = 9$$

Dus het snijpunt met de  $x$ -as is  $(9, 0)$ .

$$x = 0 \text{ geeft } 3y = 18$$

$$y = 6$$

Dus het snijpunt met de  $y$ -as is  $(0, 6)$ .

**b** Bij  $2x + 3y = 18$  het linker- en rechterlid delen door 18 geeft  $\frac{x}{9} + \frac{y}{6} = 1$ .

**c** In de noemer van de breuk  $\frac{x}{9}$  staat de  $x$ -coördinaat van het snijpunt met de  $x$ -as.

In de noemer van de breuk  $\frac{y}{6}$  staat de  $y$ -coördinaat van het snijpunt met de  $y$ -as.

#### Bladzijde 96

**6 a**  $\frac{x}{a} + \frac{y}{b} = 1$   $\left\{ \begin{array}{l} \frac{x}{a} + 0 = 1 \\ \frac{x}{a} = 1 \\ x = a \end{array} \right.$

Dus het snijpunt met de  $x$ -as is  $(a, 0)$ .

**b**  $\frac{x}{a} + \frac{y}{b} = 1$   $\left\{ \begin{array}{l} 0 + \frac{y}{b} = 1 \\ \frac{y}{b} = 1 \\ y = b \end{array} \right.$

Dus het snijpunt met de  $y$ -as is  $(0, b)$ .

**7 a**  $l: \frac{x}{p} + \frac{y}{5} = 1$

$$l: 5x + py = 5p$$

**b**  $m: \frac{x}{4} + \frac{y}{q} = 1$

$$m: qx + 4y = 4q$$

**c**  $n: \frac{x}{3r} + \frac{y}{r} = 1$

$$n: x + 3y = 3r$$

#### Bladzijde 97

**8 a**  $k: \frac{x}{p} + \frac{y}{8} = 1$

$$k: 8x + py = 8p$$

**b**  $k: 8x + py = 8p$   $\left\{ \begin{array}{l} \text{door } (1, 2) \end{array} \right. \left\{ \begin{array}{l} 8 \cdot 1 + p \cdot 2 = 8p \\ 8 + 2p = 8p \\ -6p = -8 \\ p = 1\frac{1}{3} \end{array} \right.$

**c**  $k: 8x + py = 8p$  en  $l: 2x - y = -3$

Een snijpunt, dus  $\frac{8}{2} \neq \frac{p}{-1}$ .

$$\frac{8}{2} \neq \frac{p}{-1} \text{ geeft } p \neq -4$$

Dus voor  $p \neq -4$  hebben  $k$  en  $l$  een snijpunt.

**9 a**  $k: \frac{x}{3} + \frac{y}{p} = 1$

$$k: px + 3y = 3p$$

$$l: \frac{x}{2p} + \frac{y}{5} = 1$$

$$l: 5x + 2py = 10p$$

**b**  $k: px + 3y = 3p$   $\left\{ \begin{array}{l} \text{door } A(1, 2) \end{array} \right. \left\{ \begin{array}{l} p \cdot 1 + 3 \cdot 2 = 3p \\ p + 6 = 3p \\ -2p = -6 \\ p = 3 \end{array} \right.$

$$\begin{aligned} \text{c } l: 5x + 2py = 10p \quad & \left\{ \begin{array}{l} 5 \cdot -3 + 2p \cdot 4 = 10p \\ -15 + 8p = 10p \\ -2p = 15 \\ p = -7\frac{1}{2} \end{array} \right. \\ \text{door } B(-3, 4) \end{aligned}$$

$$\text{d } k: px + 3y = 3p \text{ en } m: 4x - y = -5$$

Geen punt gemeenschappelijk, dus  $\frac{p}{4} = \frac{3}{-1} \neq \frac{3p}{-5}$ .

$$\frac{p}{4} = \frac{3}{-1} \text{ geeft } p = -12 \text{ en } \frac{3}{-1} \neq \frac{3p}{-5} \text{ geeft } p \neq 5$$

Dus voor  $p = -12$  hebben  $k$  en  $m$  geen punt gemeenschappelijk.

$$\text{e } l: 5x + 2py = 10p \text{ en } n: 2x + 3y = 10$$

Een snijpunt, dus  $\frac{5}{2} \neq \frac{2p}{3}$ .

$$\frac{5}{2} \neq \frac{2p}{3} \text{ geeft } p \neq \frac{3 \cdot 5}{2 \cdot 2} = 3\frac{3}{4}$$

Dus voor  $p \neq 3\frac{3}{4}$  hebben  $l$  en  $n$  een snijpunt.

10

$$\begin{aligned} \text{a } l_p: \frac{x}{p} + \frac{y}{p+2} = 1 \quad & \left\{ \begin{array}{l} \frac{3}{p} + \frac{4}{p+2} = 1 \\ 3(p+2) + 4p = p(p+2) \\ 3p + 6 + 4p = p^2 + 2p \\ -p^2 + 5p + 6 = 0 \\ p^2 - 5p - 6 = 0 \\ (p+1)(p-6) = 0 \\ p = -1 \vee p = 6 \end{array} \right. \\ \text{door } (3, 4) \end{aligned}$$

Dus voor  $p = -1$  en voor  $p = 6$  gaat de lijn door het punt  $(3, 4)$ .

$$\begin{aligned} \text{b } \frac{x}{p} + \frac{y}{p+2} = 1 \\ (p+2)x + py = p(p+2) \\ py = -(p+2)x + p(p+2) \\ y = -\frac{p+2}{p} \cdot x + p+2 \\ \text{rc} = 2 \text{ geeft } -\frac{p+2}{p} = 2 \\ p+2 = -2p \\ 3p = -2 \\ p = -\frac{2}{3} \end{aligned}$$

11

$$\text{a } \tan(\angle BAC) = \frac{BC}{AC} = \frac{2}{1} = 2 \text{ geeft } \angle BAC = 63,43\dots^\circ$$

$$\text{b } \text{rc}_l = -\frac{1}{4}, \text{ dus de richting van lijn } l \text{ is 4 naar rechts en 1 omlaag.}$$

Dus de tangens van de hoek tussen  $l$  en de  $x$ -as is  $\frac{1}{4}$ .

Dit geeft dat de hoek tussen  $l$  en de  $x$ -as gelijk is aan  $14,03\dots^\circ$ .

$$\text{c } \angle(k, l) = 63,43\dots^\circ + 14,03\dots^\circ \approx 77,5^\circ$$

12

**a**  $k: y = 3x + 4$

$$\tan(\alpha) = \text{rc}_k = 3 \text{ geeft } \alpha = 71,5\dots^\circ$$

$$\alpha - \beta = 71,5\dots^\circ - 63,4\dots^\circ = 8,1\dots^\circ$$

$$\text{Dus } \angle(k, l) \approx 8^\circ.$$

**b**  $m: y = 1\frac{1}{2}x + 2$

$$\tan(\alpha) = \text{rc}_m = 1\frac{1}{2} \text{ geeft } \alpha = 56,3\dots^\circ$$

$$\alpha - \beta = 56,3\dots^\circ - -26,5\dots^\circ = 82,8\dots^\circ$$

$$\text{Dus } \angle(m, n) \approx 83^\circ.$$

**c**  $p: y = 3\frac{1}{2}x - 1$

$$\tan(\alpha) = \text{rc}_p = 3\frac{1}{2} \text{ geeft } \alpha = 74,0\dots^\circ$$

$$\alpha - \beta = 74,0\dots^\circ - -51,3\dots^\circ = 125,3\dots^\circ$$

$$\text{Dus } \angle(p, q) = 180^\circ - 125,3\dots^\circ \approx 55^\circ.$$

$$l: y = 2x - 1$$

$$\tan(\beta) = \text{rc}_l = 2 \text{ geeft } \beta = 63,4\dots^\circ$$

$$n: y = -\frac{1}{2}x + 3$$

$$\tan(\beta) = \text{rc}_n = -\frac{1}{2} \text{ geeft } \beta = -26,5\dots^\circ$$

$$q: y = -1\frac{1}{4}x + 5$$

$$\tan(\beta) = \text{rc}_q = -1\frac{1}{4} \text{ geeft } \beta = -51,3\dots^\circ$$

13

**a**  $k: 3x - 2y = 5$

$$-2y = -3x + 5$$

$$y = 1\frac{1}{2}x - 2\frac{1}{2}$$

$$\tan(\alpha) = \text{rc}_k = 1\frac{1}{2} \text{ geeft } \alpha = 56,30\dots^\circ$$

$$\alpha - \beta = 56,30\dots^\circ - 53,13\dots^\circ = 3,17\dots^\circ$$

$$\text{Dus } \angle(k, l) \approx 3,2^\circ.$$

**b**  $m: 4x + y = 1$

$$y = -4x + 1$$

$$\tan(\alpha) = \text{rc}_m = -4 \text{ geeft } \alpha = -75,96\dots^\circ$$

$$\beta - \alpha = -36,86\dots^\circ - -75,96\dots^\circ = 39,09\dots^\circ$$

$$\text{Dus } \angle(m, n) \approx 39,1^\circ.$$

**c**  $p: 5x + 3y = 4$

$$3y = -5x + 4$$

$$y = -1\frac{2}{3}x + 1\frac{1}{3}$$

$$\tan(\alpha) = \text{rc}_p = -1\frac{2}{3} \text{ geeft } \alpha = -59,03\dots^\circ$$

$$\beta - \alpha = 50,19\dots^\circ - -59,03\dots^\circ = 109,23\dots^\circ$$

$$\text{Dus } \angle(p, q) = 180^\circ - 109,23\dots^\circ \approx 70,8^\circ.$$

$$l: 4x - 3y = 6$$

$$-3y = -4x + 6$$

$$y = 1\frac{1}{3}x - 2$$

$$\tan(\beta) = \text{rc}_l = 1\frac{1}{3} \text{ geeft } \beta = 53,13\dots^\circ$$

$$n: 3x + 4y = 2$$

$$4y = -3x + 2$$

$$y = -\frac{3}{4}x + \frac{1}{2}$$

$$\tan(\beta) = \text{rc}_n = -\frac{3}{4} \text{ geeft } \beta = -36,86\dots^\circ$$

$$q: 6x - 5y = 1$$

$$-5y = -6x + 1$$

$$y = 1\frac{1}{5}x - \frac{1}{5}$$

$$\tan(\beta) = \text{rc}_q = 1\frac{1}{5} \text{ geeft } \beta = 50,19\dots^\circ$$

14

**a**  $k: y = \frac{2}{3}x + 4$

$$\tan(\alpha) = \text{rc}_k = \frac{2}{3} \text{ geeft } \alpha = 33,6\dots^\circ$$

$$\beta - \alpha = 57,9\dots^\circ - 33,6\dots^\circ = 24,3\dots^\circ$$

$$\text{Dus } \angle(k, l) \approx 24^\circ.$$

**b**  $\tan(\alpha) = \text{rc}_m = \frac{0-5}{4-0} = -1\frac{1}{4} \text{ geeft } \alpha = -51,34\dots^\circ$

$$\tan(\beta) = \text{rc}_n = \frac{1-0}{0-2} = \frac{1}{2} \text{ geeft } \beta = 26,56\dots^\circ$$

$$\beta - \alpha = 26,56\dots^\circ - -51,34\dots^\circ = 77,90\dots^\circ$$

$$\text{Dus } \angle(m, n) \approx 77,9^\circ.$$

**c**  $\tan(\alpha) = \text{rc}_p = \frac{6-1}{5-2} = 1\frac{2}{3} \text{ geeft } \alpha = 59,0\dots^\circ$

$$\tan(\beta) = \text{rc}_q = \frac{-6-1}{2-3} = -1\frac{2}{5} \text{ geeft } \beta = -54,4\dots^\circ$$

$$\alpha - \beta = 59,0\dots^\circ - -54,4\dots^\circ = 113,4\dots^\circ$$

$$\text{Dus } \angle(p, q) = 180^\circ - 113,4\dots^\circ \approx 67^\circ.$$

$$l: 8x - 5y = 3$$

$$-5y = -8x + 3$$

$$y = 1\frac{3}{5}x - \frac{3}{5}$$

$$\tan(\beta) = \text{rc}_l = 1\frac{3}{5} \text{ geeft } \beta = 57,9\dots^\circ$$

**15 a**  $f(x) = 1\frac{1}{3}$  geeft  $\frac{4}{4-x^2} = \frac{4}{3}$   
 $4-x^2 = 3$   
 $x^2 = 1$   
 $x = 1 \vee x = -1$

$$f(x) = \frac{4}{4-x^2} = 4(4-x^2)^{-1} \text{ geeft } f'(x) = -4(4-x^2)^{-2} \cdot -2x = \frac{8x}{(4-x^2)^2}$$

$$x_A = -1 \text{ geeft } rc_l = f'(-1) = -\frac{8}{9} \text{ en } x_B = 1 \text{ geeft } rc_m = f'(1) = \frac{8}{9}$$

$$\tan(\alpha) = rc_l = -\frac{8}{9} \text{ geeft } \alpha = -41,63\dots^\circ$$

$$\tan(\beta) = rc_m = \frac{8}{9} \text{ geeft } \beta = 41,63\dots^\circ$$

$$\beta - \alpha = 41,63\dots^\circ - (-41,63\dots^\circ) = 83,26\dots^\circ$$

$$\text{Dus } \angle ASB = 180^\circ - 83,26\dots^\circ \approx 96,7^\circ.$$

**b**  $x_A = -1$  en  $x_B = 1$ , dus  $AB = 2$ .

$$\left. \begin{array}{l} l: y = -\frac{8}{9}x + b \\ \text{door } A(-1, 1\frac{1}{3}) \end{array} \right\} \begin{array}{l} -\frac{8}{9} \cdot -1 + b = 1\frac{1}{3} \\ \frac{8}{9} + b = 1\frac{1}{3} \\ b = \frac{4}{9} \end{array}$$

$$\text{Dus } l: y = -\frac{8}{9}x + \frac{4}{9}.$$

$$l \text{ snijden met de } x\text{-as geeft } -\frac{8}{9}x + \frac{4}{9} = 0$$

$$-\frac{8}{9}x = -\frac{4}{9}$$

$$x = \frac{1}{2}$$

$$\text{Dus } P(\frac{1}{2}, 0).$$

$$\text{Uit de symmetrie volgt } Q(-\frac{1}{2}, 0) \text{ en } PQ = 1.$$

$$\text{Dus } O(AQPB) = \frac{1}{2}(2+1) \cdot 1\frac{1}{3} = 2.$$

**16** De richtingshoek van  $k$  is  $\alpha$  en de richtingshoek van  $l$  is  $\beta$ .

$$\tan(\alpha) = rc_k = 2 \text{ geeft } \alpha = 63,4\dots^\circ$$

$$\beta = 63,4\dots^\circ + 20^\circ = 83,4\dots^\circ \text{ geeft } rc_l = \tan(83,4\dots^\circ) \approx 8,69 \text{ of}$$

$$\beta = 63,4\dots^\circ - 20^\circ = 43,4\dots^\circ \text{ geeft } rc_l = \tan(43,4\dots^\circ) \approx 0,95.$$

**17** De richtingshoek van  $k$  is  $\alpha$  en de richtingshoek van  $l$  is  $\beta$ .

$$k: 2x - 5y = 9$$

$$-5y = -2x + 9$$

$$y = \frac{2}{5}x - 1\frac{4}{5}$$

$$\tan(\alpha) = rc_k = \frac{2}{5} \text{ geeft } \alpha = 21,8\dots^\circ$$

$$\beta = 21,8\dots^\circ + 60^\circ = 81,8\dots^\circ \text{ geeft } rc_l = \tan(81,8\dots^\circ) = 6,940\dots$$

$$\beta = 21,8\dots^\circ - 60^\circ = -38,1\dots^\circ \text{ geeft } rc_l = \tan(-38,1\dots^\circ) = -0,786\dots$$

$$S(2, -1) \text{ op } l \text{ geeft } 2a - b = 3, \text{ dus } b = 2a - 3.$$

$$l: ax + (2a - 3)y = 3$$

$$(2a - 3)y = -ax + 3$$

$$y = \frac{-a}{2a-3}x + \frac{3}{2a-3}, \text{ dus } rc_l = \frac{-a}{2a-3}.$$

$$\frac{-a}{2a-3} = 6,940\dots$$

$$6,940\dots \cdot (2a - 3) = -a$$

$$13,881\dots \cdot a - 20,822\dots = -a$$

$$14,881\dots \cdot a = 20,822\dots$$

$$a = 1,399\dots$$

$$a = 1,399\dots \text{ geeft } b = 2 \cdot 1,399\dots - 3 \approx -0,20$$

$$a = 4,114\dots \text{ geeft } b = 2 \cdot 4,114\dots - 3 \approx 5,23$$

$$\text{Dus } a \approx 1,40 \text{ en } b \approx -0,20 \text{ of } a \approx 4,11 \text{ en } b \approx 5,23.$$

$$\frac{-a}{2a-3} = -0,786\dots$$

$$-0,786\dots \cdot (2a - 3) = -a$$

$$-1,573\dots \cdot a + 2,360\dots = -a$$

$$-0,573\dots \cdot a = -2,360\dots$$

$$a = 4,114\dots$$



**C1 a** Samenvallen, dus  $\frac{p}{p+1} = \frac{p+2}{p-3} = \frac{5}{q}$ .

$$\frac{p}{p+1} = \frac{p+2}{p-3} \text{ geeft } p(p-3) = (p+1)(p+2)$$

$$p^2 - 3p = p^2 + 3p + 2$$

$$-6p = 2$$

$$p = -\frac{1}{3}$$

$$p = -\frac{1}{3} \text{ geeft } \frac{-\frac{1}{3}}{-\frac{1}{3}+1} = \frac{5}{q} \text{ oftewel } \frac{-1}{2} = \frac{5}{q}, \text{ dus } q = -10.$$

Dus voor  $p = -\frac{1}{3}$  en  $q = -10$  vallen de lijnen samen.

**b** Voor  $p \neq -\frac{1}{3}$  en  $q$  elk getal van  $\mathbb{R}$  hebben de lijnen een snijpunt.

**C2 a**  $k: \frac{x}{5} + \frac{y}{p} = 1$

$$k: px + 5y = 5p$$

**b**  $k: px + 5y = 5p$   
 door  $(-3, 4)$   $\left\{ \begin{array}{l} p \cdot -3 + 5 \cdot 4 = 5p \\ -3p + 20 = 5p \\ -8p = -20 \\ p = 2\frac{1}{2} \end{array} \right.$

**c**  $k: px + 5y = 5p$  en  $l: 3x - y = 2$

Geen punt gemeenschappelijk, dus  $\frac{p}{3} = \frac{5}{-1} \neq \frac{5p}{2}$ .

$$\frac{p}{3} = \frac{5}{-1} \text{ geeft } p = -15$$

Voor  $p = -15$  is  $\frac{5}{-1} \neq \frac{5p}{2}$ .

Dus voor  $p = -15$  hebben  $k$  en  $l$  geen punt gemeenschappelijk.

**C3 a**  $k: y = 2x - 3$

$$\tan(\alpha) = \text{rc}_k = 2 \text{ geeft } \alpha = 63,4\dots^\circ$$

$$\alpha - \beta = 64,4\dots^\circ - -16,6\dots^\circ = 80,1\dots^\circ$$

$$\text{Dus } \angle(k, l) \approx 80^\circ.$$

**b**  $m: 3x + 4y = 5$

$$4y = -3x + 5$$

$$y = -\frac{3}{4}x + 1\frac{1}{4}$$

$$\tan(\alpha) = \text{rc}_m = -\frac{3}{4} \text{ geeft } \alpha = -36,86\dots^\circ$$

$$\beta - \alpha = -18,43\dots^\circ - -36,86\dots^\circ = 18,43\dots^\circ$$

$$\text{Dus } \angle(m, n) \approx 18,4^\circ.$$

**c**  $p: 3x - y = 6$

$$-y = -3x + 6$$

$$y = 3x - 6$$

$$\tan(\alpha) = \text{rc}_p = 3 \text{ geeft } \alpha = 71,56\dots^\circ$$

$$\alpha - \beta = 71,56\dots^\circ - -26,56\dots^\circ = 98,13\dots^\circ$$

$$\text{Dus } \angle(p, q) = 180^\circ - 98,13\dots^\circ \approx 81,9^\circ.$$

$$l: y = -0,3x - 1$$

$$\tan(\beta) = \text{rc}_l = -0,3 \text{ geeft } \beta = -16,6\dots^\circ$$

$$n: y = -\frac{1}{3}x + 1$$

$$\tan(\beta) = \text{rc}_n = -\frac{1}{3} \text{ geeft } \beta = -18,43\dots^\circ$$

$$q: x + 2y = 5$$

$$2y = -x + 5$$

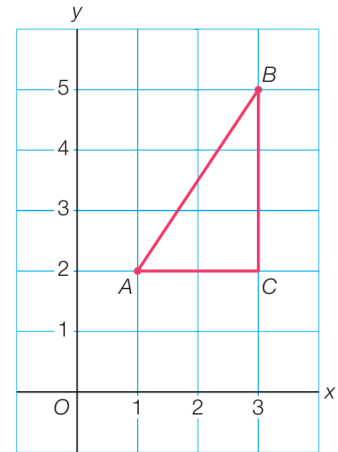
$$y = -\frac{1}{2}x + 2\frac{1}{2}$$

$$\tan(\beta) = \text{rc}_q = -\frac{1}{2} \text{ geeft } \beta = -26,56\dots^\circ$$

## 7.2 Afstanden bij punten en lijnen

### Bladzijde 101

- 18 a** Zie de figuur hiernaast.  
De stelling van Pythagoras in  $\triangle ABC$  geeft  $AB = \sqrt{2^2 + 3^2} = \sqrt{13}$ .
- b**  $x_M = \frac{1}{2}(x_A + x_B)$  en  $y_M = \frac{1}{2}(y_A + y_B)$   
 $M(\frac{1}{2}(1 + 3), \frac{1}{2}(2 + 5))$ , dus  $M(2, 3\frac{1}{2})$ .
- c**  $N(\frac{1}{2}(83 + 89), \frac{1}{2}(61 + 69))$ , dus  $N(86, 65)$ .



### Bladzijde 102

- 19 a**  $x_M = -1$  geeft  $\frac{1}{2}p + 2 = -1$   
 $\frac{1}{2}p = -3$   
 $p = -6$ , dus  $A(-6, 3)$ .
- $y_M = \frac{1}{2}$  geeft  $1\frac{1}{2} + \frac{1}{2}q = \frac{1}{2}$   
 $\frac{1}{2}q = -1$   
 $q = -2$ , dus  $B(4, -2)$ .
- $d(A, B) = \sqrt{(4 - (-6))^2 + (-2 - 3)^2} = \sqrt{125} = 5\sqrt{5}$
- b**  $d(A, B) = 5\sqrt{2}$  geeft  $\sqrt{5p^2 - 20p + 25} = 5\sqrt{2}$   
 $5p^2 - 20p + 25 = 50$   
 $5p^2 - 20p - 25 = 0$   
 $p^2 - 4p - 5 = 0$   
 $(p + 1)(p - 5) = 0$   
 $p = -1 \vee p = 5$   
 $p = -1$  geeft  $q = -2$ , dus  $A(-1, 3)$  en  $B(4, -2)$ .  
 $p = 5$  geeft  $q = 10$ , dus  $A(5, 3)$  en  $B(4, 10)$ .
- 20 a**  $A(0, 12)$  en  $B(12, 7)$  geeft  $d(A, B) = \sqrt{(12 - 0)^2 + (7 - 12)^2} = \sqrt{169} = 13$
- b**  $A(0, -2)$  en  $B(-2, 7)$  geeft  $M(\frac{1}{2}(0 + (-2)), \frac{1}{2}(-2 + 7)) = M(-1, 2\frac{1}{2})$
- c**  $A(0, 4)$  en  $B(4, 7)$  geeft  $M(\frac{1}{2}(0 + 4), \frac{1}{2}(4 + 7)) = M(2, 5\frac{1}{2})$
- $d(A, M) = \sqrt{(2 - 0)^2 + (5\frac{1}{2} - 4)^2} = \sqrt{6\frac{1}{4}} = 2\frac{1}{2}$
- d**  $M(\frac{1}{2}(0 + p), \frac{1}{2}(p + 7)) = M(\frac{1}{2}p, \frac{1}{2}p + 3\frac{1}{2})$
- e**  $d = \sqrt{(p - 0)^2 + (7 - p)^2} = \sqrt{p^2 + 49 - 14p + p^2} = \sqrt{2p^2 - 14p + 49}$
- 21 a**  $M(\frac{1}{2}(p + p + 2), \frac{1}{2}(3 + 2p)) = M(p + 1, 1\frac{1}{2} + p)$
- b**  $d = \sqrt{(p + 2 - p)^2 + (2p - 3)^2} = \sqrt{4 + 4p^2 - 12p + 9} = \sqrt{4p^2 - 12p + 13}$
- c**  $d = 2\sqrt{2}$  geeft  $\sqrt{4p^2 - 12p + 13} = 2\sqrt{2}$   
 $4p^2 - 12p + 13 = 8$   
 $4p^2 - 12p + 5 = 0$   
 $D = (-12)^2 - 4 \cdot 4 \cdot 5 = 64$ , dus  $\sqrt{D} = 8$   
 $p = \frac{12 + 8}{8} = 2\frac{1}{2} \vee p = \frac{12 - 8}{8} = \frac{1}{2}$

Bladzijde 103

22

a  $x_M = 10$  geeft  $\frac{1}{2}(3 + p + 5) = 10$   
 $\frac{1}{2}p + 4 = 10$   
 $\frac{1}{2}p = 6$   
 $p = 12$

$p = 12$  geeft  $y_M = \frac{1}{2}(4 + 12 + 2) = 9$

b  $M(\frac{1}{2}(3 + p + 5), \frac{1}{2}(4 + p + 2)) = M(\frac{1}{2}p + 4, \frac{1}{2}p + 3)$

$M$  op  $k$  geeft  $2(\frac{1}{2}p + 4) - 3 = \frac{1}{2}p + 3$

$p + 8 - 3 = \frac{1}{2}p + 3$

$p + 5 = \frac{1}{2}p + 3$

$\frac{1}{2}p = -2$

$p = -4$

c  $A(3, 4)$  en  $B(11, 8)$  geeft  $d(A, B) = \sqrt{(11 - 3)^2 + (8 - 4)^2} = \sqrt{80} = 4\sqrt{5}$

d  $d = \sqrt{(p + 5 - 3)^2 + (p + 2 - 4)^2} = \sqrt{(p + 2)^2 + (p - 2)^2} = \sqrt{p^2 + 4p + 4 + p^2 - 4p + 4} = \sqrt{2p^2 + 8}$

Dus  $m = 2$  en  $n = 8$ .

23

a  $m: 5x + 2y = 3$

$n: 2x - 5y = 12$

$2y = -5x + 3$

$-5y = -2x + 12$

$y = -2\frac{1}{2}x + 1\frac{1}{2}$

$y = \frac{2}{5}x - 2\frac{2}{5}$

Er geldt  $rc_m \cdot rc_n = -2\frac{1}{2} \cdot \frac{2}{5} = -1$ , dus  $m \perp n$ .

b  $k: ax + by = c$

$l: bx - ay = d$

$by = -ax + c$

$-ay = -bx + d$

$y = -\frac{a}{b}x + \frac{c}{b}$

$y = \frac{b}{a}x - \frac{d}{a}$

Er geldt  $rc_k \cdot rc_l = -\frac{a}{b} \cdot \frac{b}{a} = -1$ , dus  $k \perp l$ .

Bladzijde 104

24

In de theorie boven het rode kader gaat het over de breuken  $-\frac{a}{b}$  en  $\frac{b}{a}$ ,

dus moet gelden  $b \neq 0$  en  $a \neq 0$ .

In het rode kader mogen  $a$  en  $b$  wel gelijk zijn aan 0, want

- voor  $b = 0$  is  $k$  een verticale lijn en  $l$  een horizontale lijn, dus staan  $k$  en  $l$  loodrecht op elkaar
- voor  $a = 0$  is  $k$  een horizontale lijn en  $l$  een verticale lijn, dus staan  $k$  en  $l$  loodrecht op elkaar.

Dus in het rode kader zijn de voorwaarden  $b \neq 0$  en  $a \neq 0$  niet nodig.

25

a  $k \perp l$ , dus  $k: 3x + 2y = c$  }  $c = 3 \cdot 4 + 2 \cdot 1 = 14$   
 door  $A(4, 1)$

Dus  $k: 3x + 2y = 14$ .

b  $m \perp n$ , dus  $m: 5x - 4y = c$  }  $c = 5 \cdot 3 - 4 \cdot -1 = 19$   
 door  $B(3, -1)$

Dus  $m: 5x - 4y = 19$ .

c  $p \perp q$ , dus  $p: x + 2y = c$  }  $c = -4 + 2 \cdot 3 = 2$   
 door  $C(-4, 3)$

$p: x + 2y = 2$

$2y = -x + 2$

$y = -\frac{1}{2}x + 1$

26

a  $rc_k = \frac{5 - 3}{6 - 2} = \frac{1}{2}$

$l \perp k$ , dus  $rc_l \cdot \frac{1}{2} = -1$  en dit geeft  $rc_l = -2$ .

$l: y = -2x + b$  }  $-2 \cdot 4 + b = 6$   
 door  $C(4, 6)$  }  $-8 + b = 6$

$b = 14$

Dus  $l: y = -2x + 14$ .

$$\begin{aligned} \text{b } rc_m &= \frac{-6-4}{2-3} = -2 \\ n \perp m, \text{ dus } rc_n \cdot -2 &= -1 \text{ en dit geeft } rc_n = \frac{1}{2}. \\ n: y &= \frac{1}{2}x + b \left\{ \begin{array}{l} \frac{1}{2} \cdot 3 + b = 7 \\ \frac{1}{2} + b = 7 \end{array} \right. \\ \text{door } F(3, 7) & \\ b &= 5\frac{1}{2} \\ n: y &= \frac{1}{2}x + 5\frac{1}{2} \\ -\frac{1}{2}x + y &= 5\frac{1}{2} \\ x - 2y &= -11 \end{aligned}$$

### Bladzijde 105

- 27 a** Lijn  $l$  gaat door  $P$  en staat loodrecht op  $k$ .

$$l: 2x - 3y = c \left\{ \begin{array}{l} c = 2 \cdot 5 - 3 \cdot 5 = -5 \\ \text{door } P(5, 5) \end{array} \right.$$

$$\text{Dus } l: 2x - 3y = -5.$$

$k$  en  $l$  snijden geeft het punt  $P'$ .

$$\left\{ \begin{array}{l} 3x + 2y = 12 \\ 2x - 3y = -5 \end{array} \right. \left| \begin{array}{l} 3 \\ 2 \end{array} \right. \text{ geeft } \left\{ \begin{array}{l} 9x + 6y = 36 \\ 4x - 6y = -10 \end{array} \right. +$$

$$\frac{13x}{x} = \frac{26}{2} \quad x = 2$$

$$x = 2 \text{ geeft } 3 \cdot 2 + 2y = 12$$

$$6 + 2y = 12$$

$$2y = 6$$

$$y = 3$$

Dus  $P'(2, 3)$ .

$$\text{b } d(P, k) = d(P, P') = \sqrt{(2-5)^2 + (3-5)^2} = \sqrt{13}$$

- 28** Lijn  $l$  gaat door  $P$  en staat loodrecht op  $k$ .

$$l: 2x + y = c \left\{ \begin{array}{l} c = 2 \cdot 6 + 4 = 16 \\ \text{door } P(6, 4) \end{array} \right.$$

$$\text{Dus } l: 2x + y = 16.$$

$k$  en  $l$  snijden geeft het punt  $P'$ .

$$\left\{ \begin{array}{l} x - 2y = 2 \\ 2x + y = 16 \end{array} \right. \left| \begin{array}{l} 1 \\ 2 \end{array} \right. \text{ geeft } \left\{ \begin{array}{l} x - 2y = 2 \\ 4x + 2y = 32 \end{array} \right. +$$

$$\frac{5x}{x} = \frac{34}{5} \quad x = 6\frac{4}{5}$$

$$x = 6\frac{4}{5} \text{ geeft } 6\frac{4}{5} - 2y = 2$$

$$-2y = -4\frac{4}{5}$$

$$y = 2\frac{2}{5}$$

$$\text{Dus } P'(6\frac{4}{5}, 2\frac{2}{5}) \text{ en } d(P, k) = d(P, P') = \sqrt{(6\frac{4}{5}-6)^2 + (2\frac{2}{5}-4)^2} = \sqrt{3\frac{1}{5}} = \sqrt{\frac{16}{5}} = \frac{\sqrt{16}}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4}{5}\sqrt{5}.$$

### Bladzijde 106

- 29 a**  $k: ax + by = c$  door  $B(12, 0)$  geeft  $12a = c$ , dus  $k: ax + by = 12a$  oftewel

$$k: ax + by - 12a = 0.$$

Je krijgt één vergelijking met twee onbekenden.

- b**  $k: ax + by = 1$  door  $B(12, 0)$  geeft  $12a = 1$  oftewel  $a = \frac{1}{12}$ , dus  $k: \frac{1}{12}x + by = 1$  oftewel  $k: \frac{1}{12}x + by - 1 = 0$ .

Je had dus ook uit kunnen gaan van  $k: ax + by = 1$ .

$$30 \quad a \quad d(A, k) = \frac{|1 - 4 \cdot -1 + 4|}{\sqrt{1^2 + (-4)^2}} = \frac{|9|}{\sqrt{17}} = \frac{9}{\sqrt{17}} = \frac{9}{17}\sqrt{17}$$

$$b \quad d(B, k) = \frac{|2 - 4 \cdot 3\frac{1}{2} + 4|}{\sqrt{1^2 + (-4)^2}} = \frac{|-8|}{\sqrt{17}} = \frac{8}{\sqrt{17}} = \frac{8}{17}\sqrt{17}$$

$$d(C, k) = \frac{|6 - 4 \cdot 5 + 4|}{\sqrt{1^2 + (-4)^2}} = \frac{|-10|}{\sqrt{17}} = \frac{10}{\sqrt{17}} = \frac{10}{17}\sqrt{17}$$

$B$  ligt het dichtst bij  $k$ .

#### Bladzijde 107

$$31 \quad a \quad d(A, k) = \frac{|3 \cdot 2 + 4 \cdot 5 - 10|}{\sqrt{3^2 + 4^2}} = \frac{|16|}{\sqrt{25}} = \frac{16}{5} = 3\frac{1}{5}$$

$$b \quad l: y = -2x + 5 \text{ oftewel } l: 2x + y - 5 = 0$$

$$d(B, l) = \frac{|2 \cdot 2 + -1 - 5|}{\sqrt{2^2 + 1^2}} = \frac{|-2|}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{5}\sqrt{5}$$

$$c \quad m: \frac{x}{6} + \frac{y}{2} = 1 \text{ oftewel } m: x + 3y - 6 = 0$$

$$d(C, m) = \frac{|5 + 3 \cdot 3 - 6|}{\sqrt{1^2 + 3^2}} = \frac{|8|}{\sqrt{10}} = \frac{8}{\sqrt{10}} = \frac{8}{10}\sqrt{10} = \frac{4}{5}\sqrt{10}$$

$$32 \quad d(A, B) = \sqrt{(7-1)^2 + (4-0)^2} = \sqrt{52} = 2\sqrt{13}$$

$$\text{Lijn } l \text{ door } A \text{ en } B \text{ heeft } rc_l = \frac{4-0}{7-1} = \frac{2}{3}.$$

$$l: y = \frac{2}{3}x + b \quad \left\{ \begin{array}{l} \frac{2}{3} \cdot 1 + b = 0 \\ \frac{2}{3} + b = 0 \\ b = -\frac{2}{3} \end{array} \right.$$

$$l: y = \frac{2}{3}x - \frac{2}{3} \text{ oftewel } l: \frac{2}{3}x - y - \frac{2}{3} = 0, \text{ dus } l: 2x - 3y - 2 = 0.$$

$$d(C, l) = \frac{|2 \cdot 3\frac{1}{2} - 3 \cdot 6 - 2|}{\sqrt{2^2 + (-3)^2}} = \frac{|-13|}{\sqrt{13}} = \frac{13}{\sqrt{13}} = \sqrt{13}$$

$$\text{Dus } O(\triangle ABC) = \frac{1}{2} \cdot 2\sqrt{13} \cdot \sqrt{13} = 13.$$

$$33 \quad \text{Lijn } l \text{ gaat door } P \text{ en staat loodrecht op } k.$$

$$l: 4x + 3y = c \quad \left\{ \begin{array}{l} c = 4 \cdot -2 + 3 \cdot 0 = -8 \end{array} \right. \text{ door } P(-2, 0)$$

$$\text{Dus } l: 4x + 3y = -8.$$

$$d(P, R) = d(P, k) = \frac{|3 \cdot -2 - 4 \cdot 0 - 24|}{\sqrt{3^2 + (-4)^2}} = \frac{|-30|}{\sqrt{25}} = \frac{30}{5} = 6$$

$$d(Q, S) = d(Q, k) = \frac{|3 \cdot 3 - 4 \cdot 0 - 24|}{\sqrt{3^2 + (-4)^2}} = \frac{|-15|}{\sqrt{25}} = \frac{15}{5} = 3$$

$$d(R, S) = d(Q, l) = \frac{|4 \cdot 3 + 3 \cdot 0 + 8|}{\sqrt{4^2 + 3^2}} = \frac{|20|}{\sqrt{25}} = \frac{20}{5} = 4$$

$$\text{Dus } O(PRSQ) = \frac{1}{2}(6+3) \cdot 4 = 18.$$

**34 a**  $P(p, 0)$  is een punt op de  $x$ -as.

$$d(P, k) = 1 \text{ geeft } \frac{|3 \cdot p + 4 \cdot 0 - 10|}{\sqrt{3^2 + 4^2}} = 1$$

$$\frac{|3p - 10|}{5} = 1$$

$$|3p - 10| = 5$$

$$3p - 10 = 5 \vee 3p - 10 = -5$$

$$3p = 15 \vee 3p = 5$$

$$p = 3 \vee p = 1\frac{2}{3}$$

Dus  $P_1(3, 0)$  en  $P_2(1\frac{2}{3}, 0)$ .

**b**  $l: y = ax + b$  door  $A(0, 4)$  geeft  $b = 4$   
 $l: y = ax + 4$  oftewel  $l: ax - y + 4 = 0$

$$d(B, l) = 5 \text{ geeft } \frac{|5\frac{1}{2}a - 5 + 4|}{\sqrt{a^2 + (-1)^2}} = 5$$

$$|5\frac{1}{2}a - 1| = 5\sqrt{a^2 + 1}$$

kwadrateren geeft

$$30\frac{1}{4}a^2 - 11a + 1 = 25a^2 + 25$$

$$5\frac{1}{4}a^2 - 11a - 24 = 0$$

$$D = (-11)^2 - 4 \cdot 5\frac{1}{4} \cdot -24 = 625, \text{ dus } \sqrt{D} = 25$$

$$a = \frac{11 + 25}{10\frac{1}{2}} = 3\frac{3}{7} \vee a = \frac{11 - 25}{10\frac{1}{2}} = -1\frac{1}{3}$$

Dus  $l_1: y = 3\frac{3}{7}x + 4$  en  $l_2: y = -1\frac{1}{3}x + 4$ .

**35 a**  $P(p, 2p)$  is een punt op  $k$ .

$$d(P, l) = \sqrt{13} \text{ geeft } \frac{|2 \cdot p - 3 \cdot 2p - 5|}{\sqrt{2^2 + (-3)^2}} = \sqrt{13}$$

$$\frac{|-4p - 5|}{\sqrt{13}} = \sqrt{13}$$

$$|-4p - 5| = 13$$

$$-4p - 5 = 13 \vee -4p - 5 = -13$$

$$-4p = 18 \vee -4p = -8$$

$$p = -4\frac{1}{2} \vee p = 2$$

Dus  $P_1(-4\frac{1}{2}, -9)$  en  $P_2(2, 4)$ .

**b**  $m: y = ax + b$  door  $A(3, 0)$  geeft  $3a + b = 0$ , dus  $b = -3a$ .  
 $m: y = ax - 3a$  oftewel  $m: ax - y - 3a = 0$

$$d(B, m) = \sqrt{5} \text{ geeft } \frac{|6a - 4 - 3a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{5}$$

$$|3a - 4| = \sqrt{5} \cdot \sqrt{a^2 + 1}$$

$$|3a - 4| = \sqrt{5a^2 + 5}$$

kwadrateren geeft

$$9a^2 - 24a + 16 = 5a^2 + 5$$

$$4a^2 - 24a + 11 = 0$$

$$D = (-24)^2 - 4 \cdot 4 \cdot 11 = 400, \text{ dus } \sqrt{D} = 20$$

$$a = \frac{24 + 20}{8} = 5\frac{1}{2} \vee a = \frac{24 - 20}{8} = \frac{1}{2}$$

$a = 5\frac{1}{2}$  geeft  $b = -3 \cdot 5\frac{1}{2} = -16\frac{1}{2}$ , dus  $m_1: y = 5\frac{1}{2}x - 16\frac{1}{2}$ .

$a = \frac{1}{2}$  geeft  $b = -3 \cdot \frac{1}{2} = -1\frac{1}{2}$ , dus  $m_2: y = \frac{1}{2}x - 1\frac{1}{2}$ .

**36**  $P(-2, 3)$  is een punt op  $k$ .

$$\begin{aligned} d(P, l) = 3 \text{ geeft } \frac{|5 \cdot -2 + 12 \cdot 3 - c|}{\sqrt{5^2 + 12^2}} &= 3 \\ \frac{|26 - c|}{13} &= 3 \\ |26 - c| &= 39 \\ 26 - c &= 39 \vee 26 - c = -39 \\ c &= -13 \vee c = 65 \end{aligned}$$

Dus  $l_1: 5x + 12y = -13$  en  $l_2: 5x + 12y = 65$ .

**37 a**  $P(4, 0)$  is een punt op  $k$ .

$$d(k, l) = d(P, l) = \frac{|3 \cdot 4 + 4 \cdot 0 - 20|}{\sqrt{3^2 + 4^2}} = \frac{|-8|}{5} = \frac{8}{5} = 1\frac{3}{5}$$

**b**  $m: 3x + 4y = c$

$$\begin{aligned} d(P, m) = 2 \text{ geeft } \frac{|3 \cdot 4 + 4 \cdot 0 - c|}{\sqrt{3^2 + 4^2}} &= 2 \\ \frac{|12 - c|}{5} &= 2 \\ |12 - c| &= 10 \\ 12 - c &= 10 \vee 12 - c = -10 \\ c &= 2 \vee c = 22 \end{aligned}$$

Dus  $m_1: 3x + 4y = 2$  en  $m_2: 3x + 4y = 22$ .

**c**  $A(p, 0)$  is een punt op de  $x$ -as.

$$\begin{aligned} d(A, k) = 3 \text{ geeft } \frac{|3 \cdot p + 4 \cdot 0 - 12|}{\sqrt{3^2 + 4^2}} &= 3 \\ \frac{|3p - 12|}{5} &= 3 \\ |3p - 12| &= 15 \\ 3p - 12 &= 15 \vee 3p - 12 = -15 \\ 3p &= 27 \vee 3p = -3 \\ p &= 9 \vee p = -1 \end{aligned}$$

Dus  $A_1(9, 0)$  en  $A_2(-1, 0)$ .

**38**  $P(p, 2p^2)$  is een punt op de parabool.

$$\begin{aligned} d(P, k) = \sqrt{5} \text{ geeft } \frac{|p - 2 \cdot 2p^2 - 2|}{\sqrt{1^2 + (-2)^2}} &= \sqrt{5} \\ \frac{|-4p^2 + p - 2|}{\sqrt{5}} &= \sqrt{5} \\ |-4p^2 + p - 2| &= 5 \\ -4p^2 + p - 2 &= 5 & \vee -4p^2 + p - 2 &= -5 \\ -4p^2 + p - 7 &= 0 & \vee -4p^2 + p + 3 &= 0 \\ D = 1^2 - 4 \cdot -4 \cdot -7 &= -111 & D = 1^2 - 4 \cdot -4 \cdot 3 &= 49, \text{ dus } \sqrt{D} = 7 \\ \text{geen oplossing} & & p = \frac{-1 + 7}{-8} = -\frac{3}{4} \vee p &= \frac{-1 - 7}{-8} = 1 \end{aligned}$$

Dus  $P_1(-\frac{3}{4}, 1\frac{1}{8})$  en  $P_2(1, 2)$ .

39  $k: ax + by = 1$ 

$$d(A, k) = \sqrt{2} \text{ geeft } \frac{|4a + 0 - 1|}{\sqrt{a^2 + b^2}} = \sqrt{2}$$

$$|4a - 1| = \sqrt{2} \cdot \sqrt{a^2 + b^2}$$

$$d(B, k) = 2\sqrt{2} \text{ geeft } \frac{|6a + 0 - 1|}{\sqrt{a^2 + b^2}} = 2\sqrt{2}$$

$$|6a - 1| = 2\sqrt{2} \cdot \sqrt{a^2 + b^2}$$

Substitutie van  $\sqrt{2} \cdot \sqrt{a^2 + b^2} = |4a - 1|$  in  $|6a - 1| = 2\sqrt{2} \cdot \sqrt{a^2 + b^2}$  geeft

$$|6a - 1| = 2 \cdot |4a - 1|$$

$$|6a - 1| = |8a - 2|$$

$$6a - 1 = 8a - 2 \vee 6a - 1 = -8a + 2$$

$$-2a = -1 \vee 14a = 3$$

$$a = \frac{1}{2} \vee a = \frac{3}{14}$$

$$a = \frac{1}{2} \text{ en } \sqrt{2} \cdot \sqrt{a^2 + b^2} = |4a - 1| \text{ geeft } \sqrt{2} \cdot \sqrt{\frac{1}{4} + b^2} = |2 - 1|$$

$$\sqrt{\frac{1}{2} + 2b^2} = 1$$

$$\frac{1}{2} + 2b^2 = 1$$

$$2b^2 = \frac{1}{2}$$

$$b^2 = \frac{1}{4}$$

$$b = \frac{1}{2} \vee b = -\frac{1}{2}$$

$$a = \frac{3}{14} \text{ en } \sqrt{2} \cdot \sqrt{a^2 + b^2} = |4a - 1| \text{ geeft } \sqrt{2} \cdot \sqrt{\frac{9}{196} + b^2} = |\frac{6}{7} - 1|$$

$$\sqrt{\frac{9}{98} + 2b^2} = \frac{1}{7}$$

$$\frac{9}{98} + 2b^2 = \frac{1}{49}$$

$$2b^2 = -\frac{7}{98}$$

$$b^2 = -\frac{7}{196}$$

geen oplossing

Dus  $k_1: \frac{1}{2}x + \frac{1}{2}y = 1$  en  $k_2: \frac{1}{2}x - \frac{1}{2}y = 1$  oftewel  $k_1: x + y = 2$  en  $k_2: x - y = 2$ .40 a  $m \perp k$  geeft  $m: bx - ay = d$ 

$$\left. \begin{array}{l} m: bx - ay = d \\ \text{door } O(0, 0) \end{array} \right\} d = 0$$

$$\text{Dus } m: bx - ay = 0$$

$$-ay = -bx$$

$$y = \frac{b}{a}x$$

b  $k: ax + by = c$ 

$$by = -ax + c$$

$$y = -\frac{a}{b}x + \frac{c}{b}$$

$$m \text{ snijden met } k \text{ geeft } \frac{b}{a}x = -\frac{a}{b}x + \frac{c}{b}$$

$$b^2x = -a^2x + ac$$

$$a^2x + b^2x = ac$$

$$(a^2 + b^2)x = ac$$

$$x = \frac{ac}{a^2 + b^2}$$

$$x = \frac{ac}{a^2 + b^2} \text{ geeft } y = \frac{b}{a} \cdot \frac{ac}{a^2 + b^2} = \frac{bc}{a^2 + b^2}, \text{ dus } R\left(\frac{ac}{a^2 + b^2}, \frac{bc}{a^2 + b^2}\right).$$

In  $l: ax + by = ax_P + by_P$  is  $c = ax_P + by_P$ .

Gebruik het resultaat hierboven.

$$\text{Zo krijg je } S\left(\frac{a(ax_P + by_P)}{a^2 + b^2}, \frac{b(ax_P + by_P)}{a^2 + b^2}\right) = S\left(\frac{a^2x_P + aby_P}{a^2 + b^2}, \frac{abx_P + b^2y_P}{a^2 + b^2}\right).$$



$$\text{c } d(S, R) = \sqrt{(x_S - x_R)^2 + (y_S - y_R)^2} = \sqrt{\left(\frac{a^2x_P + aby_P - ac}{a^2 + b^2}\right)^2 + \left(\frac{abx_P + b^2y_P - bc}{a^2 + b^2}\right)^2}$$

$$\text{Herleiden geeft } d(S, R) = \sqrt{\frac{(a^2x_P + aby_P - ac)^2}{(a^2 + b^2)^2} + \frac{(abx_P + b^2y_P - bc)^2}{(a^2 + b^2)^2}}$$

$$= \frac{\sqrt{(a^2x_P + aby_P - ac)^2 + (abx_P + b^2y_P - bc)^2}}{\sqrt{(a^2 + b^2)^2}}$$

$$= \frac{\sqrt{(a(ax_P + by_P - c))^2 + (b(ax_P + by_P - c))^2}}{a^2 + b^2}$$

$$\text{d } d(S, R) = \frac{\sqrt{(aE)^2 + (bE)^2}}{a^2 + b^2} = \frac{\sqrt{a^2E^2 + b^2E^2}}{a^2 + b^2} = \frac{\sqrt{(a^2 + b^2)E^2}}{a^2 + b^2} = \frac{\sqrt{a^2 + b^2} \cdot \sqrt{E^2}}{\sqrt{a^2 + b^2} \cdot \sqrt{a^2 + b^2}}$$

$$= \frac{\sqrt{E^2}}{\sqrt{a^2 + b^2}} = \frac{|E|}{\sqrt{a^2 + b^2}}$$

$$\left. \begin{aligned} d(P, k) &= d(S, R) \\ d(S, R) &= \frac{|E|}{\sqrt{a^2 + b^2}} \\ E &= ax_P + by_P - c \end{aligned} \right\} d(P, k) = \frac{|ax_P + by_P - c|}{\sqrt{a^2 + b^2}}$$

#### Bladzijde 109

**C4 a**  $A(2, 7)$  en  $B(3, 3)$  geeft  $d(A, B) = \sqrt{(3 - 2)^2 + (3 - 7)^2} = \sqrt{17}$

**b**  $y_M = 6$  geeft  $\frac{1}{2}(7 + p + 2) = 6$

$$\frac{1}{2}p + 4\frac{1}{2} = 6$$

$$\frac{1}{2}p = 1\frac{1}{2}$$

$$p = 3$$

$A(6, 7)$  en  $B(3, 5)$  geeft  $d(A, B) = \sqrt{(3 - 6)^2 + (5 - 7)^2} = \sqrt{13}$

**c**  $d = \sqrt{(3 - 2p)^2 + (p + 2 - 7)^2}$

$$= \sqrt{(3 - 2p)^2 + (p - 5)^2}$$

$$= \sqrt{9 - 12p + 4p^2 + p^2 - 10p + 25}$$

$$= \sqrt{5p^2 - 22p + 34}$$

**C5 a**  $k \perp l$ , dus  $k: 3x + 2y = c$  }  $c = 3 \cdot -2 + 2 \cdot 5 = 4$   
door  $A(-2, 5)$

$k: 3x + 2y = 4$

$$2y = -3x + 4$$

$$y = -1\frac{1}{2}x + 2$$

**b**  $m \perp n$ , dus  $rc_m \cdot -3 = -1$  en dit geeft  $rc_m = \frac{1}{3}$ .

$m: y = \frac{1}{3}x + b$  }  $\frac{1}{3} \cdot 9 + b = -1$   
door  $B(9, -1)$

$$3 + b = -1$$

$$b = -4$$

$m: y = \frac{1}{3}x - 4$

$$-\frac{1}{3}x + y = -4$$

$$x - 3y = 12$$

**C6** a Lijn  $l$  gaat door  $A$  en staat loodrecht op  $k$ .

$$\left. \begin{array}{l} l: 2x - y = c \\ \text{door } A(4, -3) \end{array} \right\} c = 2 \cdot 4 - (-3) = 11$$

$$\text{Dus } l: 2x - y = 11.$$

$k$  en  $l$  snijden geeft het punt  $A'$ .

$$\left\{ \begin{array}{l} x + 2y = 3 \\ 2x - y = 11 \end{array} \right| \begin{array}{l} 1 \\ 2 \end{array} \text{ geeft } \left\{ \begin{array}{l} x + 2y = 3 \\ 4x - 2y = 22 \end{array} \right. +$$

$$\begin{array}{r} 5x \\ x = 5 \end{array} = 25$$

$$x = 5 \text{ geeft } 5 + 2y = 3$$

$$2y = -2$$

$$y = -1$$

Dus  $A'(5, -1)$ .

$$\text{b } d(A, k) = d(A, A') = \sqrt{(5-4)^2 + (-1-(-3))^2} = \sqrt{5}$$

**C7** a  $k: y = ax + b$  door  $A(-2, 0)$  geeft  $-2a + b = 0$ , dus  $b = 2a$ .

$$k: y = ax + 2a \text{ oftewel } k: ax - y + 2a = 0$$

$$d(B, k) = \sqrt{13} \text{ geeft } \frac{|3a - (-1) + 2a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{13}$$

$$|5a + 1| = \sqrt{13} \cdot \sqrt{a^2 + 1}$$

$$|5a + 1| = \sqrt{13a^2 + 13}$$

kwadrateren geeft

$$25a^2 + 10a + 1 = 13a^2 + 13$$

$$12a^2 + 10a - 12 = 0$$

$$6a^2 + 5a - 6 = 0$$

$$D = 5^2 - 4 \cdot 6 \cdot (-6) = 169, \text{ dus } \sqrt{D} = 13$$

$$a = \frac{-5 \pm 13}{12} = \frac{2}{3} \vee a = \frac{-5 - 13}{12} = -1\frac{1}{2}$$

$$a = \frac{2}{3} \text{ geeft } b = 2 \cdot \frac{2}{3} = 1\frac{1}{3}, \text{ dus } k_1: y = \frac{2}{3}x + 1\frac{1}{3}.$$

$$a = -1\frac{1}{2} \text{ geeft } b = 2 \cdot -1\frac{1}{2} = -3, \text{ dus } k_2: y = -1\frac{1}{2}x - 3.$$

$$\text{b } l: x + 2y = c$$

$P(4, 0)$  is een punt op  $m$ .

$$d(P, l) = \sqrt{5} \text{ geeft } \frac{|4 + 2 \cdot 0 - c|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$$

$$\frac{|4 - c|}{\sqrt{5}} = \sqrt{5}$$

$$|4 - c| = 5$$

$$4 - c = 5 \vee 4 - c = -5$$

$$c = -1 \vee c = 9$$

Dus  $l_1: x + 2y = -1$  en  $l_2: x + 2y = 9$ .

## 7.3 Cirkelvergelijkingen

### Bladzijde 110

**41** a Dit volgt rechtstreeks uit de formule van de afstand tussen twee punten.

b Als de afstand van  $P(x, y)$  tot  $M(1, 4)$  gelijk is aan 5, dan ligt  $P$  op de cirkel met middelpunt  $M$  en straal 5.

c Een vergelijking van de cirkel met middelpunt  $M(1, 4)$  en straal 10 is  $(x-1)^2 + (y-4)^2 = 10^2$  oftewel  $(x-1)^2 + (y-4)^2 = 100$ .

d Middelpunt  $(-2, 3)$  en straal  $\sqrt{16} = 4$ .

### Bladzijde 111

**42** a  $A(5, 3)$  op  $c$  geeft  $(5-3)^2 + (3-4)^2 = r^2$ , dus  $r^2 = 2^2 + (-1)^2 = 5$ .

b \*

43

a  $c_1: (x-2)^2 + (y+5)^2 = 9$

b Middel­punt  $M_2$  en  $c_2$  door  $O$ , dus de straal is  $d(M_2, O) = \sqrt{(0-3)^2 + (0-4)^2} = 5$ .

Dus  $c_2: (x-3)^2 + (y-4)^2 = 25$ .

c Middel­punt  $M_3$  en  $c_3$  door  $A$ , dus de straal is  $d(M_3, A) = \sqrt{(-1-3)^2 + (2-5)^2} = \sqrt{13}$ .

Dus  $c_3: (x+3)^2 + (y-5)^2 = 13$ .

d  $c_4$  raakt de  $y$ -as en  $x_{M_4} = 3$ , dus de straal is 3.

Dus  $c_4: (x-3)^2 + (y-1)^2 = 9$ .

e  $c_5$  raakt de  $x$ -as en  $y_{M_5} = 2$ , dus de straal is 2.

Dus  $c_5: (x+4)^2 + (y-2)^2 = 4$ .

44

$c$  raakt de  $y$ -as en de straal is 10, dus  $x_M = -10$ .

$x = -10$  en  $M$  op  $k$  geeft  $y = -\frac{3}{5} \cdot -10 = 6$ .

Dus  $M(-10, 6)$  en  $c: (x+10)^2 + (y-6)^2 = 100$ .

$d$  raakt de  $x$ -as en de straal is 10, dus  $y_N = -10$ .

$y = -10$  en  $N$  op  $k$  geeft  $-\frac{3}{5}x = -10$

$$x = 16\frac{2}{3}$$

Dus  $N(16\frac{2}{3}, -10)$  en  $d: (x-16\frac{2}{3})^2 + (y+10)^2 = 100$ .

45

a  $c_1$  raakt de  $x$ -as en  $y_A = 7$ , dus de straal is 7.

Dus  $c_1: (x-3)^2 + (y-7)^2 = 49$ .

b Middel­punt  $B$  en  $c_2$  door  $A$ , dus de straal is  $d(A, B) = \sqrt{(9-3)^2 + (1-7)^2} = \sqrt{72}$ .

Dus  $c_2: (x-9)^2 + (y-1)^2 = 72$ .

c Middellijn  $AB$ , dus het middel­punt van  $c_3$  is het midden van  $AB$  oftewel

$$M_3(\frac{1}{2}(3+9), \frac{1}{2}(7+1)) = M_3(6, 4).$$

De straal is  $d(A, M_3) = \sqrt{(6-3)^2 + (4-7)^2} = \sqrt{18}$ .

Dus  $c_3: (x-6)^2 + (y-4)^2 = 18$ .

d  $c_4$  en  $c_5$  raken de  $y$ -as en de straal is 2, dus  $x_M = 2$  of  $x_M = -2$ .

$x = 2$  en  $M$  op  $k$  geeft  $5 \cdot 2 + 2y = 12$

$$10 + 2y = 12$$

$$2y = 2$$

$$y = 1$$

Dus  $M_4(2, 1)$  en  $c_4: (x-2)^2 + (y-1)^2 = 4$ .

$x = -2$  en  $M$  op  $k$  geeft  $5 \cdot -2 + 2y = 12$

$$-10 + 2y = 12$$

$$2y = 22$$

$$y = 11$$

Dus  $M_5(-2, 11)$  en  $c_5: (x+2)^2 + (y-11)^2 = 4$ .

46

$c$  is de cirkel van de doorsnede met middel­punt  $M$  en straal 55.

$$y_M = 85 - 55 = 30$$

Dus  $M(0, 30)$  en  $c: x^2 + (y-30)^2 = 3025$ .

$y_A = 74$  en  $A$  op  $c$  geeft  $x^2 + (74-30)^2 = 3025$

$$x^2 + 1936 = 3025$$

$$x^2 = 1089$$

$$x = 33 \vee x = -33$$

Dus  $A(-33, 74)$ .

$d$  is de cirkel met middel­punt  $A$  en straal  $OA$ .

De straal is  $d(O, A) = \sqrt{(-33-0)^2 + (74-0)^2} = \sqrt{6565}$ .

Dus  $d: (x+33)^2 + (y-74)^2 = 6565$ .

$x_P = 33$  en  $P$  op  $d$  geeft  $(33+33)^2 + (y-74)^2 = 6565$

$$4356 + (y-74)^2 = 6565$$

$$(y-74)^2 = 2209$$

$$y-74 = 47 \vee y-74 = -47$$

$$y = 121 \vee y = 27$$

Dus  $P(33, 27)$ .

$x_Q = 41$  en  $Q$  op  $d$  geeft  $(41+33)^2 + (y-74)^2 = 6565$

$$5476 + (y-74)^2 = 6565$$

$$(y-74)^2 = 1089$$

$$y-74 = 33 \vee y-74 = -33$$

$$y = 107 \vee y = 41$$

Dus  $Q(41, 41)$ .

$d(P, Q) = \sqrt{(41-33)^2 + (41-27)^2} = \sqrt{260} \approx 16,1$

Dus de toeschouwers  $P$  en  $Q$  zitten 16,1 meter uit elkaar.

47

**a** Middellijn  $QR$ , dus het middelpunt  $M$  van  $c$  is het midden van  $QR$  oftewel

$$M(\frac{1}{2}(-2+5), \frac{1}{2}(6+7)) = M(1\frac{1}{2}, 6\frac{1}{2}).$$

$$\text{De straal is } d(Q, M) = \sqrt{(1\frac{1}{2} - -2)^2 + (6\frac{1}{2} - 6)^2} = \sqrt{12\frac{1}{2}}.$$

$$\text{Dus } c: (x - 1\frac{1}{2})^2 + (y - 6\frac{1}{2})^2 = 12\frac{1}{2}.$$

$$P(-1, p) \text{ op } c \text{ geeft } (-2\frac{1}{2})^2 + (p - 6\frac{1}{2})^2 = 12\frac{1}{2}$$

$$6\frac{1}{4} + (p - 6\frac{1}{2})^2 = 12\frac{1}{2}$$

$$(p - 6\frac{1}{2})^2 = 6\frac{1}{4}$$

$$p - 6\frac{1}{2} = 2\frac{1}{2} \vee p - 6\frac{1}{2} = -2\frac{1}{2}$$

$$p = 9 \vee p = 4$$

**b** Middellijn  $PR$ , dus het middelpunt  $M$  van  $c$  is het midden van  $PR$  oftewel

$$M(\frac{1}{2}(-1+5), \frac{1}{2}(p+7)) = M(2, \frac{1}{2}p + 3\frac{1}{2}).$$

$$\text{De straal is } d(R, M) = \sqrt{(2-5)^2 + (\frac{1}{2}p + 3\frac{1}{2} - 7)^2}$$

$$= \sqrt{9 + (\frac{1}{2}p - 3\frac{1}{2})^2}$$

$$= \sqrt{9 + \frac{1}{4}p^2 - 3\frac{1}{2}p + 12\frac{1}{4}}$$

$$= \sqrt{\frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}}$$

$$\text{Dus } c: (x-2)^2 + (y - \frac{1}{2}p - 3\frac{1}{2})^2 = \frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}.$$

$$Q(-2, 6) \text{ op } c \text{ geeft } (-4)^2 + (2\frac{1}{2} - \frac{1}{2}p)^2 = \frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}$$

$$16 + 6\frac{1}{4} - 2\frac{1}{2}p + \frac{1}{4}p^2 = \frac{1}{4}p^2 - 3\frac{1}{2}p + 21\frac{1}{4}$$

$$p = -1$$

48

$$c: (x-p)^2 + (y-q)^2 = 13$$

$$A(4, -1) \text{ op } c \text{ geeft } (4-p)^2 + (-1-q)^2 = 13$$

$$B(-1, -2) \text{ op } c \text{ geeft } (-1-p)^2 + (-2-q)^2 = 13$$

$$\text{Hieruit volgt } (4-p)^2 + (-1-q)^2 = (-1-p)^2 + (-2-q)^2$$

$$16 - 8p + p^2 + 1 + 2q + q^2 = 1 + 2p + p^2 + 4 + 4q + q^2$$

$$-2q = 10p - 12$$

$$q = -5p + 6$$

$$\text{Substitutie van } q = -5p + 6 \text{ in } (4-p)^2 + (-1-q)^2 = 13 \text{ geeft}$$

$$(4-p)^2 + (5p-7)^2 = 13$$

$$16 - 8p + p^2 + 25p^2 - 70p + 49 - 13 = 0$$

$$26p^2 - 78p + 52 = 0$$

$$p^2 - 3p + 2 = 0$$

$$(p-1)(p-2) = 0$$

$$p = 1 \vee p = 2$$

$$p = 1 \text{ geeft } q = -5 \cdot 1 + 6 = 1 \text{ en } p = 2 \text{ geeft } q = -5 \cdot 2 + 6 = -4.$$

$$\text{Dus } p = 1 \text{ en } q = 1 \text{ of } p = 2 \text{ en } q = -4.$$

49

**a**  $k: y = x$  oftewel  $k: x - y = 0$

$$r = d(M, k) = \frac{|5-3|}{\sqrt{1^2+(-1)^2}} = \frac{|2|}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\text{Dus } c: (x-5)^2 + (y-3)^2 = 2.$$

**b**  $r = d(N, m) = \frac{|-1+2 \cdot 3|}{\sqrt{1^2+2^2}} = \frac{|5|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

$$\text{Dus } d: (x+1)^2 + (y-3)^2 = 5.$$

50

$$\mathbf{a} \quad r = d(M_1, k) = \frac{|2 \cdot 8 + 3 \cdot 3 - 12|}{\sqrt{2^2 + 3^2}} = \frac{|13|}{\sqrt{13}} = \frac{13}{\sqrt{13}} = \sqrt{13}$$

$$\text{Dus } c_1: (x-8)^2 + (y-3)^2 = 13.$$

$\mathbf{b} \quad c_2$  en  $c_3$  raken de  $x$ -as en de straal is 8, dus  $y_M = 8$  of  $y_M = -8$ .

$$y = 8 \text{ en } M \text{ op } k \text{ geeft } 2x + 3 \cdot 8 = 12$$

$$2x + 24 = 12$$

$$2x = -12$$

$$x = -6$$

$$\text{Dus } M_2(-6, 8) \text{ en } c_2: (x+6)^2 + (y-8)^2 = 64.$$

$$y = -8 \text{ en } M \text{ op } k \text{ geeft } 2x + 3 \cdot -8 = 12$$

$$2x - 24 = 12$$

$$2x = 36$$

$$x = 18$$

$$\text{Dus } M_3(18, -8) \text{ en } c_3: (x-18)^2 + (y+8)^2 = 64.$$

$\mathbf{c} \quad k$  snijden met de  $x$ -as geeft  $A(6, 0)$  en  $k$  snijden met de  $y$ -as geeft  $B(0, 4)$ .

Middellijn  $AB$ , dus het middelpunt van  $c_4$  is het midden van  $AB$  oftewel

$$M_4\left(\frac{1}{2}(6+0), \frac{1}{2}(0+4)\right) = M_4(3, 2).$$

$$\text{De straal is } d(A, M_4) = \sqrt{(3-6)^2 + (2-0)^2} = \sqrt{13}.$$

$$\text{Dus } c_4: (x-3)^2 + (y-2)^2 = 13.$$

51

$\mathbf{a} \quad k: y = \frac{1}{3}x$  oftewel  $k: \frac{1}{3}x - y = 0$ , dus  $k: 4x - 3y = 0$ .

$$r = d(M, k) = \frac{|4 \cdot 4 - 3 \cdot 2|}{\sqrt{4^2 + (-3)^2}} = \frac{|10|}{5} = \frac{10}{5} = 2$$

$y_M = 2$  en  $r = 2$ , dus  $c$  raakt de  $x$ -as.

$\mathbf{b} \quad$  Zie de figuur hiernaast.

$OA = OB = 4$  (twee raaklijnen aan een cirkel vanuit punt), dus

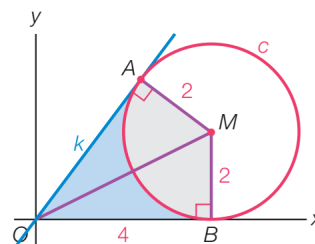
$$O(\triangle OAM) = O(\triangle OBM).$$

$$\tan(\angle OMB) = \frac{4}{2} \text{ geeft } \angle OMB = 63,43\dots^\circ$$

$$\angle AMB = 2 \cdot \angle OMB = 2 \cdot 63,43\dots^\circ = 126,86\dots^\circ$$

$$O(\text{blauwe gebied}) = O(\triangle OBM) - O(\text{grijze cirkelsector})$$

$$= 2 \cdot \frac{1}{2} \cdot 4 \cdot 2 - \frac{126,86\dots}{360} \cdot \pi \cdot 2^2 \approx 3,57$$



52

$$\mathbf{a} \quad \text{De straal van } c \text{ is } d(M, k) = \frac{|-1 - 2 \cdot 2 - 10|}{\sqrt{1^2 + (-2)^2}} = \frac{|-15|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}.$$

$$\text{Dus } c: (x+1)^2 + (y-2)^2 = (3\sqrt{5})^2 \text{ oftewel } c: (x+1)^2 + (y-2)^2 = 45.$$

$k$  snijden met de  $x$ -as geeft  $A(10, 0)$ .

Lijn  $l$  gaat door  $M$  en staat loodrecht op  $k$ .

$$\left. \begin{array}{l} l: 2x + y = m \\ \text{door } M(-1, 2) \end{array} \right\} m = 2 \cdot -1 + 2 = 0$$

$$\text{Dus } l: 2x + y = 0 \text{ oftewel } l: y = -2x.$$

$k$  en  $l$  snijden geeft het punt  $B$ .

$$x - 2y = 10 \text{ en } y = -2x \text{ geeft } x - 2 \cdot -2x = 10$$

$$5x = 10$$

$$x = 2$$

$$x = 2 \text{ geeft } y = -2 \cdot 2 = -4, \text{ dus } B(2, -4).$$

$$\text{De straal van } d \text{ is } d(A, B) = \sqrt{(2-10)^2 + (-4-0)^2} = \sqrt{80}.$$

$$\text{Dus } d: (x-10)^2 + y^2 = 80.$$

$\mathbf{b} \quad$  Middellijn  $AM$ , dus het middelpunt van  $e$  is het midden  $C$  van  $AM$  oftewel

$$C\left(\frac{1}{2}(10+1), \frac{1}{2}(0+2)\right) = C\left(4\frac{1}{2}, 1\right).$$

$$\text{De straal van } e \text{ is } d(C, A) = \sqrt{\left(10-4\frac{1}{2}\right)^2 + (0-1)^2} = \sqrt{31\frac{1}{4}}.$$

$$d(C, B) = \sqrt{\left(2-4\frac{1}{2}\right)^2 + (-4-1)^2} = \sqrt{31\frac{1}{4}}$$

$$d(C, B) = d(C, A), \text{ dus } e \text{ gaat door } B.$$

- 53** De lijnen  $l: 3x - 4y = c$  liggen op afstand 5 van  $k$ .

$$d(A, l) = 5 \text{ geeft } \frac{|3 \cdot 2 - 4 \cdot -1 - c|}{\sqrt{3^2 + (-4)^2}} = 5$$

$$\frac{|10 - c|}{5} = 5$$

$$|10 - c| = 25$$

$$10 - c = 25 \vee 10 - c = -25$$

$$c = -15 \vee c = 35$$

Dus  $l_1: 3x - 4y = -15$  en  $l_2: 3x - 4y = 35$ .

De middelpunten van de cirkels zijn de snijpunten van  $l_1$  en  $l_2$  met de lijn  $m$  door  $A$  die loodrecht staat op  $k$ .

$m \perp k$ , dus  $m: 4x + 3y = d$  }  $d = 4 \cdot 2 + 3 \cdot -1 = 5$   
door  $A(2, -1)$

Dus  $m: 4x + 3y = 5$ .

$l_1$  en  $m$  snijden geeft het eerste middelpunt.

$$\begin{cases} 3x - 4y = -15 \\ 4x + 3y = 5 \end{cases} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \text{ geeft } \begin{cases} 9x - 12y = -45 \\ 16x + 12y = 20 \end{cases} +$$

$$\frac{25x}{x} = -25$$

$$x = -1$$

$x = -1$  geeft  $3 \cdot -1 - 4y = -15$

$$-3 - 4y = -15$$

$$-4y = -12$$

$$y = 3$$

Dus het eerste middelpunt is  $(-1, 3)$ .

Dus de vergelijkingen van deze cirkels zijn  $(x + 1)^2 + (y - 3)^2 = 25$  en  $(x - 5)^2 + (y + 5)^2 = 25$ .

$l_2$  en  $m$  snijden geeft het tweede middelpunt

$$\begin{cases} 3x - 4y = 35 \\ 4x + 3y = 5 \end{cases} \begin{vmatrix} 3 \\ 4 \end{vmatrix} \text{ geeft } \begin{cases} 9x - 12y = 105 \\ 16x + 12y = 20 \end{cases} +$$

$$\frac{25x}{x} = 125$$

$$x = 5$$

$x = 5$  geeft  $3 \cdot 5 - 4y = 35$

$$15 - 4y = 35$$

$$-4y = 20$$

$$y = -5$$

Dus het tweede middelpunt is  $(5, -5)$ .

- 54**  $c: (x - 4)^2 + (y + 1)^2 = 36$

Haakjes wegwerken geeft  $c: x^2 - 8x + 16 + y^2 + 2y + 1 = 36$  oftewel  $c: x^2 + y^2 - 8x + 2y - 19 = 0$ .

### Bladzijde 115

- 55 a**  $x^2 + y^2 + 6x - 4y + 4 = 0$

$$x^2 + 6x + y^2 - 4y + 4 = 0$$

$$(x + 3)^2 - 9 + (y - 2)^2 - 4 + 4 = 0$$

$$(x + 3)^2 + (y - 2)^2 = 9$$

Dus van  $c_1$  is de straal 3 en het middelpunt  $(-3, 2)$ .

- b**  $x^2 + y^2 - 8x + 10y + 31 = 0$

$$x^2 - 8x + y^2 + 10y + 31 = 0$$

$$(x - 4)^2 - 16 + (y + 5)^2 - 25 + 31 = 0$$

$$(x - 4)^2 + (y + 5)^2 = 10$$

Dus van  $c_2$  is de straal  $\sqrt{10}$  en het middelpunt  $(4, -5)$ .

- c**  $x^2 + y^2 + 5x + 3y + 3 = 0$

$$x^2 + 5x + y^2 + 3y + 3 = 0$$

$$(x + 2\frac{1}{2})^2 - 6\frac{1}{4} + (y + 1\frac{1}{2})^2 - 2\frac{1}{4} + 3 = 0$$

$$(x + 2\frac{1}{2})^2 + (y + 1\frac{1}{2})^2 = 5\frac{1}{2}$$

Dus van  $c_3$  is de straal  $\sqrt{5\frac{1}{2}}$  en het middelpunt  $(-2\frac{1}{2}, -1\frac{1}{2})$ .

- d**  $x^2 + y^2 - 7x + 8y = 0$

$$x^2 - 7x + y^2 + 8y = 0$$

$$(x - 3\frac{1}{2})^2 - 12\frac{1}{4} + (y + 4)^2 - 16 = 0$$

$$(x - 3\frac{1}{2})^2 + (y + 4)^2 = 28\frac{1}{4}$$

Dus van  $c_4$  is de straal  $\sqrt{28\frac{1}{4}}$  en het middelpunt  $(3\frac{1}{2}, -4)$ .

- 56 a**  $x^2 + y^2 + 6x - 8y + 15 = 0$

$$x^2 + 6x + y^2 - 8y + 15 = 0$$

$$(x + 3)^2 - 9 + (y - 4)^2 - 16 + 15 = 0$$

$$(x + 3)^2 + (y - 4)^2 = 10$$

Dus  $M(-3, 4)$  en  $r = \sqrt{10}$ .

- b**  $d(A, M) = \sqrt{(-3 - -1)^2 + (4 - 5)^2} = \sqrt{5}$

$A$  ligt binnen  $c$ , want  $d(A, M) < r$ .

- c**  $d(B, M) = \sqrt{(-3 - -4)^2 + (4 - 0)^2} = \sqrt{17}$

$B$  ligt buiten  $c$ , want  $d(B, M) > r$ .

- d**  $d(C, M) = \sqrt{(-3 - -2)^2 + (4 - 1)^2} = \sqrt{10}$

$C$  ligt op  $c$ , want  $d(C, M) = r$ .

57

- a**  $x^2 + y^2 - 6x - 8y = 0$   
 $x^2 - 6x + y^2 - 8y = 0$   
 $(x-3)^2 - 9 + (y-4)^2 - 16 = 0$   
 $(x-3)^2 + (y-4)^2 = 25$   
 Dus van  $c_1$  is de straal  $r_1 = 5$  en het middelpunt  $M_1(3, 4)$ .

$$d(A, M_1) = \sqrt{(3-1)^2 + (4-2)^2} = \sqrt{20}$$

$d(A, M_1) < r_1$ , dus  $A$  ligt binnen  $c_1$ .

- b**  $x^2 + y^2 - 8x - 4y + 2 = 0$   
 $x^2 - 8x + y^2 - 4y + 2 = 0$   
 $(x-4)^2 - 16 + (y-2)^2 - 4 + 2 = 0$   
 $(x-4)^2 + (y-2)^2 = 18$   
 Dus van  $c_2$  is de straal  $r_2 = \sqrt{18}$  en het middelpunt  $M_2(4, 2)$ .

$$d(O, M_2) = \sqrt{(4-0)^2 + (2-0)^2} = \sqrt{20}$$

$d(O, M_2) > r_2$ , dus  $O$  ligt buiten  $c_2$ .

- c**  $x^2 + y^2 + 4x - 6y + 3 = 0$   
 $x^2 + 4x + y^2 - 6y + 3 = 0$   
 $(x+2)^2 - 4 + (y-3)^2 - 9 + 3 = 0$   
 $(x+2)^2 + (y-3)^2 = 10$   
 Dus van  $c_3$  is de straal  $r_3 = \sqrt{10}$  en het middelpunt  $M_3(-2, 3)$ .

$$d(B, M_3) = \sqrt{(-2-1)^2 + (3-4)^2} = \sqrt{10}$$

$d(B, M_3) = r_3$ , dus  $B$  ligt op  $c_3$ .

58

- a**  $x^2 + y^2 + 4x - 6y - 12 = 0$   
 $x^2 + 4x + y^2 - 6y - 12 = 0$   
 $(x+2)^2 - 4 + (y-3)^2 - 9 - 12 = 0$   
 $(x+2)^2 + (y-3)^2 = 25$   
 Dus van  $c_1$  is de straal  $r_1 = 5$  en het middelpunt  $M_1(-2, 3)$ .

$$d(P, M_1) = \sqrt{(-2-1)^2 + (3-p)^2} = \sqrt{9 + (3-p)^2}$$

$$\begin{aligned} d(P, M_1) = r_1 \text{ geeft } \sqrt{9 + (3-p)^2} &= 5 \\ 9 + (3-p)^2 &= 25 \\ (3-p)^2 &= 16 \\ 3-p &= 4 \vee 3-p = -4 \\ p &= -1 \vee p = 7 \end{aligned}$$

Zie de figuur hiernaast.

$$d(P, M_1) < 5 \text{ geeft } -1 < p < 7$$

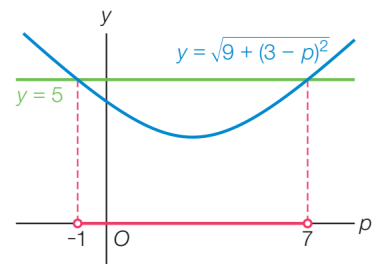
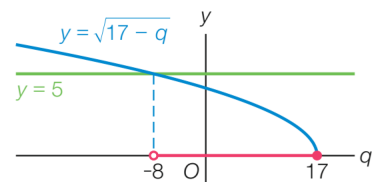
- b**  $x^2 + y^2 - 8x - 2y + q = 0$   
 $x^2 - 8x + y^2 - 2y + q = 0$   
 $(x-4)^2 - 16 + (y-1)^2 - 1 + q = 0$   
 $(x-4)^2 + (y-1)^2 = 17 - q$   
 Dus van  $c_2$  is de straal  $r_2 = \sqrt{17-q}$  en het middelpunt  $M_2(4, 1)$ .

$$d(Q, M_2) = \sqrt{(4-1)^2 + (1-5)^2} = \sqrt{25} = 5$$

$$\begin{aligned} d(Q, M_2) = r_2 \text{ geeft } \sqrt{17-q} &= 5 \\ 17-q &= 25 \\ q &= -8 \end{aligned}$$

Zie de figuur hiernaast.

$$d(Q, M_2) > r_2 \text{ geeft } 5 > \sqrt{17-q} \text{ oftewel } -8 < q \leq 17$$



$$x^2 + y^2 + ax + by = 71$$

$$x^2 + ax + y^2 + by = 71$$

$$(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + (y + \frac{1}{2}b)^2 - \frac{1}{4}b^2 = 71$$

$$(x + \frac{1}{2}a)^2 + (y + \frac{1}{2}b)^2 = \frac{1}{4}a^2 + \frac{1}{4}b^2 + 71$$

Dus de straal is  $r = \sqrt{\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71}$  en het middelpunt  $M(-\frac{1}{2}a, -\frac{1}{2}b)$ .

$$M(-\frac{1}{2}a, -\frac{1}{2}b) \text{ op } k: x + 2y = 1 \text{ geeft } -\frac{1}{2}a - b = 1$$

$$b = -\frac{1}{2}a - 1$$

$$r = 10 \text{ geeft } \sqrt{\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71} = 10$$

$$\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71 = 100$$

Substitueren van  $b = -\frac{1}{2}a - 1$  in  $\frac{1}{4}a^2 + \frac{1}{4}b^2 + 71 = 100$  geeft  $\frac{1}{4}a^2 + \frac{1}{4}(-\frac{1}{2}a - 1)^2 + 71 = 100$

$$\frac{1}{4}a^2 + \frac{1}{4}(\frac{1}{4}a^2 + a + 1) + 71 = 100$$

$$\frac{1}{4}a^2 + \frac{1}{16}a^2 + \frac{1}{4}a + \frac{1}{4} + 71 = 100$$

$$\frac{5}{16}a^2 + \frac{1}{4}a - 28\frac{3}{4} = 0$$

$$5a^2 + 4a - 460 = 0$$

$$D = 4^2 - 4 \cdot 5 \cdot -460 = 9216, \text{ dus } \sqrt{D} = 96$$

$$a = \frac{-4 \pm 96}{10} = 9\frac{1}{5} \vee a = \frac{-4 - 96}{10} = -10$$

$$a = 9\frac{1}{5} \text{ geeft } b = -\frac{1}{2} \cdot 9\frac{1}{5} - 1 = -5\frac{3}{5} \text{ en } a = -10 \text{ geeft } b = -\frac{1}{2} \cdot -10 - 1 = 4$$

Dus  $a = 9\frac{1}{5}$  en  $b = -5\frac{3}{5}$  of  $a = -10$  en  $b = 4$ .

### Bladzijde 116

C8

**a**  $c_1: (x + 1)^2 + (y - 4)^2 = 64$

**b**  $c_2$  raakt de  $y$ -as en  $x_B = 7$ , dus de straal is 7.

$$\text{Dus } c_2: (x - 7)^2 + (y - 2)^2 = 49.$$

**c** Middellijn  $AB$ , dus het middelpunt van  $c_3$  is het midden van  $AB$  oftewel

$$M_3(\frac{1}{2}(-1 + 7), \frac{1}{2}(4 + 2)) = M_3(3, 3).$$

$$\text{De straal is } d(A, M_3) = \sqrt{(3 - (-1))^2 + (3 - 4)^2} = \sqrt{17}.$$

$$\text{Dus } c_3: (x - 3)^2 + (y - 3)^2 = 17.$$

**d**  $c_4$  en  $c_5$  raken de  $x$ -as en de straal is 3, dus  $y_M = 3$  of  $y_M = -3$ .

$$y = 3 \text{ en } M \text{ op } k \text{ geeft } \frac{1}{2}x = 3$$

$$x = 2$$

$$\text{Dus } M_4(2, 3) \text{ en } c_4: (x - 2)^2 + (y - 3)^2 = 9.$$

$$y = -3 \text{ en } M \text{ op } k \text{ geeft } \frac{1}{2}x = -3$$

$$x = -2$$

$$\text{Dus } M_5(-2, -3) \text{ en } c_5: (x + 2)^2 + (y + 3)^2 = 9.$$

C9

**a**  $x^2 + y^2 + 10x - 6y - 2 = 0$

$$x^2 + 10x + y^2 - 6y - 2 = 0$$

$$(x + 5)^2 - 25 + (y - 3)^2 - 9 - 2 = 0$$

$$(x + 5)^2 + (y - 3)^2 = 36$$

$$\text{Dus } r = 6 \text{ en } M(-5, 3).$$

**b**  $x^2 + y^2 - 2x + 8y - 7 = 0$

$$x^2 - 2x + y^2 + 8y - 7 = 0$$

$$(x - 1)^2 - 1 + (y + 4)^2 - 16 - 7 = 0$$

$$(x - 1)^2 + (y + 4)^2 = 24$$

Dus van  $d$  is de straal  $r_d = \sqrt{24}$  en het middelpunt  $M_d(1, -4)$ .

$$d(A, M_d) = \sqrt{(1 - 5)^2 + (-4 - (-1))^2} = \sqrt{25} = 5$$

$d(A, M_d) > r_d$ , dus  $A$  ligt buiten  $d$ .



## 7.4 Afstanden en raaklijnen bij cirkels

### Bladzijde 117

- 60 a**  $x^2 + y^2 - 10x - 6y + 18 = 0$   
 $x^2 - 10x + y^2 - 6y + 18 = 0$   
 $(x - 5)^2 - 25 + (y - 3)^2 - 9 + 18 = 0$   
 $(x - 5)^2 + (y - 3)^2 = 16$   
Dus van  $c$  is de straal  $r = 4$  en het middelpunt  $M(5, 3)$ .  
 $d(P, M) = \sqrt{(5 - 1)^2 + (3 - 6)^2} = \sqrt{25} = 5$   
 $d(P, M) > r$ , dus  $P$  ligt buiten  $c$ .  
 $d(Q, M) = \sqrt{(5 - 6)^2 + (3 - (-2))^2} = \sqrt{26}$   
 $d(Q, M) > r$ , dus  $Q$  ligt buiten  $c$ .
- b**  $5 < \sqrt{26}$ , dus  $P$  ligt dichterbij het middelpunt  $M$  van cirkel  $c$  dan  $Q$ .  
Dus  $d(P, c) < d(Q, c)$ .

### Bladzijde 119

- 61 a**  $x^2 + y^2 - 6x - 4y + 3 = 0$   
 $x^2 - 6x + y^2 - 4y + 3 = 0$   
 $(x - 3)^2 - 9 + (y - 2)^2 - 4 + 3 = 0$   
 $(x - 3)^2 + (y - 2)^2 = 10$   
Dus van  $c$  is de straal  $r = \sqrt{10}$  en het middelpunt  $M(3, 2)$ .  
 $d(A, M) = \sqrt{(3 - 2)^2 + (2 - 1)^2} = \sqrt{2}$   
Dus  $d(A, c) = \sqrt{10} - \sqrt{2}$ .
- b**  $d(B, M) = \sqrt{(3 - (-1))^2 + (2 - 5)^2} = \sqrt{25} = 5$   
Dus  $d(B, c) = 5 - \sqrt{10}$ .
- c**  $d(C, M) = \sqrt{(3 - 9)^2 + (2 - 4)^2} = \sqrt{40} = 2\sqrt{10}$   
Dus  $d(C, c) = 2\sqrt{10} - \sqrt{10} = \sqrt{10}$ .

- 62**  $M$  op  $k$  geeft  $2x + 3 \cdot 6 = 18$   
 $2x + 18 = 18$   
 $2x = 0$   
 $x = 0$   
Dus  $M(0, 6)$  en  $r_1 = 6$ .  
 $N$  op  $k$  geeft  $2 \cdot 12 + 3y = 18$   
 $24 + 3y = 18$   
 $3y = -6$   
 $y = -2$   
Dus  $N(12, -2)$  en  $r_2 = 2$ .  
 $d(M, N) = \sqrt{(12 - 0)^2 + (-2 - 6)^2} = \sqrt{208} = 4\sqrt{13}$   
Dus  $d(c_1, c_2) = 4\sqrt{13} - 6 - 2 = 4\sqrt{13} - 8$ .

63

$$\mathbf{a} \quad x^2 + y^2 - 6x - 2y + 9 = 0$$

$$x^2 - 6x + y^2 - 2y + 9 = 0$$

$$(x-3)^2 - 9 + (y-1)^2 - 1 + 9 = 0$$

$$(x-3)^2 + (y-1)^2 = 1$$

Dus  $M(3, 1)$  en  $r_1 = 1$ .

$$x^2 + y^2 - 12x - 4y + 36 = 0$$

$$x^2 - 12x + y^2 - 4y + 36 = 0$$

$$(x-6)^2 - 36 + (y-2)^2 - 4 + 36 = 0$$

$$(x-6)^2 + (y-2)^2 = 4$$

Dus  $N(6, 2)$  en  $r_2 = 2$ .

$$d(M, N) = \sqrt{(6-3)^2 + (2-1)^2} = \sqrt{10}$$

$$\text{Dus } d(c_1, c_2) = \sqrt{10} - 1 - 2 = \sqrt{10} - 3.$$

$$\mathbf{b} \quad d(M, l) = \frac{|3 \cdot 3 - 4 \cdot 1|}{\sqrt{3^2 + (-4)^2}} = \frac{|5|}{5} = \frac{5}{5} = 1, \text{ dus } d(M, l) = r_1, \text{ dus } l \text{ raakt } c_1.$$

$$d(N, l) = \frac{|3 \cdot 6 - 4 \cdot 2|}{\sqrt{3^2 + (-4)^2}} = \frac{|10|}{5} = \frac{10}{5} = 2, \text{ dus } d(N, l) = r_2, \text{ dus } l \text{ raakt } c_2.$$

$$\mathbf{c} \quad r_3 = d(P, l) = \frac{|3 \cdot 2 - 4 \cdot 5|}{\sqrt{3^2 + (-4)^2}} = \frac{|-14|}{5} = \frac{14}{5} = 2\frac{4}{5}$$

$$\text{Dus } c_3: (x-2)^2 + (y-5)^2 = 7\frac{21}{25}.$$

64

$$x^2 + y^2 - 12x + 27 = 0$$

$$x^2 + (y-6)^2 - 36 + 27 = 0$$

$$x^2 + (y-6)^2 = 9$$

Dus  $M(0, 6)$  en  $r_1 = 3$ .

$$x^2 + y^2 - 20x + 96 = 0$$

$$x^2 - 20x + y^2 + 96 = 0$$

$$(x-10)^2 - 100 + y^2 + 96 = 0$$

$$(x-10)^2 + y^2 = 4$$

Dus  $N(10, 0)$  en  $r_2 = 2$ .

$P(2p, p)$  is een punt op  $k$ .

$$d(M, P) = \sqrt{(2p-0)^2 + (p-6)^2} = \sqrt{4p^2 + p^2 - 12p + 36} = \sqrt{5p^2 - 12p + 36}$$

$$d(P, c_1) = \sqrt{5p^2 - 12p + 36} - 3$$

$$d(N, P) = \sqrt{(2p-10)^2 + (p-0)^2} = \sqrt{4p^2 - 40p + 100 + p^2} = \sqrt{5p^2 - 40p + 100}$$

$$d(P, c_2) = \sqrt{5p^2 - 40p + 100} - 2$$

Invoeren van de formules  $y = \sqrt{5x^2 - 12x + 36} - 3$  en  $y = \sqrt{5x^2 - 40x + 100} - 2$  en gebruiken van de optie snijpunt geeft  $x = 2,702\dots$  en  $y = 3,331\dots$

Dus  $p = 2,702\dots$ ,  $P(5,40; 2,70)$  en  $d(P, c_1) = d(P, c_2) \approx 3,33$ .

65

$$x^2 + y^2 + 4x - 2y - 4 = 0$$

$$x^2 + 4x + y^2 - 2y - 4 = 0$$

$$(x+2)^2 - 4 + (y-1)^2 - 1 - 4 = 0$$

$$(x+2)^2 + (y-1)^2 = 9$$

Dus  $M(-2, 1)$  en  $r = 3$ .

$$d(M, k) = \frac{|2 \cdot -2 + 1 - 7|}{\sqrt{2^2 + 1^2}} = \frac{|-10|}{\sqrt{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5}$$

$$\text{Dus } d(k, c) = 2\sqrt{5} - 3.$$

66

$$x^2 + y^2 + 8x + 2y - 8 = 0$$

$$x^2 + 8x + y^2 + 2y - 8 = 0$$

$$(x+4)^2 - 16 + (y+1)^2 - 1 - 8 = 0$$

$$(x+4)^2 + (y+1)^2 = 25$$

Dus  $M(-4, -1)$  en  $r = 5$ .

$$l: y = -3x + 17 \text{ oftewel } l: 3x + y - 17 = 0$$

$$d(M, l) = \frac{|3 \cdot -4 - 1 - 17|}{\sqrt{3^2 + 1^2}} = \frac{|-30|}{\sqrt{10}} = \frac{30}{\sqrt{10}} = 3\sqrt{10}$$

$$d(l, c) = 3\sqrt{10} - 5 = 4,48\dots$$

Dus  $d(l, c) < r$ .

67

$$a \quad x^2 + y^2 - 8x - 6y - 20 = 0$$

$$x^2 - 8x + y^2 - 6y - 20 = 0$$

$$(x-4)^2 - 16 + (y-3)^2 - 9 - 20 = 0$$

$$(x-4)^2 + (y-3)^2 = 45$$

$$\text{Dus } M(4, 3) \text{ en } r_c = \sqrt{45} = 3\sqrt{5}.$$

$$c \text{ snijden met de } x\text{-as geeft } x^2 - 8x - 20 = 0$$

$$(x+2)(x-10) = 0$$

$$x = -2 \vee x = 10$$

Dus  $A(10, 0)$ .

$$k \text{ door } M \text{ en } A \text{ heeft } rc_k = \frac{0-3}{10-4} = -\frac{1}{2}.$$

$$k: y = -\frac{1}{2}x + b \quad \left\{ \begin{array}{l} -\frac{1}{2} \cdot 10 + b = 0 \\ -5 + b = 0 \end{array} \right.$$

$$b = 5$$

$$\text{Dus } k: y = -\frac{1}{2}x + 5 \text{ oftewel } k: \frac{1}{2}x + y - 5 = 0, \text{ dus } k: x + 2y - 10 = 0.$$

$$d(N, k) = \frac{|3 + 2 \cdot -1 - 10|}{\sqrt{1^2 + 2^2}} = \frac{|-9|}{\sqrt{5}} = \frac{9}{\sqrt{5}} = 1\frac{4}{5}\sqrt{5}$$

$$\text{Dus } d(k, d) = 1\frac{4}{5}\sqrt{5} - \sqrt{5} = \frac{4}{5}\sqrt{5}.$$

$$b \quad d(M, N) = \sqrt{(3-4)^2 + (-1-3)^2} = \sqrt{17}$$

$$\text{Dus } d(c, d) = 3\sqrt{5} - \sqrt{5} - \sqrt{17} = 2\sqrt{5} - \sqrt{17}.$$

68

$$a \quad \text{Lijn } l \text{ door } M(2, 1) \text{ en } A \text{ heeft } rc_l = \frac{2-1}{5-2} = \frac{1}{3}.$$

$$b \quad \text{Omdat geldt dat } k \perp l, \text{ kun je } rc_k \text{ berekenen met } rc_k \cdot \frac{1}{3} = -1.$$

$$c \quad k \perp l, \text{ dus } rc_k \cdot \frac{1}{3} = -1 \text{ en dit geeft } rc_k = -3.$$

$$k: y = -3x + b \quad \left\{ \begin{array}{l} -3 \cdot 5 + b = 2 \\ -15 + b = 2 \end{array} \right.$$

$$b = 17$$

$$\text{Dus } k: y = -3x + 17.$$

$$x^2 + y^2 - 6x + 2y + 5 = 0$$

$$x^2 - 6x + y^2 + 2y + 5 = 0$$

$$(x-3)^2 - 9 + (y+1)^2 - 1 + 5 = 0$$

$$(x-3)^2 + (y+1)^2 = 5$$

$$\text{Dus } N(3, -1) \text{ en } r_d = \sqrt{5}.$$

**69 a** Lijn  $n$  door  $M$  en  $B$  heeft  $rc_n = \frac{-1-1}{2-3} = 2$ .  
 $m \perp n$ , dus  $rc_m \cdot 2 = -1$  en dit geeft  $rc_m = -\frac{1}{2}$ .  
 $m: y = -\frac{1}{2}x + b$   
 door  $B(2, -1)$   $\left\{ \begin{array}{l} -\frac{1}{2} \cdot 2 + b = -1 \\ -1 + b = -1 \\ b = 0 \end{array} \right.$

Dus  $m: y = -\frac{1}{2}x$ .

**b** De straal van  $c$  is  $r = \sqrt{5}$ .  
 $p: y = 2x$  oftewel  $p: 2x - y = 0$   
 $d(M, p) = \frac{|2 \cdot 3 - 1|}{\sqrt{2^2 + (-1)^2}} = \frac{|5|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$ , dus  $d(M, p) = r$ , dus  $p$  raakt  $c$ .

**c**  $c$  snijden met de  $x$ -as geeft  $(x-3)^2 + (-1)^2 = 5$   
 $(x-3)^2 + 1 = 5$   
 $(x-3)^2 = 4$   
 $x-3 = 2 \vee x-3 = -2$   
 $x = 5 \vee x = 1$

Dus  $C(1, 0)$ .

Lijn  $r$  door  $M$  en  $C$  heeft  $rc_r = \frac{0-1}{1-3} = \frac{1}{2}$ .

$q \perp r$ , dus  $rc_q \cdot \frac{1}{2} = -1$  en dit geeft  $rc_q = -2$ .

$q: y = -2x + b$   
 door  $C(1, 0)$   $\left\{ \begin{array}{l} -2 \cdot 1 + b = 0 \\ -2 + b = 0 \\ b = 2 \end{array} \right.$

Dus  $q: y = -2x + 2$ .

**70 a**  $x = 3$  geeft  $3^2 + y^2 - 12 \cdot 3 + 11 = 0$   
 $y^2 - 16 = 0$   
 $y^2 = 16$   
 $y = 4 \vee y = -4$

Dus  $A(3, 4)$  en  $B(3, -4)$ .

$x^2 + y^2 - 12x + 11 = 0$

$x^2 - 12x + y^2 + 11 = 0$

$(x-6)^2 - 36 + y^2 + 11 = 0$

$(x-6)^2 + y^2 = 25$

Dus  $M(6, 0)$ .

Lijn  $m$  door  $M$  en  $A$  heeft  $rc_m = \frac{4-0}{3-6} = -\frac{4}{3}$ .

$k \perp m$ , dus  $rc_k \cdot -\frac{4}{3} = -1$  en dit geeft  $rc_k = \frac{3}{4}$ .

$k: y = \frac{3}{4}x + b$   
 door  $A(3, 4)$   $\left\{ \begin{array}{l} \frac{3}{4} \cdot 3 + b = 4 \\ 2\frac{1}{4} + b = 4 \\ b = 1\frac{3}{4} \end{array} \right.$

Dus  $k: y = \frac{3}{4}x + 1\frac{3}{4}$ .

**b**  $\tan(\alpha) = rc_k = \frac{3}{4}$  geeft  $\alpha = 36,8\dots^\circ$   
 $\tan(\beta) = rc_l = -\frac{3}{4}$  geeft  $\beta = -36,8\dots^\circ$   
 $\alpha - \beta = 36,8\dots^\circ - (-36,8\dots^\circ) = 73,7\dots^\circ$   
 Dus  $\angle(k, l) \approx 74^\circ$ .

**c** Zie de figuur hiernaast.  
 $m$  en  $n$  raken de cirkel in de punten  $C$  en  $D$ .  
 De straal van  $c$  is  $r = 5$ , dus  $CM = DM = 5$ .

In  $\triangle OMC$  is  $\sin(\angle COM) = \frac{5}{6}$  en dit geeft  $\angle COM = 56,4\dots^\circ$ .

$\angle COD = 2 \cdot \angle COM = 2 \cdot 56,4\dots^\circ = 112,8\dots^\circ$

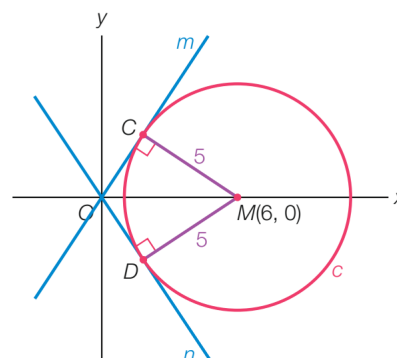
Dus  $\angle(m, n) = 180^\circ - 112,8\dots^\circ \approx 67^\circ$ .

Lijn  $n$  door  $M$  en  $B$  heeft  $rc_n = \frac{-4-0}{3-6} = \frac{4}{3}$ .

$l \perp n$ , dus  $rc_l \cdot \frac{4}{3} = -1$  en dit geeft  $rc_l = -\frac{3}{4}$ .

$l: y = -\frac{3}{4}x + b$   
 door  $B(3, -4)$   $\left\{ \begin{array}{l} -\frac{3}{4} \cdot 3 + b = -4 \\ -2\frac{1}{4} + b = -4 \\ b = -1\frac{3}{4} \end{array} \right.$

Dus  $l: y = -\frac{3}{4}x - 1\frac{3}{4}$ .



**71 a**  $c$  snijden met de  $x$ -as geeft  $x^2 + 6x - 7 = 0$   
 $(x-1)(x+7) = 0$   
 $x = 1 \vee x = -7$

Dus  $A(-7, 0)$  en  $B(1, 0)$ .

$$x^2 + y^2 + 6x - 6y - 7 = 0$$

$$x^2 + 6x + y^2 - 6y - 7 = 0$$

$$(x+3)^2 - 9 + (y-3)^2 - 9 - 7 = 0$$

$$(x+3)^2 + (y-3)^2 = 25$$

Dus  $M(-3, 3)$ .

Lijn  $m$  door  $M$  en  $A$  heeft  $rc_m = \frac{0-3}{-7-3} = \frac{3}{4}$ .

$k \perp m$ , dus  $rc_k \cdot \frac{3}{4} = -1$  en dit geeft  $rc_k = -\frac{4}{3}$ .

$$k: y = -\frac{4}{3}x + b \left\{ \begin{array}{l} -\frac{4}{3} \cdot -7 + b = 0 \\ \text{door } A(-7, 0) \end{array} \right.$$

$$9\frac{1}{3} + b = 0$$

$$b = -9\frac{1}{3}$$

Dus  $k: y = -1\frac{1}{3}x - 9\frac{1}{3}$ .

**b**  $c$  snijden met de  $y$ -as geeft  $y^2 - 6y - 7 = 0$   
 $(y+1)(y-7) = 0$   
 $y = -1 \vee y = 7$

Dus  $C(0, -1)$  en  $D(0, 7)$ .

Lijn  $s$  door  $M$  en  $C$  heeft  $rc_s = \frac{-1-3}{0-3} = -\frac{4}{3}$ .

$p \perp s$ , dus  $rc_p \cdot -\frac{4}{3} = -1$  en dit geeft  $rc_p = \frac{3}{4}$ .

$p$  door  $C(0, -1)$ , dus  $p: y = \frac{3}{4}x - 1$ .

**c**  $\tan(\alpha_1) = rc_k = -1\frac{1}{3}$  geeft  $\alpha_1 = -53,1\dots^\circ$   
 $\tan(\beta_1) = rc_l = 1\frac{1}{3}$  geeft  $\beta_1 = 53,1\dots^\circ$   
 $\beta_1 - \alpha_1 = 53,1\dots^\circ - (-53,1\dots^\circ) = 106,2\dots^\circ$   
 $\angle(k, l) = 180^\circ - 106,2\dots^\circ = 73,7\dots^\circ$

Dus de hoek tussen  $k$  en  $l$  is gelijk aan de hoek tussen  $p$  en  $q$ .

Lijn  $n$  door  $M$  en  $B$  heeft  $rc_n = \frac{0-3}{1-3} = -\frac{3}{2}$ .

$l \perp n$ , dus  $rc_l \cdot -\frac{3}{2} = -1$  en dit geeft  $rc_l = \frac{2}{3}$ .

$$l: y = \frac{2}{3}x + b \left\{ \begin{array}{l} \frac{2}{3} \cdot 1 + b = 0 \\ \text{door } B(1, 0) \end{array} \right.$$

$$1\frac{1}{3} + b = 0$$

$$b = -1\frac{1}{3}$$

Dus  $l: y = \frac{2}{3}x - 1\frac{1}{3}$ .

Lijn  $t$  door  $M$  en  $D$  heeft

$$rc_t = \frac{7-3}{0-3} = -\frac{4}{3}$$

$q \perp t$ , dus  $rc_q \cdot -\frac{4}{3} = -1$  en dit geeft  $rc_q = \frac{3}{4}$ .

$q$  door  $D(0, 7)$ , dus  $q: y = -\frac{3}{4}x + 7$ .

$\tan(\alpha_2) = rc_p = \frac{3}{4}$  geeft  $\alpha_2 = 36,8\dots^\circ$

$\tan(\beta_2) = rc_q = -\frac{3}{4}$  geeft  $\beta_2 = -36,8\dots^\circ$

$\alpha_2 - \beta_2 = 36,8\dots^\circ - (-36,8\dots^\circ) = 73,7\dots^\circ$

$\angle(p, q) = 73,7\dots^\circ$

## Bladzijde 122

**72 a**  $x^2 + y^2 - 2x + 8y + 4 = 0$   
 $x^2 - 2x + y^2 + 8y + 4 = 0$   
 $(x-1)^2 - 1 + (y+4)^2 - 16 + 4 = 0$   
 $(x-1)^2 + (y+4)^2 = 13$   
Dus  $M_1(1, -4)$ .

Lijn  $l$  door  $M_1$  en  $A$  heeft  $rc_l = \frac{-2-4}{4-1} = -\frac{2}{3}$ .

$k \perp l$ , dus  $rc_k \cdot -\frac{2}{3} = -1$  en dit geeft  $rc_k = \frac{3}{2}$ .

$$k: y = \frac{3}{2}x + b \left\{ \begin{array}{l} -\frac{3}{2} \cdot 4 + b = -2 \\ \text{door } A(4, -2) \end{array} \right.$$

$$-6 + b = -2$$

$$b = 4$$

$k: y = 1\frac{1}{2}x + 4$

$$1\frac{1}{2}x + y = 4$$

$$3x + 2y = 8$$

**b**  $m: y = 2x - 5$  oftewel  $m: 2x - y - 5 = 0$

$$r_2 = d(M_2, m) = \frac{|2 \cdot -3 - 4 - 5|}{\sqrt{2^2 + (-1)^2}} = \frac{|-15|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

Dus  $c_2: (x+3)^2 + (y-4)^2 = (3\sqrt{5})^2$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = 45$$

$$x^2 + y^2 + 6x - 8y - 20 = 0$$

**73 a**  $c$  snijden met de  $x$ -as geeft  $x^2 - 4x - 12 = 0$   
 $(x+2)(x-6) = 0$   
 $x = -2 \vee x = 6$

Dus  $A(-2, 0)$  en  $B(6, 0)$ .

$$x^2 + y^2 - 4x + 2y - 12 = 0$$

$$x^2 - 4x + y^2 + 2y - 12 = 0$$

$$(x-2)^2 - 4 + (y+1)^2 - 1 - 12 = 0$$

$$(x-2)^2 + (y+1)^2 = 17$$

Dus  $M(2, -1)$ .

Lijn  $p$  door  $M$  en  $A$  heeft  $rc_p = \frac{0 - -1}{-2 - 2} = -\frac{1}{4}$ .

$k \perp p$ , dus  $rc_k \cdot -\frac{1}{4} = -1$  en dit geeft  $rc_k = 4$ .

Lijn  $q$  door  $M$  en  $C$  heeft  $rc_q = \frac{3 - -1}{1 - 2} = -4$ .

$m \perp q$ , dus  $rc_m \cdot -4 = -1$  en dit geeft  $rc_m = \frac{1}{4}$ .

$\tan(\alpha) = rc_k = 4$  geeft  $\alpha = 75,9\dots^\circ$

$\tan(\beta) = rc_m = \frac{1}{4}$  geeft  $\beta = 14,0\dots^\circ$

$\alpha - \beta = 75,9\dots^\circ - 14,0\dots^\circ = 61,9\dots^\circ$

Dus  $\angle(k, m) \approx 62^\circ$ .

**b** Zie de figuur hiernaast.

Het midden van  $AB$  is  $F(2, 0)$ .

In  $\triangle AMF$  is  $\tan(\angle AMF) = \frac{4}{1} = 4$  en dit geeft  $\angle AMF = 75,9\dots^\circ$ .

Dus  $\angle AMB = 2 \cdot 75,9\dots^\circ \approx 152^\circ$ .

**c** De straal van  $c$  is  $r = \sqrt{17}$ .

$$\left. \begin{array}{l} m: y = \frac{1}{4}x + b \\ \text{door } C(1, 3) \end{array} \right\} \begin{array}{l} \frac{1}{4} \cdot 1 + b = 3 \\ \frac{1}{4} + b = 3 \\ b = 2\frac{3}{4} \end{array}$$

$$b = 2\frac{3}{4}$$

Dus  $m: y = \frac{1}{4}x + 2\frac{3}{4}$ .

Lijn  $s$  door  $M$  en  $B$  heeft  $rc_s = \frac{0 - -1}{6 - 2} = \frac{1}{4}$ .

$l \perp s$ , dus  $rc_l \cdot \frac{1}{4} = -1$  en dit geeft  $rc_l = -4$ .

$$\left. \begin{array}{l} l: y = -4x + b \\ \text{door } B(6, 0) \end{array} \right\} \begin{array}{l} -4 \cdot 6 + b = 0 \\ -24 + b = 0 \\ b = 24 \end{array}$$

Dus  $l: y = -4x + 24$ .

$l$  en  $m$  snijden geeft het punt  $E$ .

$$-4x + 24 = \frac{1}{4}x + 2\frac{3}{4}$$

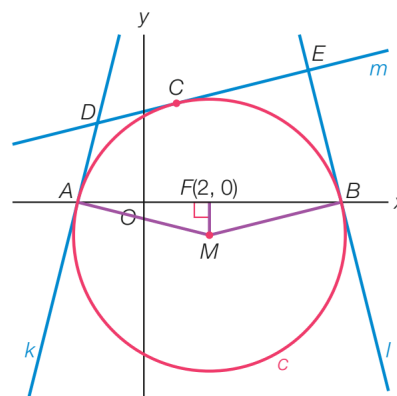
$$-4\frac{1}{4}x = -21\frac{1}{4}$$

$$x = 5$$

$x = 5$  invullen in  $y = -4x + 24$  geeft  $y = -4 \cdot 5 + 24 = 4$ , dus  $E(5, 4)$ .

$$d(E, M) = \sqrt{(2-5)^2 + (-1-4)^2} = \sqrt{34}$$

Dus  $d(E, c) = \sqrt{34} - \sqrt{17}$ .



**74**  $x = -1$  geeft  $(-1)^2 + y^2 + 6 \cdot -1 - 2y - 10 = 0$

$$y^2 - 2y - 15 = 0$$

$$(y + 3)(y - 5) = 0$$

$$y = -3 \vee y = 5$$

Dus  $A(-1, 5)$ .

$$x^2 + y^2 + 6x - 2y - 10 = 0$$

$$x^2 + 6x + y^2 - 2y - 10 = 0$$

$$(x + 3)^2 - 9 + (y - 1)^2 - 1 - 10 = 0$$

$$(x + 3)^2 + (y - 1)^2 = 20$$

Dus  $M(-3, 1)$ .

Lijn  $m$  door  $M$  en  $A$  heeft  $rc_m = \frac{5 - 1}{-1 - -3} = 2$ .

$k \perp m$ , dus  $rc_k \cdot 2 = -1$  en dit geeft  $rc_k = -\frac{1}{2}$ .

$$l \parallel k, \text{ dus } l: y = -\frac{1}{2}x + b \left\{ \begin{array}{l} -\frac{1}{2} \cdot -3 + b = 1 \\ 1\frac{1}{2} + b = 1 \end{array} \right.$$

$$b = -\frac{1}{2}$$

Dus  $l: y = -\frac{1}{2}x - \frac{1}{2}$ .

$l$  snijden met de  $x$ -as geeft  $-\frac{1}{2}x - \frac{1}{2} = 0$

$$-\frac{1}{2}x = \frac{1}{2}$$

$$x = -1$$

Dus  $B(-1, 0)$ .

$$r_2 = d(B, M) = \sqrt{(-3 - -1)^2 + (1 - 0)^2} = \sqrt{5}$$

Dus  $c_2: (x + 1)^2 + y^2 = 5$ .

**75 a** De straal van  $c$  is  $r = \sqrt{5}$ .

$k$  raakt  $c$ , dus er geldt  $d(M, k) = r = \sqrt{5}$ .

$k: y = \frac{1}{2}x + b$  oftewel  $k: \frac{1}{2}x - y + b = 0$ , dus  $k: x - 2y + 2b = 0$ .

Het middelpunt van  $c$  is  $M(2, 3)$ .

$$d(M, k) = \sqrt{5} \text{ geeft } \frac{|2 - 2 \cdot 3 + 2b|}{\sqrt{1^2 + (-2)^2}} = \sqrt{5}$$

$$\frac{|2b - 4|}{\sqrt{5}} = \sqrt{5}$$

$$|2b - 4| = 5$$

**b**  $|2b - 4| = 5$  geeft  $2b - 4 = 5 \vee 2b - 4 = -5$

$$2b = 9 \vee 2b = -1$$

$$b = 4\frac{1}{2} \vee b = -\frac{1}{2}$$

Dus  $k_1: y = \frac{1}{2}x + 4\frac{1}{2}$  en  $k_2: y = \frac{1}{2}x - \frac{1}{2}$ .

#### Bladzijde 124

**76 a** Van  $c$  is  $M(0, 0)$  en  $r = \sqrt{10}$ .

$l: y = 3x + b$  oftewel  $l: 3x - y + b = 0$

$$d(M, l) = r \text{ geeft } \frac{|3 \cdot 0 - 0 + b|}{\sqrt{3^2 + (-1)^2}} = \sqrt{10}$$

$$\frac{|b|}{\sqrt{10}} = \sqrt{10}$$

$$|b| = 10$$

$$b = 10 \vee b = -10$$

Dus  $l_1: y = 3x + 10$  en  $l_2: y = 3x - 10$ .

**b**  $m: y = ax + b$

$A(10, 0)$  op  $m$  geeft  $10a + b = 0$ , dus  $b = -10a$ .

$m: y = ax - 10a$  oftewel  $m: ax - y - 10a = 0$

$$d(M, m) = r \text{ geeft } \frac{|a \cdot 0 - 0 - 10a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{10}$$

$$|-10a| = \sqrt{10} \cdot \sqrt{a^2 + 1}$$

$$|-10a| = \sqrt{10a^2 + 10}$$

kwadrateren geeft

$$100a^2 = 10a^2 + 10$$

$$90a^2 = 10$$

$$a^2 = \frac{1}{9}$$

$$a = \frac{1}{3} \vee a = -\frac{1}{3}$$

$a = \frac{1}{3}$  geeft  $b = -10 \cdot \frac{1}{3} = -3\frac{1}{3}$ , dus  $m_1: y = \frac{1}{3}x - 3\frac{1}{3}$ .

$a = -\frac{1}{3}$  geeft  $b = -10 \cdot -\frac{1}{3} = 3\frac{1}{3}$ , dus  $m_2: y = -\frac{1}{3}x + 3\frac{1}{3}$ .

**77**

**a**  $x^2 + y^2 - 10x - 4y + 19 = 0$

$$x^2 - 10x + y^2 - 4y + 19 = 0$$

$$(x - 5)^2 - 25 + (y - 2)^2 - 4 + 19 = 0$$

$$(x - 5)^2 + (y - 2)^2 = 10$$

Dus  $M(5, 2)$ .

Lijn  $l$  door  $M$  en  $A$  heeft  $rc_l = \frac{5-2}{4-5} = -3$ .

$k \perp l$ , dus  $rc_k \cdot -3 = -1$  en dit geeft  $rc_k = \frac{1}{3}$ .

$$k: y = \frac{1}{3}x + b \left\{ \begin{array}{l} \frac{1}{3} \cdot 4 + b = 5 \\ \frac{1}{3} + b = 5 \end{array} \right.$$

$$b = 3\frac{2}{3}$$

Dus  $k: y = \frac{1}{3}x + 3\frac{2}{3}$ .

**b** De straal van  $c$  is  $r = \sqrt{10}$ .

$m: y = 3x + b$  oftewel  $m: 3x - y + b = 0$

$$d(M, m) = r \text{ geeft } \frac{|3 \cdot 5 - 2 + b|}{\sqrt{3^2 + (-1)^2}} = \sqrt{10}$$

$$\frac{|13 + b|}{\sqrt{10}} = \sqrt{10}$$

$$|13 + b| = 10$$

$$13 + b = 10 \vee 13 + b = -10$$

$$b = -3 \vee b = -23$$

Dus  $m_1: y = 3x - 3$  en  $m_2: y = 3x - 23$ .

**c**  $n: y = ax + b$

$B(9, 0)$  op  $n$  geeft  $9a + b = 0$ , dus  $b = -9a$ .

$n: y = ax - 9a$  oftewel  $n: ax - y - 9a = 0$

$$d(M, n) = r \text{ geeft } \frac{|5a - 2 - 9a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{10}$$

$$|-4a - 2| = \sqrt{10} \cdot \sqrt{a^2 + 1}$$

$$|-4a - 2| = \sqrt{10a^2 + 10}$$

kwadrateren geeft

$$16a^2 + 16a + 4 = 10a^2 + 10$$

$$6a^2 + 16a - 6 = 0$$

$$3a^2 + 8a - 3 = 0$$

$$D = 8^2 - 4 \cdot 3 \cdot -3 = 100, \text{ dus } \sqrt{D} = 10$$

$$a = \frac{-8 + 10}{6} = \frac{1}{3} \vee a = \frac{-8 - 10}{6} = -3$$

$a = \frac{1}{3}$  geeft  $b = -9 \cdot \frac{1}{3} = -3$ , dus  $n_1: y = \frac{1}{3}x - 3$ .

$a = -3$  geeft  $b = -9 \cdot -3 = 27$ , dus  $n_2: y = -3x + 27$ .



**a**  $x^2 + y^2 - 2x - 4y - 12 = 0$   
 $x^2 - 2x + y^2 - 4y - 12 = 0$   
 $(x-1)^2 - 1 + (y-2)^2 - 4 - 12 = 0$   
 $(x-1)^2 + (y-2)^2 = 17$   
Dus  $M(1, 2)$ .

Lijn  $l$  door  $M$  en  $A$  heeft  $rc_l = \frac{1-2}{-3-1} = \frac{1}{4}$ .

$k \perp l$ , dus  $rc_k \cdot \frac{1}{4} = -1$  en dit geeft  $rc_k = -4$ .

$k: y = -4x + b$   
door  $A(-3, 1)$   $\left\{ \begin{array}{l} -4 \cdot -3 + b = 1 \\ 12 + b = 1 \\ b = -11 \end{array} \right.$

Dus  $k: y = -4x - 11$ .

**b** De straal van  $c$  is  $r = \sqrt{17}$ .

$m \perp n$ , dus  $m: x + 4y = c$ .

$d(M, m) = r$  geeft  $\frac{|1 + 4 \cdot 2 - c|}{\sqrt{1^2 + 4^2}} = \sqrt{17}$

$\frac{|9 - c|}{\sqrt{17}} = \sqrt{17}$

$|9 - c| = 17$

$9 - c = 17 \vee 9 - c = -17$

$c = -8 \vee c = 26$

Dus  $m_1: x + 4y = -8$  en  $m_2: x + 4y = 26$ .

**c**  $p: y = ax + b$

$B(6, -1)$  op  $p$  geeft  $6a + b = -1$ , dus  $b = -6a - 1$ .

$p: y = ax - 6a - 1$  oftewel  $p: ax - y - 6a - 1 = 0$

$d(M, p) = r$  geeft  $\frac{|a - 2 - 6a - 1|}{\sqrt{a^2 + (-1)^2}} = \sqrt{17}$

$|-5a - 3| = \sqrt{17} \cdot \sqrt{a^2 + 1}$

$|-5a - 3| = \sqrt{17a^2 + 17}$

kwadrateren geeft

$25a^2 + 30a + 9 = 17a^2 + 17$

$8a^2 + 30a - 8 = 0$

$4a^2 + 15a - 4 = 0$

$D = 15^2 - 4 \cdot 4 \cdot -4 = 289$ , dus  $\sqrt{D} = 17$

$a = \frac{-15 \pm 17}{8} = \frac{1}{4} \vee a = \frac{-15 - 17}{8} = -4$

$a = \frac{1}{4}$  geeft  $b = -6 \cdot \frac{1}{4} - 1 = -2\frac{1}{2}$ , dus  $p_1: y = \frac{1}{4}x - 2\frac{1}{2}$ .

$a = -4$  geeft  $b = -6 \cdot -4 - 1 = 23$ , dus  $p_2: y = -4x + 23$ .

$$x^2 + y^2 - 14x + 2y + 30 = 0$$

$$x^2 - 14x + y^2 + 2y + 30 = 0$$

$$(x-7)^2 - 49 + (y+1)^2 - 1 + 30 = 0$$

$$(x-7)^2 + (y+1)^2 = 20, \text{ dus } M(7, -1) \text{ en } r = \sqrt{20}.$$

$$k: y = ax + b$$

$$P(1, -3) \text{ op } k \text{ geeft } a + b = -3, \text{ dus } b = -a - 3.$$

$$k: y = ax - a - 3 \text{ oftewel } k: ax - y - a - 3 = 0$$

$$d(M, k) = r \text{ geeft } \frac{|7a + 1 - a - 3|}{\sqrt{a^2 + (-1)^2}} = \sqrt{20}$$

$$|6a - 2| = \sqrt{20} \cdot \sqrt{a^2 + 1}$$

$$|6a - 2| = \sqrt{20a^2 + 20}$$

$$\text{kwadrateren geeft}$$

$$36a^2 - 24a + 4 = 20a^2 + 20$$

$$16a^2 - 24a - 16 = 0$$

$$2a^2 - 3a - 2 = 0$$

$$D = (-3)^2 - 4 \cdot 2 \cdot -2 = 25, \text{ dus } \sqrt{D} = 5$$

$$a = \frac{3+5}{4} = 2 \vee a = \frac{3-5}{4} = -\frac{1}{2}$$

$$\text{Dus } rc_{k_1} = -\frac{1}{2} \text{ en } rc_{k_2} = 2.$$

$$rc_{k_1} \cdot rc_{k_2} = -\frac{1}{2} \cdot 2 = -1, \text{ dus } k_1 \perp k_2, \text{ dus } \angle APB = 90^\circ.$$

$$k_1 \text{ raakt } c \text{ in } A, \text{ dus } \angle PAM = 90^\circ \text{ en } d(A, M) = d(M, k_1) = r = \sqrt{20}.$$

$$k_2 \text{ raakt } c \text{ in } B, \text{ dus } \angle PBM = 90^\circ \text{ en } d(B, M) = d(M, k_2) = r = \sqrt{20}.$$

$$\left. \begin{array}{l} \angle APB = 90^\circ \\ \angle PAM = 90^\circ \\ \angle PBM = 90^\circ \\ d(A, M) = d(B, M) \end{array} \right\} AMBP \text{ is een vierkant.}$$

### Bladzijde 125

C10

$$\text{a } x^2 + y^2 - 12x + 8y + 3 = 0$$

$$x^2 - 12x + y^2 + 8y + 3 = 0$$

$$(x-6)^2 - 36 + (y+4)^2 - 16 + 3 = 0$$

$$(x-6)^2 + (y+4)^2 = 49$$

Dus van  $c$  is de straal  $r_c = 7$  en het middelpunt  $M(6, -4)$ .

$$d(A, M) = \sqrt{(6-3)^2 + (-4-2)^2} = \sqrt{13}$$

$$\text{Dus } d(A, c) = 7 - \sqrt{13}.$$

$$\text{c } d(M, k) = \frac{|3 \cdot 6 + 4 \cdot -4 - 52|}{\sqrt{3^2 + 4^2}} = \frac{|-50|}{5} = \frac{50}{5} = 10$$

$$\text{Dus } d(k, c) = 10 - 7 = 3.$$

$$\text{b } x^2 + y^2 + 2x - 6y + 6 = 0$$

$$x^2 + 2x + y^2 - 6y + 6 = 0$$

$$(x+1)^2 - 1 + (y-3)^2 - 9 + 6 = 0$$

$$(x+1)^2 + (y-3)^2 = 4$$

Dus van  $d$  is de straal  $r_d = 2$  en het middelpunt  $N(-1, 3)$ .

$$d(M, N) = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{98} = 7\sqrt{2}$$

$$\text{Dus } d(c, d) = 7\sqrt{2} - 7 - 2 = 7\sqrt{2} - 9.$$

C11

$$x = 3 \text{ geeft } 3^2 + y^2 - 10 \cdot 3 + 2y = 3$$

$$y^2 + 2y - 24 = 0$$

$$(y-4)(y+6) = 0$$

$$y = 4 \vee y = -6$$

Dus  $A(3, 4)$ .

$$x^2 + y^2 - 10x + 2y = 3$$

$$x^2 - 10x + y^2 + 2y = 3$$

$$(x-5)^2 - 25 + (y+1)^2 - 1 = 3$$

$$(x-5)^2 + (y+1)^2 = 29, \text{ dus } M(5, -1).$$

$$\text{Lijn } l \text{ door } M \text{ en } A \text{ heeft } rc_l = \frac{4-1}{3-5} = -\frac{3}{2}.$$

$$k \perp l, \text{ dus } rc_k \cdot -\frac{3}{2} = -1 \text{ en dit geeft } rc_k = \frac{2}{3}.$$

$$k: y = \frac{2}{3}x + b \left\{ \begin{array}{l} \frac{2}{3} \cdot 3 + b = 4 \\ 1\frac{1}{3} + b = 4 \end{array} \right.$$

$$b = 2\frac{4}{3}$$

$$\text{Dus } k: y = \frac{2}{3}x + 2\frac{4}{3}.$$

**C12** a Van  $c$  is  $M(5, 0)$  en  $r = \sqrt{13}$ .

$$l: y = ax + b$$

$A(10, 1)$  op  $l$  geeft  $10a + b = 1$ , dus  $b = 1 - 10a$ .

$$l: y = ax + 1 - 10a \text{ oftewel } l: ax - y + 1 - 10a = 0$$

$$d(M, l) = r \text{ geeft } \frac{|5a - 0 + 1 - 10a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{13}$$

$$|-5a + 1| = \sqrt{13} \cdot \sqrt{a^2 + 1}$$

$$|-5a + 1| = \sqrt{13a^2 + 13}$$

kwadrateren geeft

$$25a^2 - 10a + 1 = 13a^2 + 13$$

$$12a^2 - 10a - 12 = 0$$

$$6a^2 - 5a - 6 = 0$$

$$D = (-5)^2 - 4 \cdot 6 \cdot -6 = 169, \text{ dus } \sqrt{D} = 13$$

$$a = \frac{5 + 13}{12} = 1\frac{1}{2} \vee a = \frac{5 - 13}{12} = -\frac{2}{3}$$

$a = 1\frac{1}{2}$  geeft  $b = 1 - 10 \cdot 1\frac{1}{2} = -14$ , dus  $l_1: y = 1\frac{1}{2}x - 14$ .

$a = -\frac{2}{3}$  geeft  $b = 1 - 10 \cdot -\frac{2}{3} = 7\frac{2}{3}$ , dus  $l_2: y = -\frac{2}{3}x + 7\frac{2}{3}$ .

**b**  $m: y = -1\frac{1}{2}x + b$  oftewel  $m: 1\frac{1}{2}x + y - b = 0$ , dus  $m: 3x + 2y - 2b = 0$ .

$$d(M, m) = r \text{ geeft } \frac{|3 \cdot 5 + 2 \cdot 0 - 2b|}{\sqrt{3^2 + 2^2}} = \sqrt{13}$$

$$\frac{|15 - 2b|}{\sqrt{13}} = \sqrt{13}$$

$$|15 - 2b| = 13$$

$$15 - 2b = 13 \vee 15 - 2b = -13$$

$$-2b = -2 \vee -2b = -28$$

$$b = 1 \vee b = 14$$

Dus  $m_1: y = -1\frac{1}{2}x + 1$  en  $m_2: y = -1\frac{1}{2}x + 14$ .

## Kennis- en vaardighedentest

### Bladzijde 126

**1** Samenvallen, dus  $\frac{9}{p} = \frac{p}{p+4} = \frac{q}{-2}$ .

$$\frac{9}{p} = \frac{p}{p+4} \text{ geeft } p^2 = 9(p+4)$$

$$p^2 = 9p + 36$$

$$p^2 - 9p - 36 = 0$$

$$(p+3)(p-12) = 0$$

$$p = -3 \vee p = 12$$

$$p = -3 \text{ geeft } \frac{9}{-3} = \frac{q}{-2}, \text{ dus } q = \frac{-2 \cdot 9}{-3} = 6.$$

$$p = 12 \text{ geeft } \frac{9}{12} = \frac{q}{-2}, \text{ dus } q = \frac{-2 \cdot 9}{12} = -1\frac{1}{2}.$$

Dus voor  $p = -3$  en  $q = 6$  en voor  $p = 12$  en  $q = -1\frac{1}{2}$  vallen de lijnen samen.

2

a  $k: \frac{x}{p} + \frac{y}{2} = 1$

$k: 2x + py = 2p$

$l: \frac{x}{4} + \frac{y}{2p} = 1$

$l: px + 2y = 4p$

b  $\left. \begin{array}{l} k: 2x + py = 2p \\ \text{door } A(3, 4) \end{array} \right\} \begin{array}{l} 2 \cdot 3 + p \cdot 4 = 2p \\ 6 + 4p = 2p \\ 2p = -6 \\ p = -3 \end{array}$

c Geen punt gemeenschappelijk, dus  $\frac{2}{p} = \frac{p}{2} \neq \frac{2p}{4p}$ .

$\frac{2}{p} = \frac{p}{2}$  geeft  $p^2 = 4$   
 $p = 2 \vee p = -2$

$\frac{2}{p} \neq \frac{2p}{4p}$  geeft  $2p^2 \neq 8p$   
 $2p^2 - 8p \neq 0$   
 $2p(p - 4) \neq 0$   
 $p \neq 0 \wedge p \neq 4$

Dus voor  $p = 2$  en voor  $p = -2$  hebben  $k$  en  $l$  geen punt gemeenschappelijk.

3

a  $k: y = 3x - 1$

$\tan(\alpha) = \text{rc}_k = 3$  geeft  $\alpha = 71,56\dots^\circ$

$l: 5x - 2y = 6$

$-2y = -5x + 6$

$y = 2\frac{1}{2}x - 3$

$\tan(\beta) = \text{rc}_l = 2\frac{1}{2}$  geeft  $\beta = 68,19\dots^\circ$

$\alpha - \beta = 71,56\dots^\circ - 68,19\dots^\circ = 3,36\dots^\circ$

Dus  $\angle(k, l) \approx 3,4^\circ$ .

b  $\tan(\alpha) = \text{rc}_m = \frac{4 - -5}{4 - -2} = 1\frac{1}{2}$  geeft  $\alpha = 56,30\dots^\circ$

$\tan(\beta) = \text{rc}_n = \frac{8 - 0}{0 - 3} = -2\frac{2}{3}$  geeft  $\beta = -69,44\dots^\circ$

$\alpha - \beta = 56,30\dots^\circ - -69,44\dots^\circ = 125,75\dots^\circ$

Dus  $\angle(m, n) = 180^\circ - 125,75\dots^\circ \approx 54,2^\circ$ .

c  $p: 4x + y = 8$

$y = -4x + 8$

$\tan(\alpha) = \text{rc}_p = -4$  geeft  $\alpha = -75,9\dots^\circ$

$q: 2x + 3y = 9$

$3y = -2x + 9$

$y = -\frac{2}{3}x + 3$

$\tan(\beta) = \text{rc}_q = -\frac{2}{3}$  geeft  $\beta = -33,6\dots^\circ$

$\beta - \alpha = -33,6\dots^\circ - -75,9\dots^\circ = 42,2\dots^\circ$

Dus  $\angle(p, q) \approx 42^\circ$ .

4

a  $x_M = 5$  geeft  $\frac{1}{2}(p + 3) = 5$

$p + 3 = 10$

$p = 7$

$A(7, -1)$  en  $B(3, 5)$  geeft  $d(A, B) = \sqrt{(3 - 7)^2 + (5 - -1)^2} = \sqrt{52} = 2\sqrt{13}$

b  $d(A, B) = \sqrt{(3 - p)^2 + (p - 2 - -1)^2} = \sqrt{(3 - p)^2 + (p - 1)^2} = \sqrt{9 - 6p + p^2 + p^2 - 2p + 1}$   
 $= \sqrt{2p^2 - 8p + 10}$

$d(A, B) = 10$  geeft  $\sqrt{2p^2 - 8p + 10} = 10$

$2p^2 - 8p + 10 = 100$

$2p^2 - 8p - 90 = 0$

$p^2 - 4p - 45 = 0$

$(p + 5)(p - 9) = 0$

$p = -5 \vee p = 9$

- 5 a** Lijn  $l$  gaat door  $A$  en staat loodrecht op  $k$ .

$$l: 5x + 3y = c \quad \left. \begin{array}{l} \\ \text{door } A(7, -3) \end{array} \right\} c = 5 \cdot 7 + 3 \cdot -3 = 26$$

$$\text{Dus } l: 5x + 3y = 26.$$

$k$  en  $l$  snijden geeft het punt  $A'$ .

$$\begin{cases} 3x - 5y = 2 \\ 5x + 3y = 26 \end{cases} \begin{array}{l} |3| \\ |5| \end{array} \text{ geeft } \begin{cases} 9x - 15y = 6 \\ 25x + 15y = 130 \end{cases} +$$

$$\frac{34x}{x} = 136$$

$$x = 4$$

$$x = 4 \text{ geeft } 3 \cdot 4 - 5y = 2$$

$$12 - 5y = 2$$

$$-5y = -10$$

$$y = 2$$

Dus  $A'(4, 2)$ .

**b**  $d(A, k) = d(A, A') = \sqrt{(4-7)^2 + (2-(-3))^2} = \sqrt{34}$

- 6 a**  $P(p, 0)$  is een willekeurig punt op de  $x$ -as.

$$d(P, k) = 8 \text{ geeft } \frac{|4 \cdot p - 3 \cdot 0 - 12|}{\sqrt{4^2 + (-3)^2}} = 8$$

$$\frac{|4p - 12|}{5} = 8$$

$$|4p - 12| = 40$$

$$4p - 12 = 40 \vee 4p - 12 = -40$$

$$4p = 52 \vee 4p = -28$$

$$p = 13 \vee p = -7$$

Dus  $P_1(13, 0)$  en  $P_2(-7, 0)$ .

- b**  $l: y = ax + b$  door  $A(2, 0)$  geeft  $2a + b = 0$ , dus  $b = -2a$ .

$$l: y = ax - 2a \text{ oftewel } l: ax - y - 2a = 0$$

$$d(B, k) = \sqrt{10} \text{ geeft } \frac{|-2a - 2 - 2a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{10}$$

$$|-4a - 2| = \sqrt{10} \cdot \sqrt{a^2 + 1}$$

$$|-4a - 2| = \sqrt{10a^2 + 10}$$

kwadrateren geeft

$$16a^2 + 16a + 4 = 10a^2 + 10$$

$$6a^2 + 16a - 6 = 0$$

$$3a^2 + 8a - 3 = 0$$

$$D = 8^2 - 4 \cdot 3 \cdot -3 = 100, \text{ dus } \sqrt{D} = 10$$

$$a = \frac{-8 \pm 10}{6} = \frac{1}{3} \vee a = \frac{-8 - 10}{6} = -3$$

$$a = \frac{1}{3} \text{ geeft } b = -2 \cdot \frac{1}{3} = -\frac{2}{3}, \text{ dus } l_1: y = \frac{1}{3}x - \frac{2}{3}.$$

$$a = -3 \text{ geeft } b = -2 \cdot -3 = 6, \text{ dus } l_2: y = -3x + 6.$$

- c**  $m: 2x + 3y = c$

$P(1, 1)$  is een punt op  $n$ .

$$d(P, m) = \sqrt{13} \text{ geeft } \frac{|2 \cdot 1 + 3 \cdot 1 - c|}{\sqrt{2^2 + 3^2}} = \sqrt{13}$$

$$\frac{|5 - c|}{\sqrt{13}} = \sqrt{13}$$

$$|5 - c| = 13$$

$$5 - c = 13 \vee 5 - c = -13$$

$$c = -8 \vee c = 18$$

Dus  $m_1: 2x + 3y = -8$  en  $m_2: 2x + 3y = 18$ .

- 7 a** Middelpunt  $A$  en  $c_1$  door  $B$ , dus de straal is  $d(A, B) = \sqrt{(8-2)^2 + (2-3)^2} = \sqrt{61}$ .  
Dus  $c_1: (x-2)^2 + (y+3)^2 = 61$ .
- b**  $c_2$  raakt de  $y$ -as en  $x_B = 8$ , dus  $r_2 = 8$ .  
Dus  $c_2: (x-8)^2 + (y-2)^2 = 64$ .
- c** Middellijn  $AB$ , dus het middelpunt van  $c_3$  is het midden van  $AB$  oftewel  $M_3(\frac{1}{2}(2+8), \frac{1}{2}(-3+2)) = M_3(5, -\frac{1}{2})$ .  
De straal is  $d(A, M_3) = \sqrt{(5-2)^2 + (-\frac{1}{2}-3)^2} = \sqrt{15\frac{1}{4}}$ .  
Dus  $c_3: (x-5)^2 + (y+\frac{1}{2})^2 = 15\frac{1}{4}$ .
- d**  $c_4$  en  $c_5$  raken de  $y$ -as en de straal is 3, dus  $x_M = 3$  of  $x_M = -3$ .  
 $x = 3$  en  $M$  op  $k$  geeft  $y = 3 \cdot 3 - 6 = 3$ .  
Dus  $M_4(3, 3)$  en  $c_4: (x-3)^2 + (y-3)^2 = 9$ .  
 $x = -3$  en  $M$  op  $k$  geeft  $y = 3 \cdot -3 - 6 = -15$ .  
Dus  $M_5(-3, -15)$  en  $c_5: (x+3)^2 + (y+15)^2 = 9$ .

- 8**  $k: y = 2x - 3$  oftewel  $k: 2x - y - 3 = 0$

$$r = d(M, k) = \frac{|2 \cdot 6 - 1 - 3|}{\sqrt{2^2 + (-1)^2}} = \frac{|10|}{\sqrt{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5}$$

$$\text{Dus } c: (x-6)^2 + (y+1)^2 = 20.$$

- 9 a**  $x^2 + y^2 - 6x + 8y - 11 = 0$   
 $x^2 - 6x + y^2 + 8y - 11 = 0$   
 $(x-3)^2 - 9 + (y+4)^2 - 16 - 11 = 0$   
 $(x-3)^2 + (y+4)^2 = 36$   
Dus van  $c_1$  is de straal 6 en het middelpunt  $(3, -4)$ .

- b**  $x^2 + y^2 + 3x - 7y = 0$   
 $x^2 + 3x + y^2 - 7y = 0$   
 $(x+1\frac{1}{2})^2 - 2\frac{1}{4} + (y-3\frac{1}{2})^2 - 12\frac{1}{4} = 0$   
 $(x+1\frac{1}{2})^2 + (y-3\frac{1}{2})^2 = 14\frac{1}{2}$   
Dus van  $c_2$  is de straal  $\sqrt{14\frac{1}{2}}$  en het middelpunt  $(-1\frac{1}{2}, 3\frac{1}{2})$ .

- 10**  $x^2 + y^2 - 18x + 10y + 6 = 0$   
 $x^2 - 18x + y^2 + 10y + 6 = 0$   
 $(x-9)^2 - 81 + (y+5)^2 - 25 + 6 = 0$   
 $(x-9)^2 + (y+5)^2 = 100$   
Dus van  $c$  is de straal  $r = 10$  en het middelpunt  $M(9, -5)$ .  
 $d(A, M) = \sqrt{(9-3)^2 + (-5-3)^2} = \sqrt{100} = 10$   
 $d(A, M) = r$ , dus  $A$  ligt op  $c$ .  
 $d(B, M) = \sqrt{(9-2)^2 + (-5-4)^2} = \sqrt{130}$   
 $d(B, M) > r$ , dus  $B$  ligt buiten  $c$ .  
 $d(C, M) = \sqrt{(9-15)^2 + (-5-3)^2} = \sqrt{40}$   
 $d(C, M) < r$ , dus  $C$  ligt binnen  $c$ .

- 11 a**  $x^2 + y^2 - 4x + 2y = 0$   
 $x^2 - 4x + y^2 + 2y = 0$   
 $(x-2)^2 - 4 + (y+1)^2 - 1 = 0$   
 $(x-2)^2 + (y+1)^2 = 5$   
Dus van  $c$  is de straal  $r_c = \sqrt{5}$  en het middelpunt  $M(2, -1)$ .  
 $d(A, M) = \sqrt{(2-1)^2 + (-1-2)^2} = \sqrt{2}$   
Dus  $d(A, c) = \sqrt{5} - \sqrt{2}$ .
- c**  $d(N, k) = \frac{|2 \cdot -3 + 4 - 13|}{\sqrt{2^2 + 1^2}} = \frac{|-15|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$   
Dus  $d(k, d) = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}$ .

- b**  $x^2 + y^2 + 6x - 8y + 5 = 0$   
 $x^2 + 6x + y^2 - 8y + 5 = 0$   
 $(x+3)^2 - 9 + (y-4)^2 - 16 + 5 = 0$   
 $(x+3)^2 + (y-4)^2 = 20$   
Dus van  $d$  is de straal  $r_d = \sqrt{20} = 2\sqrt{5}$  en het middelpunt  $N(-3, 4)$ .  
 $d(M, N) = \sqrt{(-3-2)^2 + (4-1)^2} = \sqrt{50} = 5\sqrt{2}$   
Dus  $d(c, d) = 5\sqrt{2} - \sqrt{5} - 2\sqrt{5} = 5\sqrt{2} - 3\sqrt{5}$ .

**12 a**  $x = 4$  geeft  $4^2 + y^2 - 4 \cdot 4 + 2y - 8 = 0$

$$y^2 + 2y - 8 = 0$$

$$(y - 2)(y + 4) = 0$$

$$y = 2 \vee y = -4$$

Dus  $A(4, 2)$  en  $B(4, -4)$ .

$$x^2 + y^2 - 4x + 2y - 8 = 0$$

$$x^2 - 4x + y^2 + 2y - 8 = 0$$

$$(x - 2)^2 - 4 + (y + 1)^2 - 1 - 8 = 0$$

$$(x - 2)^2 + (y + 1)^2 = 13$$

Dus  $M(2, -1)$ .

Lijn  $m$  door  $M$  en  $A$  heeft  $rc_m = \frac{2 - 1}{4 - 2} = \frac{1}{2}$ .

$k \perp m$ , dus  $rc_k \cdot \frac{1}{2} = -1$  en dit geeft  $rc_k = -\frac{2}{3}$ .

$$k: y = -\frac{2}{3}x + b \quad \left\{ \begin{array}{l} -\frac{2}{3} \cdot 4 + b = 2 \\ -2\frac{2}{3} + b = 2 \end{array} \right.$$

$$\text{door } A(4, 2) \quad \left\{ \begin{array}{l} -2\frac{2}{3} + b = 2 \\ b = 4\frac{2}{3} \end{array} \right.$$

$$b = 4\frac{2}{3}$$

Dus  $k: y = -\frac{2}{3}x + 4\frac{2}{3}$ .

**b**  $\tan(\alpha) = rc_k = -\frac{2}{3}$  geeft  $\alpha = -33,69\dots^\circ$

$\tan(\beta) = rc_l = \frac{2}{3}$  geeft  $\beta = 33,69\dots^\circ$

$\beta - \alpha = 33,69\dots^\circ - (-33,69\dots^\circ) = 67,38\dots^\circ$

Dus  $\angle(k, l) \approx 67,4^\circ$ .

Lijn  $n$  door  $M$  en  $B$  heeft  $rc_n = \frac{-4 - 1}{4 - 2} = -\frac{5}{2}$ .

$l \perp n$ , dus  $rc_l \cdot -\frac{5}{2} = -1$  en dit geeft  $rc_l = \frac{2}{5}$ .

$$l: y = \frac{2}{5}x + b \quad \left\{ \begin{array}{l} \frac{2}{5} \cdot 4 + b = -4 \\ 2\frac{2}{5} + b = -4 \end{array} \right.$$

$$\text{door } B(4, -4) \quad \left\{ \begin{array}{l} 2\frac{2}{5} + b = -4 \\ b = -6\frac{2}{5} \end{array} \right.$$

$$b = -6\frac{2}{5}$$

Dus  $l: y = \frac{2}{5}x - 6\frac{2}{5}$ .

**13 a** Van  $c$  is  $M(0, -2)$  en  $r = \sqrt{20}$ .

$l: y = ax + b$

$A(-6, 0)$  op  $l$  geeft  $-6a + b = 0$ , dus  $b = 6a$ .

$l: y = ax + 6a$  oftewel  $l: ax - y + 6a = 0$

$d(M, l) = r$  geeft  $\frac{|0 + 2 + 6a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{20}$

$$|6a + 2| = \sqrt{20} \cdot \sqrt{a^2 + 1}$$

$$|6a + 2| = \sqrt{20a^2 + 20}$$

kwadrateren geeft

$$36a^2 + 24a + 4 = 20a^2 + 20$$

$$16a^2 + 24a - 16 = 0$$

$$2a^2 + 3a - 2 = 0$$

$$D = 3^2 - 4 \cdot 2 \cdot -2 = 25, \text{ dus } \sqrt{D} = 5$$

$$a = \frac{-3 \pm 5}{4} = \frac{1}{2} \vee a = \frac{-3 - 5}{4} = -2$$

$a = \frac{1}{2}$  geeft  $b = 6 \cdot \frac{1}{2} = 3$ , dus  $l_1: y = \frac{1}{2}x + 3$ .

$a = -2$  geeft  $b = 6 \cdot -2 = -12$ , dus  $l_2: y = -2x - 12$ .

**b**  $m: y = -\frac{1}{2}x + b$  oftewel  $m: \frac{1}{2}x + y - b = 0$ , dus  $m: x + 2y - 2b = 0$ .

$d(M, m) = r$  geeft  $\frac{|0 + 2 \cdot -2 - 2b|}{\sqrt{1^2 + 2^2}} = \sqrt{20}$

$$\frac{|-2b - 4|}{\sqrt{5}} = \sqrt{20}$$

$$|-2b - 4| = 10$$

$$-2b - 4 = 10 \vee -2b - 4 = -10$$

$$-2b = 14 \vee -2b = -6$$

$$b = -7 \vee b = 3$$

Dus  $m_1: y = -\frac{1}{2}x - 7$  en  $m_2: y = -\frac{1}{2}x + 3$ .

## Gemengde opgaven

### Bladzijde 128

1

a  $k: \frac{x}{2p+1} + \frac{y}{5} = 1$   
 $k: 5x + (2p+1)y = 5(2p+1)$   
 $k: 5x + (2p+1)y = 10p+5$   
 door  $A(2, 3)$   $\left. \begin{array}{l} 5 \cdot 2 + 3(2p+1) = 10p+5 \\ 10 + 6p + 3 = 10p+5 \\ -4p = -8 \\ p = 2 \end{array} \right\}$

b  $l: \frac{x}{2} + \frac{y}{q+2} = 1$

$l: (q+2)x + 2y = 2(q+2)$

$l: (q+2)x + 2y = 2q+4$

$m: y = -2x + 3$  oftewel  $m: 2x + y = 3$

$k$  en  $m$  geen punt gemeenschappelijk, dus  $\frac{5}{2} = \frac{2p+1}{1} \neq \frac{10p+5}{3}$ .

$\frac{5}{2} = \frac{2p+1}{1}$  geeft  $2(2p+1) = 5$   
 $4p+2 = 5$

$4p = 3$

$p = \frac{3}{4}$

Voor  $p = \frac{3}{4}$  geldt tevens  $\frac{2p+1}{1} \neq \frac{10p+5}{3}$ , dus voor  $p = \frac{3}{4}$  hebben  $k$  en  $m$  geen punt gemeenschappelijk.

$l$  en  $m$  geen punt gemeenschappelijk, dus  $\frac{q+2}{2} = \frac{2}{1} \neq \frac{2q+4}{3}$ .

$\frac{q+2}{2} = \frac{2}{1}$  geeft  $q+2 = 4$ , dus  $q = 2$ .

Voor  $q = 2$  geldt tevens  $\frac{2}{1} \neq \frac{2q+4}{3}$ , dus voor  $q = 2$  hebben  $l$  en  $m$  geen punt gemeenschappelijk.

Dus voor  $p = \frac{3}{4}$  en  $q = 2$  hebben de lijnen  $k$  en  $l$  geen punt gemeenschappelijk met de lijn  $m$ .

c De lijnen  $k$  en  $l$  vallen samen als  $k$  door het punt  $(2, 0)$  en  $l$  door het punt  $(0, 5)$  gaat.

Dus er moet gelden  $2p+1 = 2$  en  $q+2 = 5$  oftewel  $p = \frac{1}{2}$  en  $q = 3$ .

Dus voor  $p = \frac{1}{2}$  en  $q = 3$  vallen de lijnen  $k$  en  $l$  samen.

2

a  $m: y = ax + b$  door  $B(1, 6)$  geeft  $a + b = 6$ , dus  $b = -a + 6$ .

$m: y = ax - a + 6$  oftewel  $m: ax - y - a + 6 = 0$

$d(A, m) = 2$  geeft  $\frac{|-3a - 4 - a + 6|}{\sqrt{a^2 + (-1)^2}} = 2$

$|-4a + 2| = 2\sqrt{a^2 + 1}$

kwadrateren geeft

$16a^2 - 16a + 4 = 4a^2 + 4$

$12a^2 - 16a = 0$

$4a(3a - 4) = 0$

$4a = 0 \vee 3a = 4$

$a = 0 \vee a = 1\frac{1}{3}$

$a = 0$  geeft  $b = 0 + 6$ , dus  $m_1: y = 6$ .

$a = 1\frac{1}{3}$  geeft  $b = -1\frac{1}{3} + 6 = 4\frac{2}{3}$ , dus  $m_2: y = 1\frac{1}{3}x + 4\frac{2}{3}$ .



$$\text{b } rc_k = \frac{6-4}{1-3} = \frac{1}{2}$$

$$\left. \begin{array}{l} k: y = \frac{1}{2}x + b \\ \text{door } A(-3, 4) \end{array} \right\} \begin{array}{l} \frac{1}{2} \cdot -3 + b = 4 \\ -1\frac{1}{2} + b = 4 \\ b = 5\frac{1}{2} \end{array}$$

$$k: y = \frac{1}{2}x + 5\frac{1}{2} \text{ oftewel } k: \frac{1}{2}x - y = -5\frac{1}{2}, \text{ dus } k: x - 2y = -11.$$

$$n: x - 2y = c$$

$$\begin{aligned} d(A, n) = 2\sqrt{5} \text{ geeft } \frac{|-3 - 2 \cdot 4 - c|}{\sqrt{1^2 + (-2)^2}} &= 2\sqrt{5} \\ \frac{|-11 - c|}{\sqrt{5}} &= 2\sqrt{5} \\ |-11 - c| &= 10 \\ -11 - c &= 10 \vee -11 - c = -10 \\ c &= -21 \vee c = -1 \end{aligned}$$

$$\text{Dus } n_1: x - 2y = -21 \text{ en } n_2: x - 2y = -1.$$

$$\text{c } P(2p, p + 5\frac{1}{2}) \text{ is een punt op } k.$$

$$\begin{aligned} d(P, q) = 5 \text{ geeft } \frac{|3 \cdot 2p + 4(p + 5\frac{1}{2}) - 7|}{\sqrt{3^2 + 4^2}} &= 5 \\ \frac{|10p + 15|}{5} &= 5 \\ |10p + 15| &= 25 \\ 10p + 15 &= 25 \vee 10p + 15 = -25 \\ 10p &= 10 \vee 10p = -40 \\ p &= 1 \vee p = -4 \end{aligned}$$

$$\text{Dus } P_1(2, 6\frac{1}{2}) \text{ en } P_2(-8, 1\frac{1}{2}).$$

$$\text{d } rc_l = \frac{-1-4}{0-3} = -1\frac{2}{3}$$

$$\tan(\alpha) = rc_k = \frac{1}{2} \text{ geeft } \alpha = 26,5\dots^\circ$$

$$\tan(\beta) = rc_l = -1\frac{2}{3} \text{ geeft } \beta = -59,0\dots^\circ$$

$$\alpha - \beta = 26,5\dots^\circ - (-59,0\dots^\circ) = 85,6\dots^\circ$$

$$\text{Dus } \angle(k, l) \approx 86^\circ.$$

$$\text{3 a } k: y = -3x + 1 \quad \tan(\alpha) = rc_k = -3 \text{ geeft } \alpha = -71,56\dots^\circ$$

$$l: 2x + y = 3$$

$$y = -2x + 3$$

$$\tan(\beta) = rc_l = -2 \text{ geeft } \beta = -63,43\dots^\circ$$

$$\beta - \alpha = -63,43\dots^\circ - (-71,56\dots^\circ) = 8,13\dots^\circ$$

$$\text{Dus } \angle(k, l) \approx 8,1^\circ.$$

$$\begin{aligned} \text{b } -63,43\dots^\circ + 15^\circ &= -48,43\dots^\circ \text{ geeft } p = rc_k = \tan(-48,43\dots^\circ) \approx -1,128 \text{ of} \\ -63,43\dots^\circ - 15^\circ &= -78,43\dots^\circ \text{ geeft } p = rc_k = \tan(-78,43\dots^\circ) \approx -4,887. \end{aligned}$$

$$\text{4 } k_p: \frac{x}{p} - \frac{y}{4} = 1 \text{ oftewel } k_p: 4x - py = 4p$$

$$\left. \begin{array}{l} k_p: 4x - py = 4p \\ \text{door } A(4, 4) \end{array} \right\} \begin{array}{l} 4 \cdot 4 - 4p = 4p \\ 16 - 4p = 4p \\ -8p = -16 \\ p = 2 \end{array}$$

$$\text{Dus } k_2: 4x - 2y = 8.$$

$$l \perp k_2, \text{ dus } l: 2x + 4y = c.$$

$$\left. \begin{array}{l} l: 2x + 4y = c \\ \text{door } A(4, 4) \end{array} \right\} c = 2 \cdot 4 + 4 \cdot 4 = 24$$

$$\text{Dus } l: 2x + 4y = 24 \text{ en } P(2a, 6-a) \text{ op } l.$$

$$\begin{aligned}
 d(A, P) = 5\sqrt{5} \text{ geeft } \sqrt{(2a-4)^2 + (6-a-4)^2} &= 5\sqrt{5} \\
 (2a-4)^2 + (2-a)^2 &= 125 \\
 4a^2 - 16a + 16 + 4 - 4a + a^2 &= 125 \\
 5a^2 - 20a - 105 &= 0 \\
 a^2 - 4a - 21 &= 0 \\
 (a+3)(a-7) &= 0 \\
 a = -3 \vee a = 7
 \end{aligned}$$

Dus  $P_1(-6, 9)$  en  $P_2(14, -1)$ .

**5**  $d(A, B) = \sqrt{(4-2)^2 + (4-8)^2} = \sqrt{20} = 2\sqrt{5}$

Lijn  $k$  door  $A$  en  $B$  heeft  $rc_k = \frac{4-8}{4-2} = -2$ .

$$\begin{aligned}
 k: y = -2x + b \quad & \left. \begin{array}{l} -2 \cdot 2 + b = 8 \\ -4 + b = 8 \end{array} \right\} \\
 \text{door } A(2, 8) \quad & \\
 b &= 12
 \end{aligned}$$

$k: y = -2x + 12$  oftewel  $k: 2x + y - 12 = 0$

$O(\triangle ABC) = 10$  geeft  $\frac{1}{2} \cdot d(A, B) \cdot d(C, k) = 10$

$$\frac{1}{2} \cdot 2\sqrt{5} \cdot \frac{|2 \cdot 5 + q - 12|}{\sqrt{2^2 + 1^2}} = 10$$

$$|q - 2| = 10$$

$$q - 2 = 10 \vee q - 2 = -10$$

$$q = 12 \vee q = -8$$

**6 a**  $r = d(M, k) = \frac{|3 \cdot 8 - 4 \cdot 2 - 6|}{\sqrt{3^2 + (-4)^2}} = \frac{|10|}{5} = \frac{10}{5} = 2$

Dus  $c: (x-8)^2 + (y-2)^2 = 4$ .

**b**  $l: 3x - 4y = c$

$P(2, 0)$  is een punt op  $k$ .

$$d(P, l) = 3 \text{ geeft } \frac{|3 \cdot 2 - 4 \cdot 0 - c|}{\sqrt{3^2 + (-4)^2}} = 3$$

$$\frac{|6 - c|}{5} = 3$$

$$|6 - c| = 15$$

$$6 - c = 15 \vee 6 - c = -15$$

$$c = -9 \vee c = 21$$

Dus  $l_1: 3x - 4y = -9$  en  $l_2: 3x - 4y = 21$ .

**c**  $A(a, a)$  is een punt op  $m$ .

$$d(A, k) = 4 \text{ geeft } \frac{|3a - 4a - 6|}{\sqrt{3^2 + (-4)^2}} = 4$$

$$\frac{|-a - 6|}{5} = 4$$

$$|-a - 6| = 20$$

$$-a - 6 = 20 \vee -a - 6 = -20$$

$$a = -26 \vee a = 14$$

Dus  $A_1(-26, -26)$  en  $A_2(14, 14)$ .

**7**  $O(\triangle AMD) = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

Stel  $A(0, 0)$  en  $AB$  horizontaal, dan  $C(2, 1)$ ,  $D(0, 1)$  en  $M(1, 0)$ .

$$rc_{AC} = \frac{1-0}{2-0} = \frac{1}{2}, \text{ dus } AC: y = \frac{1}{2}x.$$

$P(2p, p)$  is een punt op  $k$ .

$$rc_{DP} = \frac{p-1}{2p-0} = \frac{p-1}{2p} \text{ en } rc_{MP} = \frac{p-0}{2p-1} = \frac{p}{2p-1}$$

$DP \perp MP$  geeft  $rc_{DP} \cdot rc_{MP} = -1$

$$\frac{p-1}{2p} \cdot \frac{p}{2p-1} = -1$$

$$\frac{p(p-1)}{2p(2p-1)} = -1$$

$$\frac{p^2-p}{4p^2-2p} = -1$$

$$p^2-p = -4p^2+2p$$

$$5p^2-3p=0$$

$$p(5p-3)=0$$

$$p=0 \vee 5p=3$$

$$p=0 \vee p=\frac{3}{5}$$

$p=0$  voldoet niet, dus  $p=\frac{3}{5}$  en  $P(\frac{6}{5}, \frac{3}{5})$ .

$$DP = \sqrt{(\frac{6}{5}-0)^2 + (\frac{3}{5}-1)^2} = \sqrt{\frac{8}{5}} \text{ en } MP = \sqrt{(\frac{6}{5}-1)^2 + (\frac{3}{5}-0)^2} = \sqrt{\frac{2}{5}}$$

$$O(\triangle MPD) = \frac{1}{2} \cdot \sqrt{\frac{8}{5}} \cdot \sqrt{\frac{2}{5}} = \frac{1}{2} \cdot \sqrt{\frac{16}{25}} = \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$$

$$O(\triangle AMPD) = O(\triangle AMD) + O(\triangle MPD) = \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$$

#### Bladzijde 129

**8 a**  $x^2 + y^2 - 12x + 4y + 15 = 0$

$$x^2 - 12x + y^2 + 4y + 15 = 0$$

$$(x-6)^2 - 36 + (y+2)^2 - 4 + 15 = 0$$

$$(x-6)^2 + (y+2)^2 = 25$$

Dus  $M(6, -2)$  en  $r_c = 5$ .

$$l: y = ax + b$$

$A(-8, -4)$  op  $l$  geeft  $-8a + b = -4$ , dus  $b = 8a - 4$ .

$$l: y = ax + 8a - 4 \text{ oftewel } l: ax - y + 8a - 4 = 0$$

$$d(M, l) = r_c + 5 \text{ geeft } \frac{|6a + 2 + 8a - 4|}{\sqrt{a^2 + (-1)^2}} = 5 + 5$$

$$|14a - 2| = 10\sqrt{a^2 + 1}$$

kwadrateren geeft

$$196a^2 - 56a + 4 = 100a^2 + 100$$

$$96a^2 - 56a - 96 = 0$$

$$12a^2 - 7a - 12 = 0$$

$$D = (-7)^2 - 4 \cdot 12 \cdot -12 = 625, \text{ dus } \sqrt{D} = 25$$

$$a = \frac{7+25}{24} = 1\frac{1}{3} \vee a = \frac{7-25}{24} = -\frac{3}{4}$$

$$a = 1\frac{1}{3} \text{ geeft } b = 8 \cdot 1\frac{1}{3} - 4 = 8\frac{2}{3}, \text{ dus } l_1: y = 1\frac{1}{3}x + 8\frac{2}{3}.$$

$$a = -\frac{3}{4} \text{ geeft } b = 8 \cdot -\frac{3}{4} - 4 = -10, \text{ dus } l_2: y = -\frac{3}{4}x - 10.$$

**b**  $N(0, a)$  is een punt op de  $y$ -as.

Uit  $M(6, -2)$  en  $r_c = 5$  volgt dat  $c$  geen punt met de  $y$ -as gemeenschappelijk heeft, en dat cirkel  $d$  dus niet in cirkel  $c$  ligt.

$d(c, d) = 3$  geeft  $d(M, N) = r_c + r_d + 3$

$$\sqrt{(0-6)^2 + (a-(-2))^2} = 5 + 2 + 3$$

$$\sqrt{36 + (a+2)^2} = 10$$

$$36 + (a+2)^2 = 100$$

$$(a+2)^2 = 64$$

$$a+2 = 8 \vee a+2 = -8$$

$$a = 6 \vee a = -10$$

Dus  $d_1: x^2 + (y-6)^2 = 4$  en  $d_2: x^2 + (y+10)^2 = 4$ .

**c**  $e$  raakt de  $y$ -as in  $O$ , dus het middelpunt  $P$  van  $e$  ligt op de  $x$ -as.

$P(p, 0)$  is een punt op de  $x$ -as.

$$r_e = OP = p$$

Hieruit volgt  $e: (x-p)^2 + y^2 = p^2$ .

$M(6, -2)$  op  $e$  geeft  $(6-p)^2 + (-2)^2 = p^2$

$$36 - 12p + p^2 + 4 = p^2$$

$$-12p = -40$$

$$p = 3\frac{1}{3}$$

Dus  $e: (x - 3\frac{1}{3})^2 + y^2 = 11\frac{1}{9}$ .

**9 a** Invullen van  $x = -6$  en  $y = 4$  in  $x^2 + y^2 + 4x - 12y + 20 = 0$  geeft  $(-6)^2 + 4^2 + 4 \cdot -6 - 12 \cdot 4 + 20 = 0$  en dit klopt, dus  $A$  ligt op  $c$ .

$$x^2 + y^2 + 4x - 12y + 20 = 0$$

$$x^2 + 4x + y^2 - 12y + 20 = 0$$

$$(x+2)^2 - 4 + (y-6)^2 - 36 + 20 = 0$$

$$(x+2)^2 + (y-6)^2 = 20$$

Dus  $M(-2, 6)$ .

Lijn  $k$  is de raaklijn aan  $c$  in  $A$ .

$$\text{Lijn } l \text{ door } M \text{ en } A \text{ heeft } rc_l = \frac{4-6}{-6-(-2)} = \frac{1}{2}.$$

$k \perp l$ , dus  $rc_k \cdot \frac{1}{2} = -1$  en dit geeft  $rc_k = -2$ .

$f(-6) = 4$ , dus  $A$  ligt op de grafiek van  $f$ .

$$f(x) = \frac{2x^2}{3x+36} \text{ geeft } f'(x) = \frac{(3x+36) \cdot 4x - 2x^2 \cdot 3}{(3x+36)^2} = \frac{6x^2 + 144x}{(3x+36)^2}$$

$f'(-6) = -2$  en dit is de richtingscoëfficiënt van  $k$ .

Dus  $k$  is ook raaklijn aan de grafiek van  $f$  in  $A$ .

$k$  raakt  $c$  in  $A$  en  $k$  raakt de grafiek van  $f$  in  $A$ , dus  $c$  raakt de grafiek van  $f$  in  $A$ .

**b**  $g(-6) = 8$  geeft  $2\sqrt{-6a+b} = 8$

$$\sqrt{-6a+b} = 4$$

$$-6a+b = 16$$

$$b = 6a + 16$$

Lijn  $m$  is de raaklijn aan  $c$  in  $B$ .

$$\text{Lijn } n \text{ door } M \text{ en } B \text{ heeft } rc_n = \frac{8-6}{-6-(-2)} = -\frac{1}{2}.$$

$m \perp n$ , dus  $rc_m \cdot -\frac{1}{2} = -1$  en dit geeft  $rc_m = 2$ .

$$g(x) = 2\sqrt{ax+b} \text{ geeft } g'(x) = 2 \cdot \frac{1}{2\sqrt{ax+b}} \cdot a = \frac{a}{\sqrt{ax+b}}$$

Lijn  $p$  is de raaklijn aan de grafiek van  $g$  in  $B$ .

$$rc_p = g'(-6) = \frac{a}{\sqrt{-6a+b}}$$

$m \perp p$  geeft  $rc_m \cdot rc_p = -1$

$$2 \cdot \frac{a}{\sqrt{-6a+b}} = -1$$

$$\frac{a}{\sqrt{-6a+b}} = \frac{-1}{2}$$

$$-\sqrt{-6a+b} = 2a$$

kwadrateren geeft

$$-6a + b = 4a^2$$

$$b = 4a^2 + 6a$$

Hieruit volgt  $4a^2 + 6a = 6a + 16$

$$4a^2 = 16$$

$$a^2 = 4$$

$$a = 2 \vee a = -2$$

$a = 2$  voldoet niet (want dan is  $rc_p = \frac{a}{\sqrt{-6a+b}} > 0$ ), dus  $a = -2$ .

$a = -2$  geeft  $b = 6 \cdot -2 + 16 = 4$  en dit geeft  $g(x) = 2\sqrt{-2x+4}$

$$-2x + 4 = 0$$

$$-2x = -4$$

$$x = 2$$

$g(2) = 0$ , dus  $R(2, 0)$ .

$$d(R, M) = \sqrt{(-2-2)^2 + (6-0)^2} = \sqrt{52} = 2\sqrt{13}$$

De straal van  $c$  is  $r = \sqrt{20} = 2\sqrt{5}$ .

Dus  $d(R, c) = 2\sqrt{13} - 2\sqrt{5}$ .

**10**  $x^2 + y^2 - 8x + 30y + 191 = 0$

$$x^2 - 8x + y^2 + 30y + 191 = 0$$

$$(x-4)^2 - 16 + (y+15)^2 - 225 + 191 = 0$$

$$(x-4)^2 + (y+15)^2 = 50$$

Dus  $M(4, -15)$  en  $r = \sqrt{50}$ .

Lijn  $l$  door  $M$  en  $A$  heeft  $rc_l = \frac{-10 - (-15)}{9 - 4} = 1$ .

$k \perp l$ , dus  $rc_k \cdot 1 = -1$  en dit geeft  $rc_k = -1$ .

$$\left. \begin{array}{l} k: y = -x + b \\ \text{door } A(9, -10) \end{array} \right\} \begin{array}{l} -9 + b = -10 \\ b = -1 \end{array}$$

Dus  $k: y = -x - 1$ .

$k$  snijden met de  $x$ -as geeft  $-x - 1 = 0$

$$x = -1$$

Dus  $B(-1, 0)$ .

$m: y = ax + b$

$B(-1, 0)$  op  $m$  geeft  $-a + b = 0$ , dus  $b = a$ .

$m: y = ax + a$  oftewel  $m: ax - y + a = 0$

$$d(M, m) = r \text{ geeft } \frac{|4a + 15 + a|}{\sqrt{a^2 + (-1)^2}} = \sqrt{50}$$

$$|5a + 15| = \sqrt{50} \cdot \sqrt{a^2 + 1}$$

$$|5a + 15| = \sqrt{50a^2 + 50}$$

kwadrateren geeft

$$25a^2 + 150a + 225 = 50a^2 + 50$$

$$-25a^2 + 150a + 175 = 0$$

$$a^2 - 6a - 7 = 0$$

$$(a+1)(a-7) = 0$$

$$a = -1 \vee a = 7$$

$a = 7$  geeft  $b = 7$ , dus  $m: y = 7x + 7$ .

$$k: y = -x - 1$$

$$\tan(\alpha) = rc_k = -1 \text{ geeft } \alpha = -45^\circ$$

$$\beta - \alpha = 81,8\dots^\circ - (-45^\circ) = 126,8\dots^\circ$$

Dus  $\angle(k, m) = 180^\circ - 126,8\dots^\circ \approx 53^\circ$ .

$$m: y = 7x + 7$$

$$\tan(\beta) = rc_m = 7 \text{ geeft } \beta = 81,8\dots^\circ$$

**11 a**  $x^2 + y^2 - 8x - 4y + 15 = 0$   
 $x^2 - 8x + y^2 - 4y + 15 = 0$   
 $(x - 4)^2 - 16 + (y - 2)^2 - 4 + 15 = 0$   
 $(x - 4)^2 + (y - 2)^2 = 5$   
Dus  $M_1(4, 2)$  en  $r_1 = \sqrt{5}$ .  
 $l \perp k$ , dus  $l: x + 2y = c$ .

$$d(M_1, l) = r_1 \text{ geeft } \frac{|4 + 2 \cdot 2 - c|}{\sqrt{1^2 + 2^2}} = \sqrt{5}$$

$$\frac{|8 - c|}{\sqrt{5}} = \sqrt{5}$$

$$|8 - c| = 5$$

$$8 - c = 5 \vee 8 - c = -5$$

$$c = 3 \vee c = 13$$

Dus  $l_1: x + 2y = 3$  en  $l_2: x + 2y = 13$ .

**b**  $d(A, M_1) = \sqrt{(4 - 15)^2 + (2 - 9)^2} = \sqrt{170}$   
 $d(A, c_1) = \sqrt{170} - \sqrt{5} = 10,80\dots$   
 $d(M_1, k) = \frac{|2 \cdot 4 - 2 - 21|}{\sqrt{2^2 + (-1)^2}} = \frac{|-15|}{\sqrt{5}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$

$$d(k, c_1) = 3\sqrt{5} - \sqrt{5} = 2\sqrt{5}$$

$$2\frac{1}{2} \cdot d(k, c_1) = 2\frac{1}{2} \cdot 2\sqrt{5} = 11,18\dots$$

Dus  $d(A, c_1) > 2\frac{1}{2} \cdot d(k, c_1)$  is niet juist.

**c** Middellijn  $AB$ , dus het middelpunt van  $c_2$  is het midden van  $AB$  oftewel

$$M_2(\frac{1}{2}(15 + 1), \frac{1}{2}(9 + (-1))) = M_2(8, 4).$$

$$\text{De straal van } c_2 \text{ is } r_2 = d(A, M_2) = \sqrt{(8 - 15)^2 + (4 - 9)^2} = \sqrt{74}.$$

$$d(M_1, M_2) = \sqrt{(8 - 4)^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Dus } d(c_1, c_2) = \sqrt{74} - \sqrt{5} - 2\sqrt{5} = \sqrt{74} - 3\sqrt{5}.$$

**12**  $x^2 + y^2 + 8x = 0$   
 $x^2 + 8x + y^2 = 0$   
 $(x + 4)^2 - 16 + y^2 = 0$   
 $(x + 4)^2 + y^2 = 16$   
Dus  $M(-4, 0)$  en  $r_c = 4$ .

Lijn  $l$  gaat door  $M$  en staat loodrecht op  $k$ .

$$\left. \begin{array}{l} l: 3x - 4y = c \\ \text{door } M(-4, 0) \end{array} \right\} c = 3 \cdot -4 - 4 \cdot 0 = -12$$

$$\text{Dus } l: 3x - 4y = -12.$$

$k$  en  $l$  snijden geeft het snijpunt  $S$ , dat het middelpunt van cirkel  $d$  is.

$$\begin{cases} 4x + 3y = 9 \\ 3x - 4y = -12 \end{cases} \begin{vmatrix} 4 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} 16x + 12y = 36 \\ 9x - 12y = -36 \end{cases} +$$

$$\frac{25x}{x} = 0$$

$$x = 0$$

$$x = 0 \text{ geeft } 4 \cdot 0 + 3y = 9$$

$$3y = 9$$

$$y = 3$$

Dus  $S(0, 3)$ .

$$d(M, S) = \sqrt{(0 - (-4))^2 + (3 - 0)^2} = \sqrt{25} = 5 \text{ en } r_d = 5 - 4 = 1$$

$$\text{Dus } d: x^2 + (y - 3)^2 = 1.$$

# 8 Goniometrische functies

## 8.1 De eenheidscirkel en radialen

### Bladzijde 132

- 1 a**  $\sin(\alpha) = \frac{PQ}{OP} = \frac{PQ}{1} = PQ$ , dus  $PQ = \sin(65^\circ) \approx 0,91$ .  
 $\cos(\alpha) = \frac{OQ}{OP} = \frac{OQ}{1} = OQ$ , dus  $OQ = \cos(65^\circ) \approx 0,42$ .
- b**  $P(0,42; 0,91)$
- c**  $\angle POQ = 180^\circ - 115^\circ = 65^\circ$   
 $PQ \approx 0,91$ ,  $OQ \approx 0,42$  en  $P(-0,42; 0,91)$
- d**  $\cos(115^\circ) \approx -0,42$  en  $\sin(115^\circ) \approx 0,91$   
Er geldt  $x_P = \cos(115^\circ)$  en  $y_P = \sin(115^\circ)$ .
- e**  $\angle POQ = 245^\circ - 180^\circ = 65^\circ$  en  $P(-0,42; -0,91)$
- f** De GR geeft  $\cos(245^\circ) \approx -0,42$  en  $\sin(245^\circ) \approx -0,91$ , oftewel  $x_P = \cos(245^\circ)$  en  $y_P = \sin(245^\circ)$ .

### Bladzijde 134

- 2 a**  $\sin(0^\circ) = 0$  **e**  $\sin(270^\circ) = -1$  **i**  $\sin(450^\circ) = 1$   
**b**  $\cos(0^\circ) = 1$  **f**  $\cos(270^\circ) = 0$  **j**  $\cos(-90^\circ) = 0$   
**c**  $\sin(90^\circ) = 1$  **g**  $\sin(360^\circ) = 0$  **k**  $\tan(-540^\circ) = 0$   
**d**  $\cos(90^\circ) = 0$  **h**  $\tan(360^\circ) = 0$  **l**  $\cos(-180^\circ) = -1$
- 3 a**  $\alpha = 0^\circ$ ,  $\alpha = 180^\circ$  en  $\alpha = 360^\circ$  **c**  $\alpha = 90^\circ$  **e**  $\alpha = 45^\circ$  en  $\alpha = 225^\circ$   
**b**  $\alpha = 90^\circ$  en  $\alpha = 270^\circ$  **d**  $\alpha = 0^\circ$  en  $\alpha = 360^\circ$  **f**  $\alpha = 135^\circ$  en  $\alpha = 315^\circ$

- 4**  $P(\cos(110^\circ), \sin(110^\circ)) \approx P(-0,34; 0,94)$   
 $Q(\cos(200^\circ), \sin(200^\circ)) \approx Q(-0,94; -0,34)$   
 $R(\cos(-102^\circ), \sin(-102^\circ)) \approx R(-0,21; -0,98)$   
 $S(\cos(-50^\circ), \sin(-50^\circ)) \approx S(0,64; -0,77)$

- 5** De draaiingshoek die bij  $B$  hoort, is  $\frac{360^\circ}{5} = 72^\circ$ . Dus  $B(\cos(72^\circ), \sin(72^\circ)) \approx B(0,31; 0,95)$ .  
De draaiingshoek die bij  $C$  hoort, is  $2 \cdot 72^\circ = 144^\circ$ . Dus  $C(\cos(144^\circ), \sin(144^\circ)) \approx C(-0,81; 0,59)$ .  
De draaiingshoek die bij  $D$  hoort, is  $3 \cdot 72^\circ = 216^\circ$ . Dus  $D(\cos(216^\circ), \sin(216^\circ)) \approx D(-0,81; -0,59)$ .  
De draaiingshoek die bij  $E$  hoort, is  $4 \cdot 72^\circ = 288^\circ$ . Dus  $E(\cos(288^\circ), \sin(288^\circ)) \approx E(0,31; -0,95)$ .

- 6**  $\beta = 180^\circ - 30^\circ = 150^\circ$

### Bladzijde 136

- 7 a**  $x_P = 0,81$ , dus  $\cos(\alpha) = 0,81$ .  
De GR geeft  $35,90\dots^\circ$ .  
Dus  $\alpha \approx 35,9^\circ$ .
- b**  $y_P = 0,94$ , dus  $\sin(\alpha) = 0,94$ .  
De GR geeft  $70,05\dots^\circ$ .  
Dus  $\alpha = 180^\circ - 70,05\dots^\circ \approx 109,9^\circ$ .
- c**  $x_P = 0,26$ , dus  $\cos(\alpha) = 0,26$ .  
De GR geeft  $74,92\dots^\circ$ .  
Dus  $\alpha \approx -74,9^\circ$ .
- d**  $y_P = -0,22$ , dus  $\sin(\alpha) = -0,22$ .  
De GR geeft  $-12,70\dots^\circ$ .  
Dus  $\alpha = 180^\circ + 12,70\dots^\circ \approx 192,7^\circ$ .
- 8**  $x_P = -0,87$ , dus  $\cos(\alpha_P) = -0,87$ .  
De GR geeft  $150,45\dots^\circ$ , dus  $\alpha_P = 360^\circ - 150,45\dots^\circ = 209,54\dots^\circ$ .  
 $y_Q = 0,92$ , dus  $\sin(\alpha_Q) = 0,92$ .  
De GR geeft  $66,92\dots^\circ$ , dus  $\alpha_Q = 66,92\dots^\circ$ .  
 $\angle POQ = \alpha_P - \alpha_Q = 209,54\dots^\circ - 66,92\dots^\circ \approx 142,6^\circ$

- 9 a** omtrek cirkel  $= 2\pi r$   
 In de eenheidscirkel is  $r = 1$ , dus omtrek eenheidscirkel  $= 2\pi \cdot 1 = 2\pi$ .
- b** Bij  $\alpha = 90^\circ$  hoort een kwart cirkelboog, dus de lengte van de door  $P$  doorlopen cirkelboog is  $\frac{1}{4} \cdot 2\pi = \frac{1}{2}\pi$ .
- c**
- |                        |           |                  |             |                   |             |
|------------------------|-----------|------------------|-------------|-------------------|-------------|
| draaiingshoek $\alpha$ | $0^\circ$ | $90^\circ$       | $180^\circ$ | $270^\circ$       | $360^\circ$ |
| lengte cirkelboog $b$  | 0         | $\frac{1}{2}\pi$ | $\pi$       | $1\frac{1}{2}\pi$ | $2\pi$      |

#### Bladzijde 138

- 10 a**  $\frac{1}{6}\pi \text{ rad} = \frac{1}{6} \cdot 180^\circ = 30^\circ$       **e**  $1\frac{1}{4}\pi \text{ rad} = 1\frac{1}{4} \cdot 180^\circ = 225^\circ$   
**b**  $\frac{1}{4}\pi \text{ rad} = \frac{1}{4} \cdot 180^\circ = 45^\circ$       **f**  $1\frac{1}{4} \text{ rad} = 1\frac{1}{4} \cdot \frac{180^\circ}{\pi} \approx 71,6^\circ$   
**c**  $2\pi \text{ rad} = 2 \cdot 180^\circ = 360^\circ$       **g**  $-2\frac{1}{3}\pi \text{ rad} = -2\frac{1}{3} \cdot 180^\circ = -420^\circ$   
**d**  $2 \text{ rad} = 2 \cdot \frac{180^\circ}{\pi} \approx 114,6^\circ$       **h**  $-2\frac{1}{3} \text{ rad} = -2\frac{1}{3} \cdot \frac{180^\circ}{\pi} \approx -133,7^\circ$

- 11 a**  $360^\circ = \frac{360}{180}\pi \text{ rad} = 2\pi \text{ rad}$       **e**  $90^\circ = \frac{90}{180}\pi \text{ rad} = \frac{1}{2}\pi \text{ rad}$   
**b**  $30^\circ = \frac{30}{180}\pi \text{ rad} = \frac{1}{6}\pi \text{ rad}$       **f**  $135^\circ = \frac{135}{180}\pi \text{ rad} = \frac{3}{4}\pi \text{ rad}$   
**c**  $45^\circ = \frac{45}{180}\pi \text{ rad} = \frac{1}{4}\pi \text{ rad}$       **g**  $300^\circ = \frac{300}{180}\pi \text{ rad} = 1\frac{2}{3}\pi \text{ rad}$   
**d**  $60^\circ = \frac{60}{180}\pi \text{ rad} = \frac{1}{3}\pi \text{ rad}$       **h**  $210^\circ = \frac{210}{180}\pi \text{ rad} = 1\frac{1}{6}\pi \text{ rad}$

- 12 a**  $10^\circ = \frac{10}{180}\pi \text{ rad} \approx 0,17 \text{ rad}$       **c**  $1030^\circ = \frac{1030}{180}\pi \text{ rad} \approx 17,98 \text{ rad}$   
**b**  $57,3^\circ = \frac{57,3}{180}\pi \text{ rad} \approx 1,00 \text{ rad}$       **d**  $90^\circ = \frac{90}{180}\pi \text{ rad} \approx 1,57 \text{ rad}$

- 13 a**  $\cos(\frac{5}{8}\pi) \approx -0,38$       **c**  $\sin(\frac{4}{5}\pi) \approx 0,59$       **e**  $\cos(7,6\pi) \approx 0,31$   
**b**  $\cos(\frac{5}{8}) \approx 0,81$       **d**  $\sin(\frac{4}{5}) \approx 0,72$       **f**  $\cos(7,6) \approx 0,25$

- 14 a**  $\sin(\alpha) = 0,92$  geeft  $\alpha \approx 1,17$       **c**  $\sin(\alpha) = \frac{5}{12}$  geeft  $\alpha \approx 0,43$       **e**  $\sin(\alpha) = \frac{1}{3}\sqrt{5}$  geeft  $\alpha \approx 0,84$   
**b**  $\cos(\alpha) = 0,85$  geeft  $\alpha \approx 0,55$       **d**  $\cos(\alpha) = \frac{3}{17}$  geeft  $\alpha \approx 1,39$       **f**  $\cos(\alpha) = \frac{1}{4}\sqrt{2}$  geeft  $\alpha \approx 1,21$

#### Bladzijde 139

- 15 a**  $y_P = 0,35$ , dus  $\sin(\alpha) = 0,35$ .  
 De GR geeft  $0,357\dots$   
 Dus de bijhorende draaiingshoeken zijn  $0,36$  en  $\pi - 0,357\dots \approx 2,78$ .  
**b**  $x_Q = -0,35$ , dus  $\cos(\alpha) = -0,35$ .  
 De GR geeft  $1,928\dots$   
 Dus de bijhorende draaiingshoeken zijn  $1,93$  en  $2\pi - 1,928\dots \approx 4,35$ .

- 16**  $AB = 2\cos(\alpha)$  en  $BC = 2\sin(\alpha)$ , dus  $O(ABCD) = 2\cos(\alpha) \cdot 2\sin(\alpha) = 4\cos(\alpha)\sin(\alpha)$ .  
 $\alpha = 0,2$  geeft  $O(ABCD) = 4\cos(0,2)\sin(0,2) = 0,778\dots$   
 $\alpha = 0,4$  geeft  $O(ABCD) = 4\cos(0,4)\sin(0,4) = 1,434\dots$   
 De oppervlakte van de rechthoek neemt toe met  $\frac{1,434\dots - 0,778\dots}{0,778\dots} \cdot 100\% \approx 84,2\%$ .

- 17 a**  $\sin(\text{hoek}) = \frac{\text{overstaande rechthoekszijde}}{\text{schuine zijde}}$ , dus  $\sin(30^\circ) = \frac{1}{2}$ .  
 $\cos(\text{hoek}) = \frac{\text{aanliggende rechthoekszijde}}{\text{schuine zijde}}$ , dus  $\cos(45^\circ) = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$ .  
 $\tan(\text{hoek}) = \frac{\text{overstaande rechthoekszijde}}{\text{aanliggende rechthoekszijde}}$ , dus  $\tan(60^\circ) = \frac{\sqrt{3}}{1} = \sqrt{3}$ .



| b hoek  | 30°                   | 45°                   | 60°                   |
|---------|-----------------------|-----------------------|-----------------------|
| sinus   | $\frac{1}{2}$         | $\frac{1}{2}\sqrt{2}$ | $\frac{1}{2}\sqrt{3}$ |
| cosinus | $\frac{1}{2}\sqrt{3}$ | $\frac{1}{2}\sqrt{2}$ | $\frac{1}{2}$         |
| tangens | $\frac{1}{3}\sqrt{3}$ | 1                     | $\sqrt{3}$            |

### Bladzijde 140

- 18 a** De waarden van de sinus bij de draaiingshoeken  $0, \frac{1}{6}\pi, \frac{1}{4}\pi, \frac{1}{3}\pi, \frac{1}{2}\pi$  zijn van de vorm  $\frac{1}{2}\sqrt{A}$  met  $A = 0, 1, 2, 3, 4$ . En de waarden van de cosinus bij de draaiingshoeken  $0, \frac{1}{6}\pi, \frac{1}{4}\pi, \frac{1}{3}\pi, \frac{1}{2}\pi$  zijn van de vorm  $\frac{1}{2}\sqrt{A}$  met  $A = 4, 3, 2, 1, 0$ .

**b**  $\tan(0) = \frac{\sin(0)}{\cos(0)} = \frac{0}{1} = 0$

$$\tan\left(\frac{1}{6}\pi\right) = \frac{\sin\left(\frac{1}{6}\pi\right)}{\cos\left(\frac{1}{6}\pi\right)} = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$$

$$\tan\left(\frac{1}{4}\pi\right) = \frac{\sin\left(\frac{1}{4}\pi\right)}{\cos\left(\frac{1}{4}\pi\right)} = \frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = 1$$

$$\tan\left(\frac{1}{3}\pi\right) = \frac{\sin\left(\frac{1}{3}\pi\right)}{\cos\left(\frac{1}{3}\pi\right)} = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = \sqrt{3}$$

$$\tan\left(\frac{1}{2}\pi\right) = \frac{\sin\left(\frac{1}{2}\pi\right)}{\cos\left(\frac{1}{2}\pi\right)} = \frac{1}{0} \text{ kan niet}$$

**19 a**  $\sin\left(\frac{3}{4}\pi\right) = \frac{1}{2}\sqrt{2}$

**b**  $\cos\left(1\frac{1}{6}\pi\right) = -\frac{1}{2}\sqrt{3}$

**c**  $\cos\left(1\frac{2}{3}\pi\right) = \frac{1}{2}$

**d**  $\tan\left(\frac{2}{3}\pi\right) = \frac{\sin\left(\frac{2}{3}\pi\right)}{\cos\left(\frac{2}{3}\pi\right)} = \frac{\frac{1}{2}\sqrt{3}}{-\frac{1}{2}} = -\sqrt{3}$

**e**  $\sin\left(-\frac{1}{4}\pi\right) = -\frac{1}{2}\sqrt{2}$

**f**  $\tan\left(1\frac{3}{4}\pi\right) = \frac{\sin\left(1\frac{3}{4}\pi\right)}{\cos\left(1\frac{3}{4}\pi\right)} = \frac{-\frac{1}{2}\sqrt{2}}{\frac{1}{2}\sqrt{2}} = -1$

**20**  $\tan\left(1\frac{5}{6}\pi\right) - \tan\left(\frac{2}{3}\pi\right) = \frac{\sin\left(1\frac{5}{6}\pi\right)}{\cos\left(1\frac{5}{6}\pi\right)} - \frac{\sin\left(\frac{2}{3}\pi\right)}{\cos\left(\frac{2}{3}\pi\right)} = \frac{-\frac{1}{2}}{\frac{1}{2}\sqrt{3}} - \frac{\frac{1}{2}\sqrt{3}}{-\frac{1}{2}} = \frac{-1}{\sqrt{3}} + \sqrt{3} = -\frac{1}{3}\sqrt{3} + \sqrt{3} = \frac{2}{3}\sqrt{3}$

**21 a**  $\sin(\alpha) = \frac{1}{2}\sqrt{3}$  geeft  $\alpha = \frac{1}{3}\pi \vee \alpha = \frac{2}{3}\pi$

**b**  $\cos(\alpha) = -\frac{1}{2}$  geeft  $\alpha = \frac{2}{3}\pi \vee \alpha = 1\frac{1}{3}\pi$

**c**  $\sin(\alpha) = -\frac{1}{2}\sqrt{2}$  geeft  $\alpha = 1\frac{1}{4}\pi \vee \alpha = 1\frac{3}{4}\pi$

**d**  $\cos(\alpha) = 0$  geeft  $\alpha = \frac{1}{2}\pi \vee \alpha = 1\frac{1}{2}\pi$

**e**  $\cos(\alpha) = \frac{1}{2}\sqrt{3}$  geeft  $\alpha = \frac{1}{6}\pi \vee \alpha = 1\frac{5}{6}\pi$

**f**  $\cos(\alpha) = \frac{1}{2}\sqrt{2}$  geeft  $\alpha = \frac{1}{4}\pi \vee \alpha = 1\frac{3}{4}\pi$

**22 a**  $x = \sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}$

**b**  $\sin(x) = -\frac{1}{2}\sqrt{3}$  met  $0 \leq x \leq 1\frac{1}{2}\pi$  geeft  $x = 1\frac{1}{3}\pi$

**c**  $x = \cos\left(1\frac{3}{4}\pi\right) = \frac{1}{2}\sqrt{2}$

**d**  $\cos(x) = \frac{1}{2}$  met  $1\frac{1}{2}\pi \leq x \leq 2\pi$  geeft  $x = 1\frac{2}{3}\pi$

### Bladzijde 141

**C1**  $P(\cos(35^\circ), \sin(35^\circ)) \approx P(0,82; 0,57)$   
 $Q(\cos(105^\circ), \sin(105^\circ)) \approx Q(-0,26; 0,97)$

$R(\cos(-150^\circ), \sin(-150^\circ)) \approx R(-0,87; -0,5)$   
 $S(\cos(-90^\circ), \sin(-90^\circ)) = S(0, -1)$

**C2 a**  $y_P = 0,8$ , dus  $\sin(\alpha) = 0,8$ .

De GR geeft  $53,13\dots^\circ$ .

Dus  $\alpha = 180^\circ - 53,13\dots^\circ \approx 126,9^\circ$ .

**b**  $x_P = -0,7$ , dus  $\cos(\alpha) = -0,7$ .

De GR geeft  $134,42\dots^\circ$ .

Dus  $\alpha = 360^\circ - 134,42\dots^\circ \approx 225,6^\circ$ .

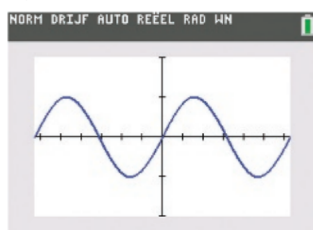
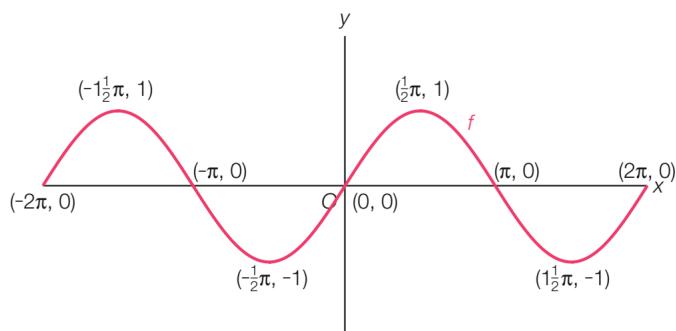
**C3** a  $\frac{2}{5}\pi \text{ rad} = \frac{2}{5} \cdot 180^\circ = 72^\circ$   
 $2\frac{1}{2}\text{ rad} = 2\frac{1}{2} \cdot \frac{180^\circ}{\pi} \approx 143,2^\circ$   
 $-1\frac{3}{4}\pi \text{ rad} = -1\frac{3}{4} \cdot 180^\circ = -315^\circ$   
 b  $120^\circ = \frac{120}{180}\pi \text{ rad} = \frac{2}{3}\pi \text{ rad}$   
 $-150^\circ = \frac{-150}{180}\pi \text{ rad} = -\frac{5}{6}\pi \text{ rad}$   
 $20^\circ = \frac{20}{180}\pi \text{ rad} = \frac{1}{9}\pi \text{ rad}$   
 c  $15^\circ = \frac{15}{180}\pi \text{ rad} \approx 0,26 \text{ rad}$   
 $400^\circ = \frac{400}{180}\pi \text{ rad} \approx 6,98 \text{ rad}$   
 $-99^\circ = \frac{-99}{180}\pi \text{ rad} \approx -1,73 \text{ rad}$

**C4**  $\cos(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$   
 $\sin(-\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}$   
 $\tan(1\frac{5}{6}\pi) = \frac{\sin(1\frac{5}{6}\pi)}{\cos(1\frac{5}{6}\pi)} = \frac{-\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \frac{-1}{\sqrt{3}} = -\frac{1}{3}\sqrt{3}$

## 8.2 Sinusoïden

Bladzijde 142

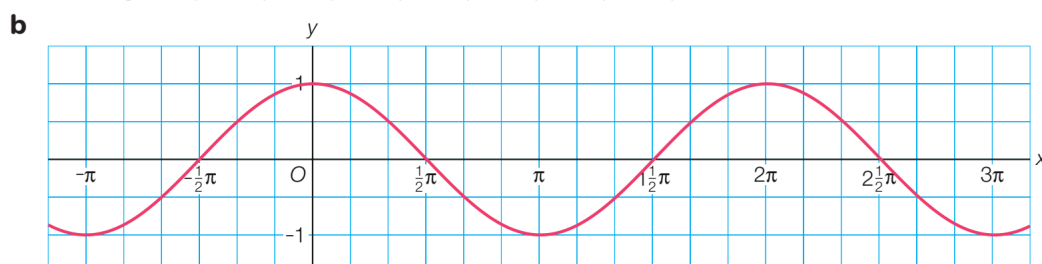
23 a, b



Bladzijde 143

24 a

| $x$       | 0 | $\frac{1}{3}\pi$ | $\frac{1}{2}\pi$ | $\frac{2}{3}\pi$ | $\pi$ | $1\frac{1}{3}\pi$ | $1\frac{1}{2}\pi$ | $1\frac{2}{3}\pi$ | $2\pi$ |
|-----------|---|------------------|------------------|------------------|-------|-------------------|-------------------|-------------------|--------|
| $\cos(x)$ | 1 | $\frac{1}{2}$    | 0                | $-\frac{1}{2}$   | -1    | $-\frac{1}{2}$    | 0                 | $\frac{1}{2}$     | 1      |



- 25 a**  $f(x) = \sin(x)$   
 $\downarrow$  translatie  $(0, 2)$   
 $g(x) = 2 + \sin(x)$   
 evenwichtsstand = 2
- b**  $f(x) = \sin(x)$   
 $\downarrow$  translatie  $(3, 0)$   
 $h(x) = \sin(x - 3)$

- c**  $f(x) = \sin(x)$   
 $\downarrow \times 4$  t.o.v. de  $x$ -as  
 $k(x) = 4 \sin(x)$   
 amplitude = 4
- d**  $f(x) = \sin(x)$   
 $\downarrow \times \frac{1}{5}$  t.o.v. de  $y$ -as  
 $l(x) = \sin(5x)$   
 periode =  $\frac{2\pi}{5} = \frac{2}{5}\pi$

#### Bladzijde 145

- 26 a** Uit de translatie  $(\frac{1}{3}\pi, \frac{1}{2})$ .  
**b** Ja, ik ben het met José eens.  
**c** Uit de vermenigvuldiging met 3 ten opzichte van de  $x$ -as gevolgd door de translatie  $(\frac{1}{3}\pi, \frac{1}{2})$ .

- 27 a**  $y = \sin(x)$   
 $\downarrow \times 2$  t.o.v. de  $x$ -as  
 $y = 2 \sin(x)$   
 $\downarrow \times 2$  t.o.v. de  $y$ -as  
 $y = 2 \sin(\frac{1}{2}x)$   
 $\downarrow$  translatie  $(-\frac{3}{4}\pi, 4)$   
 $f(x) = 4 + 2 \sin(\frac{1}{2}(x + \frac{3}{4}\pi))$

De evenwichtsstand is 4, de amplitude is 2, de periode is  $\frac{2\pi}{\frac{1}{2}} = 4\pi$  en een beginpunt is  $(-\frac{3}{4}\pi, 4)$ .

- b**  $y = \cos(x)$   
 $\downarrow \times 7$  t.o.v. de  $x$ -as  
 $y = 7 \cos(x)$   
 $\downarrow \times \frac{1}{5}$  t.o.v. de  $y$ -as  
 $y = 7 \cos(5x)$   
 $\downarrow$  translatie  $(\frac{1}{6}\pi, -1)$   
 $g(x) = -1 + 7 \cos(5(x - \frac{1}{6}\pi))$

De evenwichtsstand is -1, de amplitude is 7, de periode is  $\frac{2\pi}{5} = \frac{2}{5}\pi$  en een beginpunt is  $(\frac{1}{6}\pi, 6)$ .

- 28 a** De evenwichtsstand is 5, de amplitude is 1,2, de periode is  $2\pi$  en een beginpunt is  $(\frac{1}{6}\pi; 6,2)$ .  
**b** De evenwichtsstand is 0,4, de amplitude is 1, de periode is  $\frac{2\pi}{\frac{1}{5}} = 10\pi$  en een beginpunt is  $(-\frac{1}{3}\pi; 0,4)$ .  
**c** De evenwichtsstand is 0, de amplitude is 0,29, de periode is  $\frac{2\pi}{3} = \frac{2}{3}\pi$  en een beginpunt is  $(-1,4; 0,29)$ .  
**d** De evenwichtsstand is -0,8, de amplitude is 2, de periode is  $\frac{2\pi}{\pi} = 2$  en een beginpunt is  $(\frac{1}{2}, -0,8)$ .

#### Bladzijde 146

- 29 a**  $y = \cos(x)$   
 $\downarrow$  translatie  $(\frac{1}{4}\pi, 4)$   
 $y = 4 + \cos(x - \frac{1}{4}\pi)$   
 $\downarrow \times 3$  t.o.v. de  $x$ -as  
 $y = 3(4 + \cos(x - \frac{1}{4}\pi))$   
 Dus  $f(x) = 12 + 3 \cos(x - \frac{1}{4}\pi)$ .

- b**  $y = \cos(x)$   
 $\downarrow \times 3$  t.o.v. de  $x$ -as  
 $y = 3 \cos(x)$   
 $\downarrow$  translatie  $(\frac{1}{4}\pi, 4)$   
 $y = 4 + 3 \cos(x - \frac{1}{4}\pi)$   
 Dus  $g(x) = 4 + 3 \cos(x - \frac{1}{4}\pi)$ .

- 30** **a** De evenwichtsstand is  $-\frac{1}{2}$ , de amplitude is 1, de periode is  $2\pi$  en een beginpunt is  $(\frac{1}{4}\pi, -\frac{1}{2})$ .  
**b** De grafiek van  $f$  snijdt de lijn van de evenwichtsstand in de punten  $(\frac{1}{4}\pi, -\frac{1}{2})$ ,  $(\frac{1}{4}\pi, -\frac{1}{2})$  en  $(2\frac{1}{4}\pi, -\frac{1}{2})$ .  
**c** De toppen van de grafiek van  $f$  zijn  $(\frac{3}{4}\pi, \frac{1}{2})$ ,  $(1\frac{3}{4}\pi, -\frac{1}{2})$  en  $(2\frac{3}{4}\pi, \frac{1}{2})$ .  
**d** De afstand tussen  $A$  en  $C$  is gelijk aan de periode, dus  $d(A, C) = 2\pi$ .

- 31**  $f(x) = 3 + 4 \sin(x - \frac{1}{2}\pi)$   
 $\downarrow$  translatie  $(-\frac{3}{4}\pi, -5)$   
 $y = -5 + 3 + 4 \sin(x + \frac{3}{4}\pi - \frac{1}{2}\pi)$   
 oftewel  $y = -2 + 4 \sin(x + \frac{1}{4}\pi)$   
 Dus  $m = -\frac{3}{4}\pi$  en  $n = -5$  zijn mogelijke waarden van  $m$  en  $n$ .

- 32** **a** De evenwichtsstand is 3, de amplitude is 2 en de periode is  $\pi$ .  
**b**  $a = 3$ ,  $b = 2$  en  $d = \frac{1}{3}\pi$   
 Uit periode  $= \frac{2\pi}{c}$  volgt  $c = \frac{2\pi}{\pi} = 2$ .

#### Bladzijde 148

- 33** 1 Door de afstand af te lezen tussen twee opeenvolgende laagste punten.  
 2 Door de afstand af te lezen tussen twee opeenvolgende punten waarin de sinusoid stijgend door de evenwichtsstand gaat.

#### Bladzijde 149

- 34** **a**  $a = \frac{4\frac{1}{2} + -1\frac{1}{2}}{2} = 1\frac{1}{2}$  en  $b = 4\frac{1}{2} - 1\frac{1}{2} = 3$   
 De periode is  $\pi$ , dus  $c = \frac{2\pi}{\pi} = 2$ .  
 Stijgend door de evenwichtsstand bij  $x = \frac{1}{2}\pi$ , dus  $d = \frac{1}{2}\pi$ .  
 Dus  $y = 1\frac{1}{2} + 3 \sin(2(x - \frac{1}{2}\pi))$ .  
**b** Een hoogste punt is  $(\frac{3}{4}\pi, 4\frac{1}{2})$ , dus  $y = 1\frac{1}{2} + 3 \cos(2(x - \frac{3}{4}\pi))$ .

- 35** **a**  $a = \frac{1\frac{1}{2} + -3\frac{1}{2}}{2} = -1$  en  $b = 1\frac{1}{2} - -1 = 2\frac{1}{2}$   
 De periode is 2, dus  $c = \frac{2\pi}{2} = \pi$ .  
 Stijgend door de evenwichtsstand bij  $x = 1\frac{1}{2}$ , dus  $d = 1\frac{1}{2}$ .  
 Dus  $y = -1 + 2\frac{1}{2} \sin(\pi(x - 1\frac{1}{2}))$ .  
**b** Een hoogste punt is  $(0, 1\frac{1}{2})$ , dus  $y = -1 + 2\frac{1}{2} \cos(\pi x)$ .

- 36** **a**  $a = \frac{3\frac{1}{2} + \frac{1}{2}}{2} = 2$  en  $b = 3\frac{1}{2} - 2 = 1\frac{1}{2}$   
 halve periode  $= \frac{3}{4}\pi - \frac{1}{4}\pi = \frac{1}{2}\pi$  geeft periode  $= \pi$ , dus  $c = \frac{2\pi}{\pi} = 2$ .  
 Stijgend door de evenwichtsstand bij  $x = \frac{\frac{1}{4}\pi + \frac{3}{4}\pi}{2} = \frac{1}{2}\pi$ , dus  $d = \frac{1}{2}\pi$ .  
 Dus  $y = 2 + 1\frac{1}{2} \sin(2(x - \frac{1}{2}\pi))$ .  
**b** Een hoogste punt is  $(\frac{3}{4}\pi, 3\frac{1}{2})$ , dus  $y = 2 + 1\frac{1}{2} \cos(2(x - \frac{3}{4}\pi))$ .

- 37** **a**  $a = \frac{4 + -2}{2} = 1$  en  $b = 4 - 1 = 3$   
 halve periode  $= 2\frac{3}{4} - 1\frac{1}{4} = 1\frac{1}{2}$  geeft periode  $= 3$ , dus  $c = \frac{2\pi}{3} = \frac{2}{3}\pi$ .  
 Stijgend door de evenwichtsstand bij  $x = \frac{1\frac{1}{4} + 2\frac{3}{4}}{2} = 2$ , dus  $d = 2$ .  
 Dus  $y = 1 + 3 \sin(\frac{2}{3}\pi(x - 2))$ .  
**b** Een hoogste punt is  $(2\frac{3}{4}, 4)$ , dus  $y = 1 + 3 \cos(\frac{2}{3}\pi(x - 2\frac{3}{4}))$ .

38

- a**  $a = \frac{175 + 25}{2} = 100$  en  $b = 175 - 100 = 75$   
 halve periode =  $9 - 4 = 5$  geeft periode = 10, dus  $c = \frac{2\pi}{10} = \frac{1}{5}\pi$ .  
 Stijgend door de evenwichtsstand bij  $t = 4$ , dus  $d = 4$ .  
 Dus  $N = 100 + 75 \sin(\frac{1}{5}\pi(t - 4))$ .
- b** Er is een hoogste punt bij  $t = \frac{4 + 9}{2} = 6\frac{1}{2}$ .  
 Dus  $N = 100 + 75 \cos(\frac{1}{5}\pi(t - 6\frac{1}{2}))$ .

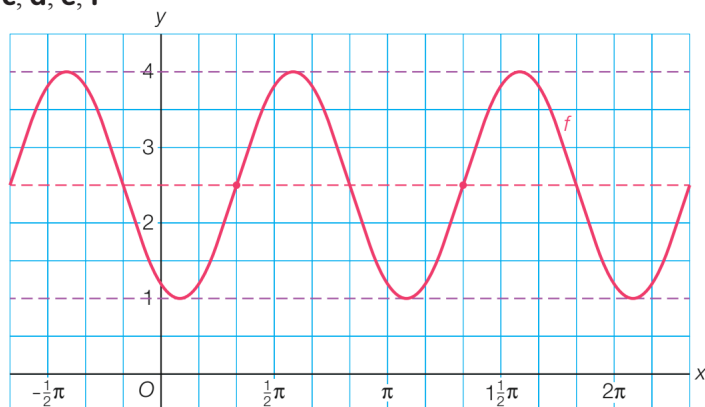
## Bladzijde 150

39

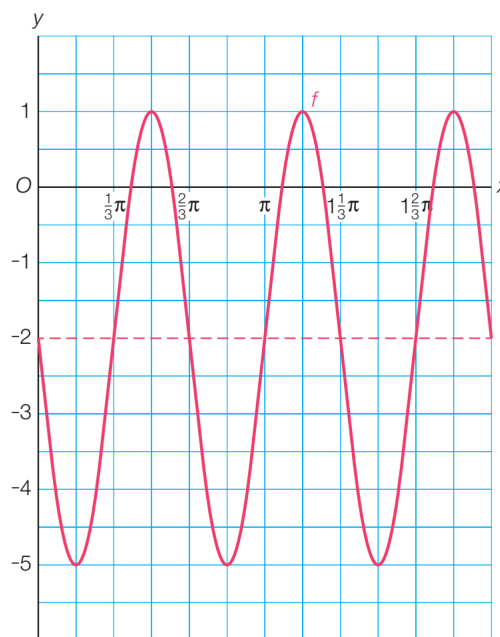
- a**  $y = a + b \cos(c(x - d))$   
 $a = 12 - 4 = 8$  en  $b = 4$   
 De periode is  $18 - 3 = 15$ , dus  $c = \frac{2\pi}{15} = \frac{2}{15}\pi$ .  
 Het punt  $(3, 12)$  is een hoogste punt, dus  $d = 3$ .  
 Dus  $y = 8 + 4 \cos(\frac{2}{15}\pi(x - 3))$ .
- b**  $y = a + b \cos(c(x - d))$   
 $a = 650$  en  $b = 812 - 650 = 162$   
 De periode is 48, dus  $c = \frac{2\pi}{48} = \frac{1}{24}\pi$ .  
 Het punt  $(16, 812)$  is een hoogste punt, dus  $d = 16$ .  
 Dus  $y = 650 + 162 \cos(\frac{1}{24}\pi(x - 16))$ .
- c**  $y = a + b \cos(c(x - d))$   
 $a = \frac{250 + 110}{2} = 180$  en  $b = 250 - 180 = 70$   
 halve periode =  $26 - 2 = 24$  geeft periode = 48, dus  $c = \frac{2\pi}{48} = \frac{1}{24}\pi$ .  
 Het punt  $(2, 250)$  is een hoogste punt, dus  $d = 2$ .  
 Dus  $y = 180 + 70 \cos(\frac{1}{24}\pi(x - 2))$ .

40

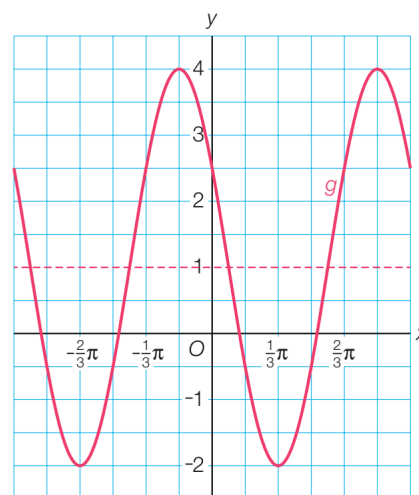
- a** De evenwichtsstand is  $2\frac{1}{2}$ , de amplitude is  $1\frac{1}{2}$ , de periode is  $\frac{2\pi}{2} = \pi$   
 en een beginpunt is  $(\frac{1}{3}\pi, 2\frac{1}{2})$ .
- b, c, d, e, f**



- 41 a**  $f(x) = -2 + 3 \sin(3x - \pi) = -2 + 3 \sin(3(x - \frac{1}{3}\pi))$   
 evenwichtsstand  $-2$   
 amplitude  $3$   
 periode  $\frac{2\pi}{3} = \frac{2}{3}\pi$   
 stijgend door de evenwichtsstand in  $(\frac{1}{3}\pi, -2)$   
 Zie de figuur hiernaast.



- b**  $g(x) = 1 + 3 \cos(2x + \frac{1}{3}\pi) = 1 + 3 \cos(2(x + \frac{1}{6}\pi))$   
 evenwichtsstand  $1$   
 amplitude  $3$   
 periode  $\frac{2\pi}{2} = \pi$   
 $(-\frac{1}{6}\pi, 4)$  is een hoogste punt.  
 Zie de figuur hiernaast.



- 42**  $f(x) = 1 + 2 \sin(\frac{1}{2}x - \frac{1}{6}\pi) = 1 + 2 \sin(\frac{1}{2}(x - \frac{1}{3}\pi))$

Van de grafiek van  $f$  is de evenwichtsstand  $1$ , de amplitude  $2$ ,  
 de periode  $\frac{2\pi}{\frac{1}{2}} = 4\pi$  en een beginpunt  $(\frac{1}{3}\pi, 1)$ .

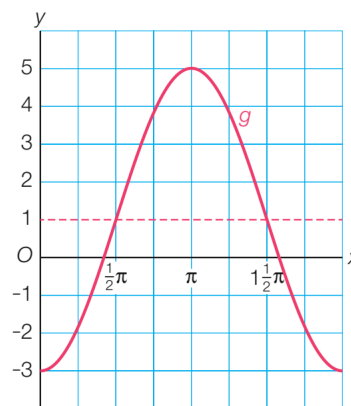
De evenwichtsstand van de grafiek van  $g$  is gelijk aan die van  
 de grafiek van  $f$ , dus  $a = 1$ .

De amplitude van de grafiek van  $g$  is twee keer zo groot als  
 die van de grafiek van  $f$ , dus  $b = 2 \cdot 2 = 4$ .

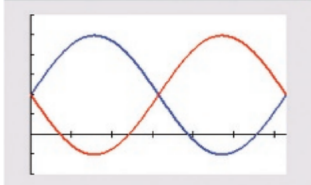
De periode van de grafiek van  $g$  is de helft van die van de  
 grafiek van  $f$ , dus periode  $= \frac{1}{2} \cdot 4\pi = 2\pi$  en  $c = \frac{2\pi}{2\pi} = 1$ .

De grafiek van  $g$  gaat bij  $x = \frac{1}{2}\pi$  stijgend door de evenwichtsstand, dus  
 de grafiek van  $g$  heeft een hoogste punt bij  $x = \frac{1}{2}\pi + \frac{1}{4} \cdot 2\pi = \pi$ , dus  $d = \pi$ .  
 Dus  $g(x) = 1 + 4 \cos(x - \pi)$ .

Zie de figuur hiernaast.



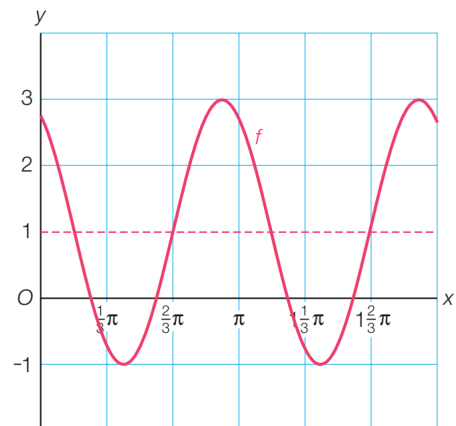
- 43 a** NORM DRIJF AUTO REËEL RAD WM



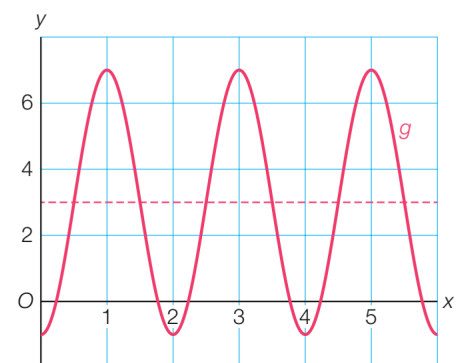
- b** Van de grafiek van  $f$  is de amplitude 3 en van de grafiek van  $g$  is de amplitude 3.  
**c** De grafiek van  $f$  gaat in het punt  $(0, 2)$  stijgend door de evenwichtsstand.  
 De grafiek van  $g$  gaat in het punt  $(0, 2)$  dalend door de evenwichtsstand.

#### Bladzijde 154

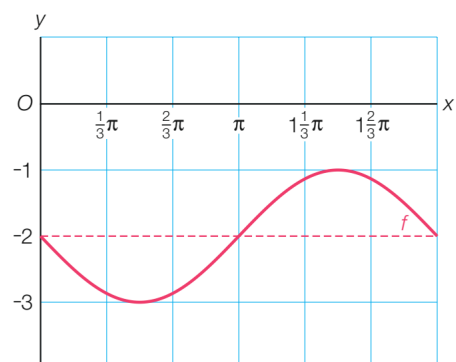
- 44 a**  $f(x) = 1 - 2 \sin(2x - \frac{1}{3}\pi) = 1 - 2 \sin(2(x - \frac{1}{6}\pi))$   
 evenwichtsstand 1  
 amplitude 2  
 periode  $\frac{2\pi}{2} = \pi$   
 $-2 < 0$ , dus grafiek dalend door de evenwichtsstand  
 in  $(\frac{1}{6}\pi, 1)$ .  
 Zie de figuur hiernaast.



- b**  $g(x) = 3 - 4 \cos(\pi x)$   
 evenwichtsstand 3  
 amplitude 4  
 periode  $\frac{2\pi}{\pi} = 2$   
 $-4 < 0$ , dus  $(0, -1)$  is een laagste punt.  
 Zie de figuur hiernaast.



- 45 a**  $f(x) = -2 - \cos(x - \frac{1}{2}\pi)$   
 evenwichtsstand -2  
 amplitude 1  
 periode  $2\pi$   
 $-1 < 0$ , dus  $(\frac{1}{2}\pi, -3)$  is een laagste punt.  
 Zie de figuur hiernaast.
- b** Van het hoogste punt is  $x = \frac{1}{2}\pi + \frac{1}{2} \cdot 2\pi = 1\frac{1}{2}\pi$   
 en  $y = -2 + 1 = -1$ .  
 $k: y = ax + b$   
 $a = \frac{-1 - (-3)}{1\frac{1}{2}\pi - \frac{1}{2}\pi} = \frac{2}{\pi}$
- $$y = \frac{2}{\pi}x + b \quad \left\{ \begin{array}{l} \frac{2}{\pi} \cdot \frac{1}{2}\pi + b = -3 \\ 1 + b = -3 \\ b = -4 \end{array} \right.$$
- Dus  $k: y = \frac{2}{\pi}x - 4$ .  
 $k$  snijden met de  $x$ -as geeft  $\frac{2}{\pi}x - 4 = 0$   
 $\frac{2}{\pi}x = 4$   
 $x = 2\pi$
- Dus  $x_S = 2\pi$ .



**46 a**  $f(x) = 5 - 3 \sin(\frac{1}{4}\pi x)$

evenwichtsstand 5

amplitude 3

periode  $\frac{2\pi}{\frac{1}{4}\pi} = 8$

$-3 < 0$ , dus grafiek dalend door de evenwichtsstand in  $(0, 5)$ .

Zie de figuur hiernaast.

**b**  $k$  door  $O$  en  $A(2, 2)$  heeft  $rc_k = \frac{2}{2} = 1$ .

$l$  door  $O$  en  $B(6, 8)$  heeft  $rc_l = \frac{8}{6} = 1\frac{1}{3}$ .

$\tan(\alpha) = rc_k = 1$  geeft  $\alpha = 45^\circ$

$\tan(\beta) = rc_l = 1\frac{1}{3}$  geeft  $\alpha = 53,13\dots^\circ$

Dus  $\angle(k, l) = 53,13\dots^\circ - 45^\circ \approx 8,1^\circ$ .

**c** Invoeren van  $y = 5 - 3 \sin(\frac{1}{4}\pi x)$  en  $y = 6,5$  en gebruiken van de optie snijpunt geeft  $x = 4,666\dots$  en  $x = 7,333\dots$

Dus  $d(C, D) = 7,333\dots - 4,666\dots \approx 2,67$ .



**47**  $y = 2 - 3 \sin(\frac{1}{2}x + \frac{1}{6}\pi) = 2 - 3 \sin(\frac{1}{2}(x + \frac{1}{3}\pi))$

evenwichtsstand 2

amplitude 3

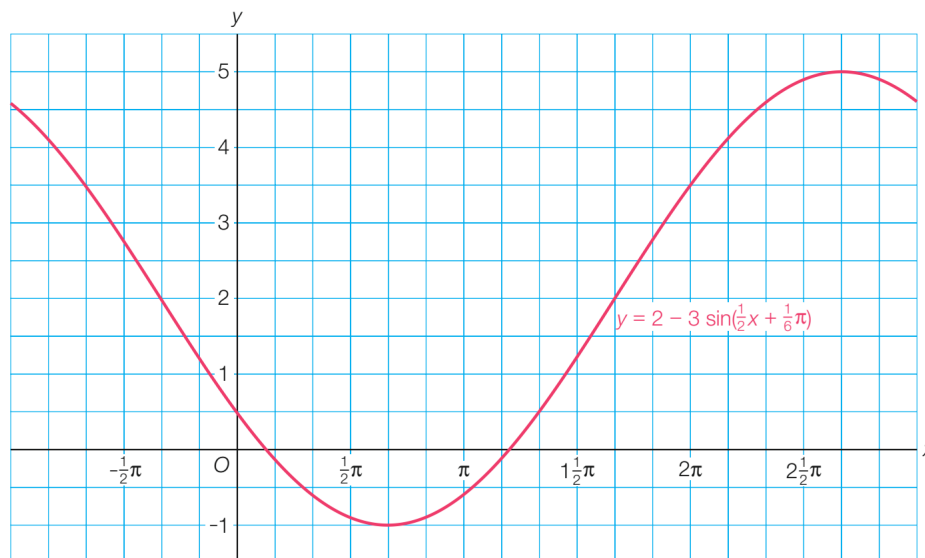
periode  $\frac{2\pi}{\frac{1}{2}} = 4\pi$

$-3 < 0$ , dus grafiek dalend door de evenwichtsstand in  $(-\frac{1}{3}\pi, 2)$ .

Tussen het laagste en het hoogste punt zitten horizontaal 12 roosterhokjes, dus  $\frac{1}{2}$  periode  $= 2\pi$  komt overeen met 12 roosterhokjes, dus horizontaal komt 1 roosterhokje overeen met  $\frac{1}{6}\pi$ .

Tussen het laagste en het hoogste punt zitten verticaal 12 roosterhokjes, dus  $2 \cdot \text{amplitude} = 6$  komt overeen met 12 roosterhokjes, dus verticaal komt 1 roosterhokje overeen met  $\frac{1}{2}$ .

Het laagste punt is  $(-\frac{1}{3}\pi + \frac{1}{4} \cdot 4\pi, 2 - 3) = (\frac{5}{3}\pi, -1)$ . Teken de verticale as 4 roosterhokjes links van het laagste punt en teken de horizontale as 2 roosterhokjes boven het laagste punt. Je krijgt de figuur hieronder.





### Bladzijde 155

C5

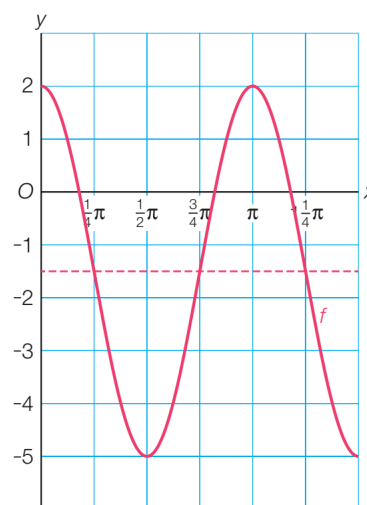
- a** De evenwichtsstand is 6, de amplitude is 7, de periode is  $\frac{2\pi}{4} = \frac{1}{2}\pi$  en een beginpunt is (2, 6).  
**b** De evenwichtsstand is 2,5, de amplitude is 0,6, de periode is  $\frac{2\pi}{0,4} = 5\pi$  en een beginpunt is  $(\frac{1}{4}\pi, 3,1)$ .

C6

- a**  $a = \frac{4 + -2}{2} = 1$  en  $b = 4 - 1 = 3$   
 halve periode  $= 3\frac{1}{4} - \frac{3}{4} = 2\frac{1}{2}$  geeft periode  $= 5$ , dus  $c = \frac{2\pi}{5} = \frac{2}{5}\pi$ .  
 Stijgend door de evenwichtsstand bij  $x = \frac{\frac{3}{4} + 3\frac{1}{4}}{2} = 2$ , dus  $d = 2$ .  
 Dus  $y = 1 + 3 \sin(\frac{2}{5}\pi(x - 2))$ .  
**b** Een hoogste punt is  $(3\frac{1}{4}, 4)$ , dus  $y = 1 + 3 \cos(\frac{2}{5}\pi(x - 3\frac{1}{4}))$ .

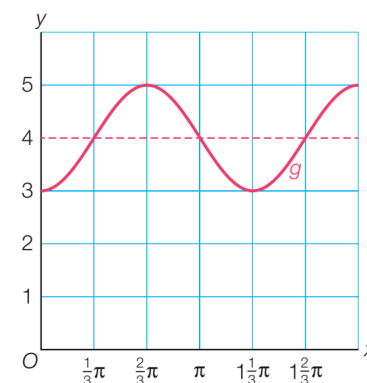
C7

- $f(x) = -1\frac{1}{2} + 3\frac{1}{2} \sin(2x - 1\frac{1}{2}\pi) = -1\frac{1}{2} + 3\frac{1}{2} \sin(2(x - \frac{3}{4}\pi))$   
 evenwichtsstand  $-1\frac{1}{2}$   
 amplitude  $3\frac{1}{2}$   
 periode  $\frac{2\pi}{2} = \pi$   
 stijgend door de evenwichtsstand in  $(\frac{3}{4}\pi, -1\frac{1}{2})$   
 Zie de figuur hiernaast.



C8

- $g(x) = 4 - \cos(1\frac{1}{2}x - 2\pi) = 4 - \cos(1\frac{1}{2}(x - 1\frac{1}{3}\pi))$   
 evenwichtsstand 4  
 amplitude 1  
 periode  $\frac{2\pi}{1\frac{1}{2}} = 1\frac{1}{3}\pi$   
 $-1 < 0$ , dus  $(1\frac{1}{3}\pi, 3)$  is een laagste punt.  
 Zie de figuur hiernaast.



## 8.3 Goniometrische vergelijkingen

### Bladzijde 156

48

- Bij (0, 1) horen de draaiingshoeken  $\dots, -3\frac{1}{2}\pi, -1\frac{1}{2}\pi, \frac{1}{2}\pi, 2\frac{1}{2}\pi, 4\frac{1}{2}\pi, \dots$   
 Bij (0, -1) horen de draaiingshoeken  $\dots, -4\frac{1}{2}\pi, -2\frac{1}{2}\pi, -\frac{1}{2}\pi, 1\frac{1}{2}\pi, 3\frac{1}{2}\pi, \dots$

### Bladzijde 157

49

- $x = 1 + k \cdot 4$  geeft de oplossingen  $-7, -3, 1, 5, 9, \dots$   
 $x = 3 + k \cdot 4$  geeft de oplossingen  $-5, -1, 3, 7, 11, \dots$   
 Samengenomen geeft dit  $-7, -5, -3, -1, 1, 3, 5, 7, 9, 11, \dots$   
 En dit is te noteren als  $x = 1 + k \cdot 2$ .

**50 a**  $\sin(3x - \frac{1}{2}\pi) = 0$

$$3x - \frac{1}{2}\pi = k \cdot \pi$$

$$3x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$$

**b**  $\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0$

$$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$$

$$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$$

$$x = 1\frac{1}{3}\pi + k \cdot 2\pi$$

**51 a**  $\cos^2(x - \frac{1}{5}\pi) = 1$

$$\cos(x - \frac{1}{5}\pi) = 1 \vee \cos(x - \frac{1}{5}\pi) = -1$$

$$x - \frac{1}{5}\pi = k \cdot 2\pi \vee x - \frac{1}{5}\pi = \pi + k \cdot 2\pi$$

$$x = \frac{1}{5}\pi + k \cdot 2\pi \vee x = 1\frac{1}{5}\pi + k \cdot 2\pi$$

$$(x = \frac{1}{5}\pi + k \cdot \pi)$$

**b**  $\sin^2(2x - \frac{1}{4}\pi) = 1$

$$\sin(2x - \frac{1}{4}\pi) = 1 \vee \sin(2x - \frac{1}{4}\pi) = -1$$

$$2x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$2x = \frac{3}{4}\pi + k \cdot 2\pi \vee 2x = 1\frac{3}{4}\pi + k \cdot 2\pi$$

$$x = \frac{3}{8}\pi + k \cdot \pi \vee x = \frac{7}{8}\pi + k \cdot \pi$$

$$(x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi)$$

**52 a**  $\sin(4x - \frac{1}{3}\pi) = 1$

$$4x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$4x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi$$

**b**  $\cos(4\pi x) = -1$

$$4\pi x = \pi + k \cdot 2\pi$$

$$x = \frac{1}{4} + k \cdot \frac{1}{2}$$

**c**  $\sin^2(x) - \sin(x) = 0$

$$\sin(x)(\sin(x) - 1) = 0$$

$$\sin(x) = 0 \vee \sin(x) = 1$$

$$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi$$

**d**  $\cos^2(2x) + \cos(2x) = 0$

$$\cos(2x)(\cos(2x) + 1) = 0$$

$$\cos(2x) = 0 \vee \cos(2x) = -1$$

$$2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = \pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = \frac{1}{2}\pi + k \cdot \pi$$

**c**  $\sin^3(x) - \sin(x) = 0$

$$\sin(x)(\sin^2(x) - 1) = 0$$

$$\sin(x) = 0 \vee \sin^2(x) = 1$$

$$\sin(x) = 0 \vee \sin(x) = 1 \vee \sin(x) = -1$$

$$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$(x = k \cdot \frac{1}{2}\pi)$$

**d**  $\cos^3(2x) - \cos(2x) = 0$

$$\cos(2x)(\cos^2(2x) - 1) = 0$$

$$\cos(2x) = 0 \vee \cos^2(2x) = 1$$

$$\cos(2x) = 0 \vee \cos(2x) = 1 \vee \cos(2x) = -1$$

$$2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = k \cdot 2\pi \vee 2x = \pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot \pi$$

$$(x = k \cdot \frac{1}{4}\pi)$$

**c**  $\sin^2(\frac{1}{4}\pi x) = 1$

$$\sin(\frac{1}{4}\pi x) = 1 \vee \sin(\frac{1}{4}\pi x) = -1$$

$$\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{4}\pi x = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$x = 2 + k \cdot 8 \vee x = 6 + k \cdot 8$$

$$(x = 2 + k \cdot 4)$$

**d**  $\sin(2x)\cos(2x) + \sin(2x) = 0$

$$\sin(2x)(\cos(2x) + 1) = 0$$

$$\sin(2x) = 0 \vee \cos(2x) = -1$$

$$2x = k \cdot \pi \vee 2x = \pi + k \cdot 2\pi$$

$$x = k \cdot \frac{1}{2}\pi \vee x = \frac{1}{2}\pi + k \cdot \pi$$

$$(x = k \cdot \frac{1}{2}\pi)$$

**53 a**  $2 \sin(x) \cos(2x) = 0$

$$\sin(x) = 0 \vee \cos(2x) = 0$$

$$x = k \cdot \pi \vee 2x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = k \cdot \pi \vee x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{4}\pi \vee x = \frac{3}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi$$

De nulpunten zijn  $0, \frac{1}{4}\pi, \frac{3}{4}\pi, \pi, 1\frac{1}{4}\pi, 1\frac{3}{4}\pi$  en  $2\pi$ .

**b**  $\text{rc}_{AB} = \frac{2 - (-2)}{1\frac{1}{2}\pi - \frac{1}{2}\pi} = \frac{4}{\pi}$

$$\left. \begin{array}{l} y = \frac{4}{\pi}x + b \\ \text{door } A(\frac{1}{2}\pi, -2) \end{array} \right\} \begin{array}{l} \frac{4}{\pi} \cdot \frac{1}{2}\pi + b = -2 \\ 2 + b = -2 \\ b = -4 \end{array}$$

Dus  $AB: y = \frac{4}{\pi}x - 4$ .

$AB$  snijden met de  $x$ -as geeft  $\frac{4}{\pi}x - 4 = 0$

$$\frac{4}{\pi}x = 4$$

$$x = \pi$$

Dus de lijn door  $A$  en  $B$  snijdt de  $x$ -as in  $(\pi, 0)$ , en dat is ook een van de snijpunten van de grafiek van  $f$  met de  $x$ -as.

**c**  $f(\frac{1}{6}\pi) = 2 \sin(\frac{1}{6}\pi) \cdot \cos(\frac{1}{3}\pi) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$ , dus  $C$  ligt op de grafiek van  $f$ .

$f(1\frac{1}{6}\pi) = 2 \sin(1\frac{1}{6}\pi) \cdot \cos(2\frac{1}{3}\pi) = 2 \cdot -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{2}$ , dus  $D$  ligt op de grafiek van  $f$ .

**d**  $\text{rc}_{CD} = \frac{-\frac{1}{2} - \frac{1}{2}}{1\frac{1}{6}\pi - \frac{1}{6}\pi} = -\frac{1}{\pi}$

$$\left. \begin{array}{l} y = -\frac{1}{\pi}x + b \\ \text{door } C(\frac{1}{6}\pi, \frac{1}{2}) \end{array} \right\} \begin{array}{l} -\frac{1}{\pi} \cdot \frac{1}{6}\pi + b = \frac{1}{2} \\ -\frac{1}{6} + b = \frac{1}{2} \end{array}$$

$$b = \frac{2}{3}$$

Dus  $CD: y = -\frac{1}{\pi}x + \frac{2}{3}$ .

$CD$  snijden met de  $x$ -as geeft  $-\frac{1}{\pi}x + \frac{2}{3} = 0$

$$-\frac{1}{\pi}x = -\frac{2}{3}$$

$$x = \frac{2}{3}\pi$$

Dus de lijn door  $C$  en  $D$  snijdt de  $x$ -as in  $(\frac{2}{3}\pi, 0)$ , en dat is geen snijpunt van de grafiek van  $f$  met de  $x$ -as.

**54 a**  $f(x) = 0$  geeft  $3 \sin(\frac{1}{2}\pi x) = 0$

$$\sin(\frac{1}{2}\pi x) = 0$$

$$\frac{1}{2}\pi x = k \cdot \pi$$

$$x = k \cdot 2$$

$$x \text{ in } [0, 4] \text{ geeft } x = 0 \vee x = 2 \vee x = 4$$

Dus  $A(2, 0)$  en  $B(4, 0)$ .

Hieruit volgt  $g(x) = ax(x - 2)(x - 4)$ .

$x_C = \frac{0+2}{2} = 1$  geeft  $y_C = f(1) = 3 \sin(\frac{1}{2}\pi) = 3 \cdot 1 = 3$ , dus  $C(1, 3)$ .

$g(1) = 3$  geeft  $a \cdot 1 \cdot (1 - 2)(1 - 4) = 3$

$$3a = 3$$

$$a = 1$$

Dus  $g(x) = x(x - 2)(x - 4) = x(x^2 - 6x + 8) = x^3 - 6x^2 + 8x$ .

$$g(x) = x^3 - 6x^2 + 8x \text{ geeft } g'(x) = 3x^2 - 12x + 8$$

$$g'(x) = 0 \text{ geeft } 3x^2 - 12x + 8 = 0$$

$$D = (-12)^2 - 4 \cdot 3 \cdot 8 = 48, \text{ dus } \sqrt{D} = 4\sqrt{3}$$

$$x = \frac{12 + 4\sqrt{3}}{6} \vee x = \frac{12 - 4\sqrt{3}}{6}$$

$$x = 2 + \frac{2}{3}\sqrt{3} \vee x = 2 - \frac{2}{3}\sqrt{3}$$

Dus de x-coördinaten van de toppen van de grafiek van  $g$  zijn  $x = 2 + \frac{2}{3}\sqrt{3}$  en  $x = 2 - \frac{2}{3}\sqrt{3}$ .

**b** Er geldt  $h(x) = ax(x - m)(x - 4) = ax(x^2 - 4x - mx + 4m) = ax^3 - 4ax^2 - amx^2 + 4amx$ .

Dit geeft  $h'(x) = 3ax^2 - 8ax - 2amx + 4am$ .

$$h'(1) = 0 \text{ geeft } 3a - 8a - 2am + 4am = 0$$

$$-5a + 2am = 0$$

$$a(-5 + 2m) = 0$$

$$a = 0 \vee 2m = 5$$

$$a = 0 \vee m = 2\frac{1}{2}$$

$a = 0$  voldoet niet, dus  $m = 2\frac{1}{2}$ .

$$\text{Dus } h(x) = ax(x - 2\frac{1}{2})(x - 4).$$

$$h(1) = 3 \text{ geeft } a \cdot 1 \cdot (1 - 2\frac{1}{2})(1 - 4) = 3$$

$$4\frac{1}{2}a = 3$$

$$a = \frac{2}{3}$$

$$m = 2\frac{1}{2} \text{ en } a = \frac{2}{3} \text{ geeft } h'(x) = 3 \cdot \frac{2}{3}x^2 - 8 \cdot \frac{2}{3}x - 2 \cdot \frac{2}{3} \cdot 2\frac{1}{2}x + 4 \cdot \frac{2}{3} \cdot 2\frac{1}{2}$$

$$= 2x^2 - 5\frac{1}{3}x - 3\frac{1}{3}x + 6\frac{2}{3}$$

$$= 2x^2 - 8\frac{2}{3}x + 6\frac{2}{3}$$

$$h'(x) = 0 \text{ geeft } 2x^2 - 8\frac{2}{3}x + 6\frac{2}{3} = 0$$

$$3x^2 - 13x + 10 = 0$$

$$D = (-13)^2 - 4 \cdot 3 \cdot 10 = 49, \text{ dus } \sqrt{D} = 7$$

$$x = \frac{13 + 7}{6} = 3\frac{1}{3} \vee x = \frac{13 - 7}{6} = 1$$

Dus de x-coördinaat van de andere top van de grafiek van  $h$  is  $x = 3\frac{1}{3}$ .

### Bladzijde 159

**55 a**  $x = \frac{1}{6}\pi$  geeft  $\sin(\frac{1}{6}\pi) = \frac{1}{2}$ , dus  $x = \frac{1}{6}\pi$  is een oplossing van  $\sin(x) = \frac{1}{2}$ .

**b**  $x = \frac{1}{6}\pi$  is een oplossing (zie vraag a). Als je de draaiingshoek met  $2\pi$  vergroot,

kom je weer in hetzelfde punt van de eenheidscirkel terecht. Dus  $x = 2\frac{1}{6}\pi$  en

$x = 4\frac{1}{6}\pi$  zijn ook oplossingen.

**c**  $x = \frac{5}{6}\pi$  geeft  $\sin(\frac{5}{6}\pi) = \frac{1}{2}$ , dus  $x = \frac{5}{6}\pi$  is een oplossing van  $\sin(x) = \frac{1}{2}$ .

**d**  $x = \frac{5}{6}\pi$  is een oplossing (zie vraag c). Als je de draaiingshoek met  $2\pi$  vergroot

of verkleint, kom je weer in hetzelfde punt van de eenheidscirkel terecht. Dus

$x = 2\frac{5}{6}\pi$  en  $x = -1\frac{1}{6}\pi$  zijn ook oplossingen.

### Bladzijde 161

**56 a**  $2\sin(\frac{1}{2}x) = 1$

$$\sin(\frac{1}{2}x) = \frac{1}{2}$$

$$\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{3}\pi + k \cdot 4\pi \vee x = 1\frac{2}{3}\pi + k \cdot 4\pi$$

**b**  $2\cos(x - \frac{1}{3}\pi) = 1$

$$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$$

$$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi$$

$$\mathbf{c} \quad 2\sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$$

$$\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$$

$$2x - \frac{1}{4}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{4}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$2x = 1\frac{7}{12}\pi + k \cdot 2\pi \vee 2x = -\frac{1}{12}\pi + k \cdot 2\pi$$

$$x = \frac{19}{24}\pi + k \cdot \pi \vee x = -\frac{1}{24}\pi + k \cdot \pi$$

$$\mathbf{d} \quad 2\cos(3x - \pi) = -1$$

$$\cos(3x - \pi) = -\frac{1}{2}$$

$$3x - \pi = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x - \pi = -\frac{2}{3}\pi + k \cdot 2\pi$$

$$3x = 1\frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

57

**a**  $2 \sin(2x - \frac{1}{6}\pi) = \sqrt{2}$

$$\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$$

$$2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$$

$$2x = \frac{5}{12}\pi + k \cdot 2\pi \vee 2x = \frac{11}{12}\pi + k \cdot 2\pi$$

$$x = \frac{5}{24}\pi + k \cdot \pi \vee x = \frac{11}{24}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{5}{24}\pi \vee x = \frac{11}{24}\pi \vee x = \frac{11}{24}\pi \vee x = \frac{11}{24}\pi$$

**b**  $2 \cos(3x - \frac{1}{2}\pi) = \sqrt{3}$

$$\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$$

$$3x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$3x = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{2}{9}\pi \vee x = \frac{8}{9}\pi \vee x = \frac{10}{9}\pi \vee x = \frac{16}{9}\pi \vee x = \frac{14}{9}\pi \vee x = \frac{20}{9}\pi$$

**c**  $\sin(\frac{2}{3}x) = -\frac{1}{2}\sqrt{2}$

$$\frac{2}{3}x = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{2}{3}x = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$x = \frac{7}{8}\pi + k \cdot 3\pi \vee x = -\frac{3}{8}\pi + k \cdot 3\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{7}{8}\pi$$

**d**  $\cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{3}$

$$\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{3}\pi + k \cdot 4\pi \vee x = -\frac{5}{3}\pi + k \cdot 4\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{5}{3}\pi$$

58

**a**  $2 \sin^2(x) = 1$

$$\sin^2(x) = \frac{1}{2}$$

$$\sin(x) = \sqrt{\frac{1}{2}} \vee \sin(x) = -\sqrt{\frac{1}{2}}$$

$$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$$

$$\text{Dus uit } 2 \sin^2(x) = 1 \text{ volgt } \sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}.$$

**b** Uit  $\sin(x) = \frac{1}{2}\sqrt{2}$  volgt  $x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi$ .

$$\text{Uit } \sin(x) = -\frac{1}{2}\sqrt{2} \text{ volgt } x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{1}{4}\pi + k \cdot 2\pi.$$

$$\text{Dus uit } \sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2} \text{ volgt}$$

$$x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{1}{4}\pi + k \cdot 2\pi.$$

**c**  $x = \frac{1}{4}\pi + k \cdot 2\pi : \dots, -\frac{3}{4}\pi, \frac{1}{4}\pi, 2\frac{1}{4}\pi, \dots$

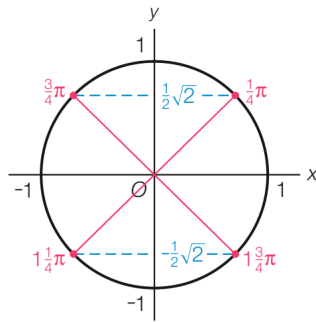
$$x = \frac{3}{4}\pi + k \cdot 2\pi : \dots, -\frac{1}{4}\pi, \frac{3}{4}\pi, 2\frac{3}{4}\pi, \dots$$

$$x = \frac{1}{4}\pi + k \cdot 2\pi : \dots, -\frac{3}{4}\pi, \frac{1}{4}\pi, 3\frac{1}{4}\pi, \dots$$

$$x = -\frac{1}{4}\pi + k \cdot 2\pi : \dots, -\frac{1}{4}\pi, \frac{1}{4}\pi, 3\frac{3}{4}\pi, \dots$$

$$\text{Dus } x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$$

d Zie de figuur hieronder.



Uit de cirkel is direct af te lezen dat  $\sin(x) = \frac{1}{2}\sqrt{2} \vee \sin(x) = -\frac{1}{2}\sqrt{2}$  geeft  $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$ .

**59 a**  $2 \cos^2(\frac{1}{2}x) = 1$

$$\cos^2(\frac{1}{2}x) = \frac{1}{2}$$

$$\cos(\frac{1}{2}x) = \sqrt{\frac{1}{2}} \vee \cos(\frac{1}{2}x) = -\sqrt{\frac{1}{2}}$$

$$\cos(\frac{1}{2}x) = \frac{1}{2}\sqrt{2} \vee \cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{2}$$

$$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

$$x = \frac{1}{2}\pi + k \cdot \pi$$

**b**  $4 \sin^2(x - \frac{1}{6}\pi) = 1$

$$\sin^2(x - \frac{1}{6}\pi) = \frac{1}{4}$$

$$\sin(x - \frac{1}{6}\pi) = \frac{1}{2} \vee \sin(x - \frac{1}{6}\pi) = -\frac{1}{2}$$

$$x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \pi + k \cdot 2\pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi$$

$$(x = \frac{1}{3}\pi + k \cdot \pi \vee x = k \cdot \pi)$$

**c**  $4 \cos^2(x + \frac{1}{4}\pi) = 3$

$$\cos^2(x + \frac{1}{4}\pi) = \frac{3}{4}$$

$$\cos(x + \frac{1}{4}\pi) = \sqrt{\frac{3}{4}} \vee \cos(x + \frac{1}{4}\pi) = -\sqrt{\frac{3}{4}}$$

$$\cos(x + \frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \vee \cos(x + \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$$

$$x + \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{12}\pi + k \cdot 2\pi \vee x = -\frac{5}{12}\pi + k \cdot 2\pi \vee x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = -\frac{1}{12}\pi + k \cdot 2\pi$$

$$(x = -\frac{1}{12}\pi + k \cdot \pi \vee x = -\frac{5}{12}\pi + k \cdot \pi)$$

**d**  $4 \sin^3(x) - \sin(x) = 0$

$$\sin(x)(4 \sin^2(x) - 1) = 0$$

$$\sin(x) = 0 \vee 4 \sin^2(x) = 1$$

$$\sin(x) = 0 \vee \sin^2(x) = \frac{1}{4}$$

$$\sin(x) = 0 \vee \sin(x) = \frac{1}{2} \vee \sin(x) = -\frac{1}{2}$$

$$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$(x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot \pi \vee x = \frac{5}{6}\pi + k \cdot \pi)$$

**60 a**  $\sin(\frac{1}{2}\pi x) = \frac{1}{2}\sqrt{3}$

$$\frac{1}{2}\pi x = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$$

$$x = \frac{2}{3} + k \cdot 4 \vee x = 1\frac{1}{3} + k \cdot 4$$

$$x \text{ in } [0, 10] \text{ geeft } x = \frac{2}{3} \vee x = 4\frac{2}{3} \vee x = 8\frac{2}{3} \vee x = 1\frac{1}{3} \vee x = 5\frac{1}{3} \vee x = 9\frac{1}{3}$$

**b**  $\cos(\frac{1}{3}\pi x) = -\frac{1}{2}\sqrt{3}$

$$\frac{1}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{3}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$x = 2\frac{1}{2} + k \cdot 6 \vee x = -2\frac{1}{2} + k \cdot 6$$

$$x \text{ in } [0, 10] \text{ geeft } x = 2\frac{1}{2} \vee x = 8\frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 9\frac{1}{2}$$

**c**  $4\sin^2(\frac{1}{5}\pi x) = 1$

$$\sin^2(\frac{1}{5}\pi x) = \frac{1}{4}$$

$$\sin(\frac{1}{5}\pi x) = \frac{1}{2} \vee \sin(\frac{1}{5}\pi x) = -\frac{1}{2}$$

$$\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{5}\pi x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{6} + k \cdot 10 \vee x = 4\frac{1}{6} + k \cdot 10 \vee x = 5\frac{5}{6} + k \cdot 10 \vee x = -\frac{5}{6} + k \cdot 10$$

$$x \text{ in } [0, 10] \text{ geeft } x = \frac{5}{6} \vee x = 4\frac{1}{6} \vee x = 5\frac{5}{6} \vee x = 9\frac{5}{6}$$

**d**  $2\cos^2(\frac{1}{10}\pi x) + \cos(\frac{1}{10}\pi x) = 0$

$$\cos(\frac{1}{10}\pi x)(2\cos(\frac{1}{10}\pi x) + 1) = 0$$

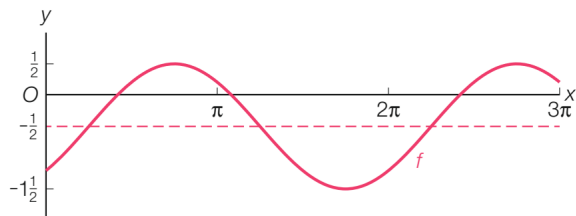
$$\cos(\frac{1}{10}\pi x) = 0 \vee \cos(\frac{1}{10}\pi x) = -\frac{1}{2}$$

$$\frac{1}{10}\pi x = \frac{1}{2}\pi + k \cdot \pi \vee \frac{1}{10}\pi x = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{1}{10}\pi x = -\frac{2}{3}\pi + k \cdot 2\pi$$

$$x = 5 + k \cdot 10 \vee x = 6\frac{2}{3} + k \cdot 20 \vee x = -6\frac{2}{3} + k \cdot 20$$

$$x \text{ in } [0, 10] \text{ geeft } x = 5 \vee x = 6\frac{2}{3}$$

**61 a**



**b**  $f(x) = -\frac{1}{2}$  geeft  $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$

$$\sin(x - \frac{1}{4}\pi) = 0$$

$$x - \frac{1}{4}\pi = k \cdot \pi$$

$$x = \frac{1}{4}\pi + k \cdot \pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{1}{4}\pi \vee x = 1\frac{1}{4}\pi \vee x = 2\frac{1}{4}\pi$$

De grafiek van  $f$  snijdt de lijn van de evenwichtsstand in de punten  $(\frac{1}{4}\pi, -\frac{1}{2})$ ,  $(1\frac{1}{4}\pi, -\frac{1}{2})$  en  $(2\frac{1}{4}\pi, -\frac{1}{2})$ .

**c**  $f(x) = \frac{1}{2}$  geeft  $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = \frac{1}{2}$

$$\sin(x - \frac{1}{4}\pi) = 1$$

$$x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = \frac{3}{4}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = 2\frac{3}{4}\pi$$

$$f(x) = -1\frac{1}{2} \text{ geeft } -\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1\frac{1}{2}$$

$$\sin(x - \frac{1}{4}\pi) = -1$$

$$x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$x = 1\frac{3}{4}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = 1\frac{3}{4}\pi$$

De toppen van de grafiek van  $f$  zijn  $(\frac{3}{4}\pi, \frac{1}{2})$ ,  $(1\frac{3}{4}\pi, -1\frac{1}{2})$  en  $(2\frac{3}{4}\pi, \frac{1}{2})$ .

**d**  $f(x) = 0$  geeft  $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = 0$

$$\sin(x - \frac{1}{4}\pi) = \frac{1}{2}$$

$$x - \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{12}\pi + k \cdot 2\pi \vee x = 1\frac{1}{12}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{5}{12}\pi \vee x = 2\frac{5}{12}\pi \vee x = 1\frac{1}{12}\pi$$

Dus  $x_A = \frac{5}{12}\pi$ ,  $x_B = 1\frac{1}{12}\pi$  en  $x_C = 2\frac{5}{12}\pi$ .

$$d(A, B) = x_B - x_A = 1\frac{1}{12}\pi - \frac{5}{12}\pi = \frac{2}{3}\pi$$

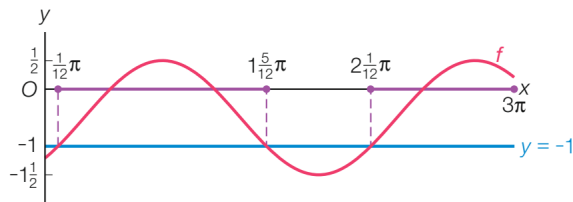
**e**  $f(x) = -1$  geeft  $-\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1$

$$\sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$$

$$x - \frac{1}{4}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = 1\frac{5}{12}\pi + k \cdot 2\pi \vee x = \frac{1}{12}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = 1\frac{5}{12}\pi \vee x = \frac{1}{12}\pi \vee x = 2\frac{1}{12}\pi$$



$$f(x) \geq -1 \text{ geeft } \frac{1}{12}\pi \leq x \leq 1\frac{5}{12}\pi \vee 2\frac{1}{12}\pi \leq x \leq 3\pi$$

**62**  $f(x) = 2$  geeft  $|2 \sin(x) - 1| = 2$

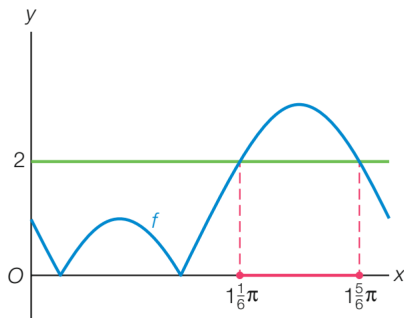
$$2 \sin(x) - 1 = 2 \vee 2 \sin(x) - 1 = -2$$

$$2 \sin(x) = 3 \vee 2 \sin(x) = -1$$

$$\sin(x) = 1\frac{1}{2} \vee \sin(x) = -\frac{1}{2}$$

$$x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$$



$$f(x) \geq 2 \text{ geeft } 1\frac{1}{6}\pi \leq x \leq 1\frac{5}{6}\pi$$



**63 a**  $\sin(3x) = \sin(\frac{1}{6}\pi)$

$$\sin(3x) = \frac{1}{2}$$

$$3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$$

**b**  $\cos(3x) = \cos(\frac{1}{6}\pi)$

$$\cos(3x) = \frac{1}{2}\sqrt{3}$$

$$3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$$

**64 a**  $\sin(x+1) = \sin(2x+3)$

$$x+1 = 2x+3 + k \cdot 2\pi \vee x+1 = \pi - (2x+3) + k \cdot 2\pi$$

$$-x = 2 + k \cdot 2\pi \vee x+1 = \pi - 2x - 3 + k \cdot 2\pi$$

$$x = -2 + k \cdot 2\pi \vee 3x = \pi - 4 + k \cdot 2\pi$$

$$x = -2 + k \cdot 2\pi \vee x = -\frac{1}{3} + \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$$

**b**  $\cos(2x-1) = \cos(x+1)$

$$2x-1 = x+1 + k \cdot 2\pi \vee 2x-1 = -(x+1) + k \cdot 2\pi$$

$$x = 2 + k \cdot 2\pi \vee 2x-1 = -x-1 + k \cdot 2\pi$$

$$x = 2 + k \cdot 2\pi \vee 3x = k \cdot 2\pi$$

$$x = 2 + k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$$

**c**  $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$

$$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$$

$$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{5}{6}\pi + k \cdot 2\pi \vee 3x = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$$

**d**  $\cos(x - \frac{1}{3}\pi) = \cos(2x)$

$$x - \frac{1}{3}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$$

$$-x = \frac{1}{3}\pi + k \cdot 2\pi \vee 3x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

**e**  $\sin(2\pi x) = \sin(\pi(x-1))$

$$\sin(2\pi x) = \sin(\pi x - \pi)$$

$$2\pi x = \pi x - \pi + k \cdot 2\pi \vee 2\pi x = \pi - (\pi x - \pi) + k \cdot 2\pi$$

$$\pi x = -\pi + k \cdot 2\pi \vee 2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$$

$$x = -1 + k \cdot 2 \vee 3\pi x = 2\pi + k \cdot 2\pi$$

$$x = -1 + k \cdot 2 \vee x = \frac{2}{3} + k \cdot \frac{2}{3}$$

$$x = -1 + k \cdot 2 \vee x = k \cdot \frac{2}{3}$$

**f**  $\cos(\frac{1}{2}\pi x) = \cos(\pi(x-2))$

$$\cos(\frac{1}{2}\pi x) = \cos(\pi x - 2\pi)$$

$$\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = -(\pi x - 2\pi) + k \cdot 2\pi$$

$$-\frac{1}{2}\pi x = -2\pi + k \cdot 2\pi \vee \frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$$

$$x = 4 + k \cdot 4 \vee \frac{1}{2}\pi x = 2\pi + k \cdot 2\pi$$

$$x = k \cdot 4 \vee x = 1\frac{1}{3} + k \cdot 1\frac{1}{3}$$

$$x = k \cdot 4 \vee x = k \cdot 1\frac{1}{3}$$

$$(x = k \cdot 1\frac{1}{3})$$

**65 a**  $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$

$$2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{7}{12}\pi \vee x = \frac{13}{36}\pi \vee x = 1\frac{1}{36}\pi \vee x = 1\frac{25}{36}\pi$$

**b**  $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$

$$3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi \vee 3x + \frac{1}{2}\pi = -(2x - \frac{1}{4}\pi) + k \cdot 2\pi$$

$$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$$

$$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee 5x = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$x = -\frac{3}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{4}\pi \vee x = \frac{7}{20}\pi \vee x = \frac{3}{4}\pi \vee x = 1\frac{3}{20}\pi \vee x = 1\frac{11}{20}\pi \vee x = 1\frac{19}{20}\pi$$

**66**  $f(x) = g(x)$  geeft  $\sin(x - \frac{1}{4}\pi) = \sin(2x - \frac{1}{3}\pi)$

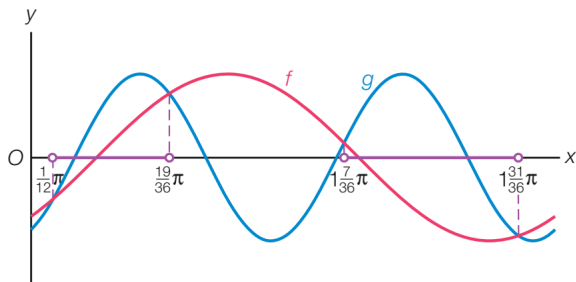
$$x - \frac{1}{4}\pi = 2x - \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \pi - (2x - \frac{1}{3}\pi) + k \cdot 2\pi$$

$$-x = -\frac{1}{12}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \pi - 2x + \frac{1}{3}\pi + k \cdot 2\pi$$

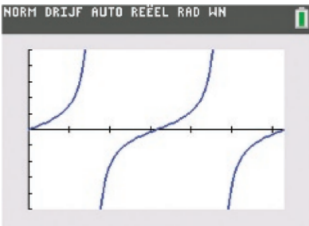
$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee 3x = 1\frac{7}{12}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot 2\pi \vee x = \frac{19}{36}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{12}\pi \vee x = \frac{19}{36}\pi \vee x = 1\frac{7}{36}\pi \vee x = 1\frac{31}{36}\pi$$



$$f(x) < g(x) \text{ geeft } \frac{1}{12}\pi < x < \frac{19}{36}\pi \vee 1\frac{7}{36}\pi < x < 1\frac{31}{36}\pi$$

**67 a** 

**b** Dat zijn de lijnen  $x = \frac{1}{2}\pi$  en  $x = 1\frac{1}{2}\pi$ .

**68 a**  $\left. \begin{array}{l} \tan(A) = \frac{\sin(A)}{\cos(A)} \\ \tan(A) = 0 \end{array} \right\} \begin{array}{l} \sin(A) = 0 \wedge \cos(A) \neq 0 \\ A = k \cdot \pi \end{array}$

**b**  $\tan(2x - \frac{1}{6}\pi) = 0$

$$2x - \frac{1}{6}\pi = k \cdot \pi$$

$$2x = \frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$$

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**69**  $3\pi x = \frac{1}{2}\pi + k \cdot \pi$

$$x = \frac{1}{6} + k \cdot \frac{1}{3}$$

$$x \text{ in } [0, 1] \text{ geeft } x = \frac{1}{6} \vee x = \frac{1}{2} \vee x = \frac{5}{6}$$

Dus de vergelijkingen van de asymptoten zijn  $x = \frac{1}{6}$ ,  $x = \frac{1}{2}$  en  $x = \frac{5}{6}$ .

**70 a**  $\tan(3x - \frac{1}{6}\pi) = \frac{1}{3}\sqrt{3}$

$$3x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot \pi$$

$$3x = \frac{1}{3}\pi + k \cdot \pi$$

$$x = \frac{1}{9}\pi + k \cdot \frac{1}{3}\pi$$

**b**  $1 - \tan(\frac{3}{4}x) = 2$

$$\tan(\frac{3}{4}x) = -1$$

$$\frac{3}{4}x = \frac{3}{4}\pi + k \cdot \pi$$

$$x = \pi + k \cdot \frac{4}{3}\pi$$

**c**  $\tan(\frac{1}{2}x) = \tan(2x - \frac{1}{6}\pi)$

$$\frac{1}{2}x = 2x - \frac{1}{6}\pi + k \cdot \pi$$

$$-\frac{1}{2}x = -\frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

**d**  $3 \tan(\frac{1}{2}\pi x) = \sqrt{3}$

$$\tan(\frac{1}{2}\pi x) = \frac{1}{3}\sqrt{3}$$

$$\frac{1}{2}\pi x = \frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{3} + k \cdot 2$$

**e**  $2 + \sqrt{3} \cdot \tan(\frac{1}{8}\pi x) = 5$

$$\sqrt{3} \cdot \tan(\frac{1}{8}\pi x) = 3$$

$$\tan(\frac{1}{8}\pi x) = \sqrt{3}$$

$$\frac{1}{8}\pi x = \frac{1}{3}\pi + k \cdot \pi$$

$$x = 2\frac{2}{3} + k \cdot 8$$

**f**  $\tan(\frac{1}{3}\pi x) = \tan(\frac{1}{4}\pi(x-1))$

$$\tan(\frac{1}{3}\pi x) = \tan(\frac{1}{4}\pi x - \frac{1}{4}\pi)$$

$$\frac{1}{3}\pi x = \frac{1}{4}\pi x - \frac{1}{4}\pi + k \cdot \pi$$

$$\frac{1}{12}\pi x = -\frac{1}{4}\pi + k \cdot \pi$$

$$x = -3 + k \cdot 12$$

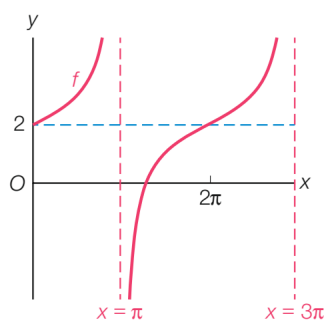
**71 a** Periode =  $\frac{\pi}{\frac{1}{2}} = 2\pi$  en beginpunt (0, 2).

$$\frac{1}{2}x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \pi \vee x = 3\pi$$

Dus de asymptoten zijn de lijnen  $x = \pi$  en  $x = 3\pi$ .



**b** Invoeren van de formule van  $f$  en gebruiken van de optie nulpunt geeft  $x = 4,068\dots$

Dus het nulpunt is 4,07.

**c**  $f(x) = 3$  geeft  $2 + \tan(\frac{1}{2}x) = 3$

$$\tan(\frac{1}{2}x) = 1$$

$$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \pi$$

$$x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 3\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 2\frac{1}{2}\pi$$

Dus de snijpunten zijn  $(\frac{1}{2}\pi, 3)$  en  $(2\frac{1}{2}\pi, 3)$ .

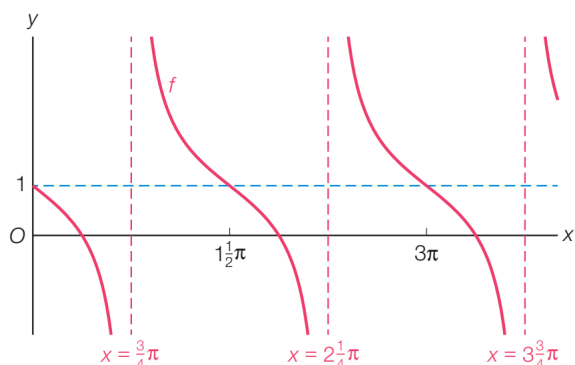
**72 a** Periode =  $\frac{\pi}{\frac{2}{3}} = 1\frac{1}{2}\pi$  en beginpunt (0, 1).

$$\frac{2}{3}x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{3}{4}\pi + k \cdot 1\frac{1}{2}\pi$$

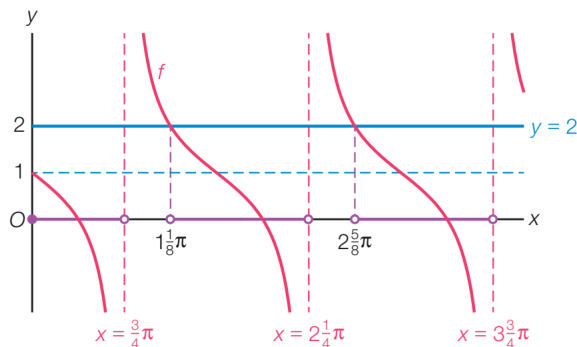
$$x \text{ in } [0, 4\pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = 2\frac{1}{4}\pi \vee x = 3\frac{3}{4}\pi$$

Dus de asymptoten zijn de lijnen  $x = \frac{3}{4}\pi$ ,  $x = 2\frac{1}{4}\pi$  en  $x = 3\frac{3}{4}\pi$ .



- b**  $f(x) = 0$  geeft  $1 - \tan(\frac{2}{3}x) = 0$   
 $\tan(\frac{2}{3}x) = 1$   
 $\frac{2}{3}x = \frac{1}{4}\pi + k \cdot \pi$   
 $x = \frac{3}{8}\pi + k \cdot 1\frac{1}{2}\pi$   
 $x$  in  $[0, 4\pi]$  geeft  $x = \frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi \vee x = 3\frac{3}{8}\pi$   
Dus de nulpunten zijn  $\frac{3}{8}\pi$ ,  $1\frac{7}{8}\pi$  en  $3\frac{3}{8}\pi$ .

- c**  $f(x) = 2$  geeft  $1 - \tan(\frac{2}{3}x) = 2$   
 $\tan(\frac{2}{3}x) = -1$   
 $\frac{2}{3}x = \frac{3}{4}\pi + k \cdot \pi$   
 $x = 1\frac{1}{8}\pi + k \cdot 1\frac{1}{2}\pi$   
 $x$  in  $[0, 4\pi]$  geeft  $x = 1\frac{1}{8}\pi \vee x = 2\frac{5}{8}\pi$

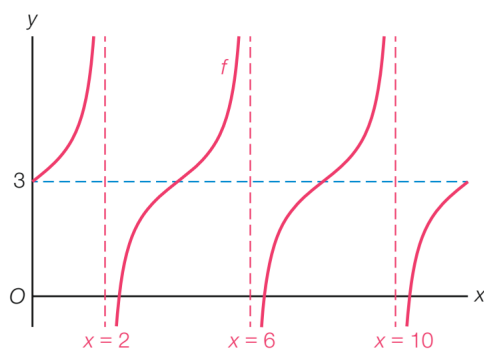


$f(x) < 2$  geeft  $0 \leq x < \frac{3}{4}\pi \vee 1\frac{1}{8}\pi < x < 2\frac{1}{4}\pi \vee 2\frac{5}{8}\pi < x < 3\frac{3}{4}\pi$

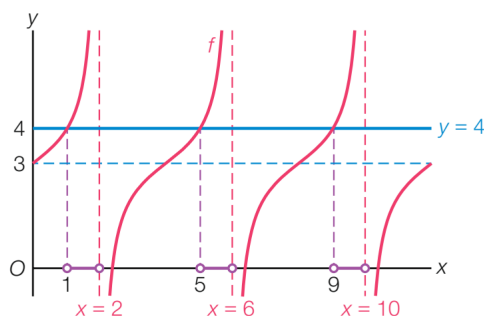
#### Bladzijde 165

- 73 a** Periode =  $\frac{\pi}{\frac{1}{4}\pi} = 4$  en beginpunt  $(0, 3)$ .

$\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot \pi$   
 $x = 2 + k \cdot 4$   
 $x$  in  $[0, 12]$  geeft  $x = 2 \vee x = 6 \vee x = 10$   
Dus de asymptoten zijn de lijnen  $x = 2$ ,  $x = 6$  en  $x = 10$ .

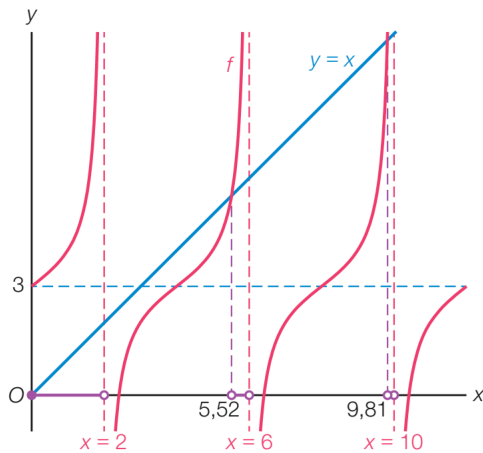


- b**  $f(x) = 4$  geeft  $3 + \tan(\frac{1}{4}\pi x) = 4$   
 $\tan(\frac{1}{4}\pi x) = 1$   
 $\frac{1}{4}\pi x = \frac{1}{4}\pi + k \cdot \pi$   
 $x = 1 + k \cdot 4$   
 $x$  in  $[0, 12]$  geeft  $x = 1 \vee x = 5 \vee x = 9$



$f(x) > 4$  geeft  $1 < x < 2 \vee 5 < x < 6 \vee 9 < x < 10$

- c Invoeren van de formule van  $f$  en de formule  $y = x$  en gebruiken van de optie snijpunt geeft  $x = 5,518\dots$  en  $x = 9,814\dots$



$f(x) > x$  geeft  $0 \leq x < 2 \vee 5,52 < x < 6 \vee 9,81 < x < 10$

**74**  $\sin(2x - \frac{1}{3}\pi) = -\cos(2x - \frac{1}{3}\pi)$

$$\frac{\sin(2x - \frac{1}{3}\pi)}{\cos(2x - \frac{1}{3}\pi)} = -1$$

$$\tan(2x - \frac{1}{3}\pi) = -1$$

$$2x - \frac{1}{3}\pi = \frac{3}{4}\pi + k \cdot \pi$$

$$2x = 1\frac{1}{12}\pi + k \cdot \pi$$

$$x = \frac{13}{24}\pi + k \cdot \frac{1}{2}\pi$$

**Bladzijde 166**

**C9 a**  $\sin(1\frac{1}{2}x - \frac{3}{4}\pi) = -1$

$$1\frac{1}{2}x - \frac{3}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$1\frac{1}{2}x = 2\frac{1}{4}\pi + k \cdot 2\pi$$

$$x = 1\frac{1}{2}\pi + k \cdot 1\frac{1}{3}\pi$$

**b**  $\cos(2x + \frac{1}{3}\pi) = 0$

$$2x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$$

$$2x = \frac{1}{6}\pi + k \cdot \pi$$

$$x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$$

**C10 a**  $2\sin(x - \frac{1}{4}\pi) = \sqrt{3}$

$$\sin(x - \frac{1}{4}\pi) = \frac{1}{2}\sqrt{3}$$

$$x - \frac{1}{4}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{4}\pi = \frac{2}{3}\pi + k \cdot 2\pi$$

$$x = \frac{7}{12}\pi + k \cdot 2\pi \vee x = \frac{11}{12}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{7}{12}\pi \vee x = \frac{11}{12}\pi$$

**b**  $\cos(2x + \frac{1}{2}\pi) = -\frac{1}{2}\sqrt{2}$

$$2x + \frac{1}{2}\pi = \frac{3}{4}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = -\frac{3}{4}\pi + k \cdot 2\pi$$

$$2x = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x = -1\frac{1}{4}\pi + k \cdot 2\pi$$

$$x = \frac{1}{8}\pi + k \cdot \pi \vee x = -\frac{5}{8}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{8}\pi \vee x = 1\frac{1}{8}\pi \vee x = \frac{3}{8}\pi \vee x = 1\frac{3}{8}\pi$$

**C11** a  $\sin(2x - \frac{1}{2}\pi) = \sin(3x - \frac{1}{6}\pi)$

$$2x - \frac{1}{2}\pi = 3x - \frac{1}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - (3x - \frac{1}{6}\pi) + k \cdot 2\pi$$

$$-x = \frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{2}\pi = \pi - 3x + \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{3}\pi + k \cdot 2\pi \vee 5x = 1\frac{2}{3}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{3}\pi + k \cdot \frac{2}{5}\pi$$

**b**  $\cos(1\frac{1}{2}\pi x) = \cos(\frac{1}{2}\pi x - \frac{1}{3}\pi)$

$$1\frac{1}{2}\pi x = \frac{1}{2}\pi x - \frac{1}{3}\pi + k \cdot 2\pi \vee 1\frac{1}{2}\pi x = -(\frac{1}{2}\pi x - \frac{1}{3}\pi) + k \cdot 2\pi$$

$$\pi x = -\frac{1}{3}\pi + k \cdot 2\pi \vee 1\frac{1}{2}\pi x = -\frac{1}{2}\pi x + \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{3} + k \cdot 2 \vee 2\pi x = \frac{1}{3}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{3} + k \cdot 2 \vee x = \frac{1}{6} + k \cdot 1$$

$$x \text{ in } [0, 3] \text{ geeft } x = 1\frac{2}{3} \vee x = \frac{1}{6} \vee x = 1\frac{1}{6} \vee x = 2\frac{1}{6}$$

**C12** a  $1 - \sqrt{3} \cdot \tan(\frac{1}{12}\pi x) = 2$

$$-\sqrt{3} \cdot \tan(\frac{1}{12}\pi x) = 1$$

$$\tan(\frac{1}{12}\pi x) = -\frac{1}{\sqrt{3}}$$

$$\tan(\frac{1}{12}\pi x) = -\frac{1}{\sqrt{3}}$$

$$\frac{1}{12}\pi x = \frac{5}{6}\pi + k \cdot \pi$$

$$x = 10 + k \cdot 12$$

**b**  $\tan(\frac{2}{3}x) = \tan(1\frac{1}{2}x - \frac{3}{4}\pi)$

$$\frac{2}{3}x = 1\frac{1}{2}x - \frac{3}{4}\pi + k \cdot \pi$$

$$-\frac{5}{6}x = -\frac{3}{4}\pi + k \cdot \pi$$

$$x = \frac{9}{10}\pi + k \cdot 1\frac{1}{5}\pi$$

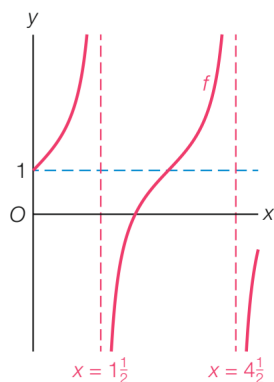
**C13** Periode =  $\frac{\pi}{\frac{1}{3}\pi} = 3$  en beginpunt (0, 1).

$$\frac{1}{3}\pi x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = 1\frac{1}{2} + k \cdot 3$$

$$x \text{ in } [0, 5] \text{ geeft } x = 1\frac{1}{2} \vee x = 4\frac{1}{2}$$

Dus de asymptoten zijn de lijnen  $x = 1\frac{1}{2}$  en  $x = 4\frac{1}{2}$ .



## 8.4 Herleiden en differentiëren

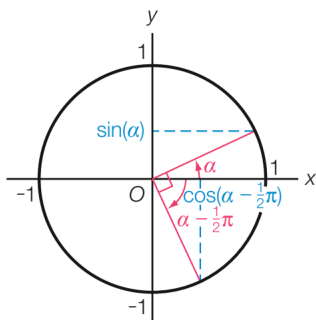
Bladzijde 167

**75**  $\sin(-\alpha) = y_Q = -y_P = -\sin(\alpha)$   
 $\cos(-\alpha) = x_Q = x_P = \cos(\alpha)$

Bladzijde 168

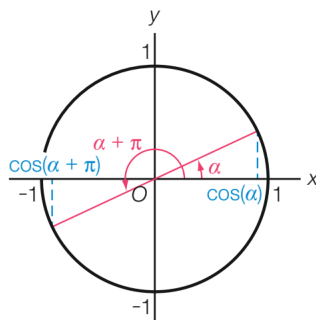
- 76 a** De formules  $-\cos(A) = \cos(A + \pi)$  en  $\cos(A) = \sin(A + \frac{1}{2}\pi)$ .  
**b**  $y = -\cos(2x + \frac{2}{3}\pi) = -\sin(2x + \frac{2}{3}\pi + \frac{1}{2}\pi) = -\sin(2x + 1\frac{1}{6}\pi)$   
 $= \sin(-2x + 1\frac{1}{6}\pi) = \sin(-2x - 1\frac{1}{6}\pi)$

**77 a**



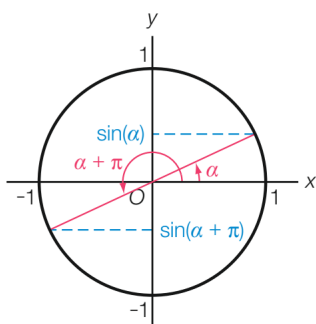
$$\cos(\alpha - \frac{1}{2}\pi) = \sin(\alpha)$$

**c**



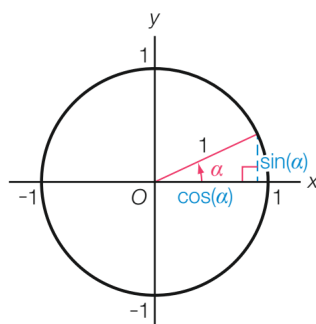
$$\cos(\alpha + \pi) = -\cos(\alpha)$$

**b**



$$\sin(\alpha + \pi) = -\sin(\alpha)$$

**d**



De stelling van Pythagoras geeft  $\sin^2(\alpha) + \cos^2(\alpha) = 1$ .

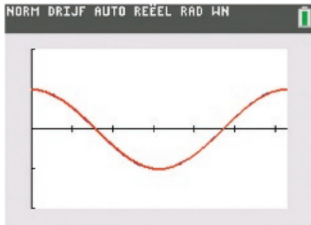
- 78 a**  $y = \sin(x + \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi - \frac{1}{2}\pi) = \cos(x - \frac{1}{3}\pi)$   
**b**  $y = \cos(2x + \frac{1}{3}\pi) = \sin(2x + \frac{1}{3}\pi + \frac{1}{2}\pi) = \sin(2x + \frac{5}{6}\pi)$   
**c**  $y = -\sin(3x - \frac{2}{3}\pi) = \sin(3x - \frac{2}{3}\pi + \pi) = \sin(3x + \frac{1}{3}\pi) = \cos(3x + \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(3x - \frac{1}{6}\pi)$   
**d**  $y = -\cos(4x + 1\frac{1}{6}\pi) = \cos(4x + 1\frac{1}{6}\pi + \pi) = \cos(4x + 2\frac{1}{6}\pi) = \sin(4x + 2\frac{1}{6}\pi + \frac{1}{2}\pi)$   
 $= \sin(4x + 2\frac{2}{3}\pi) = \sin(4x + \frac{2}{3}\pi)$

- 79 a**  $y = (\sin(x) - \cos(x))^2 = \sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x)$   
 $= \sin^2(x) + \cos^2(x) - 2\sin(x)\cos(x) = 1 - 2\sin(x)\cos(x)$

**b**  $y = \frac{2\sin^2(x) + \cos^2(x)}{\cos^2(x)} = \frac{2\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = 2\left(\frac{\sin(x)}{\cos(x)}\right)^2 + 1 = 2\tan^2(x) + 1$

Bladzijde 169

- 80 a**  $y = (\sin(x) + 1)^2 + \cos^2(x) - \sin(x) = \sin^2(x) + 2\sin(x) + 1 + \cos^2(x) - \sin(x)$   
 $= 1 + \sin(x) + 1 = \sin(x) + 2$   
**b**  $y = 2\cos^2(x) + \sin(x) - 3 = 2(1 - \sin^2(x)) + \sin(x) - 3$   
 $= 2 - 2\sin^2(x) + \sin(x) - 3 = -2\sin^2(x) + \sin(x) - 1$   
**c**  $y = 2\sin^2(x) + \cos^2(x) + \cos(x) = 2(1 - \cos^2(x)) + \cos^2(x) + \cos(x)$   
 $= 2 - 2\cos^2(x) + \cos^2(x) + \cos(x) = -\cos^2(x) + \cos(x) + 2$   
Dus  $a = -1$ ,  $b = 1$  en  $c = 2$ .



De grafieken van  $y = \frac{\sin(x + 0,01) - \sin(x)}{0,01}$  en  $y = \cos(x)$  vallen nagenoeg samen.

Dus waarschijnlijk is  $f'(x) = \cos(x)$  de afgeleide van  $f(x) = \sin(x)$ .

**b**  $g(x) = \cos(x) = \sin(x + \frac{1}{2}\pi)$  geeft  $g'(x) = \cos(x + \frac{1}{2}\pi) = \sin(x + \frac{1}{2}\pi + \frac{1}{2}\pi) = \sin(x + \pi) = -\sin(x)$

### Bladzijde 171

**a**  $f(x) = 3 + 4\sin(2x - \frac{1}{3}\pi)$  geeft  $f'(x) = 4\cos(2x - \frac{1}{3}\pi) \cdot 2 = 8\cos(2x - \frac{1}{3}\pi)$

**b**  $g(x) = x\cos(x)$  geeft  $g'(x) = 1 \cdot \cos(x) + x \cdot -\sin(x) = \cos(x) - x\sin(x)$

**c**  $h(x) = \frac{x^2 + \sin(x)}{\cos(x)}$  geeft  $h'(x) = \frac{\cos(x) \cdot (2x + \cos(x)) - (x^2 + \sin(x)) \cdot -\sin(x)}{\cos^2(x)}$   

$$= \frac{2x\cos(x) + \cos^2(x) + x^2\sin(x) + \sin^2(x)}{\cos^2(x)} = \frac{x^2\sin(x) + 2x\cos(x) + 1}{\cos^2(x)}$$

**d**  $k(x) = \cos^2(x)$  geeft  $k'(x) = 2\cos(x) \cdot -\sin(x) = -2\sin(x)\cos(x)$

**e**  $l(x) = x + 3\sin^2(x)$  geeft  $l'(x) = 1 + 6\sin(x)\cos(x)$

**f**  $m(x) = \frac{\cos(x)}{\sin(x)}$  geeft  $m'(x) = \frac{\sin(x) \cdot -\sin(x) - \cos(x) \cdot \cos(x)}{\sin^2(x)} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)}$

### Bladzijde 172

**a**  $f(x) = 10 + 16\cos(\frac{1}{2}(x-1)) = 10 + 16\cos(\frac{1}{2}x - \frac{1}{2})$  geeft  $f'(x) = -16\sin(\frac{1}{2}x - \frac{1}{2}) \cdot \frac{1}{2} = -8\sin(\frac{1}{2}x - \frac{1}{2})$

**b**  $g(x) = x^2\sin(3x)$  geeft  $g'(x) = 2x \cdot \sin(3x) + x^2 \cdot \cos(3x) \cdot 3 = 2x\sin(3x) + 3x^2\cos(3x)$

**c**  $h(x) = \frac{\sin(x)}{x^2 + \cos(x)}$  geeft  $h'(x) = \frac{(x^2 + \cos(x)) \cdot \cos(x) - \sin(x) \cdot (2x - \sin(x))}{(x^2 + \cos(x))^2}$   

$$= \frac{x^2\cos(x) + \cos^2(x) - 2x\sin(x) + \sin^2(x)}{(x^2 + \cos(x))^2} = \frac{x^2\cos(x) - 2x\sin(x) + 1}{(x^2 + \cos(x))^2}$$

**d**  $k(x) = 2\sin^2(x)$  geeft  $k'(x) = 4\sin(x)\cos(x)$

**a**  $f(x) = x\cos(2x)$  geeft  $f'(x) = 1 \cdot \cos(2x) + x \cdot -\sin(2x) \cdot 2 = \cos(2x) - 2x\sin(2x)$

**b**  $g(x) = 2x\sin(3x-1)$  geeft  $g'(x) = 2 \cdot \sin(3x-1) + 2x \cdot \cos(3x-1) \cdot 3 = 2\sin(3x-1) + 6x\cos(3x-1)$

**c**  $h(x) = \frac{x\sin(x)}{x + \sin(x)}$  geeft  $h'(x) = \frac{(x + \sin(x)) \cdot (1 \cdot \sin(x) + x \cdot \cos(x)) - x\sin(x) \cdot (1 + \cos(x))}{(x + \sin(x))^2}$   

$$= \frac{x\sin(x) + x^2\cos(x) + \sin^2(x) + x\sin(x)\cos(x) - x\sin(x) - x\sin(x)\cos(x)}{(x + \sin(x))^2}$$
  

$$= \frac{x^2\cos(x) + \sin^2(x)}{(x + \sin(x))^2}$$

**d**  $k(x) = 1 + 2\cos^2(x)$  geeft  $k'(x) = 4\cos(x) \cdot -\sin(x) = -4\sin(x)\cos(x)$



**85 a**  $f(x) = 3 \tan(2x)$  geeft  $f'(x) = 3 \cdot \frac{1}{\cos^2(2x)} \cdot 2 = \frac{6}{\cos^2(2x)}$

*Alternatieve uitwerking*

$f(x) = 3 \tan(2x)$  geeft  $f'(x) = 3(1 + \tan^2(2x)) \cdot 2 = 6 + 6 \tan^2(2x)$

**b**  $g(x) = \tan^2(x)$  geeft  $g'(x) = 2 \tan(x) \cdot \frac{1}{\cos^2(x)} = 2 \cdot \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos^2(x)} = \frac{2 \sin(x)}{\cos^3(x)}$

**86 a**  $f(x) = \cos^3(x)$  geeft  $f'(x) = 3 \cos^2(x) \cdot -\sin(x) = -3 \sin(x) \cos^2(x)$

**b**  $g(x) = \cos(x^3)$  geeft  $g'(x) = -\sin(x^3) \cdot 3x^2 = -3x^2 \sin(x^3)$

**c**  $h(x) = \sqrt{\sin(x)}$  geeft  $h'(x) = \frac{1}{2\sqrt{\sin(x)}} \cdot \cos(x) = \frac{\cos(x)}{2\sqrt{\sin(x)}}$

**d**  $k(x) = \cos^2(x) - \sin^2(x)$  geeft  $k'(x) = 2 \cos(x) \cdot -\sin(x) - 2 \sin(x) \cdot \cos(x) = -4 \sin(x) \cos(x)$

**87 a**  $f(x) = \sin^3(x) + \sin(x)$  geeft  $f'(x) = 3 \sin^2(x) \cos(x) + \cos(x)$   
 $= 3(1 - \cos^2(x)) \cos(x) + \cos(x)$   
 $= 3 \cos(x) - 3 \cos^3(x) + \cos(x)$   
 $= 4 \cos(x) - 3 \cos^3(x)$

**b**  $g(x) = \sin^2(x) \cos(x)$  geeft  $g'(x) = 2 \sin(x) \cos(x) \cdot \cos(x) + \sin^2(x) \cdot -\sin(x)$   
 $= 2 \sin(x) \cos^2(x) - \sin^3(x)$   
 $= 2 \sin(x) (1 - \sin^2(x)) - \sin^3(x)$   
 $= 2 \sin(x) - 2 \sin^3(x) - \sin^3(x)$   
 $= 2 \sin(x) - 3 \sin^3(x)$

**c**  $h(x) = \frac{\tan(x)}{\sin(x)} = \frac{\left(\frac{\sin(x)}{\cos(x)}\right)}{\sin(x)} = \frac{1}{\cos(x)} = \cos^{-1}(x)$  geeft  $h'(x) = -\cos^{-2}(x) \cdot -\sin(x) = \frac{\sin(x)}{\cos^2(x)}$

**88 a**  $f(x) = \frac{3 \cos(x)}{2 - \sin(x)}$  geeft  $f'(x) = \frac{(2 - \sin(x)) \cdot -3 \sin(x) - 3 \cos(x) \cdot -\cos(x)}{(2 - \sin(x))^2} = \frac{-6 \sin(x) + 3 \sin^2(x) + 3 \cos^2(x)}{(2 - \sin(x))^2}$   
 $= \frac{-6 \sin(x) + 3(\sin^2(x) + \cos^2(x))}{(2 - \sin(x))^2} = \frac{-6 \sin(x) + 3 \cdot 1}{(2 - \sin(x))^2} = \frac{-6 \sin(x) + 3}{(2 - \sin(x))^2}$

**b**  $f'(x) = 0$  geeft  $-6 \sin(x) + 3 = 0$   
 $-6 \sin(x) = -3$   
 $\sin(x) = \frac{1}{2}$   
 $x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$

max. is  $f(\frac{1}{6}\pi) = \frac{3 \cos(\frac{1}{6}\pi)}{2 - \sin(\frac{1}{6}\pi)} = \frac{3 \cdot \frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = \sqrt{3}$

min. is  $f(\frac{5}{6}\pi) = \frac{3 \cos(\frac{5}{6}\pi)}{2 - \sin(\frac{5}{6}\pi)} = \frac{3 \cdot -\frac{1}{2}\sqrt{3}}{2 - \frac{1}{2}} = -\sqrt{3}$

Dus  $B_f = [-\sqrt{3}, \sqrt{3}]$ .

**c**  $f(x) = 0$  geeft  $3 \cos(x) = 0$   
 $\cos(x) = 0$   
 $x = \frac{1}{2}\pi + k \cdot \pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi$

Dus  $A(\frac{1}{2}\pi, 0)$ .

$f'(\frac{1}{2}\pi) = \frac{-6 \sin(\frac{1}{2}\pi) + 3}{(2 - \sin(\frac{1}{2}\pi))^2} = \frac{-6 \cdot 1 + 3}{(2 - 1)^2} = \frac{-3}{1} = -3$ , dus  $rc_k = \frac{1}{3}$ .

$k: y = \frac{1}{3}x + b \quad \left\{ \begin{array}{l} \frac{1}{3} \cdot \frac{1}{2}\pi + b = 0 \\ \text{door } A(\frac{1}{2}\pi, 0) \end{array} \right\}$   
 $b = -\frac{1}{6}\pi$

Dus  $k: y = \frac{1}{3}x - \frac{1}{6}\pi$  en  $y_B = -\frac{1}{6}\pi$ .

**a**  $f_1(x) = \cos^2(x) - \cos(x)$  geeft  $f_1'(x) = 2 \cos(x) \cdot -\sin(x) + \sin(x) = -2 \sin(x) \cos(x) + \sin(x)$   
 $f_1'(x) = 0$  geeft  $-2 \sin(x) \cos(x) + \sin(x) = 0$   
 $\sin(x)(-2 \cos(x) + 1) = 0$   
 $\sin(x) = 0 \vee 2 \cos(x) = 1$   
 $\sin(x) = 0 \vee \cos(x) = \frac{1}{2}$   
 $x = k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$   
 $x$  in  $[0, 2\pi]$  geeft  $x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{3}\pi \vee x = \frac{5}{3}\pi$

min. is  $f_1(\frac{1}{3}\pi) = \cos^2(\frac{1}{3}\pi) - \cos(\frac{1}{3}\pi) = (\frac{1}{2})^2 - \frac{1}{2} = -\frac{1}{4}$

max. is  $f_1(\pi) = \cos^2(\pi) - \cos(\pi) = (-1)^2 - (-1) = 2$

Dus  $B_{f_1} = [-\frac{1}{4}, 2]$ .

**b**  $f_p'(x) = -2p \sin(x) \cos(x) + \sin(x)$

$f_p'(\frac{1}{6}\pi) = 0$  geeft  $-2p \sin(\frac{1}{6}\pi) \cos(\frac{1}{6}\pi) + \sin(\frac{1}{6}\pi) = 0$

$$-2p \cdot \frac{1}{2} \cdot \frac{1}{2} \sqrt{3} + \frac{1}{2} = 0$$

$$-p \cdot \frac{1}{2} \sqrt{3} = -\frac{1}{2}$$

$$p = \frac{1}{\sqrt{3}} = \frac{1}{3} \sqrt{3}$$

Dit geeft  $f_{\frac{1}{3}\sqrt{3}}(x) = \frac{1}{3}\sqrt{3} \cos^2(x) - \cos(x)$  en  $f_{\frac{1}{3}\sqrt{3}}'(x) = -\frac{2}{3}\sqrt{3} \sin(x) \cos(x) + \sin(x)$ .

$f_{\frac{1}{3}\sqrt{3}}'(x) = 0$  geeft  $-\frac{2}{3}\sqrt{3} \sin(x) \cos(x) + \sin(x) = 0$

$$\sin(x)(-\frac{2}{3}\sqrt{3} \cos(x) + 1) = 0$$

$$\sin(x) = 0 \vee \frac{2}{3}\sqrt{3} \cos(x) = 1$$

$$\sin(x) = 0 \vee \cos(x) = \frac{1}{2}\sqrt{3}$$

$$x = k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$$

max. is  $f_{\frac{1}{3}\sqrt{3}}(\pi) = \frac{1}{3}\sqrt{3} \cos^2(\pi) - \cos(\pi) = \frac{1}{3}\sqrt{3} \cdot (-1)^2 - (-1) = \frac{1}{3}\sqrt{3} + 1$

**c**  $f_p(\frac{1}{2}\pi) = p \cos^2(\frac{1}{2}\pi) - \cos(\frac{1}{2}\pi) = p \cdot 0^2 - 0 = 0$

$f_p'(\frac{1}{2}\pi) = -2p \sin(\frac{1}{2}\pi) \cos(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) = -2p \cdot 1 \cdot 0 + 1 = 1$

De grafieken van  $f_p$  gaan allemaal door het punt  $(\frac{1}{2}\pi, 0)$  en hebben in dit punt allemaal helling 1.

Dus alle grafieken van  $f_p$  raken elkaar in het punt met  $x$ -coördinaat  $\frac{1}{2}\pi$ .

$f_p(1\frac{1}{2}\pi) = p \cos^2(1\frac{1}{2}\pi) - \cos(1\frac{1}{2}\pi) = p \cdot 0^2 - 0 = 0$

$f_p'(1\frac{1}{2}\pi) = -2p \sin(1\frac{1}{2}\pi) \cos(1\frac{1}{2}\pi) + \sin(1\frac{1}{2}\pi) = -2p \cdot -1 \cdot 0 + 1 = 1$

De grafieken van  $f_p$  gaan allemaal door het punt  $(1\frac{1}{2}\pi, 0)$  en hebben in dit punt allemaal helling 1.

Dus alle grafieken van  $f_p$  raken elkaar in het punt met  $x$ -coördinaat  $1\frac{1}{2}\pi$ .

**d**  $f_p'(x) = 0$  geeft  $-2p \sin(x) \cos(x) + \sin(x) = 0$

$$\sin(x)(-2p \cos(x) + 1) = 0$$

$$\sin(x) = 0 \vee 2p \cos(x) = 1$$

$$\sin(x) = 0 \vee p = \frac{1}{2 \cos(x)}$$

Voor alle toppen van de grafieken van  $f_p$  waarvoor niet geldt  $x = 0$ ,  $x = \pi$  en  $x = 2\pi$ , geldt  $p = \frac{1}{2 \cos(x)}$ .

Dit geeft de formule  $y = \frac{1}{2 \cos(x)} \cdot \cos^2(x) - \cos(x) = \frac{1}{2} \cos(x) - \cos(x) = -\frac{1}{2} \cos(x)$ .

**a**  $f(x) = \sqrt{2 \sin(3x) + 1}$  geeft  $f'(x) = \frac{1}{2\sqrt{2 \sin(3x) + 1}} \cdot 2 \cos(3x) \cdot 3 = \frac{3 \cos(3x)}{\sqrt{2 \sin(3x) + 1}}$

**b**  $2 \sin(3x) + 1 = 0$

$$2 \sin(3x) = -1$$

$$\sin(3x) = -\frac{1}{2}$$

$$3x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{7}{18}\pi \vee x = \frac{11}{18}\pi$$

Het domein van  $f$  bestaat uit de intervallen  $[0, \frac{7}{18}\pi]$  en  $[\frac{11}{18}\pi, \pi]$ .

$$f'(x) = 0 \text{ geeft } 3 \cos(3x) = 0$$

$$\cos(3x) = 0$$

$$3x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$$

$$x \text{ in } [0, \frac{7}{18}\pi] \text{ en } [\frac{11}{18}\pi, \pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$$

$$f(\frac{1}{6}\pi) = \sqrt{2 \sin(\frac{1}{2}\pi) + 1} = \sqrt{2 \cdot 1 + 1} = \sqrt{3} \text{ en } f(\frac{5}{6}\pi) = \sqrt{2 \sin(2\frac{1}{2}\pi) + 1} = \sqrt{2 \cdot 1 + 1} = \sqrt{3}$$

Dus het maximum van  $f$  is  $\sqrt{3}$ .

$\sin(3x)$  is minimaal  $-1$ , dus  $2 \sin(3x)$  is minimaal  $-2$ , dus  $2 \sin(3x) + 1$  is minimaal  $-1$ , dus  $\sqrt{2 \sin(3x) + 1}$  is minimaal  $0$ .

Dus  $B_f = [0, \sqrt{3}]$ .

**c**  $f(x) = 1$  geeft  $\sqrt{2 \sin(3x) + 1} = 1$

$$2 \sin(3x) + 1 = 1$$

$$2 \sin(3x) = 0$$

$$\sin(3x) = 0$$

$$3x = k \cdot \pi$$

$$x = k \cdot \frac{1}{3}\pi$$

$$x \text{ in } [0, \frac{7}{18}\pi] \text{ en } [\frac{11}{18}\pi, \pi] \text{ geeft } x = 0 \vee x = \frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = \pi$$

Dus  $x_B = \frac{1}{3}\pi$ .

$$k: y = ax + b$$

$$a = f'(\frac{1}{3}\pi) = \frac{3 \cos(\pi)}{\sqrt{2 \sin(\pi) + 1}} = \frac{-3}{\sqrt{1}} = -3$$

$$\left. \begin{array}{l} y = -3x + b \\ \text{door } B(\frac{1}{3}\pi, 1) \end{array} \right\} \begin{array}{l} -3 \cdot \frac{1}{3}\pi + b = 1 \\ -\pi + b = 1 \\ b = 1 + \pi \end{array}$$

$$k: y = -3x + 1 + \pi \text{ snijden met de } x\text{-as geeft } -3x + 1 + \pi = 0$$

$$3x = 1 + \pi$$

$$x = \frac{1}{3} + \frac{1}{3}\pi$$

Dus  $x_S = \frac{1}{3} + \frac{1}{3}\pi$ .

**d** Zie vraag b.

Twee opeenvolgende laagste punten van de grafiek van  $g$  zijn  $(\frac{1}{6}\pi, \sqrt{3})$  en  $(\frac{5}{6}\pi, \sqrt{3})$ .

$$a = \frac{3\sqrt{3} + \sqrt{3}}{2} = 2\sqrt{3}$$

$$b = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

Van de grafiek van  $g$  is de periode  $\frac{5}{6}\pi - \frac{1}{6}\pi = \frac{2}{3}\pi$ , dus  $c = \frac{2\pi}{\frac{2}{3}\pi} = 3$ .

De grafiek van  $g$  gaat stijgend door de evenwichtsstand voor  $x = \frac{1}{6}\pi + \frac{1}{4} \cdot \frac{2}{3}\pi = \frac{1}{3}\pi$ ,

dus  $d = \frac{1}{3}\pi$ .

$$\text{Dus } g(x) = 2\sqrt{3} + \sqrt{3} \cdot \sin(3(x - \frac{1}{3}\pi)).$$

**91**  $f_p(x) = \cos^2(x) + 4 \sin(x) + p$  geeft  $f_p'(x) = 2 \cos(x) \cdot -\sin(x) + 4 \cos(x) = -2 \sin(x) \cos(x) + 4 \cos(x)$

$g_q(x) = q \cos(x)$  geeft  $g_q'(x) = -q \sin(x)$

$f_p'(\frac{1}{6}\pi) = -2 \sin(\frac{1}{6}\pi) \cos(\frac{1}{6}\pi) + 4 \cos(\frac{1}{6}\pi) = -2 \cdot \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} + 4 \cdot \frac{1}{2}\sqrt{3} = -\frac{1}{2}\sqrt{3} + 2\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

$g_q'(\frac{1}{6}\pi) = -q \sin(\frac{1}{6}\pi) = -\frac{1}{2}q$

$f_p'(\frac{1}{6}\pi) \cdot g_q'(\frac{1}{6}\pi) = -1$  geeft  $1\frac{1}{2}\sqrt{3} \cdot -\frac{1}{2}q = -1$

$$\frac{3}{4}\sqrt{3} \cdot q = 1$$

$$q = \frac{4}{3\sqrt{3}}$$

$f_p(\frac{1}{6}\pi) = \cos^2(\frac{1}{6}\pi) + 4 \sin(\frac{1}{6}\pi) + p = (\frac{1}{2}\sqrt{3})^2 + 4 \cdot \frac{1}{2} + p = \frac{3}{4} + 2 + p = 2\frac{3}{4} + p$

Voor  $q = \frac{4}{3\sqrt{3}}$  is  $g(\frac{1}{6}\pi) = \frac{4}{3\sqrt{3}} \cdot \cos(\frac{1}{6}\pi) = \frac{4}{3\sqrt{3}} \cdot \frac{1}{2}\sqrt{3} = \frac{2}{3}$

en  $f_p(\frac{1}{6}\pi) = g(\frac{1}{6}\pi)$  geeft  $2\frac{3}{4} + p = \frac{2}{3}$ , dus  $p = -2\frac{1}{12}$ .

Dus  $p = -2\frac{1}{12}$  en  $q = \frac{4}{3\sqrt{3}}$ .

#### Bladzijde 174

**C14** **a**  $y = -\sin(2x + \frac{1}{4}\pi) = \sin(2x + \frac{1}{4}\pi + \pi) = \sin(2x + 1\frac{1}{4}\pi) = \cos(2x + 1\frac{1}{4}\pi - \frac{1}{2}\pi) = \cos(2x + \frac{3}{4}\pi)$

**b**  $y = 2 \sin^2(x) + 3 \cos(x) = 2(1 - \cos^2(x)) + 3 \cos(x) = 2 - 2 \cos^2(x) + 3 \cos(x) = -2 \cos^2(x) + 3 \cos(x) + 2$

**C15** **a**  $f(x) = 2 \sin(4x - \frac{1}{2}\pi)$  geeft  $f'(x) = 2 \cos(4x - \frac{1}{2}\pi) \cdot 4 = 8 \cos(4x - \frac{1}{2}\pi)$

**b**  $g(x) = \cos(3x^2 + 1)$  geeft  $g'(x) = -\sin(3x^2 + 1) \cdot 6x = -6x \sin(3x^2 + 1)$

**c**  $h(x) = 4 \tan(2x)$  geeft  $h'(x) = 4 \cdot \frac{1}{\cos^2(2x)} \cdot 2 = \frac{8}{\cos^2(2x)}$

*Alternatieve uitwerking*

$h(x) = 4 \tan(2x)$  geeft  $h'(x) = 4(1 + \tan^2(2x)) \cdot 2 = 8 + 8 \tan^2(2x)$

**d**  $k(x) = x^3 \sin(x)$  geeft  $k'(x) = 3x^2 \cdot \sin(x) + x^3 \cdot \cos(x) = 3x^2 \sin(x) + x^3 \cos(x)$

**e**  $l(x) = \frac{\sin(x)}{x + \cos(x)}$  geeft  $l'(x) = \frac{(x + \cos(x)) \cdot \cos(x) - \sin(x) \cdot (1 - \sin(x))}{(x + \cos(x))^2}$

$$= \frac{x \cos(x) + \cos^2(x) - \sin(x) + \sin^2(x)}{(x + \cos(x))^2}$$

$$= \frac{x \cos(x) - \sin(x) + 1}{(x + \cos(x))^2}$$

#### Kennis- en vaardigheidentest

#### Bladzijde 175

**1**  $A(\cos(40^\circ), \sin(40^\circ)) \approx A(0,77; 0,64)$

De draaiingshoek die bij het punt  $B$  hoort, is  $40^\circ + 120^\circ = 160^\circ$ .

$B(\cos(160^\circ), \sin(160^\circ)) \approx B(-0,94; 0,34)$

De draaiingshoek die bij het punt  $C$  hoort, is  $40^\circ + 240^\circ = 280^\circ$ .

$C(\cos(280^\circ), \sin(280^\circ)) \approx C(0,17; -0,98)$

**2** **a**  $\frac{3}{4}\pi \text{ rad} = \frac{3}{4} \cdot 180^\circ = 135^\circ$

**d**  $26\pi \text{ rad} = 26 \cdot 180^\circ = 4680^\circ$

**b**  $\frac{1}{5}\pi \text{ rad} = \frac{1}{5} \cdot 180^\circ = 36^\circ$

**e**  $\frac{2}{3}\pi \text{ rad} = \frac{2}{3} \cdot 180^\circ = 120^\circ$

**c**  $0,6 \text{ rad} = 0,6 \cdot \frac{180^\circ}{\pi} \approx 34,4^\circ$

**f**  $\frac{2}{3} \text{ rad} = \frac{2}{3} \cdot \frac{180^\circ}{\pi} \approx 38,2^\circ$

**3** **a**  $270^\circ = \frac{270}{180}\pi \text{ rad} = 1\frac{1}{2}\pi \text{ rad}$

**d**  $330^\circ = \frac{330}{180}\pi \text{ rad} = 1\frac{5}{6}\pi \text{ rad}$

**b**  $-60^\circ = \frac{-60}{180}\pi \text{ rad} = -\frac{1}{3}\pi \text{ rad}$

**e**  $40^\circ = \frac{40}{180}\pi \text{ rad} = \frac{2}{9}\pi \text{ rad}$

**c**  $150^\circ = \frac{150}{180}\pi \text{ rad} = \frac{5}{6}\pi \text{ rad}$

**f**  $-70^\circ = \frac{-70}{180}\pi \text{ rad} = -\frac{7}{18}\pi \text{ rad}$

- 4**  $x_P = -0,4$ , dus  $\cos(\alpha_P) = -0,4$ .  
 De GR geeft  $1,982\dots$   
 Dus  $\alpha_P = 1,982\dots$   
 $y_Q = 0,9$ , dus  $\sin(\alpha_Q) = 0,9$ .  
 De GR geeft  $1,119\dots$   
 Dus  $\alpha_Q = 1,119\dots$   
 $\angle POQ = \alpha_P - \alpha_Q = 1,982\dots - 1,119\dots \approx 0,86$

- 5** **a**  $\sin(x) = -\frac{1}{2}$  met  $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$  geeft  $x = -\frac{1}{6}\pi$   
**b**  $x = \cos(\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$   
**c**  $\cos(x) = \frac{1}{2}$  met  $\frac{1}{2}\pi \leq x \leq 2\pi$  geeft  $x = \frac{2}{3}\pi$   
**d**  $x = \sin(\frac{2}{3}\pi) = \frac{\sqrt{3}}{2}$   
**e**  $\sin(x) = \frac{1}{2}\sqrt{3}$  met  $\frac{1}{2}\pi \leq x \leq \frac{3}{2}\pi$  geeft  $x = \frac{2}{3}\pi$   
**f**  $x = \tan(\frac{5}{6}\pi) = \frac{\sin(\frac{5}{6}\pi)}{\cos(\frac{5}{6}\pi)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{1}{3}\sqrt{3}$

- 6** **a** De evenwichtsstand is  $-2$ , de amplitude is  $3$ , de periode is  $2\pi$  en een beginpunt is  $(\frac{1}{3}\pi, -2)$ .  
**b** De evenwichtsstand is  $3$ , de amplitude is  $4$ , de periode is  $\frac{2\pi}{2} = \pi$  en een beginpunt is  $(\frac{1}{4}, 7)$ .

- 7** **a** De evenwichtsstand is  $-1$ , de amplitude is  $2$ , de periode is  $2\pi$  en een beginpunt is  $(\frac{1}{3}\pi, 1)$ .  
**b** De grafiek van  $f$  snijdt de lijn van de evenwichtsstand in de punten  $(-\frac{1}{6}\pi, -1)$ ,  $(\frac{5}{6}\pi, -1)$ ,  $(\frac{3}{2}\pi, -1)$  en  $(\frac{7}{2}\pi, -1)$ .  
**c** De toppen van de grafiek van  $f$  zijn  $(-\frac{2}{3}\pi, 3)$ ,  $(\frac{1}{3}\pi, 1)$ ,  $(\frac{4}{3}\pi, 3)$  en  $(\frac{5}{3}\pi, 1)$ .

- 8** **a**  $a = \frac{10 + -30}{2} = -10$  en  $b = 10 - -10 = 20$

halve periode  $= 17\frac{1}{2} - 2\frac{1}{2} = 15$  geeft periode  $= 30$ , dus  $c = \frac{2\pi}{30} = \frac{1}{15}\pi$ .

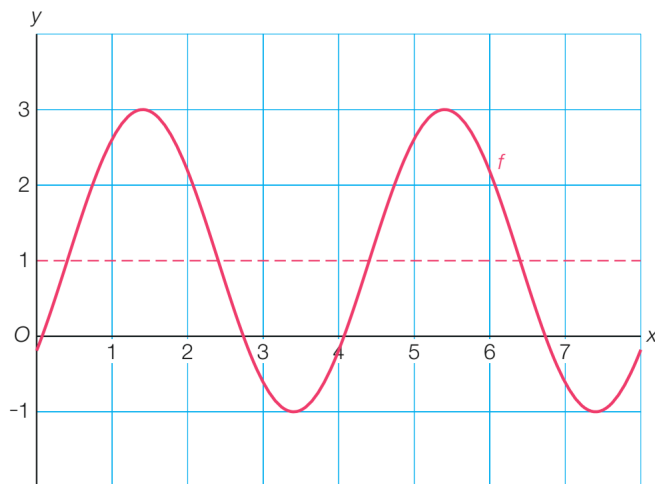
Stijgend door de evenwichtsstand bij  $x = 2\frac{1}{2} + \frac{1}{4} \cdot 30 = 10$ , dus  $d = 10$ .  
 Dus  $y = -10 + 20 \sin(\frac{1}{15}\pi(x - 10))$ .

- b** Een hoogste punt is  $(17\frac{1}{2}, 10)$ , dus  $y = -10 + 20 \cos(\frac{1}{15}\pi(x - 17\frac{1}{2}))$ .

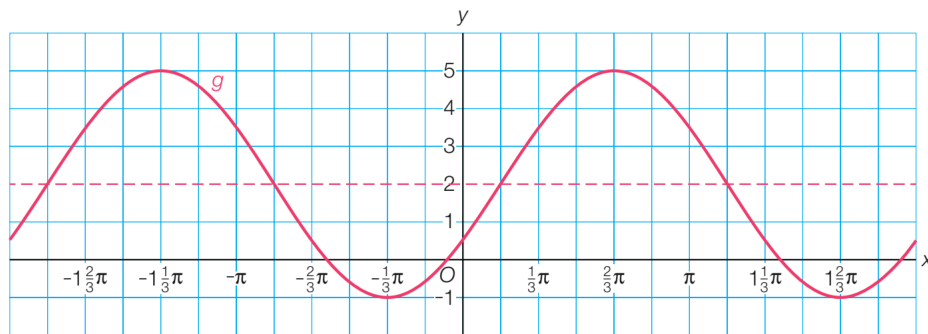
#### Bladzijde 176

- 9** **a**  $f(x) = 1 + 2 \sin(\frac{1}{2}\pi x - \frac{1}{5}\pi) = 1 + 2 \sin(\frac{1}{2}\pi(x - \frac{2}{5}))$   
 evenwichtsstand  $1$   
 amplitude  $2$   
 periode  $= \frac{2\pi}{\frac{1}{2}\pi} = 4$

stijgend door de evenwichtsstand in  $(\frac{2}{5}, 1)$



- b**  $g(x) = 2 - 3 \cos(x + \frac{1}{3}\pi)$   
 evenwichtsstand 2  
 amplitude 3  
 periode  $2\pi$   
 $-3 < 0$ , dus  $(-\frac{1}{3}\pi, -1)$  is een laagste punt.



- 10 a**  $\sin(2x - \frac{1}{2}\pi) = -1$   
 $2x - \frac{1}{2}\pi = \frac{1}{2}\pi + k \cdot 2\pi$   
 $2x = 2\pi + k \cdot 2\pi$   
 $x = \pi + k \cdot \pi$
- b**  $\cos(2x + \frac{1}{3}\pi) = 0$   
 $2x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$   
 $2x = \frac{1}{6}\pi + k \cdot \pi$   
 $x = \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi$
- c**  $\cos(\frac{1}{2}\pi x) \cdot (\cos(2\pi x) + 1) = 0$   
 $\cos(\frac{1}{2}\pi x) = 0 \vee \cos(2\pi x) = -1$   
 $\frac{1}{2}\pi x = \frac{1}{2}\pi + k \cdot \pi \vee 2\pi x = \pi + k \cdot 2\pi$   
 $x = \frac{1}{3} + k \cdot \frac{2}{3} \vee x = \frac{1}{2} + k \cdot 1$
- d**  $\sin^3(\frac{1}{3}\pi x) + \sin^2(\frac{1}{3}\pi x) = 0$   
 $\sin^2(\frac{1}{3}\pi x)(\sin(\frac{1}{3}\pi x) + 1) = 0$   
 $\sin(\frac{1}{3}\pi x) = 0 \vee \sin(\frac{1}{3}\pi x) = -1$   
 $\frac{1}{3}\pi x = k \cdot \pi \vee \frac{1}{3}\pi x = \frac{1}{2}\pi + k \cdot 2\pi$   
 $x = k \cdot 3 \vee x = 4\frac{1}{2} + k \cdot 6$

- 11 a**  $4 \sin(2x + \frac{1}{2}\pi) = 2\sqrt{2}$   
 $\sin(2x + \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$   
 $2x + \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \frac{3}{4}\pi + k \cdot 2\pi$   
 $2x = -\frac{1}{4}\pi + k \cdot 2\pi \vee 2x = \frac{1}{4}\pi + k \cdot 2\pi$   
 $x = -\frac{1}{8}\pi + k \cdot \pi \vee x = \frac{1}{8}\pi + k \cdot \pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{7}{8}\pi \vee x = \frac{1}{8}\pi \vee x = \frac{9}{8}\pi \vee x = \frac{5}{8}\pi$
- b**  $\cos(x - \frac{1}{6}\pi) = -\frac{1}{2}\sqrt{3}$   
 $x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$   
 $x = \pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \pi \vee x = \frac{1}{3}\pi$
- c**  $4 \cos^2(\frac{1}{2}x) = 2$   
 $\cos^2(\frac{1}{2}x) = \frac{1}{2}$   
 $\cos(\frac{1}{2}x) = \sqrt{\frac{1}{2}} \vee \cos(\frac{1}{2}x) = -\sqrt{\frac{1}{2}}$   
 $\cos(\frac{1}{2}x) = \frac{1}{2}\sqrt{2} \vee \cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{2}$   
 $\frac{1}{2}x = \frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{3}{4}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$   
 $x = \frac{1}{2}\pi + k \cdot 4\pi \vee x = -\frac{1}{2}\pi + k \cdot 4\pi \vee x = \frac{1}{2}\pi + k \cdot 4\pi \vee x = -\frac{1}{2}\pi + k \cdot 4\pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi$

**d**  $\sin^2(\frac{3}{4}x + 1\frac{1}{2}\pi) = \frac{3}{4}$   
 $\sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = \sqrt{\frac{3}{4}} \vee \sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = -\sqrt{\frac{3}{4}}$   
 $\sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = \frac{1}{2}\sqrt{3} \vee \sin(\frac{3}{4}x + 1\frac{1}{2}\pi) = -\frac{1}{2}\sqrt{3}$   
 $\frac{3}{4}x + 1\frac{1}{2}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{4}x + 1\frac{1}{2}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$   
 $\frac{3}{4}x = -1\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -\frac{5}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{3}{4}x = -1\frac{5}{6}\pi + k \cdot 2\pi$   
 $x = -1\frac{5}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -1\frac{1}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -\frac{2}{9}\pi + k \cdot 2\frac{2}{3}\pi \vee x = -2\frac{4}{9}\pi + k \cdot 2\frac{2}{3}\pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{9}\pi \vee x = 1\frac{5}{9}\pi \vee x = \frac{2}{9}\pi$

**12 a**  $\sin(2x + \frac{1}{2}\pi) = \sin(x - \frac{1}{3}\pi)$   
 $2x + \frac{1}{2}\pi = x - \frac{1}{3}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \pi - (x - \frac{1}{3}\pi) + k \cdot 2\pi$   
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \vee 2x + \frac{1}{2}\pi = \pi - x + \frac{1}{3}\pi + k \cdot 2\pi$   
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \vee 3x = \frac{5}{6}\pi + k \cdot 2\pi$   
 $x = -\frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

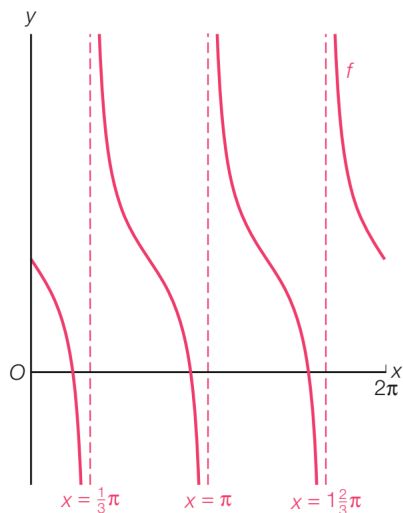
**b**  $\cos(x - \frac{1}{6}\pi) = \cos(4x + \frac{1}{4}\pi)$   
 $x - \frac{1}{6}\pi = 4x + \frac{1}{4}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -(4x + \frac{1}{4}\pi) + k \cdot 2\pi$   
 $-3x = \frac{5}{12}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -4x - \frac{1}{4}\pi + k \cdot 2\pi$   
 $x = -\frac{5}{36}\pi + k \cdot \frac{2}{3}\pi \vee 5x = -\frac{1}{12}\pi + k \cdot 2\pi$   
 $x = -\frac{5}{36}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{60}\pi + k \cdot \frac{2}{5}\pi$

**c**  $\tan(\frac{2}{3}\pi x) = -\frac{1}{3}\sqrt{3}$   
 $\frac{2}{3}\pi x = \frac{5}{6}\pi + k \cdot \pi$   
 $x = 1\frac{1}{4} + k \cdot 1\frac{1}{2}$

**d**  $\tan(2x - \frac{1}{3}\pi) = \tan(x + \frac{1}{2}\pi)$   
 $2x - \frac{1}{3}\pi = x + \frac{1}{2}\pi + k \cdot \pi$   
 $x = \frac{5}{6}\pi + k \cdot \pi$

**13 a** Periode =  $\frac{\pi}{1\frac{1}{2}} = \frac{2}{3}\pi$  en beginpunt (0, 2).

$1\frac{1}{2}x = \frac{1}{2}\pi + k \cdot \pi$   
 $x = \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$   
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{3}\pi, x = \pi \text{ en } x = 1\frac{2}{3}\pi$   
Dus de asymptoten zijn de lijnen  $x = \frac{1}{3}\pi, x = \pi$  en  $x = 1\frac{2}{3}\pi$ .



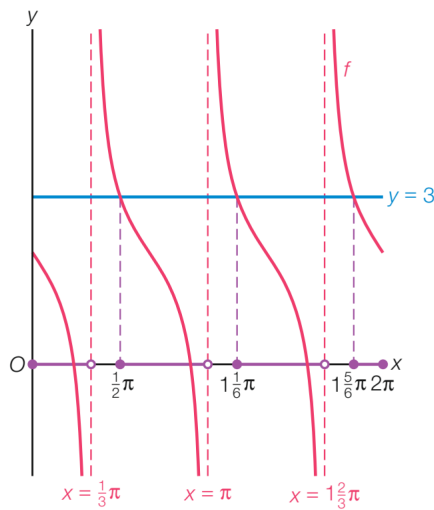
**b**  $f(x) = 3$  geeft  $2 - \tan(\frac{1}{2}x) = 3$

$$\tan(\frac{1}{2}x) = -1$$

$$\frac{1}{2}x = \frac{3}{4}\pi + k \cdot \pi$$

$$x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$$



$$f(x) \leq 3 \text{ geeft } 0 \leq x < \frac{1}{3}\pi \vee \frac{1}{2}\pi \leq x < \pi \vee 1\frac{1}{6}\pi \leq x < 1\frac{2}{3}\pi \vee 1\frac{5}{6}\pi \leq x \leq 2\pi$$

**14 a**  $y = \sin(2x - \frac{2}{3}\pi) = \cos(2x - \frac{2}{3}\pi - \frac{1}{2}\pi) = \cos(2x - 1\frac{1}{6}\pi)$

**b**  $y = -\cos(5x + 1\frac{1}{4}\pi) = \cos(5x + 1\frac{1}{4}\pi + \pi) = \cos(5x + 2\frac{1}{4}\pi) = \sin(5x + 2\frac{1}{4}\pi + \frac{1}{2}\pi)$   
 $= \sin(5x + 2\frac{3}{4}\pi) = \sin(5x + \frac{3}{4}\pi)$

**c**  $y = (\sin(x) - \cos(x)) \cdot (\sin(x) + \cos(x)) \cdot \cos(x) = (\sin^2(x) - \cos^2(x)) \cdot \cos(x)$   
 $= (1 - \cos^2(x) - \cos^2(x)) \cdot \cos(x) = (1 - 2\cos^2(x)) \cdot \cos(x) = -2\cos^3(x) + \cos(x)$

**15 a**  $f(x) = x^3 \sin(4x)$  geeft  $f'(x) = 3x^2 \cdot \sin(4x) + x^3 \cdot \cos(4x) \cdot 4 = 3x^2 \sin(4x) + 4x^3 \cos(4x)$

**b**  $g(x) = \frac{\sin(2x)}{2 + \cos(x)}$  geeft

$$g'(x) = \frac{(2 + \cos(x)) \cdot \cos(2x) \cdot 2 - \sin(2x) \cdot -\sin(x)}{(2 + \cos(x))^2} = \frac{4\cos(2x) + 2\cos(x)\cos(2x) + \sin(x)\sin(2x)}{(2 + \cos(x))^2}$$

**c**  $h(x) = 5\sin^2(x)$  geeft  $h'(x) = 10\sin(x)\cos(x)$

**d**  $k(x) = \tan(x^3)$  geeft  $k'(x) = \frac{1}{\cos^2(x^3)} \cdot 3x^2 = \frac{3x^2}{\cos^2(x^3)}$

*Alternatieve uitwerking*

$$k(x) = \tan(x^3) \text{ geeft } k'(x) = (1 + \tan^2(x^3)) \cdot 3x^2 = 3x^2 + 3x^2 \tan^2(x^3)$$

**16**  $f(x) = \frac{\sin(2x)}{2 + \sin(x)}$  geeft

$$f'(x) = \frac{(2 + \sin(x)) \cdot \cos(2x) \cdot 2 - \sin(2x) \cdot \cos(x)}{(2 + \sin(x))^2} = \frac{4\cos(2x) + 2\sin(x)\cos(2x) - \sin(2x)\cos(x)}{(2 + \sin(x))^2}$$

$k: y = ax + b$

$$a = f'(\frac{1}{2}\pi) = \frac{4\cos(\pi) + 2\sin(\frac{1}{2}\pi)\cos(\pi) - \sin(\pi)\cos(\frac{1}{2}\pi)}{(2 + \sin(\frac{1}{2}\pi))^2} = \frac{-4 - 2 - 0}{(2 + 1)^2} = -\frac{2}{3}$$

$$\left. \begin{aligned} y &= -\frac{2}{3}x + b \\ f(\frac{1}{2}\pi) &= 0, \text{ dus } A(\frac{1}{2}\pi, 0) \end{aligned} \right\} \begin{aligned} -\frac{2}{3} \cdot \frac{1}{2}\pi + b &= 0 \\ -\frac{1}{3}\pi + b &= 0 \\ b &= \frac{1}{3}\pi \end{aligned}$$

Dus  $k: y = -\frac{2}{3}x + \frac{1}{3}\pi$ .



## Gemengde opgaven

### Bladzijde 177

1

**a**  $g(x) = \cos(x) = \sin(x + \frac{1}{2}\pi)$

$$y = \sin(x + \frac{1}{2}\pi)$$

↓ translatie  $(\frac{1}{6}\pi, 0)$

$$y = \sin(x - \frac{1}{6}\pi + \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$$

↓  $\times -2$  t.o.v. de  $x$ -as

$$y = -2 \sin(x + \frac{1}{3}\pi)$$

Dus de grafiek van  $f$  kan ontstaan uit de grafiek van  $g$  door hierop de verschuiving van  $\frac{1}{6}\pi$  naar rechts en de vermenigvuldiging met  $-2$  ten opzichte van de  $x$ -as toe te passen.

**b**  $f(x) = 0$  geeft  $-2 \sin(x + \frac{1}{3}\pi) = 0$

$$\sin(x + \frac{1}{3}\pi) = 0$$

$$x + \frac{1}{3}\pi = k \cdot \pi$$

$$x = -\frac{1}{3}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{2}{3}\pi \vee x = \frac{4}{3}\pi$$

Dus  $A(\frac{2}{3}\pi, 0)$  en  $B(\frac{4}{3}\pi, 0)$ .

$$x_T = \frac{\frac{2}{3}\pi + \frac{4}{3}\pi}{2} = \frac{2}{3}\pi, \text{ dus } T(\frac{2}{3}\pi, 2).$$

$$p: y = a(x - p)^2 + q \text{ met top } T(\frac{2}{3}\pi, 2) \text{ geeft } y = a(x - \frac{2}{3}\pi)^2 + 2.$$

$$p \text{ door } A(\frac{2}{3}\pi, 0) \text{ geeft } a(\frac{2}{3}\pi - \frac{2}{3}\pi)^2 + 2 = 0$$

$$a(-\frac{1}{3}\pi)^2 = -2$$

$$a \cdot \frac{1}{9}\pi^2 = -2$$

$$a = -\frac{18}{\pi^2}$$

$$\text{Dus } p: y = -\frac{18}{\pi^2}(x - \frac{2}{3}\pi)^2 + 2.$$

$$x_S = 0 \text{ geeft } y_S = -\frac{18}{\pi^2}(0 - \frac{2}{3}\pi)^2 + 2 = -\frac{18}{\pi^2} \cdot \frac{4\pi^2}{9} + 2 = -4 + 2 = -2$$

**c**  $f(x) = -2 \sin(x + \frac{1}{3}\pi)$  geeft  $f'(x) = -2 \cos(x + \frac{1}{3}\pi)$

$$k: y = ax + b$$

$$a = f'(\pi) = -2 \cos(\pi + \frac{1}{3}\pi) = -2 \cdot -\frac{1}{2} = 1$$

$$\left. \begin{array}{l} y = x + b \\ f(\pi) = \sqrt{3}, \text{ dus } C(\pi, \sqrt{3}) \end{array} \right\} \begin{array}{l} \pi + b = \sqrt{3} \\ b = \sqrt{3} - \pi \end{array}$$

$$\text{Dus } k: y = x + \sqrt{3} - \pi \text{ en } D(0, \sqrt{3} - \pi).$$

$$g(x) = \cos(x) \text{ geeft } g'(x) = -\sin(x)$$

$$l \parallel k, \text{ dus } \text{rc}_l = \text{rc}_k = 1.$$

$$g'(x) = 1 \text{ geeft } -\sin(x) = 1$$

$$\sin(x) = -1$$

$$x = \frac{3}{2}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{3}{2}\pi$$

$$\left. \begin{array}{l} l: y = x + b \\ g(\frac{3}{2}\pi) = 0, \text{ dus } D(\frac{3}{2}\pi, 0) \end{array} \right\} \begin{array}{l} \frac{3}{2}\pi + b = 0 \\ b = -\frac{3}{2}\pi \end{array}$$

$$\text{Dus } l: y = x - \frac{3}{2}\pi \text{ en } E(0, -\frac{3}{2}\pi).$$

$$\text{Dus } d(D, E) = \sqrt{3} - \pi - (-\frac{3}{2}\pi) = \sqrt{3} + \frac{1}{2}\pi.$$

**a**  $f(x) = 2 + 3 \sin(2(x - \frac{1}{6}\pi))$

evenwichtsstand 2

amplitude 3

periode  $\frac{2\pi}{2} = \pi$

$3 > 0$ , dus stijgend door de evenwichtsstand in  $(\frac{1}{6}\pi, 2)$ .

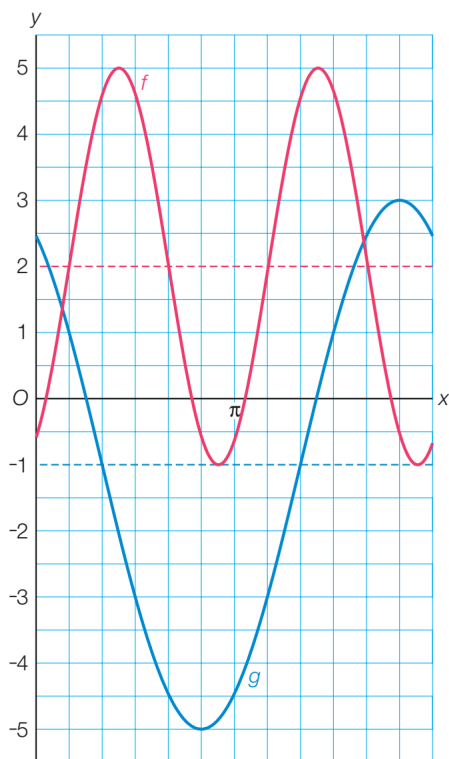
$g(x) = -1 + 4 \cos(x + \frac{1}{6}\pi)$

evenwichtsstand -1

amplitude 4

periode  $2\pi$

$4 > 0$ , dus  $(-\frac{1}{6}\pi, 3)$  is een hoogste punt en dus ook  $(\frac{5}{6}\pi, 3)$ .



**b**  $f(x) = 2 + 3 \sin(2(x - \frac{1}{6}\pi)) = 2 + 3 \sin(2x - \frac{1}{3}\pi) = 2 + 3 \cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi)$   
 $= 2 + 3 \cos(2x - \frac{5}{6}\pi) = 2 + 3 \cos(2(x - \frac{5}{12}\pi))$

**c**  $f(x) = \frac{1}{2}$  geeft  $2 + 3 \sin(2(x - \frac{1}{6}\pi)) = \frac{1}{2}$

$$3 \sin(2(x - \frac{1}{6}\pi)) = -1\frac{1}{2}$$

$$\sin(2(x - \frac{1}{6}\pi)) = -\frac{1}{2}$$

$$2(x - \frac{1}{6}\pi) = \frac{1}{6}\pi + k \cdot 2\pi \vee 2(x - \frac{1}{6}\pi) = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x - \frac{1}{6}\pi = \frac{7}{12}\pi + k \cdot \pi \vee x - \frac{1}{6}\pi = -\frac{1}{12}\pi + k \cdot \pi$$

$$x = \frac{3}{4}\pi + k \cdot \pi \vee x = \frac{1}{12}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = \frac{1}{12}\pi \vee x = \frac{1}{12}\pi \vee x = 1\frac{1}{12}\pi$$

**d**  $g(x) = 1$  geeft  $-1 + 4 \cos(x + \frac{1}{6}\pi) = 1$

$$4 \cos(x + \frac{1}{6}\pi) = 2$$

$$\cos(x + \frac{1}{6}\pi) = \frac{1}{2}$$

$$x + \frac{1}{6}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x + \frac{1}{6}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{2}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = 1\frac{1}{2}\pi$$

Zie de figuur van vraag a.

$g(x) < 1$  geeft  $\frac{1}{6}\pi < x < 1\frac{1}{2}\pi$

$$\begin{aligned}
 \text{e } y_A &= s\left(\frac{1}{3}\pi\right) = f\left(\frac{1}{3}\pi\right) + g\left(\frac{1}{3}\pi\right) = 2 + 3 \sin\left(2\left(\frac{1}{3}\pi - \frac{1}{6}\pi\right)\right) + -1 + 4 \cos\left(\frac{1}{3}\pi + \frac{1}{6}\pi\right) \\
 &= 2 + 3 \sin\left(2 \cdot \frac{1}{6}\pi\right) - 1 + 4 \cos\left(\frac{1}{2}\pi\right) \\
 &= 2 + 3 \sin\left(\frac{1}{3}\pi\right) - 1 + 4 \cdot 0 \\
 &= 2 + 3 \cdot \frac{1}{2}\sqrt{3} - 1 + 0 \\
 &= 1 + 1\frac{1}{2}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 y_B &= s(\pi) = f(\pi) + g(\pi) = 2 + 3 \sin\left(2\left(\pi - \frac{1}{6}\pi\right)\right) + -1 + 4 \cos\left(\pi + \frac{1}{6}\pi\right) \\
 &= 2 + 3 \sin\left(2 \cdot \frac{5}{6}\pi\right) - 1 + 4 \cos\left(1\frac{1}{6}\pi\right) \\
 &= 2 + 3 \sin\left(1\frac{2}{3}\pi\right) - 1 + 4 \cos\left(1\frac{1}{6}\pi\right) \\
 &= 2 + 3 \cdot -\frac{1}{2}\sqrt{3} - 1 + 4 \cdot -\frac{1}{2}\sqrt{3} \\
 &= 2 - 1\frac{1}{2}\sqrt{3} - 1 - 2\sqrt{3} \\
 &= 1 - 3\frac{1}{2}\sqrt{3}
 \end{aligned}$$

$$\text{Dus } y_A - y_B = 1 + 1\frac{1}{2}\sqrt{3} - (1 - 3\frac{1}{2}\sqrt{3}) = 1 + 1\frac{1}{2}\sqrt{3} - 1 + 3\frac{1}{2}\sqrt{3} = 5\sqrt{3}.$$

$$\begin{aligned}
 \text{f } f(x) &= 2 + 3 \sin\left(2\left(x - \frac{1}{6}\pi\right)\right) = 2 + 3 \sin\left(2x - \frac{1}{3}\pi\right) \text{ geeft } f'(x) = 3 \cos\left(2x - \frac{1}{3}\pi\right) \cdot 2 = 6 \cos\left(2x - \frac{1}{3}\pi\right) \\
 g(x) &= -1 + 4 \cos\left(x + \frac{1}{6}\pi\right) \text{ geeft } g'(x) = -4 \sin\left(x + \frac{1}{6}\pi\right) \\
 s(x) &= f(x) + g(x) \text{ geeft } s'(x) = f'(x) + g'(x) = 6 \cos\left(2x - \frac{1}{3}\pi\right) - 4 \sin\left(x + \frac{1}{6}\pi\right) \\
 \text{Invoeren van de formule van } s' \text{ en gebruiken van de optie nulpunt geeft} \\
 x &= 0,946\dots, x = 2,814\dots, x = 4,750\dots \text{ en } x = 6,149\dots \\
 s(0) &= s(2\pi) = 1,866\dots \\
 s(0,946\dots) &= 3,647\dots \\
 s(2,814\dots) &= -5,897\dots \\
 s(4,750\dots) &= 5,606\dots \\
 s(6,149\dots) &= 1,797\dots
 \end{aligned}$$

De vergelijking  $s(x) = p$  heeft drie oplossingen voor  $p \approx 3,65$  en voor  $p \approx 1,80$ .

### Bladzijde 178

$$\begin{aligned}
 3 \text{ a } f(x) &= 0 \text{ geeft } 1 + 2 \sin\left(\frac{1}{2}\pi\left(x - \frac{1}{2}\right)\right) = 0 \\
 2 \sin\left(\frac{1}{2}\pi\left(x - \frac{1}{2}\right)\right) &= -1 \\
 \sin\left(\frac{1}{2}\pi\left(x - \frac{1}{2}\right)\right) &= -\frac{1}{2} \\
 \frac{1}{2}\pi\left(x - \frac{1}{2}\right) &= 1\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi\left(x - \frac{1}{2}\right) = -\frac{1}{6}\pi + k \cdot 2\pi \\
 x - \frac{1}{2} &= 2\frac{1}{3} + k \cdot 4 \vee x - \frac{1}{2} = -\frac{1}{3} + k \cdot 4 \\
 x &= 2\frac{5}{6} + k \cdot 4 \vee x = \frac{1}{6} + k \cdot 4 \\
 x \text{ in } [0, 5] &\text{ geeft } x = 2\frac{5}{6} \vee x = \frac{1}{6} \vee x = 4\frac{1}{6}
 \end{aligned}$$

Dus  $B(2\frac{5}{6}, 0)$ .

$$f \text{ is maximaal } 1 + 2 \cdot 1 = 3 \text{ voor } x = \frac{1}{2} + \frac{1}{4} \cdot \frac{2\pi}{\frac{1}{2}\pi} = \frac{1}{2} + \frac{1}{4} \cdot 4 = 1\frac{1}{2}.$$

Dus  $D(1\frac{1}{2}, 3)$ .

$$f \text{ is minimaal } 1 - 2 \cdot 1 = -1 \text{ voor } x = \frac{1}{2} + \frac{3}{4} \cdot 4 = 3\frac{1}{2}.$$

Dus  $E(3\frac{1}{2}, -1)$ .

$$k: y = ax + b$$

$$a = \frac{-1 - 3}{3\frac{1}{2} - 1\frac{1}{2}} = -2$$

$$\left. \begin{aligned} y &= -2x + b \\ \text{door } D(1\frac{1}{2}, 3) \end{aligned} \right\} \begin{aligned} -2 \cdot 1\frac{1}{2} + b &= 3 \\ -3 + b &= 3 \\ b &= 6 \end{aligned}$$

$$k: y = -2x + 6 \text{ snijden met de } x\text{-as geeft } -2x + 6 = 0$$

$$-2x = -6$$

$$x = 3$$

Dus  $F(3, 0)$ .

$$d(B, F) = x_F - x_B = 3 - 2\frac{5}{6} = \frac{1}{6}$$

**b**  $g(x) = 0$  geeft  $3 \cos(\frac{1}{2}\pi(x + \frac{1}{3})) = 0$

$$\cos(\frac{1}{2}\pi(x + \frac{1}{3})) = 0$$

$$\frac{1}{2}\pi(x + \frac{1}{3}) = \frac{1}{2}\pi + k \cdot \pi$$

$$x + \frac{1}{3} = 1 + k \cdot 2$$

$$x = \frac{2}{3} + k \cdot 2$$

$$x \text{ in } [0, 5] \text{ geeft } x = \frac{2}{3} \vee x = 2\frac{2}{3} \vee x = 4\frac{2}{3}$$

Dus  $H(2\frac{2}{3}, 0)$ .

De  $x$ -coördinaat van het midden van  $HF$  is  $\frac{2\frac{2}{3} + 3}{2} = 2\frac{5}{6}$  en dat is ook de  $x$ -coördinaat van  $B$ .

Dus  $B$  is het midden van lijnstuk  $HF$ .

**c**  $f(x) = 1 + 2 \sin(\frac{1}{2}\pi(x - \frac{1}{2})) = 1 + 2 \sin(\frac{1}{2}\pi x - \frac{1}{4}\pi)$  geeft  $f'(x) = 2 \cos(\frac{1}{2}\pi x - \frac{1}{4}\pi) \cdot \frac{1}{2}\pi = \pi \cos(\frac{1}{2}\pi x - \frac{1}{4}\pi)$

$$f'(x) = \pi \text{ geeft } \pi \cos(\frac{1}{2}\pi x - \frac{1}{4}\pi) = \pi$$

$$\cos(\frac{1}{2}\pi x - \frac{1}{4}\pi) = 1$$

$$\frac{1}{2}\pi x - \frac{1}{4}\pi = k \cdot 2\pi$$

$$\frac{1}{2}\pi x = \frac{1}{4}\pi + k \cdot 2\pi$$

$$x = \frac{1}{2} + k \cdot 4$$

$$x \text{ in } [0, 5] \text{ geeft } x = \frac{1}{2} \vee x = 4\frac{1}{2}$$

Dus  $x_P = \frac{1}{2}$  en  $y_P = f(\frac{1}{2}) = 1 + 2 \sin(\frac{1}{2}\pi(\frac{1}{2} - \frac{1}{2})) = 1 + 2 \sin(0) = 1 + 2 \cdot 0 = 1$ .

$$\left. \begin{array}{l} l: y = \pi x + b \\ \text{door } P(\frac{1}{2}, 1) \end{array} \right\} \begin{array}{l} \pi \cdot \frac{1}{2} + b = 1 \\ b = 1 - \frac{1}{2}\pi \end{array}$$

Dus  $l: y = \pi x + 1 - \frac{1}{2}\pi$  en  $y_R = 1 - \frac{1}{2}\pi$ .

**4 a**  $f(x) = \frac{5 \cos(x)}{2 + \sin(x)}$  geeft

$$f'(x) = \frac{(2 + \sin(x)) \cdot -5 \sin(x) - 5 \cos(x) \cdot \cos(x)}{(2 + \sin(x))^2} = \frac{-10 \sin(x) - 5 \sin^2(x) - 5 \cos^2(x)}{(2 + \sin(x))^2}$$

$$= \frac{-10 \sin(x) - 5(\sin^2(x) + \cos^2(x))}{(2 + \sin(x))^2} = \frac{-10 \sin(x) - 5 \cdot 1}{(2 + \sin(x))^2} = \frac{-5 - 10 \sin(x)}{(2 + \sin(x))^2}$$

**b**  $f(0) = \frac{5 \cdot 1}{2 + 0} = 2\frac{1}{2}$

$$f'(x) = 0 \text{ geeft } -5 - 10 \sin(x) = 0$$

$$-10 \sin(x) = 5$$

$$\sin(x) = -\frac{1}{2}$$

$$x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$$

$$\text{min. is } f(1\frac{1}{6}\pi) = \frac{5 \cos(1\frac{1}{6}\pi)}{2 + \sin(1\frac{1}{6}\pi)} = \frac{5 \cdot -\frac{1}{2}\sqrt{3}}{2 + -\frac{1}{2}} = \frac{-2\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = -1\frac{2}{3}\sqrt{3}$$

$$\text{max. is } f(1\frac{5}{6}\pi) = \frac{5 \cos(1\frac{5}{6}\pi)}{2 + \sin(1\frac{5}{6}\pi)} = \frac{5 \cdot \frac{1}{2}\sqrt{3}}{2 + \frac{1}{2}} = \frac{2\frac{1}{2}\sqrt{3}}{1\frac{1}{2}} = 1\frac{2}{3}\sqrt{3}$$

Dus  $B_f = [-1\frac{2}{3}\sqrt{3}, 1\frac{2}{3}\sqrt{3}]$ .

$$\text{c } f'(\frac{1}{6}\pi) = \frac{-5 - 10 \sin(\frac{1}{6}\pi)}{(2 + \sin(\frac{1}{6}\pi))^2} = \frac{-5 - 10 \cdot \frac{1}{2}}{(2 + \frac{1}{2})^2} = \frac{-10}{6\frac{1}{4}} = -1\frac{3}{5}$$

$$g(x) = a \cos(x) + b \text{ geeft } g'(x) = -a \sin(x)$$

$$g'(\frac{1}{6}\pi) = -a \sin(\frac{1}{6}\pi) = -\frac{1}{2}a$$

$$f'(\frac{1}{6}\pi) \cdot g'(\frac{1}{6}\pi) = -1 \text{ geeft } -1\frac{3}{5} \cdot -\frac{1}{2}a = -1$$

$$\frac{4}{5}a = -1$$

$$a = -1\frac{1}{4}$$

$$f(\frac{1}{6}\pi) = \frac{5 \cos(\frac{1}{6}\pi)}{2 + \sin(\frac{1}{6}\pi)} = \frac{5 \cdot \frac{1}{2}\sqrt{3}}{2 + \frac{1}{2}} = \frac{2\frac{1}{2}\sqrt{3}}{2\frac{1}{2}} = \sqrt{3}$$

$$g(\frac{1}{6}\pi) = -1\frac{1}{4} \cos(\frac{1}{6}\pi) + b = -1\frac{1}{4} \cdot \frac{1}{2}\sqrt{3} + b = -\frac{5}{8}\sqrt{3} + b$$

$$f(\frac{1}{6}\pi) = g(\frac{1}{6}\pi) \text{ geeft } \sqrt{3} = -\frac{5}{8}\sqrt{3} + b$$

$$b = 1\frac{5}{8}\sqrt{3}$$

Dus  $a = -1\frac{1}{4}$  en  $b = 1\frac{5}{8}\sqrt{3}$ .

$$\text{5 a } f_p(\frac{1}{6}\pi) = 0 \text{ geeft } (\sin(\frac{1}{6}\pi) + p)(\sin(\frac{1}{6}\pi) + 3) = 0$$

$$(\frac{1}{2} + p)(\frac{1}{2} + 3) = 0$$

$$p = -\frac{1}{2}$$

$$f_{-\frac{1}{2}}(x) = (\sin(x) - \frac{1}{2})(\sin(x) + 3) = \sin^2(x) + 2\frac{1}{2}\sin(x) - 1\frac{1}{2} \text{ geeft}$$

$$f'_{-\frac{1}{2}}(x) = 2 \sin(x) \cos(x) + 2\frac{1}{2} \cos(x)$$

$$f'_{-\frac{1}{2}}(x) = 0 \text{ geeft } 2 \sin(x) \cos(x) + 2\frac{1}{2} \cos(x) = 0$$

$$\cos(x)(2 \sin(x) + 2\frac{1}{2}) = 0$$

$$\cos(x) = 0 \vee 2 \sin(x) = -2\frac{1}{2}$$

$$\cos(x) = 0 \vee \sin(x) = -1\frac{1}{4}$$

$$x = \frac{1}{2}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$$

Zie de schets hiernaast.

$$\text{max. is } f_{-\frac{1}{2}}(\frac{1}{2}\pi) = (\sin(\frac{1}{2}\pi) - \frac{1}{2})(\sin(\frac{1}{2}\pi) + 3) = (1 - \frac{1}{2})(1 + 3) = 2$$

$$\text{min. is } f_{-\frac{1}{2}}(1\frac{1}{2}\pi) = (\sin(1\frac{1}{2}\pi) - \frac{1}{2})(\sin(1\frac{1}{2}\pi) + 3) = (-1 - \frac{1}{2})(-1 + 3) = -3$$

Dus in het geval  $\frac{1}{6}\pi$  een nulpunt van  $f_p$  is, is het bereik  $[-3, 2]$ .

$$\text{b } f_p(x) = (\sin(x) + p)(\sin(x) + 3) = \sin^2(x) + (p + 3)\sin(x) + 3p \text{ geeft}$$

$$f'_p(x) = 2 \sin(x) \cos(x) + (p + 3)\cos(x)$$

$$f'_p(x) = 0 \text{ geeft } 2 \sin(x) \cos(x) + (p + 3)\cos(x) = 0$$

$$\cos(x)(2 \sin(x) + p + 3) = 0$$

$$\cos(x) = 0 \vee 2 \sin(x) = -p - 3$$

$$\cos(x) = 0 \vee \sin(x) = -\frac{1}{2}p - 1\frac{1}{2}$$

$$p > -1, \text{ dus } -\frac{1}{2}p < \frac{1}{2}, \text{ dus } -\frac{1}{2}p - 1\frac{1}{2} < -1.$$

Dus voor  $p > -1$  heeft  $\sin(x) = -\frac{1}{2}p - 1\frac{1}{2}$  geen oplossing.

$$\text{Dus } f'_p(x) = 0 \text{ met } x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi.$$

Uit vraag a volgt dat het maximum optreedt bij  $x = \frac{1}{2}\pi$  en het minimum bij  $x = 1\frac{1}{2}\pi$ .

$$f_p(1\frac{1}{2}\pi) = -2 + \sqrt{3} \text{ geeft } (\sin(1\frac{1}{2}\pi) + p)(\sin(1\frac{1}{2}\pi) + 3) = -2 + \sqrt{3}$$

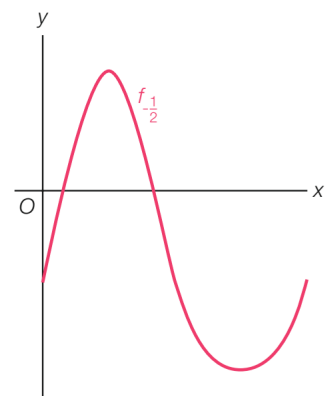
$$(-1 + p)(-1 + 3) = -2 + \sqrt{3}$$

$$(-1 + p) \cdot 2 = -2 + \sqrt{3}$$

$$-1 + p = -1 + \frac{1}{2}\sqrt{3}$$

$$p = \frac{1}{2}\sqrt{3}$$

$$\text{max. is } f_{\frac{1}{2}\sqrt{3}}(\frac{1}{2}\pi) = (\sin(\frac{1}{2}\pi) + \frac{1}{2}\sqrt{3})(\sin(\frac{1}{2}\pi) + 3) = (1 + \frac{1}{2}\sqrt{3})(1 + 3) = 4 + 2\sqrt{3}$$



c Zie vraag b.

$p = 1$  geeft  $f_1(x) = \sin^2(x) + 4 \sin(x) + 3$  en  $f_1'(x) = 2 \sin(x) \cos(x) + 4 \cos(x)$

$f_1'(0) = 2 \sin(0) \cos(0) + 4 \cos(0) = 4$ , dus  $rc_{AM} = -\frac{1}{4}$ .

$AM: y = -\frac{1}{4}x + b$   
 $f_1(0) = 3$ , dus  $A(0, 3)$  }  $AM: y = -\frac{1}{4}x + 3$

$f_1'(\pi) = 2 \sin(\pi) \cos(\pi) + 4 \cos(\pi) = -4$ , dus  $rc_{BM} = \frac{1}{4}$ .

$BM: y = \frac{1}{4}x + b$   
 $f_1(\pi) = 3$ , dus  $B(\pi, 3)$  }  $\frac{1}{4} \cdot \pi + b = 3$   
 $b = 3 - \frac{1}{4}\pi$

Dus  $BM: y = \frac{1}{4}x + 3 - \frac{1}{4}\pi$ , en ook  $BN: y = \frac{1}{4}x + 3 - \frac{1}{4}\pi$ .

$AM$  snijden met  $BM$  geeft  $-\frac{1}{4}x + 3 = \frac{1}{4}x + 3 - \frac{1}{4}\pi$

$$-\frac{1}{2}x = -\frac{1}{4}\pi$$

$$x = \frac{1}{2}\pi$$

$x = \frac{1}{2}\pi$  geeft  $y = -\frac{1}{4} \cdot \frac{1}{2}\pi + 3 = 3 - \frac{1}{8}\pi$ , dus  $M(\frac{1}{2}\pi, 3 - \frac{1}{8}\pi)$ .

$f_1'(2\pi) = 2 \sin(2\pi) \cos(2\pi) + 4 \cos(2\pi) = 4$ , dus  $rc_{CN} = -\frac{1}{4}$ .

$CN: y = -\frac{1}{4}x + b$   
 $f_1(2\pi) = 3$ , dus  $C(2\pi, 3)$  }  $-\frac{1}{4} \cdot 2\pi + b = 3$   
 $b = 3 + \frac{1}{2}\pi$

Dus  $CN: y = -\frac{1}{4}x + 3 + \frac{1}{2}\pi$ .

$BN$  snijden met  $CN$  geeft  $\frac{1}{4}x + 3 - \frac{1}{4}\pi = -\frac{1}{4}x + 3 + \frac{1}{2}\pi$

$$\frac{1}{2}x = \frac{3}{4}\pi$$

$$x = 1\frac{1}{2}\pi$$

$x = 1\frac{1}{2}\pi$  geeft  $y = \frac{1}{4} \cdot 1\frac{1}{2}\pi + 3 - \frac{1}{4}\pi = \frac{3}{8}\pi + 3 - \frac{1}{4}\pi = 3 + \frac{1}{8}\pi$ , dus  $N(1\frac{1}{2}\pi, 3 + \frac{1}{8}\pi)$ .

$$d(M, N) = \sqrt{(1\frac{1}{2}\pi - \frac{1}{2}\pi)^2 + (3 + \frac{1}{8}\pi - (3 - \frac{1}{8}\pi))^2} = \sqrt{\pi^2 + (\frac{1}{4}\pi)^2} = \sqrt{\pi^2 + \frac{1}{16}\pi^2} = \sqrt{1\frac{1}{16}\pi^2} = \frac{1}{4}\pi\sqrt{17}$$

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$$\begin{aligned} \mathbf{a} \quad f(x) &= \frac{2 \cos(x + 1\frac{2}{3}\pi) - \cos(x - \frac{1}{3}\pi)}{2 \sin(x + \frac{1}{3}\pi) + \sin(x - \frac{2}{3}\pi)} = \frac{2 \sin(x + 1\frac{2}{3}\pi + \frac{1}{2}\pi) - \sin(x - \frac{1}{3}\pi + \frac{1}{2}\pi)}{2 \cos(x + \frac{1}{3}\pi - \frac{1}{2}\pi) + \cos(x - \frac{2}{3}\pi - \frac{1}{2}\pi)} \\ &= \frac{2 \sin(x + 2\frac{1}{6}\pi) - \sin(x + \frac{1}{6}\pi)}{2 \cos(x - \frac{1}{6}\pi) + \cos(x - 1\frac{1}{6}\pi)} = \frac{2 \sin(x + \frac{1}{6}\pi) - \sin(x + \frac{1}{6}\pi)}{2 \cos(x - \frac{1}{6}\pi) - \cos(x - 1\frac{1}{6}\pi + \pi)} \\ &= \frac{\sin(x + \frac{1}{6}\pi)}{2 \cos(x - \frac{1}{6}\pi) - \cos(x - \frac{1}{6}\pi)} = \frac{\sin(x + \frac{1}{6}\pi)}{\cos(x - \frac{1}{6}\pi)} \end{aligned}$$

$$g(x) = \frac{\cos(x - \frac{1}{3}\pi)}{\sin(x + \frac{2}{3}\pi)} = \frac{\sin(x - \frac{1}{3}\pi + \frac{1}{2}\pi)}{\cos(x + \frac{2}{3}\pi - \frac{1}{2}\pi)} = \frac{\sin(x + \frac{1}{6}\pi)}{\cos(x + \frac{1}{6}\pi)} = \tan(x + \frac{1}{6}\pi)$$

$$\mathbf{b} \quad f(x) = g(x) \text{ geeft } \frac{\sin(x + \frac{1}{6}\pi)}{\cos(x - \frac{1}{6}\pi)} = \frac{\sin(x + \frac{1}{6}\pi)}{\cos(x + \frac{1}{6}\pi)}$$

$$\sin(x + \frac{1}{6}\pi) = 0 \vee \cos(x - \frac{1}{6}\pi) = \cos(x + \frac{1}{6}\pi)$$

$$x + \frac{1}{6}\pi = k \cdot \pi \vee x - \frac{1}{6}\pi = x + \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{6}\pi = -(x + \frac{1}{6}\pi) + k \cdot 2\pi$$

$$x = -\frac{1}{6}\pi + k \cdot \pi \vee x - \frac{1}{6}\pi = -x - \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{6}\pi + k \cdot \pi \vee 2x = k \cdot 2\pi$$

$$x = -\frac{1}{6}\pi + k \cdot \pi \vee x = k \cdot \pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{5}{6}\pi \vee x = 0 \vee x = \pi$$

$$g(\frac{5}{6}\pi) = \tan(\pi) = 0, g(0) = \tan(\frac{1}{6}\pi) = \frac{1}{3}\sqrt{3} \text{ en } g(\pi) = \tan(\frac{1}{6}\pi) = \frac{1}{3}\sqrt{3}$$

Dus de snijpunten zijn  $(0, \frac{1}{3}\sqrt{3})$ ,  $(\frac{5}{6}\pi, 0)$  en  $(\pi, \frac{1}{3}\sqrt{3})$ .

$$\text{c } x + \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{3}\pi + k \cdot \pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{3}\pi$$

Dus de verticale asymptoot van de grafiek van  $g$  is de lijn  $x = \frac{1}{3}\pi$ .

$$f\left(\frac{1}{3}\pi\right) = \frac{\sin\left(\frac{1}{2}\pi\right)}{\cos\left(\frac{1}{6}\pi\right)} = \frac{1}{\frac{1}{2}\sqrt{3}} = \frac{2}{3}\sqrt{3}, \text{ dus } A\left(\frac{1}{3}\pi, \frac{2}{3}\sqrt{3}\right).$$

$$\begin{aligned} f(x) = \frac{\sin\left(x + \frac{1}{6}\pi\right)}{\cos\left(x - \frac{1}{6}\pi\right)} \text{ geeft } f'(x) &= \frac{\cos\left(x - \frac{1}{6}\pi\right) \cdot \cos\left(x + \frac{1}{6}\pi\right) - \sin\left(x + \frac{1}{6}\pi\right) \cdot -\sin\left(x - \frac{1}{6}\pi\right)}{\cos^2\left(x - \frac{1}{6}\pi\right)} \\ &= \frac{\cos\left(x - \frac{1}{6}\pi\right) \cdot \cos\left(x + \frac{1}{6}\pi\right) + \sin\left(x + \frac{1}{6}\pi\right) \cdot \sin\left(x - \frac{1}{6}\pi\right)}{\cos^2\left(x - \frac{1}{6}\pi\right)} \end{aligned}$$

$$k: y = ax + b$$

$$a = f'\left(\frac{1}{3}\pi\right) = \frac{\cos\left(\frac{1}{6}\pi\right) \cdot \cos\left(\frac{1}{2}\pi\right) + \sin\left(\frac{1}{2}\pi\right) \cdot \sin\left(\frac{1}{6}\pi\right)}{\cos^2\left(\frac{1}{6}\pi\right)} = \frac{\frac{1}{2}\sqrt{3} \cdot 0 + 1 \cdot \frac{1}{2}}{\left(\frac{1}{2}\sqrt{3}\right)^2} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$$\begin{aligned} y = \frac{2}{3}x + b \\ \text{door } A\left(\frac{1}{3}\pi, \frac{2}{3}\sqrt{3}\right) \Bigg\} \frac{2}{3} \cdot \frac{1}{3}\pi + b = \frac{2}{3}\sqrt{3} \\ \frac{2}{9}\pi + b = \frac{2}{3}\sqrt{3} \\ b = \frac{2}{3}\sqrt{3} - \frac{2}{9}\pi \end{aligned}$$

$$\text{Dus } k: y = \frac{2}{3}x + \frac{2}{3}\sqrt{3} - \frac{2}{9}\pi.$$

$$\text{d } g(x) = \sqrt{3} \text{ geeft } \tan\left(x + \frac{1}{6}\pi\right) = \sqrt{3}$$

$$x + \frac{1}{6}\pi = \frac{1}{3}\pi + k \cdot \pi$$

$$x = \frac{1}{6}\pi + k \cdot \pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{6}\pi$$

$$\text{Dus } B\left(\frac{1}{6}\pi, \sqrt{3}\right).$$

$$g(x) = \tan\left(x + \frac{1}{6}\pi\right) \text{ geeft } g'(x) = \frac{1}{\cos^2\left(x + \frac{1}{6}\pi\right)}$$

$$l: y = ax + b$$

$$a = g'\left(\frac{1}{6}\pi\right) = \frac{1}{\cos^2\left(\frac{1}{3}\pi\right)} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

$$\begin{aligned} y = 4x + b \\ \text{door } B\left(\frac{1}{6}\pi, \sqrt{3}\right) \Bigg\} 4 \cdot \frac{1}{6}\pi + b = \sqrt{3} \\ \frac{2}{3}\pi + b = \sqrt{3} \\ b = \sqrt{3} - \frac{2}{3}\pi \end{aligned}$$

$$\text{Dus } l: y = 4x + \sqrt{3} - \frac{2}{3}\pi \text{ en } y_C = \sqrt{3} - \frac{2}{3}\pi.$$

# Verantwoording

## Beeld

Technisch tekenwerk: Integra Software Services, India

## Colofon

Omslagontwerp: Shootmedia, Groningen

Ontwerp binnenwerk: Ebel Kuipers grafisch ontwerp, Sappemeer

Lay-out: Integra Software Services

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