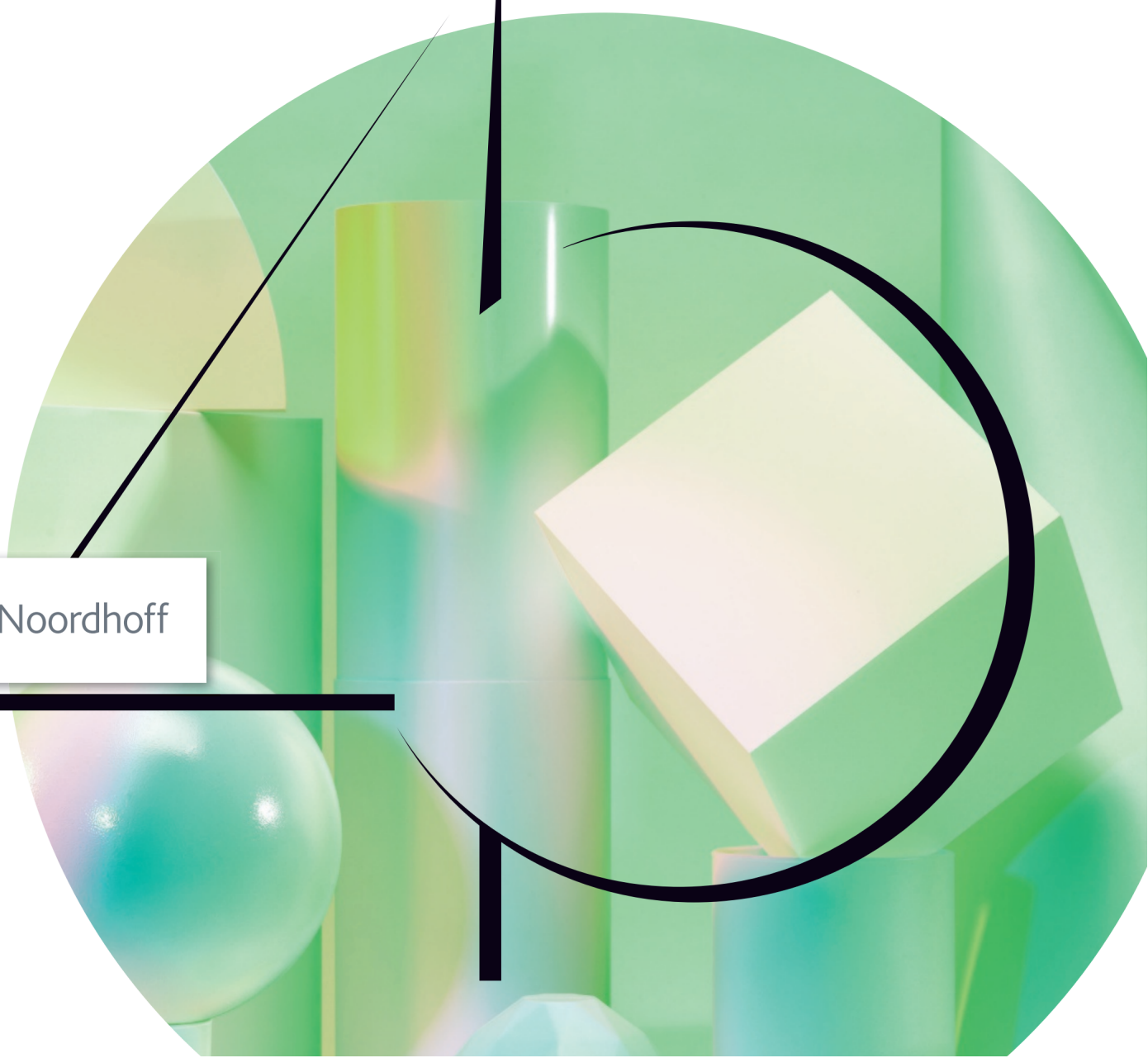


getal & ruimte

B



Noordhoff



Getal & Ruimte

Uitwerkingen vwo B deel 3

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9 Exponentiële en logaritmische functies

9.1 Groeifactoren en groeipercentages

Bladzijde 8

V1

a ${}^2\log(64) = {}^2\log(2^6) = 6$

b ${}^3\log(\frac{1}{27}) = {}^3\log(3^{-3}) = -3$

c ${}^6\log(6\sqrt{6}) = {}^6\log(6^{\frac{1}{2}}) = \frac{1}{2}$

d $\frac{1}{3}\log(1) = \frac{1}{3}\log((\frac{1}{3})^0) = 0$

e $\frac{1}{2}\log(\frac{1}{8}) = \frac{1}{2}\log((\frac{1}{2})^3) = 3$

f ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$

V2

a ${}^3\log(x) = 5$

$$x = 3^5 = 243$$

b ${}^2\log(3x + 7) = 4$

$$3x + 7 = 2^4$$

$$3x + 7 = 16$$

$$3x = 9$$

$$x = 3$$

c $5 + {}^5\log(x + 3) = 7$

$${}^5\log(x + 3) = 2$$

$$x + 3 = 5^2$$

$$x + 3 = 25$$

$$x = 22$$

d $\frac{1}{2}\log(3x + 4) = -2$

$$3x + 4 = (\frac{1}{2})^{-2}$$

$$3x + 4 = 4$$

$$3x = 0$$

$$x = 0$$

e $3 + {}^2\log(5 - 1\frac{1}{2}x) = 0$

$${}^2\log(5 - 1\frac{1}{2}x) = -3$$

$$5 - 1\frac{1}{2}x = 2^{-3}$$

$$5 - 1\frac{1}{2}x = \frac{1}{8}$$

$$-1\frac{1}{2}x = -4\frac{7}{8}$$

$$x = 3\frac{1}{4}$$

f $4 - 2 \cdot {}^3\log(x) = 3$

$$2 \cdot {}^3\log(x) = 1$$

$${}^3\log(x) = \frac{1}{2}$$

$$x = 3^{\frac{1}{2}} = \sqrt{3}$$

1 $3^x = 7$ geeft $x = {}^3\log(7)$

Bladzijde 9

2

$$3^{x+1} = 90$$

$$3 \cdot 3^x = 90$$

$$3^x = 30$$

$$x = {}^3\log(30)$$

$$5 + 2^{3x} = 25$$

$$2^{3x} = 20$$

$$(2^3)^x = 20$$

$$8^x = 20$$

$$x = {}^8\log(20)$$

3

a $3^x - 5 = 7$

$$3^x = 12$$

$$x = {}^3\log(12)$$

b $2^{x-1} = 15$

$$x - 1 = {}^2\log(15)$$

$$x = 1 + {}^2\log(15)$$

c $1 + 2^x = 15$

$$2^x = 14$$

$$x = {}^2\log(14)$$

d $4 + 3^{x+1} = 25$

$$3^{x+1} = 21$$

$$x + 1 = {}^3\log(21)$$

$$x = -1 + {}^3\log(21)$$

e $14 - 2^{x+3} = 2$

$$2^{x+3} = 12$$

$$x + 3 = {}^2\log(12)$$

$$x = -3 + {}^2\log(12)$$

f $7 + 4^{2x} = 12$

$$4^{2x} = 5$$

$$2x = {}^4\log(5)$$

$$x = \frac{1}{2} \cdot {}^4\log(5)$$

4 a $5 + 2^{x-3} = 20$

$$2^{x-3} = 15$$

$$x - 3 = {}^2\log(15)$$

$$x = 3 + {}^2\log(15)$$

b $7 + 3 \cdot 5^{2x+1} = 67$

$$3 \cdot 5^{2x+1} = 60$$

$$5^{2x+1} = 20$$

$$2x + 1 = {}^5\log(20)$$

$$2x = -1 + {}^5\log(20)$$

$$x = -\frac{1}{2} + \frac{1}{2} \cdot {}^5\log(20)$$

c $3^{x+2} + 3^x = 600$

$$3^2 \cdot 3^x + 3^x = 600$$

$$9 \cdot 3^x + 3^x = 600$$

$$10 \cdot 3^x = 600$$

$$3^x = 60$$

$$x = {}^3\log(60)$$

d $2^{1+2x} = 4^x + 6$

$$2 \cdot 2^{2x} = (2^2)^x + 6$$

$$2 \cdot 2^{2x} - 2^{2x} = 6$$

$$2^{2x} = 6$$

$$2x = {}^2\log(6)$$

$$x = \frac{1}{2} \cdot {}^2\log(6)$$

5 $7 + p \cdot 3^{x-1} = 57$

$$p \cdot 3^{-1} \cdot 3^x = 50$$

$$\frac{1}{3}p \cdot 3^x = 50$$

$$3^x = \frac{150}{p}$$

$$x = {}^3\log\left(\frac{150}{p}\right) \left\} \frac{150}{p} = 10 \right.$$

$$x = {}^3\log(10) \left\} p = 15 \right.$$

Bladzijde 10

V3 a $N = 87 \cdot 1,05^t$

b $t = 4$ geeft $N = 87 \cdot 1,05^4 \approx 106$

c $t = 9$ geeft $N = 87 \cdot 1,05^9 = 134,96\dots$

$$t = 10 \text{ geeft } N = 87 \cdot 1,05^{10} = 141,71\dots$$

$$141,71\dots - 134,96\dots \approx 7$$

N neemt in het tiende jaar met 7 toe.

V4 a $N = 725 \cdot 0,87^t$

b $t = 5$ geeft $N = 725 \cdot 0,87^5 \approx 361$

c $t = 7$ geeft $N = 725 \cdot 0,87^7 = 273,50\dots$

$$t = 8 \text{ geeft } N = 725 \cdot 0,87^8 = 237,95\dots$$

$$273,50\dots - 237,95\dots \approx 36$$

N neemt in het achtste jaar met 36 af.

6 a Jaarlijkse toename van 5%, dus de vermenigvuldigingsfactor per jaar is 1,05.

Het bedrag op 1 januari 2026 is $1000 \cdot 1,05 = 1050$ euro.

Het bedrag op 1 januari 2027 is $1000 \cdot 1,05^2 = 1102,50$ euro.

b $g = 1,05$

c Het bedrag op 1 januari 2030 is $1000 \cdot 1,05^5 \approx 1276,28$ euro.

$$\text{procentuele toename} = \frac{1276,28 - 1000}{1000} \cdot 100\% \approx 27,6\%$$

Bladzijde 12

7 a $g_{\text{kwartier}} = 1,12$

$$g_{\text{uur}} = 1,12^4 \approx 1,574$$

De toename is 57,4% per uur.

b $g_{\text{kwartier}} = 1,12$

$$g_{5 \text{ minuten}} = 1,12^{\frac{1}{3}} \approx 1,038$$

De procentuele toename per vijf minuten is 3,8%.

c $g_{\text{kwartier}} = 1,12$

$$g_{5 \text{ uur}} = 1,12^{20} \approx 9,65$$

De toename is 865% per vijf uur.

- 8 a** $g_{\text{dag}} = 1,3$
 $g_{\text{week}} = 1,3^7 \approx 6,27$
 Het groeipercentage per week is 527%.
- b** $g_{\text{dag}} = 1,3$
 $g_{4 \text{ uur}} = 1,3^{\frac{1}{6}} \approx 1,045$
 Het groeipercentage per vier uur is 4,5%.

- 9 a** $g_{\text{dag}} = 0,84$
 $g_{\text{week}} = 0,84^7 \approx 0,295$
- b** $g_{\text{dag}} = 0,84$
 $g_{\text{uur}} = 0,84^{\frac{1}{24}} \approx 0,993$
 De hoeveelheid neemt met 0,7% per uur af.

- 10 a** $g_{\text{uur}} = 0,805$
 $g_{\text{kwartier}} = 0,805^{\frac{1}{4}} \approx 0,947$
 De afname is 5,3% per kwartier.
- b** $g_{\text{jaar}} = 1,086$
 $g_{25 \text{ jaar}} = 1,086^{25} \approx 7,87$
 De toename is 687% per 25 jaar.

Bladzijde 13

- 11 a** De hoeveelheid N neemt in $10 - 3 = 7$ uur toe van 1600 tot 4100, dus $g_{7 \text{ uur}} = \frac{4100}{1600} = 2,5625$.
 $g_{\text{uur}} = 2,5625^{\frac{1}{7}} = 1,1438\dots$

b
$$\left. \begin{array}{l} N = b \cdot 1,1438\dots^t \\ t = 3 \text{ en } N = 1600 \end{array} \right\} \begin{array}{l} b \cdot 1,1438\dots^3 = 1600 \\ b = \frac{1600}{1,1438\dots^3} \approx 1070 \end{array}$$

Dus op $t = 0$ is $N \approx 1070$.

De formule is $N = 1070 \cdot 1,144^t$.

- 12 a** $N = b \cdot g^t$
 $g_{6 \text{ uur}} = \frac{1250}{150} = 8,333\dots$
 $g_{\text{uur}} = 8,333\dots^{\frac{1}{6}} = 1,4238\dots$

$$\left. \begin{array}{l} N = b \cdot 1,4238\dots^t \\ t = 2 \text{ en } N = 150 \end{array} \right\} \begin{array}{l} b \cdot 1,4238\dots^2 = 150 \\ b = \frac{150}{1,4238\dots^2} \approx 74 \end{array}$$

Dus $N = 74 \cdot 1,424^t$.

- b** $H = b \cdot g^t$
 $g_{3 \text{ dagen}} = \frac{0,47}{0,60} = 0,7833\dots$
 $g_{\text{dag}} = 0,7833\dots^{\frac{1}{3}} = 0,9218\dots$

$$\left. \begin{array}{l} H = b \cdot 0,9218\dots^t \\ t = 5 \text{ en } H = 0,60 \end{array} \right\} \begin{array}{l} b \cdot 0,9218\dots^5 = 0,60 \\ b = \frac{0,60}{0,9218\dots^5} \approx 0,90 \end{array}$$

Dus $H = 0,90 \cdot 0,922^t$.

- 13** $g_{4 \text{ dagen}} = \frac{11}{31} = 0,3548\dots$

$g_{\text{dag}} = 0,3548\dots^{\frac{1}{4}} = 0,7718\dots$
 $60 \text{ uur} = \frac{60}{24} \text{ dag} = 2,5 \text{ dag}$

Na 60 uur was de oppervlakte van de wond $\frac{31}{0,7718\dots^{0,5}} \approx 35 \text{ mm}^2$.

- 14 a** Los op $17,32 \cdot 1,027^t = 34,64$.
Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 26,01\dots$
Na 26 jaar is het aantal inwoners verdubbeld.
- b** Los op $12,53 \cdot 1,027^t = 25,06$.
Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 26,01\dots$
Na 26 jaar is het aantal inwoners verdubbeld.

Bladzijde 14

- 15 a** $g_{\text{jaar}} = 1,131$
 $1,131^T = 2$
 $T = {}^{1,131}\log(2) = 5,630\dots$
De verdubbelingstijd is 5 jaar en $0,630\dots \cdot 12 \approx 8$ maanden.
- b** $g_{\text{week}} = 0,915$
 $0,915^T = \frac{1}{2}$
 $T = {}^{0,915}\log(\frac{1}{2}) = 7,802\dots$
De halveringstijd is 7 weken en $0,802\dots \cdot 7 \approx 6$ dagen.

- 16 a** $g_{\text{jaar}} = 1,032$
 $1,032^T = 2$
 $T = {}^{1,032}\log(2) = 22,0\dots$
De verdubbelingstijd is 22 jaar.
- b** $g_{10 \text{ jaar}} = 1,062$
 $1,062^T = 2$
 $T = {}^{1,062}\log(2) = 11,522\dots$
De verdubbelingstijd is $11,522\dots \cdot 10 \text{ jaar} \approx 115 \text{ jaar}$.

Bladzijde 15

- 17 a** $g_{25 \text{ jaar}} = 2$
 $g_{\text{jaar}} = 2^{\frac{1}{25}} \approx 1,028$
Het groeipercentage per jaar is 2,8%.
- b** $g_{28 \text{ jaar}} = \frac{1}{2}$
 $g_{\text{jaar}} = (\frac{1}{2})^{\frac{1}{28}} \approx 0,976$
Strontium-90 neemt per jaar met 2,4% af.
- c** $g_{10 \text{ uur}} = \frac{1}{2}$
 $g_{\text{uur}} = (\frac{1}{2})^{\frac{1}{10}} = 0,9330\dots$
 $0,9330\dots^T = 0,1$
 $T = {}^{0,9330\dots}\log(0,1) = 33,219\dots$
Na 33 uur en $0,219\dots \cdot 60 \approx 13$ minuten is 10% van de beginhoeveelheid over.

Alternatieve uitwerking

$$g_{10 \text{ uur}} = \frac{1}{2}$$

$$(\frac{1}{2})^T = 0,1$$

$$T = {}^{\frac{1}{2}}\log(0,1) = 3,3219\dots, \text{ met } T \text{ in tientallen uren.}$$

Dus na 33 uur en $0,219\dots \cdot 60 \approx 13$ minuten is 10% van de beginhoeveelheid over.

- 18 a** $g_{\text{dag}} = 0,917$
 $0,917^T = \frac{1}{2}$
 $T = {}^{0,917}\log(\frac{1}{2}) = 7,9\dots$
De halveringstijd is 8 dagen.
- b** $g_{\text{dag}} = 0,917$
 $0,917^T = 0,1$
 $T = {}^{0,917}\log(0,1) = 26,5\dots$
Na 27 dagen is nog 10% van de beginhoeveelheid over.

- 19 a** $0,999879^T = \frac{1}{2}$
 $T = {}_{0,999879}\log(\frac{1}{2}) = 5728,1\dots$
 De halveringstijd is 5730 jaar.
- b** $t = 3000$ geeft $N = 12,5 \cdot 0,999879^{3000} = 8,69\dots$
 De radioactiviteit is 8,7 cpm.
- c** $0,999879^t = 0,773$
 $t = {}_{0,999879}\log(0,773) = 2127,7\dots$
 In 2006 was de Romeinse soldaat $2006 + 217 = 2223$ jaar geleden gesneuveld.
 Het meetresultaat geeft een afwijking van $2223 - 2127,7\dots \approx 95$ jaar.

Bladzijde 16

- C1 a** $3^{x-2} = 10$
 $x - 2 = {}^3\log(10)$
 $x = 2 + {}^3\log(10)$
- b** $3 \cdot 5^{2x} = 18$
 $5^{2x} = 6$
 $2x = {}^5\log(6)$
 $x = \frac{1}{2} \cdot {}^5\log(6)$
- c** $15 - 6^{x+1} = 8$
 $6^{x+1} = 7$
 $x + 1 = {}^6\log(7)$
 $x = -1 + {}^6\log(7)$

- C2 a** $g_{\text{kwartier}} = 1,032$
 $g_{\text{uur}} = 1,032^4 \approx 1,134$
 De toename is 13,4% per uur.
- b** $g_{\text{dag}} = 0,235$
 $g_{\text{uur}} = 0,235^{\frac{1}{24}} \approx 0,941$
 De afname is 5,9% per uur.

- C3 a** $g_{\text{jaar}} = 1,153$
 $1,153^T = 2$
 $T = {}_{1,153}\log(2) = 4,868\dots$
 De verdubbelingstijd is 4 jaar en $0,868\dots \cdot 12 \approx 10$ maanden.
- b** $g_{\text{dag}} = 0,327$
 $0,327^T = \frac{1}{2}$
 $T = {}_{0,327}\log(\frac{1}{2}) = 0,620\dots$
 $0,620\dots \cdot 24 = 14,882\dots$
 De halveringstijd is 14 uur en $0,882\dots \cdot 60 \approx 53$ minuten.
- c** $g_{10 \text{ jaar}} = 2$
 $g_{\text{jaar}} = 2^{\frac{1}{10}} \approx 1,072$
 Het groeipercentage per jaar is 7,2%.
- d** $g_{6 \text{ uur}} = \frac{1}{2}$
 $g_{\text{dag}} = (\frac{1}{2})^4 = 0,0625$
 De hoeveelheid neemt met 93,8% per dag af.

9.2 Rekenregels voor logaritmen

Bladzijde 17

- 20** ${}^g\log(x) = y$ oftewel $y = {}^g\log(x)$ substitueren in $x = g^y$ geeft $x = g^{{}^g\log(x)}$, dus $g^{{}^g\log(x)} = x$.

Bladzijde 18

- 21** $y = {}^2\log(804x) = {}^2\log(4 \cdot 201x) = {}^2\log(4) + {}^2\log(201x) = 2 + {}^2\log(201x)$
 Dus $y = 2 + {}^2\log(201x)$.

- 22** a ${}^2\log(6) + {}^2\log(10) = {}^2\log(60)$
 b ${}^3\log(30) - {}^3\log(6) = {}^3\log(5)$
 c $2 \cdot {}^5\log(3) + {}^5\log(\frac{1}{2}) = {}^5\log(3^2) + {}^5\log(\frac{1}{2}) = {}^5\log(9) + {}^5\log(\frac{1}{2}) = {}^5\log(4\frac{1}{2})$
 d $\frac{1}{2}\log(15) - 4 \cdot \frac{1}{2}\log(3) = \frac{1}{2}\log(15) - \frac{1}{2}\log(3^4) = \frac{1}{2}\log(15) - \frac{1}{2}\log(81) = \frac{1}{2}\log(\frac{5}{27})$
 e $-2 \cdot {}^4\log(6) + {}^4\log(12) = {}^4\log(6^{-2}) + {}^4\log(12) = {}^4\log(\frac{1}{36}) + {}^4\log(12) = {}^4\log(\frac{1}{3})$
 f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(50) - \log(25) = \log(2)$

- 23** a $y = \log(600x) = \log(100 \cdot 6x) = \log(100) + \log(6x) = 2 + \log(6x)$
 Dus $y = 2 + \log(6x)$.
 b $y = {}^2\log(24x) = {}^2\log(8 \cdot 3x) = {}^2\log(8) + {}^2\log(3x) = 3 + {}^2\log(3x)$
 Dus $y = 3 + {}^2\log(3x)$.
 c $y = 4 + {}^3\log(54x) = 4 + {}^3\log(27 \cdot 2x) = 4 + {}^3\log(27) + {}^3\log(2x) = 4 + 3 + {}^3\log(2x) = 7 + {}^3\log(2x)$
 Dus $y = 7 + {}^3\log(2x)$.

- 24** a $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16) + {}^2\log(3) = {}^2\log(48)$
 b $2 - \log(5) = \log(10^2) - \log(5) = \log(100) - \log(5) = \log(20)$
 c $3 - \frac{1}{2}\log(10) = \frac{1}{2}\log((\frac{1}{2})^3) - \frac{1}{2}\log(10) = \frac{1}{2}\log(\frac{1}{8}) - \frac{1}{2}\log(10) = \frac{1}{2}\log(\frac{1}{80})$
 d ${}^2\log(12) - {}^3\log(9) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(12) - {}^2\log(4) = {}^2\log(3)$
 e $\frac{1}{2} \cdot {}^3\log(16) + \frac{1}{2}\log(8) = {}^3\log(16^{\frac{1}{2}}) - 3 = {}^3\log(4) - {}^3\log(3^3) = {}^3\log(4) - {}^3\log(27) = {}^3\log(\frac{4}{27})$
 f $\log(500) - {}^5\log(125) = \log(500) - 3 = \log(500) - \log(10^3) = \log(500) - \log(1000) = \log(\frac{1}{2})$

- 25** a $g^{{}^g\log(a) - {}^g\log(b)} = \frac{g^{{}^g\log(a)}}{g^{{}^g\log(b)}} = \frac{a}{b} = g^{{}^g\log(\frac{a}{b})}$, dus ${}^g\log(a) - {}^g\log(b) = {}^g\log(\frac{a}{b})$.
 b $g^{p \cdot {}^g\log(a)} = (g^{{}^g\log(a)})^p = a^p = g^{{}^g\log(a^p)}$, dus $p \cdot {}^g\log(a) = {}^g\log(a^p)$.

- 26** a ${}^2\log(a) + 3 \cdot {}^2\log(b) = {}^2\log(a) + {}^2\log(b^3) = {}^2\log(ab^3)$
 b $5 \cdot {}^3\log(a) - 2 \cdot {}^3\log(b) = {}^3\log(a^5) - {}^3\log(b^2) = {}^3\log(\frac{a^5}{b^2})$
 c $2 + {}^5\log(a) = {}^5\log(2^2) + {}^5\log(a) = {}^5\log(25) + {}^5\log(a) = {}^5\log(25a)$
 d $2 - {}^3\log(a) = {}^3\log(3^2) - {}^3\log(a) = {}^3\log(9) - {}^3\log(a) = {}^3\log(\frac{9}{a})$
 e ${}^6\log(a) - 1 = {}^6\log(a) - {}^6\log(6) = {}^6\log(\frac{a}{6}) = {}^6\log(\frac{1}{6}a)$
 f $2 \cdot {}^5\log(b) + \frac{1}{2} \cdot {}^5\log(a) = {}^5\log(b^2) + {}^5\log(a^{\frac{1}{2}}) = {}^5\log(b^2) + {}^5\log(\sqrt{a}) = {}^5\log(b^2 \cdot \sqrt{a})$

- 27** a $y = 2 + {}^5\log(1250x) = 2 + {}^5\log(625 \cdot 2x) = 2 + {}^5\log(625) + {}^5\log(2x) = 2 + 4 + {}^5\log(2x) = 6 + {}^5\log(2x)$
 Dus $y = 6 + {}^5\log(2x)$.
 b $y = \log(1600x) - 1 = \log(100 \cdot 16x) - 1 = \log(100) + \log(16x) - 1 = 2 + \log(16x) - 1 = 1 + \log(16x)$
 Dus $y = 1 + \log(16x)$.
 c $y = 7 + {}^2\log(1600x) = 7 + {}^2\log(64 \cdot 25x) = 7 + {}^2\log(64) + {}^2\log(25x) = 7 + 6 + {}^2\log(25x) = 13 + {}^2\log(25x)$
 Dus $y = 13 + {}^2\log(25x)$.

28 $y = {}^2\log(x) \xrightarrow{\times a \text{ t.o.v. de } x\text{-as}} y = a \cdot {}^2\log(x) \xrightarrow{\times b \text{ t.o.v. de } y\text{-as}} y = a \cdot {}^2\log(\frac{1}{b}x) = a \cdot {}^2\log(\frac{x}{b})$

Er geldt $a \cdot {}^2\log(\frac{x}{b}) = a \cdot ({}^2\log(x) - {}^2\log(b)) = -a \cdot {}^2\log(b) + a \cdot {}^2\log(x)$.

$$\left. \begin{aligned} g(x) &= -a \cdot {}^2\log(b) + a \cdot {}^2\log(x) \\ g(x) &= -5 + 10 \cdot {}^2\log(x) \end{aligned} \right\} a = 10 \text{ en } -10 \cdot {}^2\log(b) = -5$$

$$\begin{aligned} {}^2\log(b) &= \frac{1}{2} \\ b &= 2^{\frac{1}{2}} = \sqrt{2} \end{aligned}$$

Dus $a = 10$ en $b = \sqrt{2}$.

Bladzijde 19

- 29 a** Uit $p \cdot {}^g\log(a) = {}^g\log(a^p)$ volgt $2 \cdot \log(x) = \log(x^2)$.
Dus uit $2 \cdot \log(x) = \log(x+2)$ volgt $\log(x^2) = \log(x+2)$.
- b** $x^2 = x + 2$
 $x^2 - x - 2 = 0$
 $(x+1)(x-2) = 0$
 $x = -1 \vee x = 2$
- c** $x = -1$ voldoet niet aan de vergelijking $2 \cdot \log(x) = \log(x+2)$ omdat $2 \cdot \log(-1) \neq \log(-1+2)$, want $\log(-1)$ bestaat niet.
 $x = 2$ voldoet aan de vergelijking $2 \cdot \log(x) = \log(x+2)$, want $2 \cdot \log(2) = \log(4)$.

- 30 a** ${}^2\log(x+3) = 3 + {}^2\log(x)$
 ${}^2\log(x+3) = {}^2\log(2^3) + {}^2\log(x)$
 ${}^2\log(x+3) = {}^2\log(8) + {}^2\log(x)$
 ${}^2\log(x+3) = {}^2\log(8x)$
 $x+3 = 8x$
 $-7x = -3$
 $x = \frac{3}{7}$
- b** ${}^3\log(2x) = 1 + {}^3\log(x+1)$
 ${}^3\log(2x) = {}^3\log(3) + {}^3\log(x+1)$
 ${}^3\log(2x) = {}^3\log(3(x+1))$
 ${}^3\log(2x) = {}^3\log(3x+3)$
 $2x = 3x+3$
 $-x = 3$
 $x = -3$
 $x = -3$ voldoet niet, dus de vergelijking heeft geen oplossingen.

- c** $\frac{1}{2}\log(2x-1) = 2 + \frac{1}{2}\log(x+2)$
 $\frac{1}{2}\log(2x-1) = \frac{1}{2}\log((\frac{1}{2})^2) + \frac{1}{2}\log(x+2)$
 $\frac{1}{2}\log(2x-1) = \frac{1}{2}\log(\frac{1}{4}) + \frac{1}{2}\log(x+2)$
 $\frac{1}{2}\log(2x-1) = \frac{1}{2}\log(\frac{1}{4}(x+2))$
 $\frac{1}{2}\log(2x-1) = \frac{1}{2}\log(\frac{1}{4}x + \frac{1}{2})$
 $2x-1 = \frac{1}{4}x + \frac{1}{2}$
 $\frac{3}{4}x = 1\frac{1}{2}$
 $x = \frac{6}{7}$
- d** ${}^3\log(x+2) = 1 - {}^3\log(x)$
 ${}^3\log(x+2) = {}^3\log(3) - {}^3\log(x)$
 ${}^3\log(x+2) = {}^3\log(\frac{3}{x})$
 $x+2 = \frac{3}{x}$
 $x^2 + 2x = 3$
 $x^2 + 2x - 3 = 0$
 $(x-1)(x+3) = 0$
 $x = 1 \vee x = -3$
 $x = -3$ voldoet niet, dus $x = 1$.

Bladzijde 20

- 31 a** $2 \cdot {}^3\log(x) + 1 = {}^3\log(5x-2)$
 ${}^3\log(x^2) + {}^3\log(3) = {}^3\log(5x-2)$
 ${}^3\log(3x^2) = {}^3\log(5x-2)$
 $3x^2 = 5x-2$
 $3x^2 - 5x + 2 = 0$
 $D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1$, dus $\sqrt{D} = 1$
 $x = \frac{5+1}{6} = 1 \vee x = \frac{5-1}{6} = \frac{2}{3}$
- b** ${}^3\log(x+4) + 1 = 2 \cdot {}^3\log(x-2)$
 ${}^3\log(x+4) + {}^3\log(3) = {}^3\log((x-2)^2)$
 ${}^3\log(3(x+4)) = {}^3\log(x^2 - 4x + 4)$
 ${}^3\log(3x+12) = {}^3\log(x^2 - 4x + 4)$
 $3x+12 = x^2 - 4x + 4$
 $x^2 - 7x - 8 = 0$
 $(x+1)(x-8) = 0$
 $x = -1 \vee x = 8$
 $x = -1$ voldoet niet, dus $x = 8$.
- c** ${}^2\log(2x) - {}^2\log(x+3) = {}^2\log(x) - 2$
 ${}^2\log(\frac{2x}{x+3}) = {}^2\log(x) - {}^2\log(2^2)$
 ${}^2\log(\frac{2x}{x+3}) = {}^2\log(\frac{x}{4})$
 $\frac{2x}{x+3} = \frac{x}{4}$
 $x^2 + 3x = 8x$
 $x^2 - 5x = 0$
 $x(x-5) = 0$
 $x = 0 \vee x = 5$
 $x = 0$ voldoet niet, dus $x = 5$.
- d** ${}^3\log(x) = 2 - {}^3\log(x-1)$
 ${}^3\log(x) = {}^3\log(3^2) - {}^3\log(x-1)$
 ${}^3\log(x) = {}^3\log(\frac{9}{x-1})$
 $x = \frac{9}{x-1}$
 $x^2 - x = 9$
 $x^2 - x - 9 = 0$
 $D = (-1)^2 - 4 \cdot 1 \cdot -9 = 37$
 $x = \frac{1 + \sqrt{37}}{2} \vee x = \frac{1 - \sqrt{37}}{2}$
 $x = \frac{1}{2} + \frac{1}{2}\sqrt{37} \vee x = \frac{1}{2} - \frac{1}{2}\sqrt{37}$
 $x = \frac{1}{2} - \frac{1}{2}\sqrt{37}$ voldoet niet, dus $x = \frac{1}{2} + \frac{1}{2}\sqrt{37}$.

32 a ${}^2\log(8^{x-1}) = 2x$
 $8^{x-1} = 2^{2x}$
 $(2^3)^{x-1} = 2^{2x}$
 $2^{3x-3} = 2^{2x}$
 $3x - 3 = 2x$
 $x = 3$

b ${}^3\log(\sqrt{x^2 - 10}) = 1 + {}^3\log(\sqrt{x})$
 ${}^3\log(\sqrt{x^2 - 10}) = {}^3\log(3) + {}^3\log(\sqrt{x})$
 ${}^3\log(\sqrt{x^2 - 10}) = {}^3\log(3\sqrt{x})$
 $\sqrt{x^2 - 10} = 3\sqrt{x}$
 $x^2 - 10 = 9x$
 $x^2 - 9x - 10 = 0$
 $(x + 1)(x - 10) = 0$
 $x = -1 \vee x = 10$
 $x = -1$ voldoet niet, dus $x = 10$.

C4 a $2 \cdot {}^3\log(5) + {}^3\log(2) = {}^3\log(5^2) + {}^3\log(2) = {}^3\log(25) + {}^3\log(2) = {}^3\log(50)$
b ${}^6\log(40) - 3 \cdot {}^6\log(2) = {}^6\log(40) - {}^6\log(2^3) = {}^6\log(40) - {}^6\log(8) = {}^6\log(5)$
c $1 + 2 \cdot \log(3) = \log(10) + \log(3^2) = \log(10) + \log(9) = \log(90)$
d ${}^2\log(56) - {}^3\log(27) = {}^2\log(56) - 3 = {}^2\log(56) - {}^2\log(2^3) = {}^2\log(56) - {}^2\log(8) = {}^2\log(7)$

C5 a $y = 3 + {}^5\log(500x) = 3 + {}^5\log(125 \cdot 4x) = 3 + {}^5\log(125) + {}^5\log(4x) = 3 + 3 + {}^5\log(4x) = 6 + {}^5\log(4x)$
Dus $y = 6 + {}^5\log(4x)$.
b $y = 8 - {}^3\log(810x) = 8 - {}^3\log(81 \cdot 10x) = 8 - {}^3\log(81) - {}^3\log(10x) = 8 - 4 - {}^3\log(10x) = 4 - {}^3\log(10x)$
Dus $y = 4 - {}^3\log(10x)$.

C6 a ${}^3\log(x - 5) = {}^3\log(4x) - 2$
 ${}^3\log(x - 5) = {}^3\log(4x) - {}^3\log(3^2)$
 ${}^3\log(x - 5) = {}^3\log\left(\frac{4x}{9}\right)$
 $x - 5 = \frac{4x}{9}$
 $9x - 45 = 4x$
 $5x = 45$
 $x = 9$
b $2 \cdot \log(x) - 1 = \log(3x + 40)$
 $\log(x^2) - \log(10) = \log(3x + 40)$
 $\log\left(\frac{x^2}{10}\right) = \log(3x + 40)$
 $\frac{x^2}{10} = 3x + 40$
 $x^2 = 30x + 400$
 $x^2 - 30x - 400 = 0$
 $(x + 10)(x - 40) = 0$
 $x = -10 \vee x = 40$
 $x = -10$ voldoet niet, dus $x = 40$.

9.3 Overgaan op ander grondtal

Bladzijde 21

33 a $p^{\log(g)} = g$
 $(p^{\log(g)})^{\log(a)} = g^{\log(a)}$
 $p^{\log(g) \cdot \log(a)} = a$
b $p^{\log(g) \cdot \log(a)} = p^{\log(a)}$
 $\log(g) \cdot \log(a) = \log(a)$
 $\log(a) = \frac{\log(a)}{\log(g)}$

34 ${}^4\log(x) = {}^2\log(x-2)$
 ${}^4\log(x) = \frac{{}^4\log(x-2)}{{}^4\log(2)}$
 ${}^4\log(x) = \frac{{}^4\log(x-2)}{\frac{1}{2}}$
 ${}^4\log(x) = 2 \cdot {}^4\log(x-2)$
 ${}^4\log(x) = {}^4\log((x-2)^2)$
 $x = (x-2)^2$
 $x = x^2 - 4x + 4$
 $x^2 - 5x + 4 = 0$
 $(x-1)(x-4) = 0$
 $x = 1 \vee x = 4$
 $x = 1$ voldoet niet, dus $x = 4$.

35 a ${}^3\log(3x-5) + \frac{1}{3}\log(x-1) = 0$
 ${}^3\log(3x-5) - {}^3\log(x-1) = 0$
 ${}^3\log(3x-5) = {}^3\log(x-1)$
 $3x-5 = x-1$
 $2x = 4$
 $x = 2$
b ${}^5\log(3x) + 2 \cdot \frac{1}{5}\log(x) = 0$
 ${}^5\log(3x) + \frac{1}{5}\log(x^2) = 0$
 ${}^5\log(3x) - {}^5\log(x^2) = 0$
 ${}^5\log(3x) = {}^5\log(x^2)$
 $3x = x^2$
 $x^2 - 3x = 0$
 $x(x-3) = 0$
 $x = 0 \vee x = 3$
 $x = 0$ voldoet niet, dus $x = 3$.

36 a $-2 \cdot \frac{1}{2}\log(x) = 2 + {}^2\log(3-x)$
 $2 \cdot {}^2\log(x) = {}^2\log(2^2) + {}^2\log(3-x)$
 ${}^2\log(x^2) = {}^2\log(4(3-x))$
 $x^2 = 4(3-x)$
 $x^2 = 12 - 4x$
 $x^2 + 4x - 12 = 0$
 $(x-2)(x+6) = 0$
 $x = 2 \vee x = -6$
 $x = -6$ voldoet niet, dus $x = 2$.
b ${}^9\log(2x) = {}^3\log(x-4)$
 $\frac{{}^3\log(2x)}{{}^3\log(9)} = {}^3\log(x-4)$
 $\frac{{}^3\log(2x)}{2} = {}^3\log(x-4)$
 ${}^3\log(2x) = 2 \cdot {}^3\log(x-4)$
 ${}^3\log(2x) = {}^3\log((x-4)^2)$
 $2x = (x-4)^2$
 $2x = x^2 - 8x + 16$
 $x^2 - 10x + 16 = 0$
 $(x-2)(x-8) = 0$
 $x = 2 \vee x = 8$
 $x = 2$ voldoet niet, dus $x = 8$.

c $2x \cdot \frac{1}{3}\log(3x+5) = \frac{1}{3}\log(3x+5)$
 $\frac{1}{3}\log(3x+5) = 0 \vee 2x = 1$
 $3x+5 = 1 \vee x = \frac{1}{2}$
 $3x = -4 \vee x = \frac{1}{2}$
 $x = -\frac{4}{3} \vee x = \frac{1}{2}$
d ${}^2\log(x) = {}^4\log(x+20)$
 ${}^2\log(x) = \frac{{}^2\log(x+20)}{{}^2\log(4)}$
 ${}^2\log(x) = \frac{{}^2\log(x+20)}{2}$
 $2 \cdot {}^2\log(x) = {}^2\log(x+20)$
 ${}^2\log(x^2) = {}^2\log(x+20)$
 $x^2 = x+20$
 $x^2 - x - 20 = 0$
 $(x+4)(x-5) = 0$
 $x = -4 \vee x = 5$
 $x = -4$ voldoet niet, dus $x = 5$.

c $4x \cdot {}^4\log(2x-1) + 3 \cdot {}^4\log(2x-1) = 0$
 $(4x+3) \cdot {}^4\log(2x-1) = 0$
 $4x+3 = 0 \vee {}^4\log(2x-1) = 0$
 $4x = -3 \vee 2x-1 = 1$
 $x = -\frac{3}{4} \vee 2x = 2$
 $x = -\frac{3}{4} \vee x = 1$
 $x = -\frac{3}{4}$ voldoet niet, dus $x = 1$.
d $x^2 \cdot {}^5\log(2x+1) + 9 \cdot \frac{1}{5}\log(2x+1) = 0$
 $x^2 \cdot {}^5\log(2x+1) - 9 \cdot {}^5\log(2x+1) = 0$
 $(x^2-9) \cdot {}^5\log(2x+1) = 0$
 $x^2-9 = 0 \vee {}^5\log(2x+1) = 0$
 $x^2 = 9 \vee 2x+1 = 1$
 $x = 3 \vee x = -3 \vee 2x = 0$
 $x = 3 \vee x = -3 \vee x = 0$
 $x = -3$ voldoet niet, dus $x = 0 \vee x = 3$.

37 ${}^3\log^2(x) = 2 \cdot {}^3\log(x) + 15$
 Stel ${}^3\log(x) = u$.
 $u^2 = 2u + 15$
 $u^2 - 2u - 15 = 0$
 $(u + 3)(u - 5) = 0$
 $u = -3 \vee u = 5$
 ${}^3\log(x) = -3 \vee {}^3\log(x) = 5$
 $x = 3^{-3} = \frac{1}{27} \vee x = 3^5 = 243$

38 a ${}^2\log^2(x) = 3 - 2 \cdot {}^{\frac{1}{2}}\log(x)$
 ${}^2\log^2(x) = 3 + 2 \cdot {}^2\log(x)$
 Stel ${}^2\log(x) = u$.
 $u^2 = 3 + 2u$
 $u^2 - 2u - 3 = 0$
 $(u + 1)(u - 3) = 0$
 $u = -1 \vee u = 3$
 ${}^2\log(x) = -1 \vee {}^2\log(x) = 3$
 $x = 2^{-1} = \frac{1}{2} \vee x = 2^3 = 8$

b ${}^3\log^2(x) = 6 \cdot {}^9\log(x)$
 ${}^3\log^2(x) = 6 \cdot \frac{{}^3\log(x)}{{}^3\log(9)}$
 ${}^3\log^2(x) = 6 \cdot \frac{{}^3\log(x)}{2}$
 ${}^3\log^2(x) - 3 \cdot {}^3\log(x) = 0$
 Stel ${}^3\log(x) = u$.
 $u^2 - 3u = 0$
 $u(u - 3) = 0$
 $u = 0 \vee u = 3$
 ${}^3\log(x) = 0 \vee {}^3\log(x) = 3$
 $x = 1 \vee x = 3^3 = 27$

Bladzijde 23

39 a $4^x = 2^{x+2} - 3$
 $(2^2)^x = 2^2 \cdot 2^x - 3$
 $2^{2x} = 4 \cdot 2^x - 3$
 $(2^x)^2 = 4 \cdot 2^x - 3$
b Stel $2^x = u$.
 $u^2 = 4u - 3$
 $u^2 - 4u + 3 = 0$
 $(u - 1)(u - 3) = 0$
 $u = 1 \vee u = 3$
 $2^x = 1 \vee 2^x = 3$
 $x = 0 \vee x = {}^2\log(3)$

40 a $9^x = 5 \cdot 3^x - 6$
 $(3^2)^x = 5 \cdot 3^x - 6$
 $(3^x)^2 = 5 \cdot 3^x - 6$
 Stel $3^x = u$.
 $u^2 = 5u - 6$
 $u^2 - 5u + 6 = 0$
 $(u - 2)(u - 3) = 0$
 $u = 2 \vee u = 3$
 $3^x = 2 \vee 3^x = 3$
 $x = {}^3\log(2) \vee x = 1$
b $4^x = 2^x + 42$
 $(2^2)^x = 2^x + 42$
 $(2^x)^2 = 2^x + 42$
 Stel $2^x = u$.
 $u^2 = u + 42$
 $u^2 - u - 42 = 0$
 $(u + 6)(u - 7) = 0$
 $u = -6 \vee u = 7$
 $2^x = -6 \vee 2^x = 7$
 $x = {}^2\log(7)$

c $9^x + 2 = 3^{x+1}$
 $(3^2)^x + 2 = 3 \cdot 3^x$
 $(3^x)^2 + 2 = 3 \cdot 3^x$
 Stel $3^x = u$.
 $u^2 + 2 = 3u$
 $u^2 - 3u + 2 = 0$
 $(u - 1)(u - 2) = 0$
 $u = 1 \vee u = 2$
 $3^x = 1 \vee 3^x = 2$
 $x = 0 \vee x = {}^3\log(2)$
d $2^x = 24 - 2^{2x-1}$
 $2^x = 24 - 2^{-1} \cdot 2^{2x}$
 $2^x = 24 - \frac{1}{2} \cdot (2^x)^2$
 Stel $2^x = u$.
 $u = 24 - \frac{1}{2}u^2$
 $\frac{1}{2}u^2 + u - 24 = 0$
 $u^2 + 2u - 48 = 0$
 $(u - 6)(u + 8) = 0$
 $u = 6 \vee u = -8$
 $2^x = 6 \vee 2^x = -8$
 $x = {}^2\log(6)$

41 a $5^{x-1} + 5^{2x-1} = 4$
 $5^x \cdot 5^{-1} + 5^{2x} \cdot 5^{-1} = 4$
 $\frac{1}{5} \cdot 5^x + \frac{1}{5} \cdot (5^x)^2 = 4$
 Stel $5^x = u$.
 $\frac{1}{5}u + \frac{1}{5}u^2 = 4$
 $u^2 + u - 20 = 0$
 $(u-4)(u+5) = 0$
 $u = 4 \vee u = -5$
 $5^x = 4 \vee 5^x = -5$

$x = {}^5\log(4)$
b $\frac{1}{2}\log^2(x+2) - 3 \cdot {}^2\log(x+2) = 0$
 $\frac{1}{2}\log^2(x+2) + 3 \cdot \frac{1}{2}\log(x+2) = 0$
 Stel $\frac{1}{2}\log(x+2) = u$.
 $u^2 + 3u = 0$
 $u(u+3) = 0$
 $u = 0 \vee u = -3$
 $\frac{1}{2}\log(x+2) = 0 \vee \frac{1}{2}\log(x+2) = -3$
 $x+2 = 1 \vee x+2 = 8$
 $x = -1 \vee x = 6$

42 a $3^x + 2 \cdot (\frac{1}{3})^{x-2} = 9$
 $3^x + 2 \cdot (\frac{1}{3})^x \cdot (\frac{1}{3})^{-2} = 9$
 $3^x + 18 \cdot 3^{-x} = 9$
 $(3^x)^2 + 18 = 9 \cdot 3^x$
 Stel $3^x = u$.
 $u^2 + 18 = 9u$
 $u^2 - 9u + 18 = 0$
 $(u-3)(u-6) = 0$
 $u = 3 \vee u = 6$
 $3^x = 3 \vee 3^x = 6$
 $x = 1 \vee x = {}^3\log(6)$

43 $y = 2^{x-1}$
 $2^{x-1} = y$
 $x-1 = {}^2\log(y)$
 $x = 1 + {}^2\log(y)$

Bladzijde 24

44 a $t = 1 + {}^2\log(0,2N)$
 $t = {}^2\log(2) + {}^2\log(0,2N)$
 $t = {}^2\log(0,4N)$
 $t = \frac{\log(0,4N)}{\log(2)}$
 $t \approx 3,32 \cdot \log(0,4N)$
 Dus $t = 3,32 \cdot \log(0,4N)$.
b $t = 1 + {}^2\log(0,2N)$
 $t = 1 + {}^2\log(0,2) + {}^2\log(N)$
 $t = 1 + {}^2\log(0,2) + \frac{{}^3\log(N)}{{}^3\log(2)}$
 $t \approx -1,32 + 1,58 \cdot {}^3\log(N)$
 Dus $t = -1,32 + 1,58 \cdot {}^3\log(N)$.
c $x = 64 \cdot 8^y$
 $x = 64 \cdot (3^{{}^3\log(8)})^y$
 $x \approx 64 \cdot (3^{1,893})^y$
 $x \approx 64 \cdot 3^{1,893y}$
 Dus $x = 64 \cdot 3^{1,893y}$.

c $2^x = 6 - 5 \cdot (\frac{1}{2})^x$
 $2^x = 6 - 5 \cdot 2^{-x}$
 $(2^x)^2 = 6 \cdot 2^x - 5$
 Stel $2^x = u$.
 $u^2 = 6u - 5$
 $u^2 - 6u + 5 = 0$
 $(u-1)(u-5) = 0$
 $u = 1 \vee u = 5$
 $2^x = 1 \vee 2^x = 5$
 $x = 0 \vee x = {}^2\log(5)$
d ${}^5\log^2(x) + 3 \cdot \frac{1}{5}\log(x) + 2 = 0$
 ${}^5\log^2(x) - 3 \cdot {}^5\log(x) + 2 = 0$
 Stel ${}^5\log(x) = u$.
 $u^2 - 3u + 2 = 0$
 $(u-1)(u-2) = 0$
 $u = 1 \vee u = 2$
 ${}^5\log(x) = 1 \vee {}^5\log(x) = 2$
 $x = 5 \vee x = 25$

b ${}^2\log(2^x - 4) = 2x - 4$
 $2^x - 4 = 2^{2x-4}$
 $2^x - 4 = 2^{2x} \cdot 2^{-4}$
 $2^x - 4 = \frac{1}{16} \cdot (2^x)^2$
 Stel $2^x = u$.
 $u - 4 = \frac{1}{16}u^2$
 $u^2 - 16u + 64 = 0$
 $(u-8)^2 = 0$
 $u = 8$
 $2^x = 8$
 $x = 3$

45

$$\begin{aligned}
 \text{a } N &= 50 \cdot 3^{2t+3} \\
 50 \cdot 3^{2t+3} &= N \\
 3^{2t+3} &= 0,02N \\
 2t+3 &= {}^3\log(0,02N) \\
 2t &= -3 + {}^3\log(0,02) + {}^3\log(N) \\
 t &= -1\frac{1}{2} + \frac{1}{2} \cdot {}^3\log(0,02) + \frac{1}{2} \cdot {}^3\log(N) \\
 t &\approx -3,28 + 0,5 \cdot {}^3\log(N) \\
 \text{Dus } t &= -3,28 + 0,5 \cdot {}^3\log(N).
 \end{aligned}$$

$$\begin{aligned}
 \text{b } y &= 20 \cdot 2^{3x-1} \\
 20 \cdot 2^{3x-1} &= y \\
 2^{3x-1} &= \frac{1}{20}y \\
 3x-1 &= {}^2\log\left(\frac{1}{20}y\right) \\
 3x &= 1 + {}^2\log\left(\frac{1}{20}y\right) \\
 3x &= {}^2\log(2) + {}^2\log\left(\frac{1}{20}y\right) \\
 3x &= {}^2\log\left(\frac{1}{10}y\right) \\
 x &= \frac{1}{3} \cdot {}^2\log\left(\frac{1}{10}y\right) \\
 \text{Dus } x &= \frac{1}{3} \cdot {}^2\log\left(\frac{1}{10}y\right).
 \end{aligned}$$

46

$$\begin{aligned}
 \text{a } {}^2\log(4t-1) &= 5N+3 \\
 4t-1 &= 2^{5N+3} \\
 4t &= 1 + (2^5)^N \cdot 2^3 \\
 4t &= 1 + 8 \cdot 32^N \\
 4t &= 1 + 8 \cdot (10^{\log(32)})^N \\
 4t &\approx 1 + 8 \cdot (10^{1,51})^N \\
 t &\approx \frac{1}{4} + 2 \cdot 10^{1,51N} \\
 \text{Dus } t &= \frac{1}{4} + 2 \cdot 10^{1,51N}.
 \end{aligned}$$

$$\begin{aligned}
 \text{c } y &= 3 \cdot \log(x) - 5 \\
 3 \cdot \log(x) &= y + 5 \\
 \log(x) &= \frac{1}{3}y + \frac{5}{3} \\
 x &= 10^{\frac{1}{3}y + \frac{5}{3}} = 10^{\frac{1}{3}y} \cdot 10^{\frac{5}{3}} \approx 46,42 \cdot 10^{0,33y} \\
 \text{Dus } x &= 46,42 \cdot 10^{0,33y}. \\
 \text{d } K &= 1 - \frac{1}{2} \cdot {}^3\log(2v) \\
 \frac{1}{2} \cdot {}^3\log(2v) &= 1 - K \\
 {}^3\log(2v) &= 2 - 2K \\
 2v &= 3^{2-2K} \\
 v &= \frac{1}{2} \cdot 3^2 \cdot 3^{-2K} = \frac{1}{2} \cdot 9 \cdot (3^{-2})^K = 4\frac{1}{2} \cdot \left(\frac{1}{9}\right)^K \\
 \text{Dus } v &= 4\frac{1}{2} \cdot \left(\frac{1}{9}\right)^K.
 \end{aligned}$$

$$\begin{aligned}
 \text{b } M &= 14 \cdot 1,3^{4N-3} \\
 14 \cdot 1,3^{4N-3} &= M \\
 1,3^{4N-3} &= \frac{1}{14}M \\
 4N-3 &= {}^{1,3}\log\left(\frac{1}{14}M\right) \\
 4N &= 3 + {}^{1,3}\log\left(\frac{1}{14}\right) + {}^{1,3}\log(M) \\
 4N &= 3 + {}^{1,3}\log\left(\frac{1}{14}\right) + \frac{\log(M)}{\log(1,3)} \\
 N &= \frac{3 + {}^{1,3}\log\left(\frac{1}{14}\right)}{4} + \frac{\log(M)}{4 \cdot \log(1,3)} \\
 N &\approx -1,76 + 2,19 \cdot \log(M) \\
 \text{Dus } N &= -1,76 + 2,19 \cdot \log(M).
 \end{aligned}$$

Bladzijde 25

47

$$\text{a } \text{Voor } f_a \text{ geldt } y = {}^2\log(4^{y+a} + 1), \text{ dus voor } f_a^{\text{inv}} \text{ geldt } x = {}^2\log(4^{y+a} + 1).$$

$$\begin{aligned}
 x = {}^2\log(4^{y+a} + 1) &\text{ geeft } 4^{y+a} + 1 = 2^x \\
 4^{y+a} &= 2^x - 1 \\
 y + a &= {}^4\log(2^x - 1) \\
 y &= {}^4\log(2^x - 1) - a
 \end{aligned}$$

$$\text{Dus } f_a^{\text{inv}}(x) = {}^4\log(2^x - 1) - a = g_a(x).$$

$$\text{b } f_0(x) = {}^2\log(4^x + 1) \text{ en } g_0(x) = {}^4\log(2^x - 1)$$

$$y_A = f_0(0) = {}^2\log(4^0 + 1) = {}^2\log(2) = 1$$

$$y_B = y_A \text{ geeft } {}^4\log(2^x - 1) = 1$$

$$2^x - 1 = 4$$

$$2^x = 5$$

$$2^x = 5 \text{ geeft } 4^x = (2^x)^2 = 5^2 = 25, \text{ dus } y_C = {}^2\log(25 + 1) = {}^2\log(26).$$

$$BC = y_C - y_B = {}^2\log(26) - 1 = {}^2\log(26) - {}^2\log(2) = {}^2\log(13)$$

$$\text{c } f_{-2}(x) = {}^2\log(4^{x-2} + 1) \text{ en } g_{-2}(x) = {}^4\log(2^x - 1) + 2$$

$$RS = g_{-2}(p) - f_{-2}(p) = {}^4\log(2^p - 1) + 2 - {}^2\log(4^{p-2} + 1)$$

Invoeren van de formule en gebruiken van de optie maximum geeft dat RS maximaal 1,86 is voor $x \approx 1,62$.

48 $f_a(x) = x$ geeft ${}^2\log(4^{x+a} + 1) = x$

$$4^{x+a} + 1 = 2^x$$

$$4^x \cdot 4^a + 1 = 2^x$$

$$4^a \cdot (2^x)^2 - 2^x + 1 = 0$$

$$\text{Stel } 2^x = u.$$

$$4^a \cdot u^2 - u + 1 = 0$$

$$D = (-1)^2 - 4 \cdot 4^a \cdot 1 = 1 - 4 \cdot 4^a$$

Raken, dus $D = 0$ geeft $1 - 4 \cdot 4^a = 0$

$$4^a = \frac{1}{4}$$

$$a = -1$$

Dus voor $a = -1$ hebben de grafieken van f_a en g_a één punt gemeenschappelijk met de lijn $y = x$.

49 a $f(x) = g_a(x)$ geeft ${}^2\log(x) = {}^2\log(x\sqrt{x}) + a$

$${}^2\log(x) = {}^2\log(x^{1\frac{1}{2}}) + a$$

$${}^2\log(x) = 1\frac{1}{2} \cdot {}^2\log(x) + a$$

$$\frac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -2a$$

$$x = 2^{-2a} = (2^{-2})^a = \left(\frac{1}{4}\right)^a$$

$$\left. \begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -2a \end{array} \right\} y = -2a$$

Dus $S\left(\left(\frac{1}{4}\right)^a, -2a\right)$.

b Uit $AB = 4$ en $y_A = 0$ volgt $y_B = 4 \vee y_B = -4$.

$$g_a(x) = 0 \text{ geeft } {}^2\log(x\sqrt{x}) + a = 0$$

$${}^2\log(x^{1\frac{1}{2}}) = -a$$

$$1\frac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -\frac{2}{3}a$$

$$\left. \begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -\frac{2}{3}a \end{array} \right\} y_B = -\frac{2}{3}a$$

Dus $-\frac{2}{3}a = 4 \vee -\frac{2}{3}a = -4$ en dit geeft $a = -6 \vee a = 6$.

c De grafiek van g_a^{inv} snijdt de y -as in $(0, 64)$, dus de grafiek van g_a snijdt de x -as in $(64, 0)$.

$$g_a(64) = 0 \text{ geeft } {}^2\log(64\sqrt{64}) + a = 0$$

$$a = -{}^2\log(64 \cdot 8)$$

$$a = -{}^2\log(512)$$

$$a = -{}^2\log(2^9)$$

$$a = -9$$

$$\text{Dus } g_{-9}(x) = {}^2\log(x\sqrt{x}) - 9.$$

Voor g_{-9} geldt $y = {}^2\log(x\sqrt{x}) - 9$, dus voor g_{-9}^{inv} geldt $x = {}^2\log(y\sqrt{y}) - 9$.

$$x = {}^2\log(y\sqrt{y}) - 9 \text{ geeft } {}^2\log(y\sqrt{y}) = x + 9$$

$${}^2\log(y^{1\frac{1}{2}}) = x + 9$$

$$1\frac{1}{2} \cdot {}^2\log(y) = x + 9$$

$${}^2\log(y) = \frac{2}{3}x + 6$$

$$y = 2^{\frac{2}{3}x + 6}$$

$$\text{Dus } g_{-9}^{\text{inv}}(x) = 2^{\frac{2}{3}x + 6}.$$

50

$${}^2\log(N) = -1,8 + 0,3 \cdot {}^3\log(t)$$

$$0,3 \cdot {}^3\log(t) = 1,8 + {}^2\log(N)$$

$${}^3\log(t) = 6 + \frac{1}{0,3} \cdot {}^2\log(N)$$

$$t = 3^{6 + \frac{1}{0,3} \cdot {}^2\log(N)}$$

$$t = 3^6 \cdot 3^{\frac{1}{0,3} \cdot \frac{{}^2\log(N)}{{}^3\log(2)}}$$

$$t = 3^6 \cdot (3^{{}^3\log(N)})^{\frac{1}{0,3 \cdot {}^3\log(2)}}$$

$$t = 3^6 \cdot N^{\frac{1}{0,3 \cdot {}^3\log(2)}}$$

$$t \approx 729 \cdot N^{5,28}$$

$$\text{Dus } t = 729 \cdot N^{5,28}.$$

C7

$$\text{a } {}^2\log(x) = {}^4\log(x+12)$$

$${}^2\log(x) = \frac{{}^2\log(x+12)}{{}^2\log(4)}$$

$${}^2\log(x) = \frac{{}^2\log(x+12)}{2}$$

$$2 \cdot {}^2\log(x) = {}^2\log(x+12)$$

$${}^2\log(x^2) = {}^2\log(x+12)$$

$$x^2 = x + 12$$

$$x^2 - x - 12 = 0$$

$$(x+3)(x-4) = 0$$

$$x = -3 \vee x = 4$$

$$x = -3 \text{ voldoet niet, dus } x = 4.$$

$$\text{b } {}^3\log(2x) + 2 \cdot \frac{1}{3} \log(x-4) = 0$$

$${}^3\log(2x) - 2 \cdot {}^3\log(x-4) = 0$$

$${}^3\log(2x) = {}^3\log((x-4)^2)$$

$$2x = (x-4)^2$$

$$2x = x^2 - 8x + 16$$

$$x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0$$

$$x = 2 \vee x = 8$$

$$x = 2 \text{ voldoet niet, dus } x = 8.$$

C8

$$\text{a } {}^5\log^2(x) = {}^5\log(x) + 2$$

$$\text{Stel } {}^5\log(x) = u.$$

$$u^2 = u + 2$$

$$u^2 - u - 2 = 0$$

$$(u+1)(u-2) = 0$$

$$u = -1 \vee u = 2$$

$${}^5\log(x) = -1 \vee {}^5\log(x) = 2$$

$$x = \frac{1}{5} \vee x = 25$$

$$\text{b } 3^x = 18 - 3^{2x-1}$$

$$3^x = 18 - 3^{2x} \cdot 3^{-1}$$

$$3^x = 18 - \frac{1}{3} \cdot (3^x)^2$$

$$\text{Stel } 3^x = u.$$

$$u = 18 - \frac{1}{3}u^2$$

$$u^2 + 3u - 54 = 0$$

$$(u-6)(u+9) = 0$$

$$u = 6 \vee u = -9$$

$$3^x = 6 \vee 3^x = -9$$

$$x = {}^3\log(6)$$

$$\text{c } \log^2(x) + 3 \cdot \frac{1}{10} \log(x) + 2 = 0$$

$$\log^2(x) - 3 \cdot \log(x) + 2 = 0$$

$$\text{Stel } \log(x) = u.$$

$$u^2 - 3u + 2 = 0$$

$$(u-1)(u-2) = 0$$

$$u = 1 \vee u = 2$$

$$\log(x) = 1 \vee \log(x) = 2$$

$$x = 10 \vee x = 100$$

$$\text{d } 4^x + 8 = 6 \cdot 2^x$$

$$(2^2)^x + 8 = 6 \cdot 2^x$$

$$(2^x)^2 + 8 = 6 \cdot 2^x$$

$$\text{Stel } 2^x = u.$$

$$u^2 + 8 = 6u$$

$$u^2 - 6u + 8 = 0$$

$$(u-2)(u-4) = 0$$

$$u = 2 \vee u = 4$$

$$2^x = 2 \vee 2^x = 4$$

$$x = 1 \vee x = 2$$

C9

$$\text{a } N = 10 \cdot 2^{3t-1}$$

$$10 \cdot 2^{3t-1} = N$$

$$2^{3t-1} = 0,1N$$

$$3t-1 = {}^2\log(0,1N)$$

$$3t = 1 + {}^2\log(0,1N)$$

$$3t = 1 + {}^2\log(0,1) + {}^2\log(N)$$

$$3t = 1 + {}^2\log(0,1) + \frac{\log(N)}{\log(2)}$$

$$t = \frac{1 + {}^2\log(0,1)}{3} + \frac{\log(N)}{3 \cdot \log(2)}$$

$$t \approx -0,774 + 1,107 \cdot \log(N)$$

$$\text{Dus } t = -0,774 + 1,107 \cdot \log(N).$$

$$\text{b } y = 2 - \frac{1}{2} \cdot {}^3\log(2x)$$

$$\frac{1}{2} \cdot {}^3\log(2x) = 2 - y$$

$${}^3\log(2x) = 4 - 2y$$

$$2x = 3^{4-2y}$$

$$x = \frac{1}{2} \cdot 3^4 \cdot (3^{-2})^y$$

$$x = 40 \frac{1}{2} \cdot \left(\frac{1}{9}\right)^y$$

$$x = 40 \frac{1}{2} \cdot (10^{\log(\frac{1}{9})})^y$$

$$x \approx 40 \frac{1}{2} \cdot 10^{-0,954y}$$

$$\text{Dus } x = 40 \frac{1}{2} \cdot 10^{-0,954y}.$$

9.4 Het grondtal e

Bladzijde 27

- 51 a** $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h \rightarrow 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim_{h \rightarrow 0} \frac{2^x \cdot (2^h - 1)}{h} = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^x$
- b** $\frac{2^{0,01} - 1}{0,01} \approx 0,6956$
 $\frac{2^{0,001} - 1}{0,001} \approx 0,6934$
 $\frac{2^{0,0001} - 1}{0,0001} \approx 0,6932$
- c** $a \approx 0,693$
- d** $k(x) = 3^x$ geeft
 $k'(x) = \lim_{h \rightarrow 0} \frac{k(x+h) - k(x)}{h} = \lim_{h \rightarrow 0} \frac{3^{x+h} - 3^x}{h} = \lim_{h \rightarrow 0} \frac{3^x \cdot 3^h - 3^x}{h} = \lim_{h \rightarrow 0} \frac{3^x \cdot (3^h - 1)}{h} = \lim_{h \rightarrow 0} \frac{3^h - 1}{h} \cdot 3^x$
 $k'(x) = b \cdot 3^x$ met $b \approx \frac{3^{0,0001} - 1}{0,0001} \approx 1,099$

Bladzijde 28



- b** Omdat voor $h = 0$ de exponent $\frac{1}{h}$ niet bestaat.

c

h	y
0,01	2,7048
0,001	2,7169
0,0001	2,7181
0,00001	2,7183

- d** $g \approx 2,718$

Bladzijde 29

- 53 a** De regel ' $AB = AC$ geeft $A = 0 \vee B = C$ ' is gebruikt.
b Omdat $e^x > 0$ voor elke x , geeft $e^x = 0$ geen oplossing.

c $3xe^x = 10e^x$
 $3xe^x - 10e^x = 0$
 $e^x \cdot (3x - 10) = 0$
 $e^x = 0 \vee 3x - 10 = 0$
 $3x = 10$
 $x = 3\frac{1}{3}$

- 54 a** $2e^2 - e^2 = e^2$
b $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$
c $5e^2 \cdot 3e^3 = 15e^5$
d $\frac{12e^6}{4e^2} = 3e^4$
e $e^{5x} \cdot e^x = e^{6x}$
f $e^x \cdot e^2 = e^{x+2}$
g $5e^x - 3e^x = 2e^x$
h $e^x(e^2 + 1) = e^{x+2} + e^x$
i $e^x(e^x + 1) = e^{2x} + e^x$
j $(e^x + 1)^2 = e^{2x} + 2e^x + 1$

$$\mathbf{k} \quad (e^{3x} + 3)^2 = e^{6x} + 6e^{3x} + 9$$

$$\mathbf{l} \quad \frac{6e^{2x} - e^x}{e^x} = 6e^x - 1$$

Bladzijde 30

$$\mathbf{55} \quad \mathbf{a} \quad (2 + 3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$$

$$\mathbf{b} \quad (e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$$

$$\mathbf{c} \quad \frac{e^{2x} - 4}{e^x + 2} = \frac{(e^x + 2)(e^x - 2)}{e^x + 2} = e^x - 2$$

$$\mathbf{56} \quad \mathbf{a} \quad (2x + 4)e^x = 0$$

$$2x + 4 = 0 \vee e^x = 0$$

$$2x = -4$$

$$x = -2$$

$$\mathbf{b} \quad x^2 e^x = 3x e^x$$

$$e^x = 0 \vee x^2 = 3x$$

$$x = 0 \vee x = 3$$

$$\mathbf{c} \quad x^2 e^x = e^x$$

$$e^x = 0 \vee x^2 = 1$$

$$x = 1 \vee x = -1$$

$$\mathbf{d} \quad e^{3x} - e^x = 0$$

$$e^{3x} = e^x$$

$$3x = x$$

$$x = 0$$

$$\mathbf{e} \quad e^{4x} - 1 = 0$$

$$e^{4x} = 1$$

$$4x = 0$$

$$x = 0$$

$$\mathbf{f} \quad e^x \cdot e^x = e^6$$

$$e^{2x} = e^6$$

$$2x = 6$$

$$x = 3$$

$$\mathbf{57} \quad \mathbf{a} \quad e^x + e^x = 2e^6$$

$$2e^x = 2e^6$$

$$e^x = e^6$$

$$x = 6$$

$$\mathbf{b} \quad \frac{e^{5x}}{e^x} = e$$

$$e^{4x} = e^1$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$\mathbf{c} \quad 2xe^x + e^x = 0$$

$$e^x(2x + 1) = 0$$

$$e^x = 0 \vee 2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$\mathbf{d} \quad e^{x+2} - \sqrt{e} = 0$$

$$e^{x+2} = \sqrt{e}$$

$$e^{x+2} = e^{\frac{1}{2}}$$

$$x + 2 = \frac{1}{2}$$

$$x = -1\frac{1}{2}$$

$$\mathbf{e} \quad e^x + 3 = 2e^{-x} + 2$$

$$e^{2x} + 3e^x = 2 + 2e^x$$

$$(e^x)^2 + e^x - 2 = 0$$

$$\text{Stel } e^x = u.$$

$$u^2 + u - 2 = 0$$

$$(u - 1)(u + 2) = 0$$

$$u = 1 \vee u = -2$$

$$e^x = 1 \vee e^x = -2$$

$$x = 0$$

$$\mathbf{f} \quad e^{6x} + 1 = 2e^{3x}$$

$$(e^{3x})^2 - 2e^{3x} + 1 = 0$$

$$\text{Stel } e^{3x} = u.$$

$$u^2 - 2u + 1 = 0$$

$$(u - 1)^2 = 0$$

$$u = 1$$

$$e^{3x} = 1$$

$$3x = 0$$

$$x = 0$$

$$\mathbf{58} \quad e^{2x} + e^3 = (e + 1)e^{x+1}$$

$$e^{2x} + e^3 = (e + 1)e^x \cdot e$$

$$e^{2x} + e^3 = (e^2 + e)e^x$$

$$(e^x)^2 - (e^2 + e)e^x + e^3 = 0$$

$$\text{Stel } e^x = u.$$

$$u^2 - (e^2 + e)u + e^3 = 0$$

$$(u - e)(u - e^2) = 0$$

$$u = e \vee u = e^2$$

$$e^x = e \vee e^x = e^2$$

$$x = 1 \vee x = 2$$

59 $g(x) = 2e^x$ geeft $g'(x) = 2e^x$
 $h(x) = e^{2x}$ geeft $h'(x) = e^{2x} \cdot 2 = 2e^{2x}$
 $k(x) = \frac{1}{e^x} = e^{-x}$ geeft $k'(x) = e^{-x} \cdot -1 = -e^{-x} = -\frac{1}{e^x}$
 $l(x) = \sqrt{e^x} = e^{\frac{1}{2}x}$ geeft $l'(x) = e^{\frac{1}{2}x} \cdot \frac{1}{2} = \frac{1}{2}e^{\frac{1}{2}x} = \frac{1}{2}\sqrt{e^x}$

Bladzijde 32

- 60 a** In voorbeeld c zijn de productregel en de kettingregel gebruikt.
 In voorbeeld d zijn de quotiëntregel en de kettingregel gebruikt.

b $f'(x) = \frac{e^{x^2} \cdot 3x^2 - x^3 \cdot e^{x^2} \cdot 2x}{(e^{x^2})^2} = \frac{e^{x^2}(3x^2 - 2x^4)}{e^{x^2} \cdot e^{x^2}} = \frac{3x^2 - 2x^4}{e^{x^2}}$

c $f(x) = \frac{x^3}{e^{x^2}} = x^3 \cdot e^{-x^2}$ geeft

$$f'(x) = 3x^2 \cdot e^{-x^2} + x^3 \cdot e^{-x^2} \cdot -2x = 3x^2 \cdot e^{-x^2} - 2x^4 \cdot e^{-x^2} = \frac{3x^2}{e^{x^2}} - \frac{2x^4}{e^{x^2}} = \frac{3x^2 - 2x^4}{e^{x^2}}$$

Dus ja, je krijgt hetzelfde antwoord.

61 a $f(x) = e^x + 2$ geeft $f'(x) = e^x$

b $f(x) = 2e^x + \frac{1}{x} = 2e^x + x^{-1}$ geeft $f'(x) = 2e^x - x^{-2} = 2e^x - \frac{1}{x^2}$

c $f(x) = xe^x + 4$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (x+1)e^x$

d $f(x) = \frac{x}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \frac{1-x}{e^x}$

e $f(x) = (2x-4)e^x$ geeft $f'(x) = 2 \cdot e^x + (2x-4) \cdot e^x = (2x-2)e^x$

f $f(x) = \frac{2e^x}{x-1}$ geeft $f'(x) = \frac{(x-1) \cdot 2e^x - 2e^x \cdot 1}{(x-1)^2} = \frac{2xe^x - 2e^x - 2e^x}{(x-1)^2} = \frac{(2x-4)e^x}{(x-1)^2}$

62 a $f(x) = e^{x^2+x}$ geeft $f'(x) = e^{x^2+x} \cdot (2x+1) = (2x+1)e^{x^2+x}$

b $f(x) = x^2 + 2e^{3x}$ geeft $f'(x) = 2x + 2e^{3x} \cdot 3 = 2x + 6e^{3x}$

c $f(x) = xe^{x^2}$ geeft $f'(x) = 1 \cdot e^{x^2} + x \cdot e^{x^2} \cdot 2x = (2x^2+1)e^{x^2}$

d $f(x) = \frac{2e^{-x-1}}{x^2}$ geeft

$$f'(x) = \frac{x^2 \cdot 2e^{-x-1} \cdot -1 - 2e^{-x-1} \cdot 2x}{x^4} = \frac{-2xe^{-x-1} - 4e^{-x-1}}{x^3} = \frac{(-2x-4)e^{-x-1}}{x^3}$$

e $f(x) = 3xe^{2x-1}$ geeft $f'(x) = 3 \cdot e^{2x-1} + 3x \cdot e^{2x-1} \cdot 2 = (6x+3)e^{2x-1}$

f $f(x) = \frac{e^{2x}}{e^{2x}+1}$ geeft $f'(x) = \frac{(e^{2x}+1) \cdot e^{2x} \cdot 2 - e^{2x} \cdot e^{2x} \cdot 2}{(e^{2x}+1)^2} = \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x}+1)^2} = \frac{2e^{2x}}{(e^{2x}+1)^2}$

63 a $f(x) = xe^e$ geeft $f'(x) = ex^{e-1}$

b $f(x) = \sqrt{e^{2x} - e^2}$ geeft $f'(x) = \frac{1}{2\sqrt{e^{2x} - e^2}} \cdot e^{2x} \cdot 2 = \frac{e^{2x}}{\sqrt{e^{2x} - e^2}}$

c $f(x) = \frac{x+e}{ex+2}$ geeft $f'(x) = \frac{(ex+2) \cdot 1 - (x+e) \cdot e}{(ex+2)^2} = \frac{ex+2-ex-e^2}{(ex+2)^2} = \frac{2-e^2}{(ex+2)^2}$

d $f(x) = \frac{1}{e(ex-1)^2} = \frac{1}{e}(ex-1)^{-2}$ geeft $f'(x) = -\frac{2}{e}(ex-1)^{-3} \cdot e = -\frac{2}{(ex-1)^3}$

64 a $f(x) = -xe^x$ geeft $f'(x) = -1 \cdot e^x - x \cdot e^x = (-x-1)e^x$
 $f'(x) = 0$ geeft $(-x-1)e^x = 0$

$$-x-1 = 0 \vee e^x = 0$$

$$-x = 1$$

$$x = -1$$

max. is $f(-1) = e^{-1} = \frac{1}{e}$

$$\begin{aligned} \text{b } f'(x) &= (-x-1)e^x \text{ geeft } f''(x) = -1 \cdot e^x + (-x-1) \cdot e^x = (-x-2)e^x \\ f''(x) &= 0 \text{ geeft } (-x-2)e^x = 0 \\ -x-2 &= 0 \vee e^x = 0 \\ -x &= 2 \\ x &= -2 \\ f(-2) &= 2e^{-2} = \frac{2}{e^2}, \text{ dus de coördinaten van het buigpunt zijn } \left(-2, \frac{2}{e^2}\right). \end{aligned}$$

$$k: y = ax + b \text{ met } a = f'(-2) = (2-1)e^{-2} = \frac{1}{e^2}$$

$$y = \frac{1}{e^2}x + b \quad \left\{ \begin{array}{l} \frac{1}{e^2} \cdot -2 + b = \frac{2}{e^2} \\ \frac{-2}{e^2} + b = \frac{2}{e^2} \\ b = \frac{4}{e^2} \end{array} \right.$$

$$\text{Dus } k: y = \frac{1}{e^2}x + \frac{4}{e^2}.$$

$$\begin{aligned} 65 \text{ a } f(x) &= 0 \text{ geeft } (x^2-3)e^x = 0 \\ x^2-3 &= 0 \vee e^x = 0 \\ x^2 &= 3 \\ x &= \sqrt{3} \vee x = -\sqrt{3} \end{aligned}$$

De nulpunten zijn $\sqrt{3}$ en $-\sqrt{3}$.

$$\begin{aligned} \text{b } f(x) &= (x^2-3)e^x \text{ geeft } f'(x) = 2x \cdot e^x + (x^2-3) \cdot e^x = (x^2+2x-3)e^x \\ f'(x) &= 0 \text{ geeft } (x^2+2x-3)e^x = 0 \\ x^2+2x-3 &= 0 \vee e^x = 0 \\ (x-1)(x+3) &= 0 \\ x &= 1 \vee x = -3 \end{aligned}$$

$$\text{max. is } f(-3) = (9-3)e^{-3} = \frac{6}{e^3} \text{ en min. is } f(1) = (1-3)e^1 = -2e$$

$$\begin{aligned} \text{c } f'(x) &= (x^2+2x-3)e^x \text{ geeft } f''(x) = (2x+2) \cdot e^x + (x^2+2x-3) \cdot e^x = (x^2+4x-1)e^x \\ f''(x) &= 0 \text{ geeft } (x^2+4x-1)e^x = 0 \\ x^2+4x-1 &= 0 \vee e^x = 0 \\ D &= 4^2 - 4 \cdot 1 \cdot -1 = 20, \text{ dus } \sqrt{D} = 2\sqrt{5} \\ x &= \frac{-4+2\sqrt{5}}{2} \vee x = \frac{-4-2\sqrt{5}}{2} \\ x &= -2+\sqrt{5} \vee x = -2-\sqrt{5} \end{aligned}$$

De x-coördinaten van de buigpunten zijn $-2+\sqrt{5}$ en $-2-\sqrt{5}$.

Bladzijde 33

$$\begin{aligned} 66 \text{ a } f(x) &= g(x) \text{ geeft } \frac{1}{e^{x+3}} = e^{2x} \\ e^{-x-3} &= e^{2x} \\ -x-3 &= 2x \\ -3x &= 3 \\ x &= -1 \end{aligned}$$

$$f(-1) = \frac{1}{e^2}, \text{ dus } S\left(-1, \frac{1}{e^2}\right).$$

b $f(x) = \frac{1}{e^{x+3}} = e^{-x-3}$ geeft $f'(x) = e^{-x-3} \cdot -1 = -e^{-x-3}$

$k: y = ax + b$ met $a = f'(-1) = -e^{-2} = -\frac{1}{e^2}$

$$\left. \begin{array}{l} y = -\frac{1}{e^2}x + b \\ \text{door } S\left(-1, \frac{1}{e^2}\right) \end{array} \right\} \begin{array}{l} -\frac{1}{e^2} \cdot -1 + b = \frac{1}{e^2} \\ \frac{1}{e^2} + b = \frac{1}{e^2} \\ b = 0 \end{array}$$

Dus k gaat door de oorsprong.

c $g(x) = e^{2x}$ geeft $g'(x) = e^{2x} \cdot 2 = 2e^{2x}$

$l: y = ax + b$ met $a = g'(-1) = 2e^{-2} = \frac{2}{e^2}$

$$\left. \begin{array}{l} y = \frac{2}{e^2}x + b \\ \text{door } S\left(-1, \frac{1}{e^2}\right) \end{array} \right\} \begin{array}{l} \frac{2}{e^2} \cdot -1 + b = \frac{1}{e^2} \\ -\frac{2}{e^2} + b = \frac{1}{e^2} \\ b = \frac{3}{e^2} \end{array}$$

Dus $l: y = \frac{2}{e^2}x + \frac{3}{e^2}$.

$y = 0$ geeft $\frac{2}{e^2}x + \frac{3}{e^2} = 0$

$$\begin{aligned} \frac{2}{e^2}x &= -\frac{3}{e^2} \\ x &= -\frac{3}{e^2} \cdot \frac{e^2}{2} = -1\frac{1}{2} \end{aligned}$$

Dus $x_A = -1\frac{1}{2}$.

67

a $rc_{AB} = \frac{\Delta y}{\Delta x} = \frac{f_a(2) - f_a(1)}{2 - 1} = 2e^{2a} - e^a$

$rc_{AB} = 6$ geeft $2e^{2a} - e^a = 6$

$$2 \cdot (e^a)^2 - e^a - 6 = 0$$

Stel $e^a = u$.

$$2u^2 - u - 6 = 0$$

$$u^2 - \frac{1}{2}u - 3 = 0$$

$$(u + 1\frac{1}{2})(u - 2) = 0$$

$$u = -1\frac{1}{2} \vee u = 2$$

$$e^a = -1\frac{1}{2} \vee e^a = 2$$

$e^a = 2$ geeft $y_A = e^a = 2$ en $y_B = 2e^{2a} = 2 \cdot (e^a)^2 = 2 \cdot 2^2 = 8$.

b $f_a(x) = x e^{ax}$ geeft $f'_a(x) = 1 \cdot e^{ax} + x \cdot e^{ax} \cdot a = (1 + ax)e^{ax}$

$f'_a(p) = 0$ geeft $(1 + ap)e^{ap} = 0$

$$1 + ap = 0 \vee e^{ap} = 0$$

$$ap = -1$$

$$p = -\frac{1}{a}$$

$f'_a(x) = (1 + ax)e^{ax}$ geeft $f''_a(x) = a \cdot e^{ax} + (1 + ax) \cdot e^{ax} \cdot a = (2 + ax)a e^{ax}$

$f''_a(q) = 0$ geeft $(2 + aq)a e^{aq} = 0$

$$2 + aq = 0 \vee a = 0 \text{ (voldoet niet)} \vee e^{aq} = 0$$

$$aq = -2$$

$$q = -\frac{2}{a}$$

Dus $q = 2p$.

68 a $f(x) = x e^{-\frac{1}{2}x^2}$ geeft $f'(x) = 1 \cdot e^{-\frac{1}{2}x^2} + x \cdot e^{-\frac{1}{2}x^2} \cdot -x = (1 - x^2) e^{-\frac{1}{2}x^2}$
 $f'(x) = 0$ geeft $(1 - x^2) e^{-\frac{1}{2}x^2} = 0$
 $1 - x^2 = 0 \vee e^{-\frac{1}{2}x^2} = 0$
 $x^2 = 1$
 $x = 1 \vee x = -1$

$$y_A = f(-1) = -1 \cdot e^{-\frac{1}{2}} = -\frac{1}{e^{\frac{1}{2}}} = -\frac{1}{\sqrt{e}}$$

$$y_B = f(1) = 1 \cdot e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}}$$

$$y_B - y_A = \frac{1}{\sqrt{e}} - \left(-\frac{1}{\sqrt{e}}\right) = \frac{2}{\sqrt{e}} = \sqrt{\frac{4}{e}}$$

Dus $c = \frac{4}{e}$.

b $y = ax + b$ met $a = f'(p) = (1 - p^2) e^{-\frac{1}{2}p^2}$
 $y = (1 - p^2) e^{-\frac{1}{2}p^2} \cdot x + b$
 door $P(p, p e^{-\frac{1}{2}p^2})$ $\left\{ \begin{array}{l} (1 - p^2) e^{-\frac{1}{2}p^2} \cdot p + b = p e^{-\frac{1}{2}p^2} \\ p e^{-\frac{1}{2}p^2} - p^3 e^{-\frac{1}{2}p^2} + b = p e^{-\frac{1}{2}p^2} \end{array} \right.$
 $b = p^3 e^{-\frac{1}{2}p^2}$

Dus $y = (1 - p^2) e^{-\frac{1}{2}p^2} \cdot x + p^3 e^{-\frac{1}{2}p^2}$.

$y = 0$ geeft $(1 - p^2) e^{-\frac{1}{2}p^2} \cdot x + p^3 e^{-\frac{1}{2}p^2} = 0$
 $(1 - p^2) e^{-\frac{1}{2}p^2} \cdot x = -p^3 e^{-\frac{1}{2}p^2}$

$$x = \frac{-p^3 e^{-\frac{1}{2}p^2}}{(1 - p^2) e^{-\frac{1}{2}p^2}}$$

$$x = \frac{p^3}{p^2 - 1}$$

c $x = \frac{p^3}{p^2 - 1}$ geeft $x' = \frac{(p^2 - 1) \cdot 3p^2 - p^3 \cdot 2p}{(p^2 - 1)^2} = \frac{3p^4 - 3p^2 - 2p^4}{(p^2 - 1)^2} = \frac{p^4 - 3p^2}{(p^2 - 1)^2}$

$x' = 0$ geeft $p^4 - 3p^2 = 0$

$$p^2(p^2 - 3) = 0$$

$$p = 0 \vee p^2 = 3$$

$$p = 0 \vee p = \sqrt{3} \vee p = -\sqrt{3}$$

Er geldt dat $p > x_B$ oftewel $p > 1$, dus $p = \sqrt{3}$.

De minimale waarde van de x -coördinaat van Q is $x = \frac{(\sqrt{3})^3}{(\sqrt{3})^2 - 1} = \frac{3\sqrt{3}}{3 - 1} = 1\frac{1}{2}\sqrt{3}$.

69 $f_a(x) = x e^{ax^3}$ geeft $f'_a(x) = 1 \cdot e^{ax^3} + x \cdot e^{ax^3} \cdot 3ax^2 = (1 + 3ax^3) e^{ax^3}$
 $f'_a(2) = 0$ geeft $(1 + 3a \cdot 2^3) e^{a \cdot 2^3} = 0$

$$(1 + 24a) e^{8a} = 0$$

$$1 + 24a = 0 \vee e^{8a} = 0$$

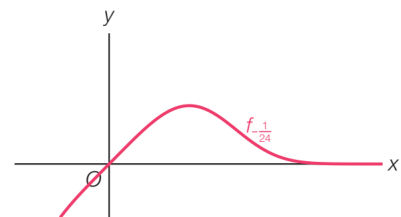
$$24a = -1$$

$$a = -\frac{1}{24}$$

$a = -\frac{1}{24}$ geeft $f_{-\frac{1}{24}}(x) = x e^{-\frac{1}{24}x^3}$

Zie de figuur hiernaast.

De extreme waarde voor $x = 2$ is een maximum.



Bladzijde 34**C10**

- a** $5e^3 - 3e^3 = 2e^3$
b $2e^5 \cdot 5e^2 = 10e^7$
c $\frac{15e^{15}}{5e^5} = 3e^{10}$
d $e^{2x} \cdot e^{3x} = e^{5x}$
e $e^x(e^{2x} + e) = e^{3x} + e^{x+1}$
f $(e^{3x} - 5)^2 = e^{6x} - 10e^{3x} + 25$

C11

- a** $e^{3x} - e^{x-1} = 0$
 $e^{3x} = e^{x-1}$
 $3x = x - 1$
 $2x = -1$
 $x = -\frac{1}{2}$
b $x^2 e^x = 4e^x$
 $e^x = 0 \vee x^2 = 4$
 $x = 2 \vee x = -2$
c $e^{6x} + 5e^{3x} = 6$
 $(e^{3x})^2 + 5e^{3x} - 6 = 0$
 Stel $e^{3x} = u$.
 $u^2 + 5u - 6 = 0$
 $(u - 1)(u + 6) = 0$
 $u = 1 \vee u = -6$
 $e^{3x} = 1 \vee e^{3x} = -6$
 $3x = 0$
 $x = 0$

C12

- a** $f(x) = x^2 e^x$ geeft $f'(x) = 2x \cdot e^x + x^2 \cdot e^x = (x^2 + 2x)e^x$
b $f(x) = \frac{x^3}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 3x^2 - x^3 \cdot e^x}{(e^x)^2} = \frac{3x^2 - x^3}{e^x}$
c $f(x) = 2e^{x^2+1}$ geeft $f'(x) = 2e^{x^2+1} \cdot 2x = 4xe^{x^2+1}$

9.5 De natuurlijke logaritme**Bladzijde 35****70**

- a** $f(x) = 2^x$ geeft $f^{\text{inv}}(x) = {}^2\log(x)$
b $g(x) = e^x$ geeft $g^{\text{inv}}(x) = {}^e\log(x)$

Bladzijde 36**71**

- a** $\ln(e) = 1$
b $\ln(e\sqrt{e}) = \ln(e^{1\frac{1}{2}}) = 1\frac{1}{2}$
c $\ln\left(\frac{1}{e}\right) = \ln(e^{-1}) = -1$
d $\ln(1) = 0$
e $3\ln(e \cdot \sqrt[3]{e}) = 3\ln(e^{1\frac{1}{3}}) = 3 \cdot 1\frac{1}{3} = 4$
f $\ln^2(e^3) = 3^2 = 9$
g $e^{\ln(7)} + e^{2\ln(7)} = 7 + e^{\ln(7^2)} = 7 + 49 = 56$
h $e^{\frac{1}{2}\ln(5)} = e^{\ln(5^{\frac{1}{2}})} = 5^{\frac{1}{2}} = \sqrt{5}$
i $e^{\ln(10)} \cdot e^{\ln(3)} = 10 \cdot 3 = 30$

72 a $5e^{3x} = 60$
 $e^{3x} = 12$
 $3x = \ln(12)$
 $x = \frac{1}{3}\ln(12) \approx 0,828$

b $6 + e^{0,5x+2} = 10$
 $e^{0,5x+2} = 4$
 $0,5x + 2 = \ln(4)$
 $0,5x = -2 + \ln(4)$
 $x = -4 + 2\ln(4) \approx -1,227$

c $\frac{3}{5e^{2x-1}} = 10$
 $50e^{2x-1} = 3$
 $e^{2x-1} = 0,06$
 $2x - 1 = \ln(0,06)$
 $2x = 1 + \ln(0,06)$
 $x = \frac{1}{2} + \frac{1}{2}\ln(0,06) \approx -0,907$

73 a $2\ln(3) + \ln(4) = \ln(3^2) + \ln(4) = \ln(9) + \ln(4) = \ln(36)$
b $\ln(20) - 3\ln(2) = \ln(20) - \ln(2^3) = \ln(20) - \ln(8) = \ln(2\frac{1}{2})$
c $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$
d $1 + \ln(10) = \ln(e) + \ln(10) = \ln(10e)$
e $\frac{1}{2} + 2\ln(6) = \ln(e^{\frac{1}{2}}) + \ln(6^2) = \ln(\sqrt{e}) + \ln(36) = \ln(36\sqrt{e})$
f $e + \ln(2) = \ln(e^e) + \ln(2) = \ln(2e^e)$

74 a $\ln(x) = -1$
 $x = e^{-1} = \frac{1}{e}$
b $4\ln(x) = 2$
 $\ln(x) = \frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e}$

c $\ln(3x) = 3$
 $3x = e^3$
 $x = \frac{1}{3}e^3$
d $\ln(-x+2) = -2$
 $-x+2 = e^{-2}$
 $-x = -2 + \frac{1}{e^2}$
 $x = 2 - \frac{1}{e^2}$

e $\ln^2(x) = \frac{1}{4}$
 $\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e} \vee x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$
f $\ln(x) = 1 + \ln(5)$
 $\ln(x) = \ln(e) + \ln(5)$
 $\ln(x) = \ln(5e)$
 $x = 5e$

75 a $x^2 \ln(x+1) = 4\ln(x+1)$
 $\ln(x+1) = 0 \vee x^2 = 4$
 $x+1 = 1 \vee x = 2 \vee x = -2$
 $x = 0 \vee x = 2 \vee x = -2$
 $x = -2$ voldoet niet, dus $x = 0 \vee x = 2$.
b $\ln^2(x) - \ln(x) = 0$
 $\ln(x)(\ln(x) - 1) = 0$
 $\ln(x) = 0 \vee \ln(x) = 1$
 $x = 1 \vee x = e$
c $4e^{1-3x} = 20$
 $e^{1-3x} = 5$
 $1 - 3x = \ln(5)$
 $-3x = -1 + \ln(5)$
 $x = \frac{1}{3} - \frac{1}{3}\ln(5)$
d $\ln^2(x) - 2\ln(x) - 3 = 0$
Stel $\ln(x) = u$.
 $u^2 - 2u - 3 = 0$
 $(u+1)(u-3) = 0$
 $u = -1 \vee u = 3$
 $\ln(x) = -1 \vee \ln(x) = 3$
 $x = e^{-1} = \frac{1}{e} \vee x = e^3$
e $e^{x^2} = 100$
 $x^2 = \ln(100)$
 $x = \sqrt{\ln(100)} \vee x = -\sqrt{\ln(100)}$

f $2\ln(x) = \ln(2) + \ln(x+4)$
 $\ln(x^2) = \ln(2(x+4))$
 $x^2 = 2x + 8$
 $x^2 - 2x - 8 = 0$
 $(x+2)(x-4) = 0$
 $x = -2 \vee x = 4$
 $x = -2$ voldoet niet, dus $x = 4$.
g $2e^{3x} - 6e^{2x} = 0$
 $2e^{2x}(e^x - 3) = 0$
 $e^{2x} = 0 \vee e^x = 3$
 $x = \ln(3)$
h $\ln(x+3) - \ln(x-1) = \ln(2)$
 $\ln\left(\frac{x+3}{x-1}\right) = \ln(2)$
 $\frac{x+3}{x-1} = 2$
 $2x - 2 = x + 3$
 $x = 5$
i $e^{4x} - 8 = 2e^{2x}$
 $(e^{2x})^2 - 8 = 2e^{2x}$
Stel $e^{2x} = u$.
 $u^2 - 8 = 2u$
 $u^2 - 2u - 8 = 0$
 $(u+2)(u-4) = 0$
 $u = -2 \vee u = 4$
 $e^{2x} = -2 \vee e^{2x} = 4$
 $2x = \ln(4)$
 $x = \frac{1}{2}\ln(4) = \ln(2)$

Bladzijde 37

- 76 a** $G = 100$ geeft $F = 16(0,6 + \ln(100)) \approx 83$
De hartslagfrequentie is 83 slagen per minuut.
- b** $F = 78$ geeft $16(0,6 + \ln(G)) = 78$
 $0,6 + \ln(G) = 4,875$
 $\ln(G) = 4,275$
 $G = e^{4,275} \approx 72$

Dus het hiermee corresponderende gewicht is 72 kg.

- c** $F = 16(0,6 + \ln(G))$
 $16(0,6 + \ln(G)) = F$
 $0,6 + \ln(G) = \frac{1}{16}F$
 $\ln(G) = \frac{1}{16}F - 0,6$
 $G = e^{\frac{1}{16}F - 0,6}$
 $G = e^{\frac{1}{16}F} \cdot e^{-0,6}$
 $G = e^{-0,6} \cdot (e^{\frac{1}{16}})^F$
 $G \approx 0,549 \cdot 1,064^F$
Dus $G = 0,549 \cdot 1,064^F$.

- 77 a** $2^x = (e^{\ln(2)})^x = e^{\ln(2) \cdot x}$
- b** $f(x) = 2^x = e^{\ln(2) \cdot x}$ geeft $f'(x) = e^{\ln(2) \cdot x} \cdot \ln(2) = 2^x \cdot \ln(2)$

- 78 a** $e^{\ln(x)} = x$ geeft $[e^{\ln(x)}]' = x'$
 $e^{\ln(x)} \cdot [\ln(x)]' = 1$
 $e^{\ln(x)} \cdot f'(x) = 1$
- b** $e^{\ln(x)} \cdot f'(x) = 1$
 $x \cdot f'(x) = 1$
 $f'(x) = \frac{1}{x}$
- c** $h(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)} \cdot \ln(x)$ geeft $h'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{x} = \frac{1}{x \ln(2)}$

Bladzijde 38

- 79 a** $f(x) = \ln(ax)$ geeft $f'(x) = \frac{1}{ax} \cdot a = \frac{1}{x}$
- b** $f(x) = \ln(2x)$ geeft $f'(x) = \frac{1}{x}$
 $g(x) = \ln(x\sqrt{2})$ geeft $g'(x) = \frac{1}{x}$
- c** $f(x) = \ln(x^n)$ geeft $f'(x) = \frac{1}{x^n} \cdot nx^{n-1} = \frac{nx^{n-1}}{x \cdot x^{n-1}} = \frac{n}{x}$
- d** $f(x) = \ln(x^2)$ geeft $f'(x) = \frac{2}{x}$
 $g(x) = \ln\left(\frac{1}{x}\right) = \ln(x^{-1})$ geeft $g'(x) = \frac{-1}{x}$
 $h(x) = \ln(\sqrt{x}) = \ln(x^{\frac{1}{2}})$ geeft $h'(x) = \frac{1}{2x}$
- 80 a** $f(x) = 3^{4x-2}$ geeft $f'(x) = 3^{4x-2} \cdot \ln(3) \cdot 4$
- b** $f(x) = \frac{1 - \ln(x)}{x}$ geeft $f'(x) = \frac{x \cdot -\frac{1}{x} - (1 - \ln(x)) \cdot 1}{x^2} = \frac{-1 - 1 + \ln(x)}{x^2} = \frac{\ln(x) - 2}{x^2}$
- c** $f(x) = {}^2\log(4x - 1)$ geeft $f'(x) = \frac{1}{(4x - 1)\ln(2)} \cdot 4 = \frac{4}{(4x - 1)\ln(2)}$
- d** $f(x) = x \ln(x^3)$ geeft $f'(x) = 1 \cdot \ln(x^3) + x \cdot \frac{3}{x} = \ln(x^3) + 3$

$$\text{e } f(x) = \frac{\ln(3x)}{x} \text{ geeft } f'(x) = \frac{x \cdot \frac{1}{x} - \ln(3x) \cdot 1}{x^2} = \frac{1 - \ln(3x)}{x^2}$$

$$\text{f } f(x) = \ln(x^2 + x) \text{ geeft } f'(x) = \frac{1}{x^2 + x} \cdot (2x + 1) = \frac{2x + 1}{x^2 + x}$$

$$\text{81 a } f(x) = \ln(2^x) = x \ln(2) \text{ geeft } f'(x) = \ln(2)$$

$$\text{b } f(x) = {}^2\log(x^2 + 1) \text{ geeft } f'(x) = \frac{1}{(x^2 + 1) \ln(2)} \cdot 2x = \frac{2x}{(x^2 + 1) \ln(2)}$$

$$\text{c } f(x) = x \ln^2(x) \text{ geeft } f'(x) = 1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} = \ln^2(x) + 2 \ln(x)$$

$$\text{d } f(x) = x^2 \cdot {}^3\log(4x) \text{ geeft } f'(x) = 2x \cdot {}^3\log(4x) + x^2 \cdot \frac{1}{4x \ln(3)} \cdot 4 = 2x \cdot {}^3\log(4x) + \frac{x}{\ln(3)}$$

$$\text{e } f(x) = (2x - 1) \cdot 2^x \text{ geeft } f'(x) = 2 \cdot 2^x + (2x - 1) \cdot 2^x \cdot \ln(2) = (2x \ln(2) + 2 - \ln(2)) \cdot 2^x$$

$$\text{f } f(x) = \ln^2(4x) \text{ geeft } f'(x) = 2 \ln(4x) \cdot \frac{1}{x} = \frac{2 \ln(4x)}{x}$$

Bladzijde 39

$$\text{82 a } x^n = (e^{\ln(x)})^n = e^{n \ln(x)}$$

$$\text{b } f(x) = e^{n \ln(x)} \text{ geeft } f'(x) = e^{n \ln(x)} \cdot [n \ln(x)]' = e^{n \ln(x)} \cdot \frac{n}{x}$$

$$\text{c } e^{n \ln(x)} \cdot \frac{n}{x} = x^n \cdot \frac{n}{x} = n \cdot \frac{x^n}{x} = nx^{n-1}$$

Er is geen gebruik gemaakt van enige beperking voor n , dus de regel geldt ook voor elke niet-gehele n van $\mathbb{R}$.

$$\text{83 a } f(x) = \frac{x}{\ln(x)} \text{ geeft } f'(x) = \frac{\ln(x) \cdot 1 - x \cdot \frac{1}{x}}{\ln^2(x)} = \frac{\ln(x) - 1}{\ln^2(x)}$$

$$k: y = ax + b \text{ met } a \cdot f'\left(\frac{1}{e}\right) = -1$$

$$f'\left(\frac{1}{e}\right) = \frac{\ln\left(\frac{1}{e}\right) - 1}{\ln^2\left(\frac{1}{e}\right)} = \frac{-1 - 1}{(-1)^2} = -2, \text{ dus } a = \frac{1}{2}.$$

$$\left. \begin{aligned} y &= \frac{1}{2}x + b \\ f\left(\frac{1}{e}\right) &= \frac{\frac{1}{e}}{\ln\left(\frac{1}{e}\right)} = \frac{\frac{1}{e}}{-1} = -\frac{1}{e}, \text{ dus } A\left(\frac{1}{e}, -\frac{1}{e}\right) \end{aligned} \right\} \begin{aligned} \frac{1}{2} \cdot \frac{1}{e} + b &= -\frac{1}{e} \\ b &= -\frac{1}{e} - \frac{1}{2e} \\ b &= -\frac{3}{2e} \end{aligned}$$

$$\text{Dus } k: y = \frac{1}{2}x - \frac{3}{2e}.$$

$$y = 0 \text{ geeft } \frac{1}{2}x - \frac{3}{2e} = 0$$

$$\frac{1}{2}x = \frac{3}{2e}$$

$$x = \frac{3}{e}$$

$$\text{Dus } x_B = \frac{3}{e}.$$

$$\begin{aligned}
 \text{b } f'(x) = -6 \text{ geeft } \frac{\ln(x) - 1}{\ln^2(x)} &= -6 \\
 \ln(x) - 1 &= -6\ln^2(x) \\
 \text{Stel } \ln(x) &= u. \\
 u - 1 &= -6u^2 \\
 6u^2 + u - 1 &= 0 \\
 D = 1^2 - 4 \cdot 6 \cdot (-1) &= 25, \text{ dus } \sqrt{D} = 5 \\
 u = \frac{-1 \pm 5}{12} &= \frac{1}{3} \vee u = \frac{-1 - 5}{12} = -\frac{1}{2} \\
 \ln(x) = \frac{1}{3} \vee \ln(x) &= -\frac{1}{2} \\
 x = e^{\frac{1}{3}} \vee x &= e^{-\frac{1}{2}}
 \end{aligned}$$

$$f(e^{\frac{1}{3}}) = \frac{e^{\frac{1}{3}}}{\frac{1}{3}} = 3e^{\frac{1}{3}}, \text{ dus raakpunt } (e^{\frac{1}{3}}, 3e^{\frac{1}{3}}) = (\sqrt[3]{e}, 3 \cdot \sqrt[3]{e}).$$

$$f(e^{-\frac{1}{2}}) = \frac{e^{-\frac{1}{2}}}{-\frac{1}{2}} = -2e^{-\frac{1}{2}}, \text{ dus raakpunt } (e^{-\frac{1}{2}}, -2e^{-\frac{1}{2}}) = \left(\frac{1}{\sqrt{e}}, -\frac{2}{\sqrt{e}}\right).$$

$$\begin{aligned}
 \text{84 a } f(x) &= 2^{2x} - 2^x \text{ geeft } f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = (2^{2x+1} - 2^x) \ln(2) \\
 f'(x) &= 0 \text{ geeft } (2^{2x+1} - 2^x) \ln(2) = 0 \\
 2^{2x+1} - 2^x &= 0 \\
 2^{2x+1} &= 2^x \\
 2x + 1 &= x \\
 x &= -1
 \end{aligned}$$

$$\text{min. is } f(-1) = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}, \text{ dus } B_f = [-\frac{1}{4}, \rightarrow).$$

$$\begin{aligned}
 \text{b } f'(x) &= (2^{2x+1} - 2^x) \ln(2) \text{ geeft } f''(x) = (2^{2x+1} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2)) \ln(2) = (2^{2x+2} - 2^x) \ln^2(2) \\
 f''(x) &= 0 \text{ geeft } (2^{2x+2} - 2^x) \ln^2(2) = 0 \\
 2^{2x+2} - 2^x &= 0 \\
 2^{2x+2} &= 2^x \\
 2x + 2 &= x \\
 x &= -2 \\
 f(-2) &= 2^{-4} - 2^{-2} = \frac{1}{16} - \frac{1}{4} = -\frac{3}{16}, \text{ dus buigpunt } (-2, -\frac{3}{16}).
 \end{aligned}$$

$$\begin{aligned}
 \text{85 a } \text{Voor } f_p \text{ geldt } y &= e^{-px}, \text{ dus voor } f_p^{\text{inv}} \text{ geldt } x = e^{-py}. \\
 x = e^{-py} \text{ geeft } -py &= \ln(x)
 \end{aligned}$$

$$y = -\frac{1}{p} \ln(x)$$

$$\text{Dus } f_p^{\text{inv}}(x) = -\frac{1}{p} \ln(x) = g_p(x).$$

Dus de functies f_p en g_p zijn elkaars inverse.

$$\begin{aligned}
 \text{b } \text{De lijn } y &= x \text{ is de gemeenschappelijke raaklijn, dus in het raakpunt geldt } f_p(x) = g_p(x) = x \wedge f'_p(x) = g'_p(x) = 1. \\
 f_p(x) &= e^{-px} \text{ geeft } f'_p(x) = -p e^{-px} \\
 g_p(x) &= -\frac{1}{p} \ln(x) \text{ geeft } g'_p(x) = -\frac{1}{px} \\
 f'_p(x) &= 1 \text{ geeft } -p e^{-px} = 1 \\
 g'_p(x) &= 1 \text{ geeft } -\frac{1}{px} = 1, \text{ dus } px = -1. \\
 \left. \begin{aligned} -p e^{-px} &= 1 \\ px &= -1 \end{aligned} \right\} & -p e^1 = 1 \\
 & p = -\frac{1}{e}
 \end{aligned}$$

86 a $f_2(x) = \ln\left(\frac{2x}{2x+1}\right)$ geeft

$$f_2'(x) = \frac{1}{\left(\frac{2x}{2x+1}\right)} \cdot \frac{(2x+1) \cdot 2 - 2x \cdot 2}{(2x+1)^2} = \frac{2x+1}{2x} \cdot \frac{4x+2-4x}{(2x+1)^2} = \frac{2(2x+1)}{2x(2x+1)^2} = \frac{1}{x(2x+1)}$$

$rc_k = \frac{1}{3}$, dus $f_2'(x) = \frac{1}{3}$

$$\frac{1}{x(2x+1)} = \frac{1}{3}$$

$$x(2x+1) = 3$$

$$2x^2 + x - 3 = 0$$

$$D = 1^2 - 4 \cdot 2 \cdot -3 = 25, \text{ dus } \sqrt{D} = 5$$

$$x = \frac{-1+5}{4} = 1 \vee x = \frac{-1-5}{4} = -1\frac{1}{2}$$

$x = -1\frac{1}{2}$ voldoet niet, dus $x = 1$.

$$f_2(1) = \ln\left(\frac{2}{2+1}\right) = \ln\left(\frac{2}{3}\right), \text{ dus } A\left(1, \ln\left(\frac{2}{3}\right)\right).$$

b Voor f_a geldt $y = \ln\left(\frac{ax}{2x+1}\right)$, dus voor f_a^{inv} geldt $x = \ln\left(\frac{ay}{2y+1}\right)$.

$$\begin{aligned} x = \ln\left(\frac{ay}{2y+1}\right) \text{ geeft } \frac{ay}{2y+1} &= e^x \\ ay &= (2y+1)e^x \\ ay &= 2ye^x + e^x \\ ay - 2ye^x &= e^x \\ y(a - 2e^x) &= e^x \\ y &= \frac{e^x}{a - 2e^x} \end{aligned}$$

$$\text{Dus } f_a^{\text{inv}}(x) = \frac{e^x}{a - 2e^x}.$$

Dus voor $a = 3$ is $g(x) = \frac{e^x}{3 - 2e^x}$ de inverse van f_a .

87 a $f(x) = 1$ geeft $\frac{1}{|\ln(x)|} = 1$

$$|\ln(x)| = 1$$

$$\ln(x) = 1 \vee \ln(x) = -1$$

$$x = e \vee x = e^{-1} = \frac{1}{e}$$

$x_A < x_B$, dus $x_A = \frac{1}{e}$ en $x_B = e$.

$$|\ln(x)| = \begin{cases} \ln(x) & \text{als } x \geq 1 \\ -\ln(x) & \text{als } 0 < x < 1 \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{\ln(x)} = \ln^{-1}(x) & \text{als } x > 1 \\ -\frac{1}{\ln(x)} = -\ln^{-1}(x) & \text{als } 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} -\ln^{-2}(x) \cdot \frac{1}{x} = -\frac{1}{x \ln^2(x)} & \text{als } x > 1 \\ \ln^{-2}(x) \cdot \frac{1}{x} = \frac{1}{x \ln^2(x)} & \text{als } 0 < x < 1 \end{cases}$$

$$rc_k = f'\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e} \cdot \ln^2\left(\frac{1}{e}\right)} = \frac{1}{\frac{1}{e} \cdot (-1)^2} = e$$

$$rc_l = f'(e) = -\frac{1}{e \cdot \ln^2(e)} = -\frac{1}{e \cdot 1^2} = -\frac{1}{e}$$

$rc_k \cdot rc_l = e \cdot -\frac{1}{e} = -1$, dus k en l staan loodrecht op elkaar.

b $k: y = ax + b$ met $a = e$
 $y = ex + b$
 door $A\left(\frac{1}{e}, 1\right)$ $\left\{ \begin{array}{l} e \cdot \frac{1}{e} + b = 1 \\ b = 0 \end{array} \right.$

Dus $k: y = ex$.

$l: y = ax + b$ met $a = -\frac{1}{e}$

$y = -\frac{1}{e}x + b$
 door $B(e, 1)$ $\left\{ \begin{array}{l} -\frac{1}{e} \cdot e + b = 1 \\ b = 2 \end{array} \right.$

Dus $l: y = -\frac{1}{e}x + 2$.

k snijden met l geeft $ex = -\frac{1}{e}x + 2$

$$ex + \frac{1}{e}x = 2$$

$$\left(\frac{e^2 + 1}{e}\right)x = 2$$

$$x = \frac{2e}{e^2 + 1}$$

$x = \frac{2e}{e^2 + 1}$ geeft $y = e \cdot \frac{2e}{e^2 + 1} = \frac{2e^2}{e^2 + 1}$, dus $S\left(\frac{2e}{e^2 + 1}, \frac{2e^2}{e^2 + 1}\right)$.

88 a $f(x) = 1$ geeft $e^{\ln^2(x) + 2\ln(x) - 3} = 1$

$$\ln^2(x) + 2\ln(x) - 3 = 0$$

Stel $\ln(x) = u$.

$$u^2 + 2u - 3 = 0$$

$$(u - 1)(u + 3) = 0$$

$$u = 1 \vee u = -3$$

$$\ln(x) = 1 \vee \ln(x) = -3$$

$$x = e \vee x = e^{-3} = \frac{1}{e^3}$$

$f(x) \leq 1$ geeft $\frac{1}{e^3} \leq x \leq e$

b $f(x) = e^{\ln^2(x) + 2\ln(x) - 3}$ geeft $f'(x) = e^{\ln^2(x) + 2\ln(x) - 3} \cdot \left(2\ln(x) \cdot \frac{1}{x} + \frac{2}{x}\right) = e^{\ln^2(x) + 2\ln(x) - 3} \cdot \left(\frac{2\ln(x) + 2}{x}\right)$

$f'(x) = 0$ geeft $e^{\ln^2(x) + 2\ln(x) - 3} \cdot \left(\frac{2\ln(x) + 2}{x}\right) = 0$

$$\frac{2\ln(x) + 2}{x} = 0$$

$$2\ln(x) + 2 = 0$$

$$\ln(x) = -1$$

$$x = e^{-1}$$

min. is $f(e^{-1}) = e^{(-1)^2 + 2 \cdot (-1) - 3} = e^{-4} = \frac{1}{e^4}$, dus $B_f = \left[\frac{1}{e^4}, \rightarrow\right)$.

89 a $f(x) = x^x = (e^{\ln(x)})^x = e^{x\ln(x)}$ geeft $f'(x) = e^{x\ln(x)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x}\right) = (\ln(x) + 1) \cdot x^x$

b $g_p(x) = x^{\frac{p}{x}} = (e^{\ln(x)})^{\frac{p}{x}} = e^{\frac{p}{x}\ln(x)} = e^{px^{-1} \cdot \ln(x)}$ geeft

$$g_p'(x) = e^{px^{-1} \cdot \ln(x)} \cdot \left(-px^{-2} \cdot \ln(x) + px^{-1} \cdot \frac{1}{x}\right) = x^{\frac{p}{x}} \cdot \left(\frac{-p\ln(x)}{x^2} + \frac{p}{x^2}\right) = \left(\frac{p - p\ln(x)}{x^2}\right) \cdot x^{\frac{p}{x}}$$

In het raakpunt A geldt $f(x) = g_p(x) \wedge f'(x) = g_p'(x)$.

$f(x) = g_p(x)$ geeft $x^x = x^{\frac{p}{x}}$

$$x = \frac{p}{x}$$

$$p = x^2$$

$$f'(x) = g_p'(x) \text{ met } p = x^2 \text{ geeft } (\ln(x) + 1) \cdot x^x = \left(\frac{x^2 - x^2 \ln(x)}{x^2} \right) \cdot x^{\frac{x^2}{x}}$$

$$(\ln(x) + 1) \cdot x^x = \left(\frac{x^2(1 - \ln(x))}{x^2} \right) \cdot x^x$$

$$\ln(x) + 1 = 1 - \ln(x) \vee x^x = 0$$

$$2 \ln(x) = 0$$

$$x = 1$$

Dus $p = 1^2 = 1$.

k: $y = ax + b$ met $a = f'(1) = (\ln(1) + 1) \cdot 1^1 = 1$

$$\left. \begin{array}{l} y = x + b \\ f(1) = 1, \text{ dus } A(1, 1) \end{array} \right\} \begin{array}{l} 1 + b = 1 \\ b = 0 \end{array}$$

Dus k: $y = x$.

N.B. Van de vergelijking $x^x = x^{\frac{p}{x}}$ is $x = 1$ ook een oplossing. Deze oplossing is hierboven ook gevonden.

Bladzijde 41

- C13** a $\ln^3(e^2) = 2^3 = 8$
b $\ln(\sqrt{e}) = \ln(e^{\frac{1}{2}}) = \frac{1}{2}$
c $e^{\ln(7)} + e^{\ln(3)} = 7 + 3 = 10$
d $e^{3 \ln(2)} \cdot e^{\ln(5)} = e^{\ln(2^3)} \cdot 5 = 8 \cdot 5 = 40$

- C14** a $5 - \ln(3) = \ln(e^5) - \ln(3) = \ln\left(\frac{e^5}{3}\right) = \ln\left(\frac{1}{3}e^5\right)$
b $2 \ln(3) + \ln(6) = \ln(3^2) + \ln(6) = \ln(9) + \ln(6) = \ln(54)$

- C15** a $e^{3x} = 15$
 $3x = \ln(15)$
 $x = \frac{1}{3} \ln(15) \approx 0,903$
b $e^{2x+3} = 7$
 $2x + 3 = \ln(7)$
 $2x = -3 + \ln(7)$
 $x = -1\frac{1}{2} + \frac{1}{2} \ln(7) \approx -0,527$

- C16** a $x^2 \ln(x) = 4 \ln(x)$
 $\ln(x) = 0 \vee x^2 = 4$
 $x = 1 \vee x = 2 \vee x = -2$
 $x = -2$ voldoet niet, dus $x = 1 \vee x = 2$.
b $2 \ln(x) + \ln(9) = 2$
 $\ln(x^2) + \ln(9) = 2$
 $\ln(9x^2) = 2$
 $9x^2 = e^2$
 $x^2 = \frac{1}{9}e^2$
 $x = \frac{1}{3}e \vee x = -\frac{1}{3}e$
 $x = -\frac{1}{3}e$ voldoet niet, dus $x = \frac{1}{3}e$.
c $\ln^2(x) + \ln(x) - 6 = 0$
Stel $\ln(x) = u$.
 $u^2 + u - 6 = 0$
 $(u - 2)(u + 3) = 0$
 $u = 2 \vee u = -3$
 $\ln(x) = 2 \vee \ln(x) = -3$
 $x = e^2 \vee x = e^{-3} = \frac{1}{e^3}$

- C17** a $f(x) = \frac{2^x}{2^x + 1}$ geeft $f'(x) = \frac{(2^x + 1) \cdot 2^x \cdot \ln(2) - 2^x \cdot 2^x \cdot \ln(2)}{(2^x + 1)^2} = \frac{(2^{2x} + 2^x - 2^{2x}) \cdot \ln(2)}{(2^x + 1)^2} = \frac{2^x \cdot \ln(2)}{(2^x + 1)^2}$
b $g(x) = x^2 \cdot \ln(2x)$ geeft $g'(x) = 2x \cdot \ln(2x) + x^2 \cdot \frac{1}{x} = 2x \ln(2x) + x$
c $h(x) = {}^2\log(x^2 + 4)$ geeft $h'(x) = \frac{1}{(x^2 + 4) \cdot \ln(2)} \cdot 2x = \frac{2x}{(x^2 + 4) \ln(2)}$

Bladzijde 42

- 1** **a** $4^{x-1} = 25$
 $x - 1 = {}^4\log(25)$
 $x = 1 + {}^4\log(25)$
- b** $2 + 5^{2x} = 17$
 $5^{2x} = 15$
 $2x = {}^5\log(15)$
 $x = \frac{1}{2} \cdot {}^5\log(15)$
- c** $3 + 5 \cdot 2^{3x-1} = 68$
 $5 \cdot 2^{3x-1} = 65$
 $2^{3x-1} = 13$
 $3x - 1 = {}^2\log(13)$
 $3x = 1 + {}^2\log(13)$
 $x = \frac{1}{3} + \frac{1}{3} \cdot {}^2\log(13)$
- d** $2^{x+3} + 2^{x+1} = 50$
 $2^3 \cdot 2^x + 2 \cdot 2^x = 50$
 $8 \cdot 2^x + 2 \cdot 2^x = 50$
 $10 \cdot 2^x = 50$
 $2^x = 5$
 $x = {}^2\log(5)$
- 2** **a** $g_{\text{dag}} = 1,1$
 $g_{\text{week}} = 1,1^7 \approx 1,949$
 Het groeipercentage per week is 94,9%.
- b** $g_{\text{dag}} = 1,1$
 $g_{8 \text{ uur}} = 1,1^{\frac{1}{3}} \approx 1,032$
 Het groeipercentage per 8 uur is 3,2%.
- 3** **a** $g_{\text{jaar}} = 0,64$
 $g_{\text{maand}} = 0,64^{\frac{1}{12}} \approx 0,963$
 De afname is 3,7% per maand.
- b** $g_{\text{jaar}} = 0,64$
 $g_{5 \text{ jaar}} = 0,64^5 \approx 0,107$
 De afname is 89,3% per 5 jaar.
- 4** **a** $g_{\text{maand}} = 1,002$
 $1,002^T = 2$
 $T = {}^{1,002}\log(2) = 346,920\dots$
 De verdubbelingstijd is $346,920\dots : 12 \approx 29$ jaar.
- b** $g_{\text{week}} = 0,8$
 $0,8^T = \frac{1}{2}$
 $T = {}^{0,8}\log(\frac{1}{2}) = 3,106\dots$
 De halveringstijd is 3 weken en $0,106\dots \cdot 7 \approx 1$ dag.
- c** $g_{36 \text{ uur}} = 2$
 $g_{\text{uur}} = 2^{\frac{1}{36}} = 1,019\dots$
 $1,019\dots^t = 5$
 $t = {}^{1,019\dots}\log(5) = 83,589\dots$
 Na 84 uur is de hoeveelheid vijf keer zoveel.
- 5** **a** $4 + {}^2\log(a) + \log(100) = 4 + {}^2\log(a) + 2 = 6 + {}^2\log(a) = {}^2\log(2^6) + {}^2\log(a) = {}^2\log(64) + {}^2\log(a) = {}^2\log(64a)$
- b** $2 - {}^3\log(6a) = {}^3\log(3^2) - {}^3\log(6a) = {}^3\log(9) - {}^3\log(6a) = {}^3\log\left(\frac{9}{6a}\right) = {}^3\log\left(\frac{3}{2a}\right)$
- 6** **a** $y = 4 + {}^3\log(36x) = 4 + {}^3\log(9 \cdot 4x) = 4 + {}^3\log(9) + {}^3\log(4x) = 4 + 2 + {}^3\log(4x) = 6 + {}^3\log(4x)$
 Dus $y = 6 + {}^3\log(4x)$.
- b** $y = 5 - \log(500x) = 5 - \log(100 \cdot 5x) = 5 - \log(100) - \log(5x) = 5 - 2 - \log(5x) = 3 - \log(5x)$
 Dus $y = 3 - \log(5x)$.

7 a ${}^4\log(12x+2) = 2 + {}^4\log(x-1)$
 ${}^4\log(12x+2) = {}^4\log(4^2) + {}^4\log(x-1)$
 ${}^4\log(12x+2) = {}^4\log(16(x-1))$
 ${}^4\log(12x+2) = {}^4\log(16x-16)$
 $12x+2 = 16x-16$
 $-4x = -18$
 $x = 4\frac{1}{2}$

8 a ${}^3\log(2x-1) + \frac{1}{3}\log(x+2) = 0$
 ${}^3\log(2x-1) - {}^3\log(x+2) = 0$
 ${}^3\log(2x-1) = {}^3\log(x+2)$
 $2x-1 = x+2$
 $x = 3$

b ${}^2\log(2x-1) = {}^4\log(x)$
 ${}^2\log(2x-1) = \frac{{}^2\log(x)}{{}^2\log(4)}$
 ${}^2\log(2x-1) = \frac{{}^2\log(x)}{2}$
 $2 \cdot {}^2\log(2x-1) = {}^2\log(x)$
 ${}^2\log((2x-1)^2) = {}^2\log(x)$
 $(2x-1)^2 = x$
 $4x^2 - 4x + 1 = x$
 $4x^2 - 5x + 1 = 0$
 $D = (-5)^2 - 4 \cdot 4 \cdot 1 = 9$, dus $\sqrt{D} = 3$
 $x = \frac{5+3}{8} = 1 \vee x = \frac{5-3}{8} = \frac{1}{4}$
 $x = \frac{1}{4}$ voldoet niet, dus $x = 1$.

9 a $4^x - 2^{x+4} = 80$
 $(2^2)^x - 2^x \cdot 2^4 = 80$
 $(2^x)^2 - 16 \cdot 2^x = 80$
Stel $2^x = u$.
 $u^2 - 16u = 80$
 $u^2 - 16u - 80 = 0$
 $(u+4)(u-20) = 0$
 $u = -4 \vee u = 20$
 $2^x = -4 \vee 2^x = 20$
 $x = {}^2\log(20)$

10 a $y = 4 \cdot \log(3x+12) - 1$
 $4 \cdot \log(3x+12) = y+1$
 $\log(3x+12) = 0,25y + 0,25$
 $3x+12 = 10^{0,25y+0,25}$
 $3x+12 = 10^{0,25y} \cdot 10^{0,25}$
 $3x = -12 + (10^{0,25})^y \cdot 10^{0,25}$
 $x = -4 + \frac{1}{3} \cdot 10^{0,25} \cdot (10^{0,25})^y$
 $x \approx -4 + 0,59 \cdot 1,78^y$
Dus $x = -4 + 0,59 \cdot 1,78^y$.

Bladzijde 43

11 a $e^x(2e^x + e^3) = 2e^{2x} + e^{x+3}$
b $\frac{e^{3x} - e^x}{e^x} = e^{2x} - 1$
c $(e^{3x} - 5)^2 = e^{6x} - 10e^{3x} + 25$

b $2 \cdot {}^3\log(x+1) = {}^3\log(2x+14)$
 ${}^3\log((x+1)^2) = {}^3\log(2x+14)$
 $(x+1)^2 = 2x+14$
 $x^2 + 2x + 1 = 2x + 14$
 $x^2 = 13$
 $x = \sqrt{13} \vee x = -\sqrt{13}$
 $x = -\sqrt{13}$ voldoet niet, dus $x = \sqrt{13}$.

c $\log^2(x) + \frac{1}{10}\log(x) = 2$
 $\log^2(x) - \log(x) = 2$
Stel $\log(x) = u$.
 $u^2 - u = 2$
 $u^2 - u - 2 = 0$
 $(u+1)(u-2) = 0$
 $u = -1 \vee u = 2$
 $\log(x) = -1 \vee \log(x) = 2$
 $x = \frac{1}{10} \vee x = 100$
d ${}^2\log^2(x) - 4 \cdot \frac{1}{2}\log(x) = 0$
 ${}^2\log^2(x) + 4 \cdot {}^2\log(x) = 0$
Stel ${}^2\log(x) = u$.
 $u^2 + 4u = 0$
 $u(u+4) = 0$
 $u = 0 \vee u = -4$
 ${}^2\log(x) = 0 \vee {}^2\log(x) = -4$
 $x = 1 \vee x = \frac{1}{16}$

b $5^x - 6 \cdot (\frac{1}{5})^x = 5$
 $5^x - 6 \cdot 5^{-x} = 5$
 $(5^x)^2 - 6 = 5 \cdot 5^x$
Stel $5^x = u$.
 $u^2 - 6 = 5u$
 $u^2 - 5u - 6 = 0$
 $(u+1)(u-6) = 0$
 $u = -1 \vee u = 6$
 $5^x = -1 \vee 5^x = 6$
 $x = {}^5\log(6)$

b $p = 50 \cdot 1,2^{5q+2}$
 $50 \cdot 1,2^{5q+2} = p$
 $1,2^{5q+2} = 0,02p$
 $5q+2 = {}^{1,2}\log(0,02p)$
 $5q+2 = {}^{1,2}\log(0,02) + {}^{1,2}\log(p)$
 $5q = -2 + {}^{1,2}\log(0,02) + \frac{\log(p)}{\log(1,2)}$
 $q = -0,4 + 0,2 \cdot {}^{1,2}\log(0,02) + \frac{\log(p)}{5 \log(1,2)}$
 $q \approx -4,69 + 2,53 \cdot \log(p)$
Dus $q = -4,69 + 2,53 \cdot \log(p)$.

12 a $3xe^x - e^x = 0$
 $(3x - 1)e^x = 0$
 $3x - 1 = 0 \vee e^x = 0$
 $3x = 1$
 $x = \frac{1}{3}$

b $e^{2x-1} - \sqrt[3]{e^2} = 0$
 $e^{2x-1} = \sqrt[3]{e^2}$
 $e^{2x-1} = e^{\frac{2}{3}}$
 $2x - 1 = \frac{2}{3}$
 $2x = 1\frac{2}{3}$
 $x = \frac{5}{6}$

c $e^{2x} + 3e^x = 4$
 $(e^x)^2 + 3e^x - 4 = 0$
 Stel $e^x = u$.
 $u^2 + 3u - 4 = 0$
 $(u - 1)(u + 4) = 0$
 $u = 1 \vee u = -4$
 $e^x = 1 \vee e^x = -4$
 $x = 0$

13 a $f(x) = 2e^x - 3x^2$ geeft $f'(x) = 2e^x - 6x$

b $f(x) = \frac{x^2 + 1}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 2x - (x^2 + 1) \cdot e^x}{(e^x)^2} = \frac{-x^2 + 2x - 1}{e^x}$

c $f(x) = (x^2 + 1)e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2 + 1) \cdot e^x = (x^2 + 2x + 1)e^x$

d $f(x) = \sqrt{2e^x}$ geeft $f'(x) = \frac{1}{2\sqrt{2e^x}} \cdot 2e^x = \frac{e^x}{\sqrt{2e^x}} = \sqrt{\frac{e^{2x}}{2}} = \sqrt{\frac{1}{2}}e^x$

e $f(x) = x^2 \cdot e^{2x-1}$ geeft $f'(x) = 2x \cdot e^{2x-1} + x^2 \cdot e^{2x-1} \cdot 2 = (2x^2 + 2x)e^{2x-1}$

f $f(x) = e^{x^2+9}$ geeft $f'(x) = e^{x^2+9} \cdot 2x = 2xe^{x^2+9}$

14 a $f(x) = (x^2 - x + 1)e^x$ geeft $f'(x) = (2x - 1) \cdot e^x + (x^2 - x + 1) \cdot e^x = (x^2 + x)e^x$

$f'(x) = 0$ geeft $(x^2 + x)e^x = 0$
 $x^2 + x = 0 \vee e^x = 0$

$x(x + 1) = 0$

$x = 0 \vee x = -1$

max. is $f(-1) = \frac{3}{e}$ en min. is $f(0) = 1$

b $f'(x) = (x^2 + x)e^x$ geeft $f''(x) = (2x + 1) \cdot e^x + (x^2 + x) \cdot e^x = (x^2 + 3x + 1)e^x$

$f''(x) = 0$ geeft $(x^2 + 3x + 1)e^x = 0$
 $x^2 + 3x + 1 = 0 \vee e^x = 0$

$D = 3^2 - 4 \cdot 1 \cdot 1 = 5$

$x = \frac{-3 + \sqrt{5}}{2} \vee x = \frac{-3 - \sqrt{5}}{2}$

$x = -1\frac{1}{2} + \frac{1}{2}\sqrt{5} \vee x = -1\frac{1}{2} - \frac{1}{2}\sqrt{5}$

15 a $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$

b $\ln(10) - 4\ln(2) = \ln(10) - \ln(2^4) = \ln(10) - \ln(16) = \ln(\frac{5}{8})$

16 a $2e^{5x} = 16$

$e^{5x} = 8$

$5x = \ln(8)$

$x = \frac{1}{5}\ln(8)$

b $\ln^2(5x) = 16$

$\ln(5x) = 4 \vee \ln(5x) = -4$

$5x = e^4 \vee 5x = e^{-4}$

$x = \frac{1}{5}e^4 \vee x = \frac{1}{5e^4}$

c $2\ln^2(x) - \ln(x) = 0$

$\ln(x)(2\ln(x) - 1) = 0$

$\ln(x) = 0 \vee 2\ln(x) = 1$

$x = 1 \vee \ln(x) = \frac{1}{2}$

$x = 1 \vee x = e^{\frac{1}{2}} = \sqrt{e}$

- 17** **a** $f(x) = 2^{3x-4}$ geeft $f'(x) = 2^{3x-4} \cdot \ln(2) \cdot 3$
b $f(x) = x \cdot 3^x$ geeft $f'(x) = 1 \cdot 3^x + x \cdot 3^x \cdot \ln(3) = (1 + x \ln(3)) \cdot 3^x$
c $f(x) = \ln(x \cdot \sqrt[3]{x}) = \ln(x^{1\frac{1}{3}}) = \frac{1}{3} \ln(x)$ geeft $f'(x) = \frac{\frac{1}{3}}{x} = \frac{4}{3x}$
d $f(x) = \frac{2 \ln(x) - 3}{x}$ geeft $f'(x) = \frac{x \cdot \frac{2}{x} - (2 \ln(x) - 3) \cdot 1}{x^2} = \frac{2 - 2 \ln(x) + 3}{x^2} = \frac{5 - 2 \ln(x)}{x^2}$
e $f(x) = {}^3\log(5x - 6)$ geeft $f'(x) = \frac{1}{(5x - 6) \ln(3)} \cdot 5 = \frac{5}{(5x - 6) \ln(3)}$
f $f(x) = \ln(3x^2 + 3)$ geeft $f'(x) = \frac{1}{3x^2 + 3} \cdot 6x = \frac{6x}{3x^2 + 3} = \frac{2x}{x^2 + 1}$

- 18** **a** $f(x) = x \ln(x) - \frac{1}{2}x$ geeft $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - \frac{1}{2} = \ln(x) + 1 - \frac{1}{2} = \ln(x) + \frac{1}{2}$
 $f'(x) = 0$ geeft $\ln(x) + \frac{1}{2} = 0$
 $\ln(x) = -\frac{1}{2}$
 $x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$
 $f\left(\frac{1}{\sqrt{e}}\right) = \frac{1}{\sqrt{e}} \cdot -\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{\sqrt{e}} = -\frac{1}{\sqrt{e}}$, dus $A\left(\frac{1}{\sqrt{e}}, -\frac{1}{\sqrt{e}}\right)$.
b $f(x) = 0$ geeft $x \ln(x) - \frac{1}{2}x = 0$
 $x(\ln(x) - \frac{1}{2}) = 0$
 $x = 0$ (voldoet niet) $\vee \ln(x) = \frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e}$

$$k: y = ax + b \text{ met } a = f'(\sqrt{e}) = \frac{1}{2} + \frac{1}{2} = 1$$

$$\left. \begin{array}{l} y = x + b \\ \text{door } B(\sqrt{e}, 0) \end{array} \right\} \sqrt{e} + b = 0$$

$$b = -\sqrt{e}$$

$$\text{Dus } k: y = x - \sqrt{e}.$$

Gemengde opgaven

Bladzijde 44

- 1** **a** $g_{\text{jaar}} = 1,096$
 $1,096^T = 2$
 $T = {}^{1,096}\log(2) = 7,561\dots$
De verdubbelingstijd is 7 jaar en $0,561\dots \cdot 12 \approx 7$ maanden.
b $g_{\text{dag}} = 0,83$
 $0,83^T = \frac{1}{2}$
 $T = {}^{0,83}\log(\frac{1}{2}) = 3,720\dots$
De halveringstijd is 3 dagen en $0,720\dots \cdot 24 \approx 17$ uren.
c $g_{\text{week}} = 2$
 $g_{\text{dag}} = 2^{\frac{1}{7}} \approx 1,104$
De toename is 10,4% per dag.
d $g_{8,3 \text{ dagen}} = \frac{1}{2}$
 $g_{\text{dag}} = (\frac{1}{2})^{\frac{1}{8,3}} = 0,919\dots$
 $0,919\dots^T = 0,01$
 $T = {}^{0,919\dots}\log(0,01) = 55,1\dots$
Na 55 dagen is er nog 1% over.

2 a $f(x) = g(x)$ geeft $3 \cdot 2^x = 4 + 4 \cdot (\frac{1}{2})^x$
 $3 \cdot 2^x = 4 + 4 \cdot 2^{-x}$
 $3 \cdot 2^{2x} = 4 \cdot 2^x + 4$
 $3 \cdot (2^x)^2 - 4 \cdot 2^x - 4 = 0$
Stel $2^x = u$.
 $3u^2 - 4u - 4 = 0$
 $D = (-4)^2 - 4 \cdot 3 \cdot -4 = 64$, dus $\sqrt{D} = 8$
 $u = \frac{4+8}{6} = 2 \vee u = \frac{4-8}{6} = -\frac{2}{3}$
 $2^x = 2 \vee 2^x = -\frac{2}{3}$
 $x = 1$

$f(x) \leq g(x)$ geeft $x \leq 1$

b $f(x) = 9$ geeft $3 \cdot 2^x = 9$
 $2^x = 3$
 $x = {}^2\log(3)$

$g(x) = 9$ geeft $4 + 4 \cdot (\frac{1}{2})^x = 9$
 $4 \cdot (\frac{1}{2})^x = 5$
 $(\frac{1}{2})^x = 1\frac{1}{4}$
 $x = {}^{\frac{1}{2}}\log(1\frac{1}{4})$

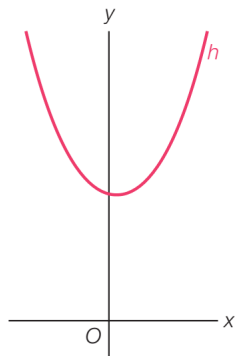
$AB = {}^2\log(3) - {}^{\frac{1}{2}}\log(1\frac{1}{4}) = {}^2\log(3) + {}^2\log(1\frac{1}{4}) = {}^2\log(3\frac{3}{4})$

c $y_C = g(0) = 4 + 4 = 8$
 $f(x) = 8$ geeft $3 \cdot 2^x = 8$
 $2^x = 2\frac{2}{3}$

$2^x = 2\frac{2}{3}$ invullen bij $y = 4 + 4 \cdot (\frac{1}{2})^x$ ofwel $y = 4 + 4 \cdot \frac{1}{2^x}$ geeft $y_E = 4 + 4 \cdot \frac{1}{2\frac{2}{3}} = 5\frac{1}{2}$.

$DE = 8 - 5\frac{1}{2} = 2\frac{1}{2}$

d $h(x) = f(x) + g(x) = 3 \cdot 2^x + 4 + 4 \cdot (\frac{1}{2})^x = 3 \cdot 2^x + 4 + 4 \cdot 2^{-x}$ geeft
 $h'(x) = 3 \cdot 2^x \cdot \ln(2) + 4 \cdot 2^{-x} \cdot \ln(2) \cdot -1 = (3 \cdot 2^x - 4 \cdot 2^{-x}) \ln(2)$
 $h'(1 - {}^2\log(\sqrt{3})) = h'({}^2\log(2) - {}^2\log(\sqrt{3})) = h'({}^2\log(\frac{2}{\sqrt{3}})) = \left(3 \cdot \frac{2}{\sqrt{3}} - 4 \cdot \frac{\sqrt{3}}{2}\right) \ln(2) = (2\sqrt{3} - 2\sqrt{3}) \ln(2) = 0$



Dus $h(x) = f(x) + g(x)$ heeft een minimum voor $x = 1 - {}^2\log(\sqrt{3})$.

3 a $f(x) = 0$ geeft $2 - {}^3\log(x+1) = 0$
 ${}^3\log(x+1) = 2$
 $x+1 = 3^2$
 $x+1 = 9$
 $x = 8$

Dus $AB = 8 - 4 = 4$.

b $f(x) = 1$ geeft $2 - {}^3\log(x+1) = 1$
 ${}^3\log(x+1) = 1$
 $x+1 = 3$
 $x = 2$

$CD = 4\frac{2}{3} - 2 = 2\frac{2}{3}$

$f(2) = 2 - {}^3\log(3) = 2 - 1 = 1$ en $g(2) = {}^{\frac{1}{3}}\log(3) = {}^{\frac{1}{3}}\log((\frac{1}{3})^{-1}) = -1$

$EF = 1 - (-1) = 2$

Dus CD is $\frac{2\frac{2}{3}}{2} = 1\frac{1}{3}$ keer zo lang als EF .

$g(x) = 0$ geeft ${}^{\frac{1}{3}}\log(5-x) = 0$
 $5-x = 1$
 $x = 4$

$g(x) = 1$ geeft ${}^{\frac{1}{3}}\log(5-x) = 1$
 $5-x = \frac{1}{3}$
 $x = 4\frac{2}{3}$

c $x + 1 > 0$ geeft $x > -1$, dus $D_f = \langle -1, \rightarrow \rangle$.

$$f(x) = g(x) \text{ geeft } 2 - {}^3\log(x+1) = {}^{\frac{1}{3}}\log(5-x)$$

$${}^{\frac{1}{3}}\log\left(\frac{1}{9}\right) + {}^{\frac{1}{3}}\log(x+1) = {}^{\frac{1}{3}}\log(5-x)$$

$${}^{\frac{1}{3}}\log\left(\frac{1}{9}(x+1)\right) = {}^{\frac{1}{3}}\log(5-x)$$

$$\frac{1}{9}(x+1) = 5-x$$

$$x+1 = 45-9x$$

$$10x = 44$$

$$x = 4\frac{2}{5}$$

$$f(x) \geq g(x) \text{ geeft } -1 < x \leq 4\frac{2}{5}$$

d $f(x) = 2 - {}^3\log(x+1)$ geeft $f'(x) = -\frac{1}{(x+1)\ln(3)}$

$$f'\left(\frac{1-\ln(3)}{\ln(3)}\right) = -\frac{1}{\left(\frac{1-\ln(3)}{\ln(3)}+1\right)\ln(3)} = -\frac{1}{\left(\frac{1-\ln(3)+\ln(3)}{\ln(3)}\right)\ln(3)} = -\frac{1}{\frac{1}{\ln(3)}\cdot\ln(3)} = -1$$

$$\text{Dus } x_P = \frac{1-\ln(3)}{\ln(3)}.$$

4 a $p = 10e^{-\frac{1}{2}t+1}$

$$e^{-\frac{1}{2}t+1} = \frac{1}{10}p$$

$$-\frac{1}{2}t+1 = \ln\left(\frac{1}{10}p\right)$$

$$-\frac{1}{2}t = -1 + \ln\left(\frac{1}{10}p\right)$$

$$t = 2 - 2\ln\left(\frac{1}{10}p\right)$$

$$t = \ln(e^2) + \ln\left(\left(\frac{1}{10}p\right)^{-2}\right)$$

$$t = \ln(e^2) + \ln(100p^{-2})$$

$$t = \ln(100e^2p^{-2})$$

b $y = 3\ln\left(\frac{2}{x}\right) + 2\ln(5x)$

$$y = 3(\ln(2) - \ln(x)) + 2(\ln(5) + \ln(x))$$

$$y = 3\ln(2) - 3\ln(x) + 2\ln(5) + 2\ln(x)$$

$$y = \ln(2^3) + \ln(5^2) - \ln(x)$$

$$\ln(x) = \ln(2^3 \cdot 5^2) - y$$

$$\ln(x) = \ln(200) + \ln(e^{-y})$$

$$\ln(x) = \ln(200e^{-y})$$

$$x = 200e^{-y}$$

Bladzijde 45

5 a $f(x) = (x-3)^2e^x = (x^2-6x+9)e^x$ geeft $f'(x) = (2x-6) \cdot e^x + (x^2-6x+9) \cdot e^x = (x^2-4x+3)e^x$

$$k: y = ax + b \text{ met } a \cdot f'(0) = -1$$

$$a \cdot f'(0) = -1 \text{ geeft } a \cdot 3 = -1, \text{ dus } a = -\frac{1}{3}.$$

$$\left. \begin{array}{l} y = -\frac{1}{3}x + b \\ f(0) = 9, \text{ dus } A(0, 9) \end{array} \right\} b = 9$$

$$\text{Dus } k: y = -\frac{1}{3}x + 9.$$

$$y = 0 \text{ geeft } -\frac{1}{3}x + 9 = 0$$

$$-\frac{1}{3}x = -9$$

$$x = 27$$

$$\text{Dus } x_B = 27.$$

b $f'(x) = 0$ geeft $(x^2-4x+3)e^x = 0$

$$x^2-4x+3 = 0 \vee e^x = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1 \vee x = 3$$

$$f(1) = (1-3)^2e^1 = 4e \text{ en } f(3) = (3-3)^2e^3 = 0$$

De toppen zijn $(1, 4e)$ en $(3, 0)$.

$$y = ax + b \text{ met } a = \frac{\Delta y}{\Delta x} = \frac{0-4e}{3-1} = -2e$$

$$\left. \begin{array}{l} y = -2ex + b \\ \text{door } (1, 4e) \end{array} \right\} \begin{array}{l} -2e + b = 4e \\ b = 6e \end{array}$$

$$\text{Dus } y_C = 6e.$$

c $f'(x) = (x^2 - 4x + 3)e^x$ geeft $f''(x) = (2x - 4) \cdot e^x + (x^2 - 4x + 3) \cdot e^x = (x^2 - 2x - 1)e^x$

$f''(x) = 0$ geeft $(x^2 - 2x - 1)e^x = 0$

$x^2 - 2x - 1 = 0 \vee e^x = 0$

$D = (-2)^2 - 4 \cdot 1 \cdot -1 = 8$, dus $\sqrt{D} = 2\sqrt{2}$

$x = \frac{2 + 2\sqrt{2}}{2} \vee x = \frac{2 - 2\sqrt{2}}{2}$

$x = 1 + \sqrt{2} \vee x = 1 - \sqrt{2}$

$x_D < x_E$, dus $x_E = 1 + \sqrt{2}$ en $x_D = 1 - \sqrt{2}$.

Dus $x_E - x_D = 1 + \sqrt{2} - (1 - \sqrt{2}) = 2\sqrt{2}$.

d $f(x) = g(x)$ geeft $(x^2 - 6x + 9)e^x = \frac{1}{10}xe^x$

$e^x = 0 \vee x^2 - 6x + 9 = \frac{1}{10}x$

$10x^2 - 61x + 90 = 0$

$D = (-61)^2 - 4 \cdot 10 \cdot 90 = 121$, dus $\sqrt{D} = 11$

$x = \frac{61 + 11}{20} = 3\frac{3}{5} \vee x = \frac{61 - 11}{20} = 2\frac{1}{2}$

$x_F < x_G$, dus $x_F = 2\frac{1}{2}$ en $x_G = 3\frac{3}{5}$.

Dus $x_G - x_F = 3\frac{3}{5} - 2\frac{1}{2} = 1\frac{1}{10}$.

e $x_F = 2\frac{1}{2}$ geeft $y_F = g(2\frac{1}{2}) = \frac{1}{10} \cdot 2\frac{1}{2}e^{2\frac{1}{2}} = \frac{1}{4}e^{2\frac{1}{2}}$

Los op $(x - 3)^2 e^x = \frac{1}{4}e^{2\frac{1}{2}}$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft

$x = -2,173... \vee x = 2,5 \vee x = 3,330...$

$FH = 2,5 - -2,173... = 4,673...$

$FI = 3,330... - 2,5 = 0,830...$

Het lijnstuk FH is $\frac{4,673...}{0,830...} \approx 5,63$ keer zo lang als het lijnstuk FI .

6 a $f(x) = g(x)$ geeft $\ln(4x) = \ln\left(\frac{1}{x}\right)$

$4x = \frac{1}{x}$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \frac{1}{2} \vee x = -\frac{1}{2}$

$x = -\frac{1}{2}$ voldoet niet, dus $x = \frac{1}{2}$.

$f(\frac{1}{2}) = \ln(2)$, dus $A(\frac{1}{2}, \ln(2))$.

$f(x) = \ln(4x)$ geeft $f'(x) = \frac{1}{x}$

$y = ax + b$ met $a = f'(\frac{1}{2}) = 2$

$\left. \begin{array}{l} y = 2x + b \\ \text{door } A(\frac{1}{2}, \ln(2)) \end{array} \right\} \begin{array}{l} 2 \cdot \frac{1}{2} + b = \ln(2) \\ b = \ln(2) - 1 \end{array}$

De lijn $y = 2x + \ln(2) - 1$ snijdt de y -as in $(0, \ln(2) - 1)$.

$g(x) = \ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = -\ln(x)$ geeft $g'(x) = -\frac{1}{x}$

$y = ax + b$ met $a = g'(\frac{1}{2}) = -2$

$\left. \begin{array}{l} y = -2x + b \\ \text{door } A(\frac{1}{2}, \ln(2)) \end{array} \right\} \begin{array}{l} -2 \cdot \frac{1}{2} + b = \ln(2) \\ b = \ln(2) + 1 \end{array}$

De lijn $y = -2x + \ln(2) + 1$ snijdt de y -as in $(0, \ln(2) + 1)$.

De gevraagde lengte is $\ln(2) + 1 - (\ln(2) - 1) = 2$.

b $f(x) = px \wedge f'(x) = p$

$$f'(x) = p \text{ geeft } \frac{1}{x} = p$$

$$f(x) = px \text{ met } p = \frac{1}{x} \text{ geeft } \ln(4x) = 1$$

$$4x = e$$

$$x = \frac{1}{4}e$$

$$\text{Dus } p = \frac{1}{\frac{1}{4}e} = \frac{4}{e} \text{ en } k: y = \frac{4}{e}x.$$

$$g(x) = qx + 2 \wedge g'(x) = q$$

$$g'(x) = q \text{ geeft } -\frac{1}{x} = q$$

$$g(x) = qx + 2 \text{ met } q = -\frac{1}{x} \text{ geeft } \ln\left(\frac{1}{x}\right) = -1 + 2$$

$$\ln\left(\frac{1}{x}\right) = 1$$

$$\frac{1}{x} = e$$

$$x = \frac{1}{e}$$

$$\text{Dus } q = -\frac{1}{\left(\frac{1}{e}\right)} = -e \text{ en } l: y = -ex + 2.$$

$$k \text{ snijden met } l \text{ geeft } \frac{4}{e}x = -ex + 2$$

$$\frac{4}{e}x + ex = 2$$

$$\frac{4 + e^2}{e}x = 2$$

$$x = \frac{2e}{e^2 + 4}$$

$$x = \frac{2e}{e^2 + 4} \text{ geeft } y = \frac{4}{e} \cdot \frac{2e}{e^2 + 4} = \frac{8}{e^2 + 4}$$

$$\text{Dus de coördinaten van het snijpunt van } k \text{ en } l \text{ zijn } \left(\frac{2e}{e^2 + 4}, \frac{8}{e^2 + 4}\right).$$

c Stel $x_P = p$.

$$PS = g(p) = \ln\left(\frac{1}{p}\right)$$

$$PQRS \text{ is een vierkant, dus } PQ = PS = \ln\left(\frac{1}{p}\right) \left. \vphantom{\begin{matrix} PQ = PS = \ln\left(\frac{1}{p}\right) \\ x_Q = p + \ln\left(\frac{1}{p}\right) \end{matrix}} \right\} x_Q = p + \ln\left(\frac{1}{p}\right)$$

$$x_P = p$$

$$x_Q = p + \ln\left(\frac{1}{p}\right) \text{ geeft } QR = f\left(p + \ln\left(\frac{1}{p}\right)\right) = \ln\left(4\left(p + \ln\left(\frac{1}{p}\right)\right)\right)$$

$$PQRS \text{ is een vierkant, dus } PS = QR$$

$$\ln\left(\frac{1}{p}\right) = \ln\left(4\left(p + \ln\left(\frac{1}{p}\right)\right)\right)$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p = 0,1065...$

Dus $x_P \approx 0,107$.

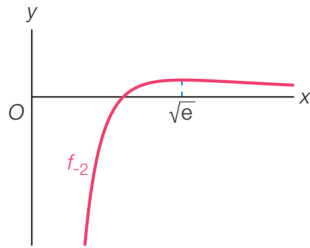
a $f_{-2}(x) = x^{-2} \cdot \ln(x)$ geeft $f_{-2}'(x) = -2x^{-3} \cdot \ln(x) + x^{-2} \cdot \frac{1}{x} = \frac{-2 \ln(x)}{x^3} + \frac{1}{x^3} = \frac{-2 \ln(x) + 1}{x^3}$

$$f_{-2}'(x) = 0 \text{ geeft } -2 \ln(x) + 1 = 0$$

$$-2 \ln(x) = -1$$

$$\ln(x) = \frac{1}{2}$$

$$x = e^{\frac{1}{2}} = \sqrt{e}$$



$$\text{max. is } f_{-2}(e^{\frac{1}{2}}) = (e^{\frac{1}{2}})^{-2} \cdot \frac{1}{2} = e^{-1} \cdot \frac{1}{2} = \frac{1}{2e}$$

b $f_3(x) = x^3 \cdot \ln(x)$ geeft $f_3'(x) = 3x^2 \cdot \ln(x) + x^3 \cdot \frac{1}{x} = 3x^2 \cdot \ln(x) + x^2$

$$f_3'(x) = 3x^2 \cdot \ln(x) + x^2 \text{ geeft } f_3''(x) = 6x \cdot \ln(x) + 3x^2 \cdot \frac{1}{x} + 2x = 6x \ln(x) + 5x$$

$$f_3''(x) = 0 \text{ geeft } 6x \ln(x) + 5x = 0$$

$$x(6 \ln(x) + 5) = 0$$

$$x = 0 \vee 6 \ln(x) = -5$$

$$x = 0 \vee \ln(x) = -\frac{5}{6}$$

$$x = 0 \vee x = e^{-\frac{5}{6}}$$

$x = 0$ voldoet niet, dus de x -coördinaat van het buigpunt van de grafiek van f_3 is $x = e^{-\frac{5}{6}}$.

c $f_p(x) = x^p \cdot \ln(x)$ geeft $f_p'(x) = px^{p-1} \cdot \ln(x) + x^p \cdot \frac{1}{x} = px^{p-1} \cdot \ln(x) + x^{p-1}$

$$f_p'(x) = 0 \text{ geeft } px^{p-1} \cdot \ln(x) + x^{p-1} = 0$$

$$x^{p-1}(p \ln(x) + 1) = 0$$

$$x = 0 \vee p \ln(x) = -1$$

$$x = 0 \vee \ln(x) = -\frac{1}{p}$$

$$x = 0 \vee x = e^{-\frac{1}{p}}$$

$x = 0$ voldoet niet, dus $x = e^{-\frac{1}{p}}$.

$$f_p(e^{-\frac{1}{p}}) = (e^{-\frac{1}{p}})^p \cdot -\frac{1}{p} = e^{-1} \cdot -\frac{1}{p} = -\frac{1}{pe}$$

$$y_{\text{top}} = \frac{1}{3}e \text{ geeft } -\frac{1}{pe} = \frac{1}{3}e$$

$$-\frac{1}{pe} = \frac{e}{3}$$

$$pe^2 = -3$$

$$p = -\frac{3}{e^2}$$

d $f_p'(x) = px^{p-1} \cdot \ln(x) + x^{p-1} = x^{p-1} \cdot (p \ln(x) + 1)$ geeft

$$f_p''(x) = (p-1)x^{p-2} \cdot (p \ln(x) + 1) + x^{p-1} \cdot \frac{p}{x} = (p-1)x^{p-2} \cdot (p \ln(x) + 1) + px^{p-2}$$

$$= x^{p-2}(p^2 \cdot \ln(x) - p \ln(x) + 2p - 1)$$

$$f_p''(e^2) = 0 \text{ geeft } e^{2(p-2)}(p^2 \ln(e^2) - p \ln(e^2) + 2p - 1) = 0$$

$$p^2 \ln(e^2) - p \ln(e^2) + 2p - 1 = 0$$

$$2p^2 - 2p + 2p - 1 = 0$$

$$2p^2 = 1$$

$$p^2 = \frac{1}{2}$$

$$p = \frac{1}{2}\sqrt{2} \vee p = -\frac{1}{2}\sqrt{2}$$

10 Meetkunde met vectoren

10.1 Vectoren en rotaties

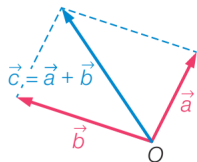
Bladzijde 48

- 1 a** De wandeling van A naar B is -3 in de x -richting en 4 in de y -richting en kan dus worden aangegeven met $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.
- b** In de x -richting: $5 + -3 = 2$.
In de y -richting: $2 + 4 = 6$.

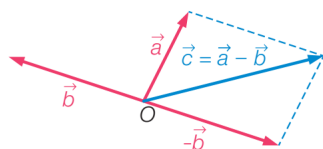
Bladzijde 51

- 2 a** $|\vec{a}| = \left| \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$
- b** $|\vec{b}| = \left| \begin{pmatrix} \sqrt{3} \\ \sqrt{6} \end{pmatrix} \right| = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} = \sqrt{9} = 3$
- c** $|\vec{c}| = \left| \begin{pmatrix} 0 \\ 5 \end{pmatrix} \right| = \sqrt{0^2 + 5^2} = \sqrt{25} = 5$
- d** $|\vec{d}| = \left| \begin{pmatrix} -0,3 \\ 0,4 \end{pmatrix} \right| = \sqrt{(-0,3)^2 + (0,4)^2} = \sqrt{0,25} = 0,5$
- e** $\vec{e} = 2 \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$
 $|\vec{e}| = \left| \begin{pmatrix} 2 \\ 6 \end{pmatrix} \right| = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$
- f** $\vec{f} = 0,6 \cdot \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 3,6 \\ 4,8 \end{pmatrix}$
 $|\vec{f}| = \left| \begin{pmatrix} 3,6 \\ 4,8 \end{pmatrix} \right| = \sqrt{3,6^2 + 4,8^2} = \sqrt{36} = 6$
- 3 a** $\vec{c} = 2\vec{a} + 3\vec{b} = 2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \end{pmatrix}$
- b** $\vec{d} = \vec{a} - 2\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$
- c** $\vec{e} = 4\vec{a} - 3\vec{b} = 4 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$
- d** $\vec{f} = 2(\vec{a} + \vec{b}) = 2 \cdot \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} \right) = 2 \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix}$

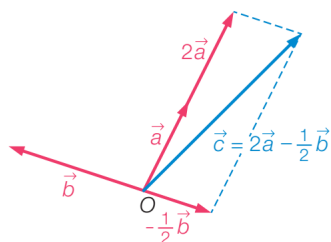
4 a



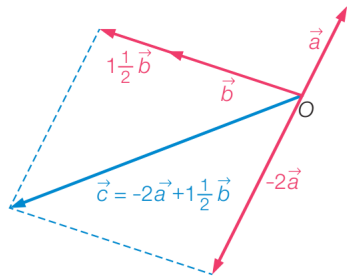
b



c



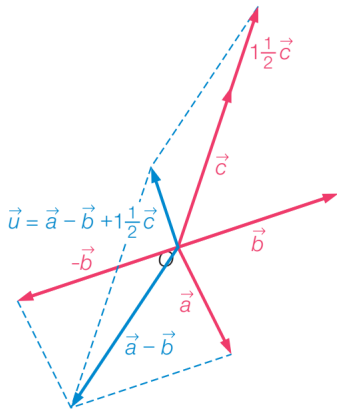
d



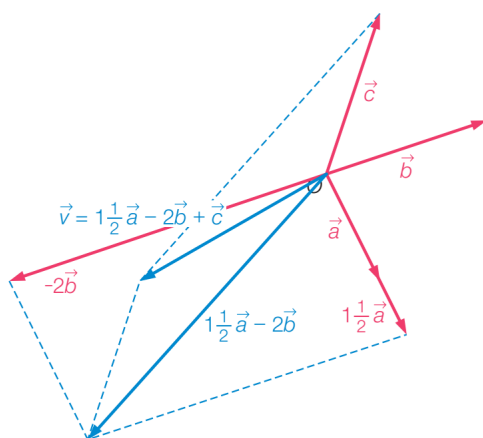
Bladzijde 52

5

a

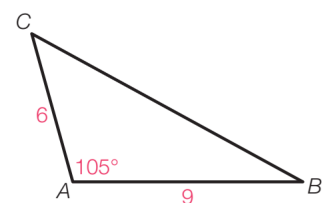


b



V1 Zie de schets hiernaast.

De cosinusregel geeft $BC^2 = 9^2 + 6^2 - 2 \cdot 9 \cdot 6 \cdot \cos(105^\circ) = 144,952...$
 $BC = \sqrt{144,952...} \approx 12,04$

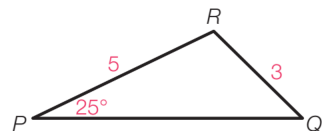


V2 Zie de schets hiernaast.

De sinusregel geeft $\frac{5}{\sin(\angle Q)} = \frac{3}{\sin(25^\circ)}$
 $\sin(\angle Q) = \frac{3 \sin(25^\circ)}{5}$

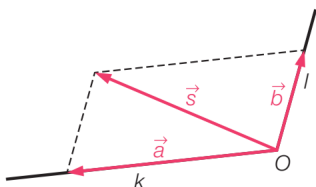
De GR geeft een hoek van $44,778...^\circ$, dus $\angle Q = 44,778...^\circ$.
 $\angle R = 180^\circ - 25^\circ - 44,778...^\circ = 110,221...^\circ$

De sinusregel geeft $\frac{3}{\sin(25^\circ)} = \frac{PQ}{\sin(110,221...^\circ)}$
 $PQ = \frac{3 \sin(110,221...^\circ)}{\sin(25^\circ)} \approx 6,66$



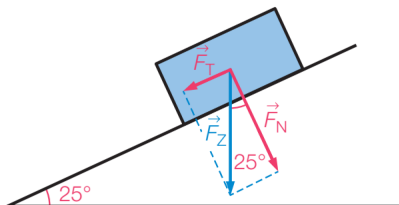
Bladzijde 53

6



Bladzijde 55

7



De grootte van de krachtcomponent langs de helling is $F_T = 750 \sin(25^\circ) \approx 317$ newton.

De grootte van de krachtcomponent loodrecht op de helling is $F_N = 750 \cos(25^\circ) \approx 680$ newton.

8 Zie de figuur hiernaast.

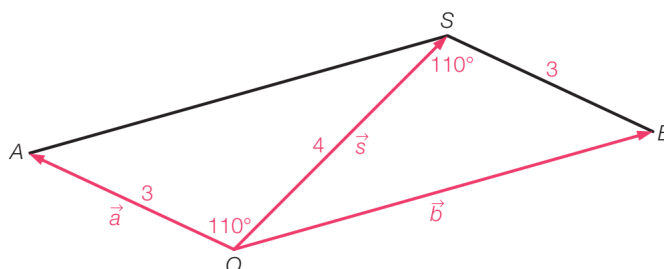
In $\triangle OBS$ is $BS = |\vec{a}| = 3$ en

$\angle OSB = \angle AOS = 110^\circ$ (Z-hoeken).

De cosinusregel in $\triangle OBS$ geeft

$$|\vec{b}|^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \cos(110^\circ) = 33,208\dots$$

$$|\vec{b}| \approx 5,76$$



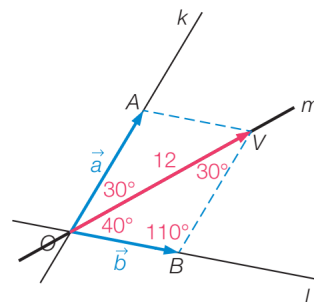
9 Zie de figuur hiernaast.

In $\triangle OBV$ is $\angle OVB = \angle AOV = 30^\circ$ (Z-hoeken) en

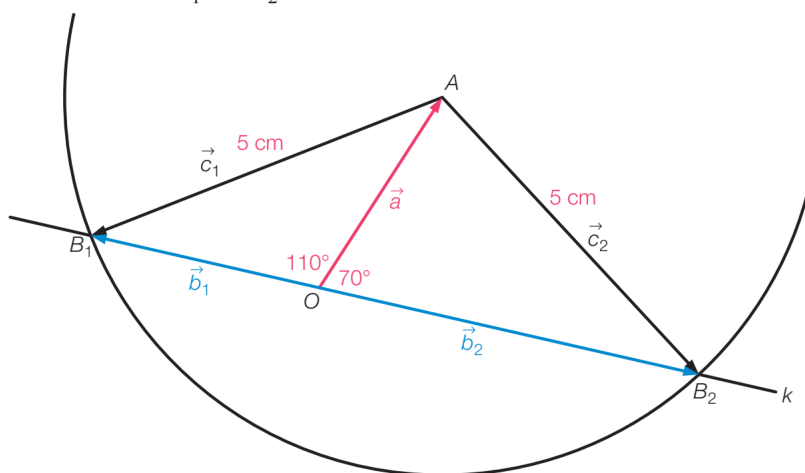
$\angle OBV = 180^\circ - 30^\circ - 40^\circ = 110^\circ$.

De sinusregel in $\triangle OBV$ geeft $\frac{12}{\sin(110^\circ)} = \frac{|\vec{b}|}{\sin(30^\circ)} = \frac{|\vec{a}|}{\sin(40^\circ)}$,

$$\text{dus } |\vec{a}| = \frac{12 \sin(40^\circ)}{\sin(110^\circ)} \approx 8,21 \text{ en } |\vec{b}| = \frac{12 \sin(30^\circ)}{\sin(110^\circ)} \approx 6,39.$$



10 a De cirkel met middelpunt A en straal 5 cm geeft de punten B_1 en B_2 , de vectoren $\vec{b}_1$ en $\vec{b}_2$ en de vectoren $\vec{c}_1$ and $\vec{c}_2$.



$$\frac{3}{\sin(\angle B_1)} = \frac{5}{\sin(110^\circ)}$$

$$\sin(\angle B_1) = \frac{3 \sin(110^\circ)}{5}$$

$$\angle OAB_1 = 180^\circ - 110^\circ - 34,32\dots^\circ = 35,67\dots^\circ$$

De sinusregel in $\triangle OAB_1$ geeft $\frac{OB_1}{\sin(35,67\dots^\circ)} = \frac{5}{\sin(110^\circ)}$.

$$\text{Dus } |\vec{b}_1| = OB_1 = \frac{5 \sin(35,67...^\circ)}{\sin(110^\circ)} \approx 3,10 \text{ cm.}$$

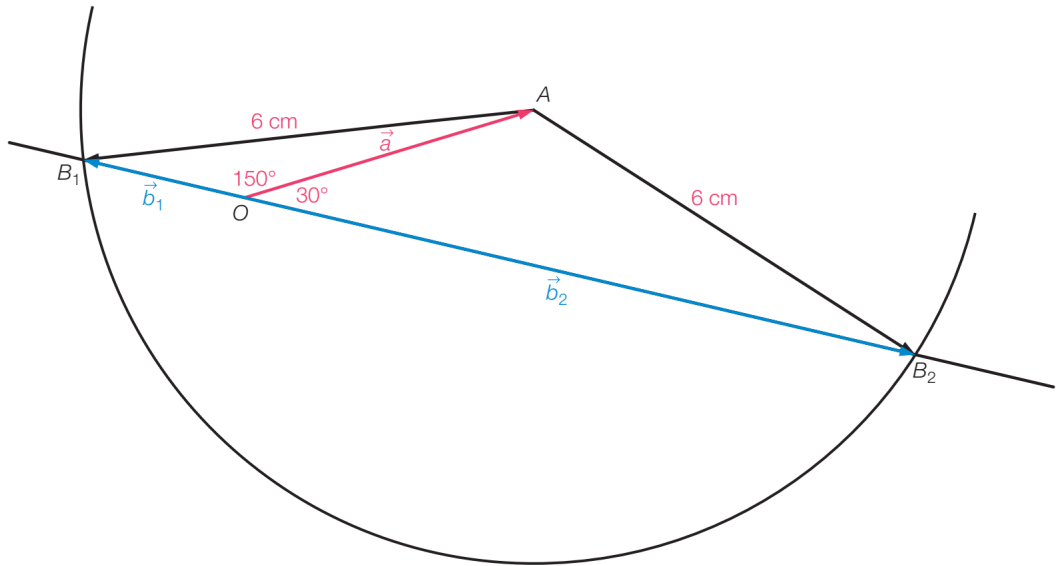
$$\angle B_2 = \angle B_1 = 34,32\dots^\circ$$
$$\angle OAB_2 = 180^\circ - 70^\circ - 34,32\dots^\circ = 75,67\dots^\circ$$

De sinusregel in $\triangle OAB_2$ geeft $\frac{OB_2}{\sin(75,67\dots^\circ)} = \frac{5}{\sin(70^\circ)}$.

$$\text{Dus } |\vec{b}_2| = OB_2 = \frac{5 \sin(75,67\dots^\circ)}{\sin(70^\circ)} \approx 5,16 \text{ cm.}$$

Bladzijde 56

11 Zie de figuur hieronder.



De sinusregel in $\triangle OAB_1$ geeft $\frac{4}{\sin(\angle B_1)} = \frac{6}{\sin(150^\circ)}$
 $\sin(\angle B_1) = \frac{4 \sin(150^\circ)}{6}$

$$\angle B_1 = 19,47...^\circ$$

$$\angle OAB_1 = 180^\circ - 150^\circ - 19,47...^\circ = 10,52...^\circ$$

De sinusregel in $\triangle OAB_1$ geeft $\frac{OB_1}{\sin(10,52\dots^\circ)} = \frac{6}{\sin(150^\circ)}$.

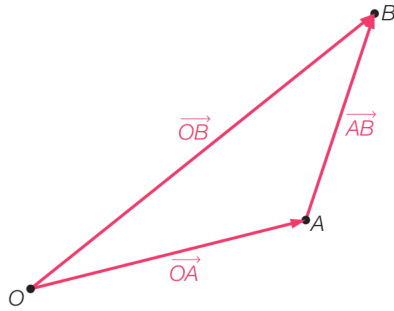
$$\text{Dus } |\vec{b}_1| = OB_1 = \frac{6 \sin(10,52\dots^\circ)}{\sin(150^\circ)} \approx 2,19 \text{ cm.}$$

$$\angle B_2 = \angle B_1 = 19,47...^\circ$$

$$\angle OAB_2 = 180^\circ - 30^\circ - 19,47\dots^\circ = 130,52\dots^\circ$$

De sinusregel in $\triangle OAB_2$ geeft $\frac{OB_2}{\sin(130,52\dots^\circ)} = \frac{6}{\sin(30^\circ)}$.

$$\text{Dus } |\vec{b}_2| = OB_2 = \frac{6 \sin(130,52\dots^\circ)}{\sin(30^\circ)} \approx 9,12 \text{ cm.}$$



b $\vec{OA} + \vec{AB} = \vec{OB}$, dus $\vec{AB} = \vec{OB} - \vec{OA} = \vec{b} - \vec{a}$.

c $\vec{OM} = \vec{OA} + \vec{AM}$

d $\vec{m} = \vec{OM} = \vec{OA} + \vec{AM} = \vec{OA} + \frac{1}{2}\vec{AB} = \vec{a} + \frac{1}{2}(\vec{b} - \vec{a}) = \vec{a} + \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} = \frac{1}{2}(\vec{a} + \vec{b})$

13 a $\vec{OA}_R = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ en $\vec{OA}_L = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

b $\vec{OB}_R = \begin{pmatrix} q \\ -p \end{pmatrix}$

Bladzijde 58

14 $\vec{OD} = \vec{OM} + \vec{MD} = \vec{OM} + \vec{MA}_R = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

Dus $D(-1, 5)$.

15 a $\vec{OB} = \vec{OA} + \vec{AB} = \vec{OA} + \vec{AD}_R$
 $\vec{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, dus $\vec{AD}_R = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

Dit geeft $\vec{OB} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$, dus $B(9, 5)$.

b $\vec{ON} = \vec{OB} + \vec{BN} = \vec{OB} + \vec{BM}_R$
 $\vec{BM} = \frac{1}{2}\vec{BD} = \frac{1}{2}(\vec{d} - \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ 5 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} -7 \\ 1 \end{pmatrix} = \begin{pmatrix} -3\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$, dus $\vec{BM}_R = \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix}$.

Dit geeft $\vec{ON} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 9\frac{1}{2} \\ 8\frac{1}{2} \end{pmatrix}$, dus $N(9\frac{1}{2}, 8\frac{1}{2})$.

Bladzijde 59

16 $\vec{ON} = \vec{n} = \frac{1}{2}(\vec{b} + \vec{d})$ met $\vec{d} = \vec{OD} = \vec{OM} + \vec{MD} = \vec{OM} + \vec{MA}_R$

$\vec{OM} = \vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\vec{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$, dus $\vec{MA}_R = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

Dit geeft $\vec{d} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ en $\vec{ON} = \frac{1}{2} \cdot \left(\begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 11 \\ 11 \end{pmatrix} = \begin{pmatrix} 5\frac{1}{2} \\ 5\frac{1}{2} \end{pmatrix}$.

Dus $N(5\frac{1}{2}, 5\frac{1}{2})$.

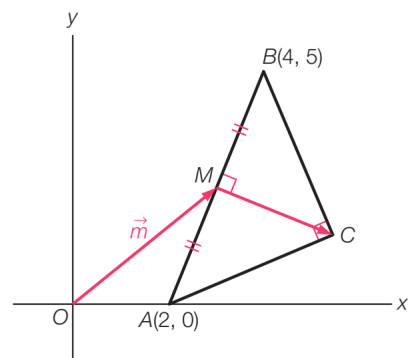
17 M is het midden van zijde AB .

$\vec{OC} = \vec{OM} + \vec{MC} = \vec{OM} + \vec{MA}_L$

$\vec{OM} = \vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 2\frac{1}{2} \end{pmatrix}$

$\vec{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 2\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ -2\frac{1}{2} \end{pmatrix}$, dus $\vec{MA}_L = \begin{pmatrix} 2\frac{1}{2} \\ -1 \end{pmatrix}$.

Dit geeft $\vec{OC} = \begin{pmatrix} 3 \\ 2\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 2\frac{1}{2} \\ -1 \end{pmatrix} = \begin{pmatrix} 5\frac{1}{2} \\ 1\frac{1}{2} \end{pmatrix}$, dus $C(5\frac{1}{2}, 1\frac{1}{2})$.



$$18 \quad \overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BP} + \overrightarrow{PQ} = \overrightarrow{OA} + \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{AB} = \overrightarrow{OA} + 1\frac{1}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC}$$

$$\overrightarrow{BC} = \overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \text{ en } \overrightarrow{AB} = \overrightarrow{AD}_R = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\overrightarrow{OQ} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 1\frac{1}{3} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \frac{1}{3} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 5\frac{1}{3} \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix}, \text{ dus } Q(7\frac{1}{3}, 5\frac{1}{3}).$$

$$\overrightarrow{OR} = \overrightarrow{OQ} + \frac{1}{3}\overrightarrow{BC} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \frac{1}{3} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \begin{pmatrix} -1 \\ 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 6\frac{1}{3} \\ 6\frac{2}{3} \end{pmatrix}, \text{ dus } R(6\frac{1}{3}, 6\frac{2}{3}).$$

Bladzijde 60

$$19 \quad \overrightarrow{OM} = \vec{m} = \frac{1}{2}(\vec{a} + \vec{c})$$

$$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{DA}_L$$

$$\overrightarrow{DA} = \vec{a} - \vec{d} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ d \end{pmatrix} = \begin{pmatrix} a \\ -d \end{pmatrix}, \text{ dus } \overrightarrow{DA}_L = \begin{pmatrix} d \\ a \end{pmatrix}.$$

$$\overrightarrow{OC} = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} d \\ a+d \end{pmatrix}$$

$$\overrightarrow{OM} = \frac{1}{2} \cdot \left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} d \\ a+d \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} a+d \\ a+d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}d \\ \frac{1}{2}a + \frac{1}{2}d \end{pmatrix}$$

$$\text{Dus } M(\frac{1}{2}a + \frac{1}{2}d, \frac{1}{2}a + \frac{1}{2}d).$$

Voor alle waarden van a en d geldt $x_M = y_M$, dus M ligt op de lijn $y = x$.

20 a M is het midden van zijde AB .

$$\overrightarrow{OC} = \overrightarrow{OM} + \overrightarrow{MC} = \overrightarrow{OM} + \overrightarrow{MA}_L$$

$$\overrightarrow{OM} = \vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} a+b \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b \\ 2 \end{pmatrix}$$

$$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a - \frac{1}{2}b \\ -2 \end{pmatrix}, \text{ dus}$$

$$\overrightarrow{MA}_L = \begin{pmatrix} 2 \\ \frac{1}{2}a - \frac{1}{2}b \end{pmatrix}.$$

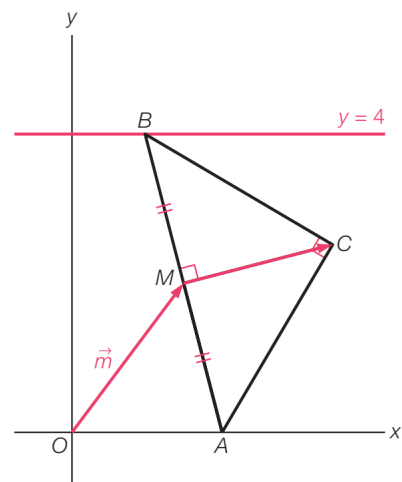
$$\text{Dit geeft } \overrightarrow{OC} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ \frac{1}{2}a - \frac{1}{2}b \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b + 2 \\ \frac{1}{2}a - \frac{1}{2}b + 2 \end{pmatrix}, \text{ dus}$$

$$C(\frac{1}{2}a + \frac{1}{2}b + 2, \frac{1}{2}a - \frac{1}{2}b + 2).$$

b $y = -x$ geeft $\frac{1}{2}a - \frac{1}{2}b + 2 = -(\frac{1}{2}a + \frac{1}{2}b + 2)$

$$\frac{1}{2}a - \frac{1}{2}b + 2 = -\frac{1}{2}a - \frac{1}{2}b - 2$$

$$a = -4$$



Bladzijde 61

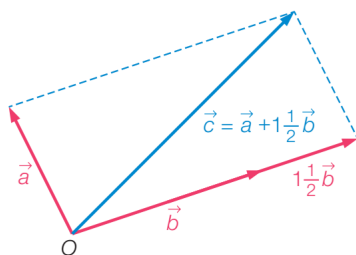
C1 a $|\vec{a}| = \left| \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

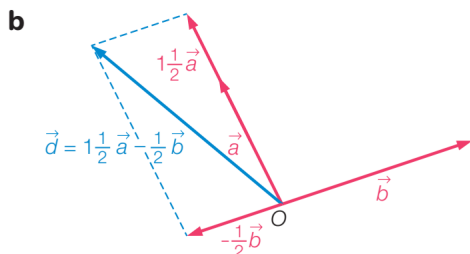
$$|\vec{b}| = \left| \begin{pmatrix} -2 \\ 4 \end{pmatrix} \right| = \sqrt{(-2)^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$$

b $\vec{c} = 2\vec{a} + \vec{b} = 2 \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

$$\vec{d} = 3\vec{a} + 2\frac{1}{2}\vec{b} = 3 \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} + 2\frac{1}{2} \cdot \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \end{pmatrix} + \begin{pmatrix} -5 \\ 10 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

C2 a

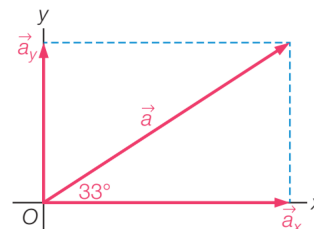




C3 a Zie de figuur hiernaast.

$$|\vec{a}_x| = 6 \cos(33^\circ) \approx 5,03$$

$$|\vec{a}_y| = 6 \sin(33^\circ) \approx 3,27$$

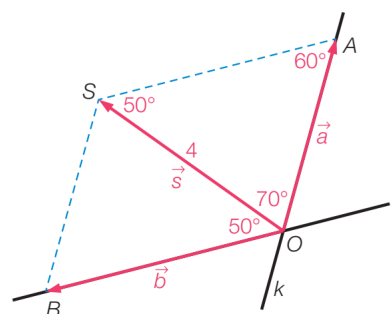


b Zie de figuur hiernaast.

In $\triangle OAS$ is $\angle OSA = \angle BOS = 50^\circ$ (Z-hoeken) en $\angle OAS = 180^\circ - 70^\circ - 50^\circ = 60^\circ$.

De sinusregel in $\triangle OAS$ geeft $\frac{4}{\sin(60^\circ)} = \frac{|\vec{a}|}{\sin(50^\circ)} = \frac{|\vec{b}|}{\sin(70^\circ)}$,

dus $|\vec{a}| = \frac{4 \sin(50^\circ)}{\sin(60^\circ)} \approx 3,54$ en $|\vec{b}| = \frac{4 \sin(70^\circ)}{\sin(60^\circ)} \approx 4,34$.



C4 $\vec{OM} = \vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 7 \\ 3 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1\frac{1}{2} \end{pmatrix}$, dus $M(6, 1\frac{1}{2})$.

$$\vec{OP} = \vec{OM} + \vec{MP} = \vec{OM} + \vec{MA}_L$$

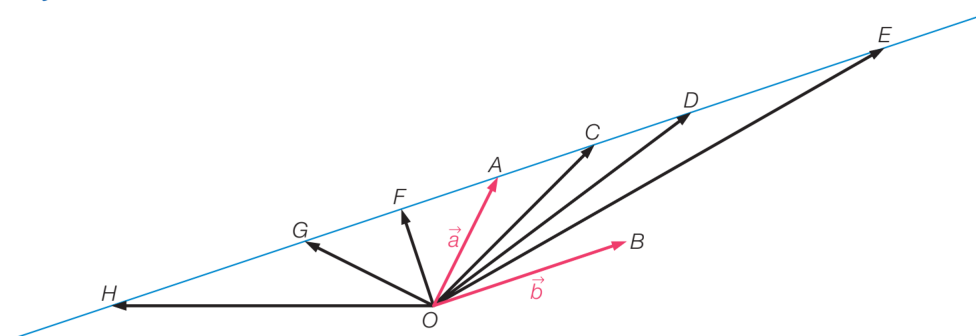
$$\vec{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ -1\frac{1}{2} \end{pmatrix}, \text{ dus } \vec{MA}_L = \begin{pmatrix} 1\frac{1}{2} \\ -1 \end{pmatrix}.$$

$$\text{Dit geeft } \vec{OP} = \begin{pmatrix} 6 \\ 1\frac{1}{2} \end{pmatrix} + \begin{pmatrix} 1\frac{1}{2} \\ -1 \end{pmatrix} = \begin{pmatrix} 7\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \text{ dus } P(7\frac{1}{2}, \frac{1}{2}).$$

10.2 Vectoren en lijnen

Bladzijde 62

21 a



b De eindpunten van de in vraag a getekende vectoren liggen op de lijn door het eindpunt van $\vec{a}$ en die evenwijdig is met $\vec{b}$.

- 22 a** Je kunt ook $\vec{q}$ als steunvector nemen in plaats van $\vec{p}$.

Dan krijg je $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

- b** $\begin{pmatrix} 2 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ is ook een richtingsvector van l .

$t = -\frac{1}{2}$ geeft $\begin{pmatrix} 5 \\ 6 \end{pmatrix} + -\frac{1}{2} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, dus $(4, 5)$ is een punt van l en dus is ook $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ een steunvector van l .

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

- 23 a** $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{b} - \vec{a})$
 $\vec{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ en $\vec{b} - \vec{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix} \triangleq \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ } $k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

- b** $l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t(\vec{d} - \vec{c})$
 $\vec{c} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ en $\vec{d} - \vec{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ } $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

- c** $m: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{e} + t(\vec{f} - \vec{e})$
 $\vec{e} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ en $\vec{f} - \vec{e} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ } $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

- 24** $x = 7$ geeft $-2 + 3t = 7$

$$3t = 9$$

$$t = 3$$

$t = 3$ geeft $y = 5 + 3 \cdot -4 = -7$

Dus A ligt op k .

$x = -13$ geeft $-2 + 3t = -13$

$$3t = -11$$

$$t = -3\frac{2}{3}$$

$t = -3\frac{2}{3}$ geeft $y = 5 - 3\frac{2}{3} \cdot -4 = 19\frac{2}{3}$

Dus B ligt niet op k .

$x = -3\frac{1}{2}$ geeft $-2 + 3t = -3\frac{1}{2}$

$$3t = -1\frac{1}{2}$$

$$t = -\frac{1}{2}$$

$t = -\frac{1}{2}$ geeft $y = 5 - \frac{1}{2} \cdot -4 = 7$

Dus C ligt op k .

25 lijn k

t	0	1
punt	(1, 2)	(4, 3)

lijn l

u	0	1
punt	(2, -1)	(4, 2)

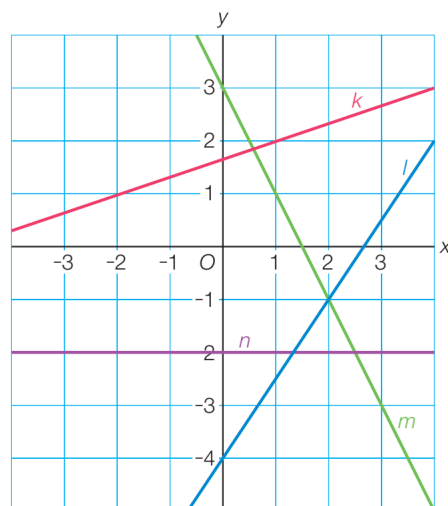
lijn m

v	0	1
punt	(0, 3)	(1, 1)

lijn n

w	0	1
punt	(1, -2)	(2, -2)

Zie de figuur hiernaast.



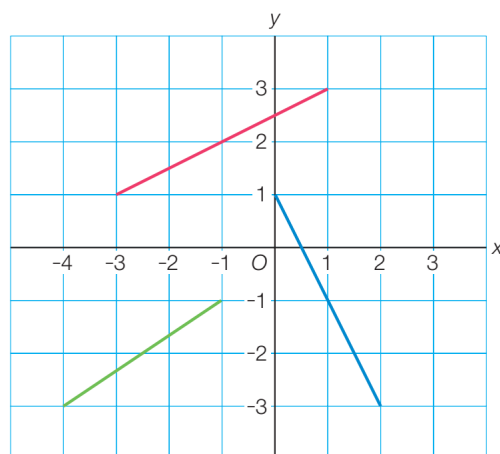
26

t	-1	1
punt	(-3, 1)	(1, 3)

u	1	3
punt	(0, 1)	(2, -3)

v	-1	0
punt	(-4, -3)	(-1, -1)

Zie de figuur hiernaast.



27

a $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{b} - \vec{c})$

$$\left. \begin{aligned} \vec{a} &= \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ 3 \end{pmatrix} \end{aligned} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

b $\vec{m} = \frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ -2 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

$l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + t(\vec{b} - \vec{m})$

$$\left. \begin{aligned} \vec{b} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{m} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ 2 \end{pmatrix} \end{aligned} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

c $\vec{n} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$m: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{n} + t(\vec{a} - \vec{c})$

$$\left. \begin{aligned} \vec{n} &= \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \right\} m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

d $\vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{c} - \vec{b} = \begin{pmatrix} -5 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$

Dus een vectorvoorstelling van het lijnstuk BC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} -8 \\ -6 \end{pmatrix} \wedge 0 \leq t \leq 1$.

Bladzijde 65

- 28 a** De lijn door de middens van AD en BC is evenwijdig met AB en CD en heeft dus dezelfde richting als AB en CD . Dus een richtingsvector van deze lijn is $\overrightarrow{DC} = \vec{c} - \vec{d}$. De vector vanuit O naar het midden van BC is $\frac{1}{2}(\vec{b} + \vec{c})$. Dus $\frac{1}{2}(\vec{b} + \vec{c})$ is een steunvector van de lijn door de middens van AD en BC . Dus $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}(\vec{b} + \vec{c}) + t(\vec{c} - \vec{d})$ is een vectorvoorstelling van de lijn door de middens van AD en BC .
- b**
- $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{d} - \vec{b})$
 - $l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + u(\frac{1}{2}(\vec{c} + \vec{d}) - \vec{b})$
 - $m: \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}(\vec{a} + \vec{c}) + v(\vec{c} - \vec{b})$
- c**
- I gaat door B en is evenwijdig met AC .
 - II gaat door het midden van AB en is evenwijdig met BD .
 - III gaat door A en het midden van BC .

- 29 a** Substitutie van $x = 1 - 3t$ en $y = 2 + t$ in $y = 4x - 28$ geeft $2 + t = 4(1 - 3t) - 28$
 $2 + t = 4 - 12t - 28$
 $13t = -26$
 $t = -2$

$t = -2$ geeft $x = 1 - 3 \cdot -2 = 7$ en $y = 2 - 2 = 0$
 Dus $S(7, 0)$.

- b** Substitutie van $x = 7 + 3u$ en $y = -3 + u$ in $2x - 5y = 11$ geeft $2(7 + 3u) - 5(-3 + u) = 11$
 $14 + 6u + 15 - 5u = 11$
 $u = -18$
 $u = -18$ geeft $x = 7 + 3 \cdot -18 = -47$ en $y = -3 - 18 = -21$
 Dus $T(-47, -21)$.

- 30 a** Substitutie van $x = 3t$ en $y = 2 + 2t$ in $2x - 5y = 6$ geeft $2 \cdot 3t - 5(2 + 2t) = 6$
 $6t - 10 - 10t = 6$
 $-4t = 16$
 $t = -4$

$t = -4$ geeft $x = 3 \cdot -4 = -12$ en $y = 2 + 2 \cdot -4 = -6$
 Dus het snijpunt is $(-12, -6)$.

- b** Substitutie van $x = 2 - u$ en $y = -5 + 3u$ in $3x + 4y = 10$ geeft $3(2 - u) + 4(-5 + 3u) = 10$
 $6 - 3u - 20 + 12u = 10$
 $9u = 24$
 $u = 2\frac{2}{3}$
 $u = 2\frac{2}{3}$ geeft $x = 2 - 2\frac{2}{3} = -\frac{2}{3}$ en $y = -5 + 3 \cdot 2\frac{2}{3} = 3$
 Dus het snijpunt is $(-\frac{2}{3}, 3)$.

- 31** $\begin{cases} x = 4 + 2t \\ y = 1 - 5t \end{cases} \begin{matrix} |5 \\ |2 \end{matrix}$ geeft $\begin{cases} 5x = 20 + 10t \\ 2y = 2 - 10t \end{cases} +$
 $5x + 2y = 22$
 Dus $k: 5x + 2y = 22$.

Bladzijde 67

- 32** Een vectorvoorstelling van k is $k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 Een vectorvoorstelling van l is $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + u \cdot \begin{pmatrix} 8 \\ 1 \end{pmatrix}$.

- 33 a** Vul je $x = 4t$ in $y = 2x + 3$ in, dan krijg je $y = 2 \cdot 4t + 3$ oftewel $y = 8t + 3$.
b $x = t + 5$ geeft $y = 2(t + 5) + 3$
 $y = 2t + 10 + 3$
 $y = 2t + 13$
 Dus $k: x = t + 5 \wedge y = 2t + 13$.

$$\begin{aligned} \text{c } x = -3t + 2 \text{ geeft } y &= 2(-3t + 2) + 3 \\ y &= -6t + 4 + 3 \\ y &= -6t + 7 \end{aligned}$$

$$\text{Dus } k: x = -3t + 2 \wedge y = -6t + 7.$$

$$\begin{aligned} \text{d } y = 4t \text{ geeft } 4t &= 2x + 3 \\ -2x &= -4t + 3 \\ x &= 2t - 1\frac{1}{2} \end{aligned}$$

$$\text{Dus } k: x = 2t - 1\frac{1}{2} \wedge y = 4t.$$

$$34 \quad \text{a } \begin{cases} x = 5 + 4t \\ y = 3 - 2t \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} x = 5 + 4t \\ 2y = 6 - 4t \end{cases} + \frac{x + 2y = 11}{}$$

$$\text{Dus } k: x + 2y = 11.$$

$$\text{b Stel } x = 2t. \text{ Dan is } 6t - 2y = 6, \text{ dus } -2y = -6t + 6 \text{ oftewel } y = 3t - 3.$$

$$\text{Dus } l: x = 2t \wedge y = 3t - 3.$$

$$\text{c } m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -9 \\ 2 \end{pmatrix}$$

$$\text{d } n: x = 4 + 3t \wedge y = -1 + 2t$$

$$\text{e } x = 2t - 1 \text{ geeft } 2t = x + 1, \text{ dus } t = \frac{1}{2}x + \frac{1}{2}.$$

$$\text{Substitutie van } t = \frac{1}{2}x + \frac{1}{2} \text{ in } y = -3t + 5 \text{ geeft } y = -3(\frac{1}{2}x + \frac{1}{2}) + 5 = -1\frac{1}{2}x - 1\frac{1}{2} + 5 = -1\frac{1}{2}x + 3\frac{1}{2}.$$

$$\text{Dus } p: y = -1\frac{1}{2}x + 3\frac{1}{2}.$$

$$35 \quad \text{a Substitutie van } x = 4 + 2t \text{ en } y = 5 + 3t \text{ in } x - 2y = 10 \text{ geeft } 4 + 2t - 2(5 + 3t) = 10$$

$$4 + 2t - 10 - 6t = 10$$

$$-4t = 16$$

$$t = -4$$

$$t = -4 \text{ geeft } x = 4 + 2 \cdot -4 = -4 \text{ en } y = 5 + 3 \cdot -4 = -7$$

$$\text{Dus } S(-4, -7).$$

$$\text{b } \begin{cases} -2 + 3u = -3a + 1 \\ 5 - 4u = 2a \end{cases} \begin{vmatrix} 4 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} -8 + 12u = -12a + 4 \\ 15 - 12u = 6a \end{cases} + \frac{7 = -6a + 4}{}$$

$$6a = -3$$

$$a = -\frac{1}{2}$$

$$\text{Dus } A(-3 \cdot -\frac{1}{2} + 1, 2 \cdot -\frac{1}{2}) = A(2\frac{1}{2}, -1).$$

$$\text{c } \begin{cases} 2v + q = 5 \\ -v - 2q = 2 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 2v + q = 5 \\ -2v - 4q = 4 \end{cases} + \frac{-3q = 9}{}$$

$$q = -3$$

Bladzijde 68

$$36 \quad x = -t - 3 \wedge y = pt - 8 \text{ geeft } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ p \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ p \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 5 \end{pmatrix} \text{ geeft } p = -2\frac{1}{2}$$

$$(1, q) \text{ op } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -2\frac{1}{2} \end{pmatrix} \text{ geeft } -3 - t = 1$$

$$-t = 4$$

$$t = -4$$

$$t = -4 \text{ geeft } q = -8 - 4 \cdot -2\frac{1}{2} = 2$$

$$\text{Dus } p = -2\frac{1}{2} \text{ en } q = 2.$$

37 Stel $x = t$, dan is $y = t$.

Dus $P(t, t)$ en $M(\frac{1}{2}t, \frac{1}{2}t)$.

$$\overrightarrow{PM} = \vec{m} - \vec{p} = \begin{pmatrix} \frac{1}{2}t \\ \frac{1}{2}t \end{pmatrix} - \begin{pmatrix} t \\ t \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}t \\ -\frac{1}{2}t \end{pmatrix}$$

$$\overrightarrow{PQ} = \overrightarrow{PM}_R = \begin{pmatrix} -\frac{1}{2}t \\ \frac{1}{2}t \end{pmatrix}$$

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix} t \\ t \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}t \\ \frac{1}{2}t \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t \\ 1\frac{1}{2}t \end{pmatrix}$$

Dus $l: y = 3x$.

38 Stel $x = 2t$, dan is $y = t + 2$ en $P(2t, t + 2)$.

$$\overrightarrow{PO} = \vec{o} - \vec{p} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2t \\ t + 2 \end{pmatrix} = \begin{pmatrix} -2t \\ -t - 2 \end{pmatrix}$$

$$\overrightarrow{PQ} = \overrightarrow{PO}_L = \begin{pmatrix} t + 2 \\ -2t \end{pmatrix}$$

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix} 2t \\ t + 2 \end{pmatrix} + \begin{pmatrix} t + 2 \\ -2t \end{pmatrix} = \begin{pmatrix} 3t + 2 \\ -t + 2 \end{pmatrix}$$

$$\begin{cases} x = 3t + 2 & |1| \\ y = -t + 2 & |3| \end{cases} \text{ geeft } \begin{cases} x = 3t + 2 \\ 3y = -3t + 6 \end{cases} +$$

$$x + 3y = 8$$

Dus $m: x + 3y = 8$.

39 $k: \frac{x}{3} + \frac{y}{4} = 1$ oftewel $k: 4x + 3y = 12$

Stel $x = -3t$. Dan is $-12t + 3y = 12$, dus $3y = 12t + 12$ oftewel $y = 4t + 4$.

Dus $P(-3t, 4t + 4)$.

$$\overrightarrow{PO} = \vec{o} - \vec{p} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -3t \\ 4t + 4 \end{pmatrix} = \begin{pmatrix} 3t \\ -4t - 4 \end{pmatrix}, \text{ dus } 2 \cdot \overrightarrow{PO} = 2 \cdot \begin{pmatrix} 3t \\ -4t - 4 \end{pmatrix} = \begin{pmatrix} 6t \\ -8t - 8 \end{pmatrix}$$

$$\overrightarrow{PQ} = (2 \cdot \overrightarrow{PO})_L = \begin{pmatrix} 8t + 8 \\ 6t \end{pmatrix}$$

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix} -3t \\ 4t + 4 \end{pmatrix} + \begin{pmatrix} 8t + 8 \\ 6t \end{pmatrix} = \begin{pmatrix} 5t + 8 \\ 10t + 4 \end{pmatrix}$$

$$\begin{cases} x = 5t + 8 & |2| \\ y = 10t + 4 & |1| \end{cases} \text{ geeft } \begin{cases} 2x = 10t + 16 \\ y = 10t + 4 \end{cases} -$$

$$2x - y = 12$$

Dus $m: 2x - y = 12$.

Bladzijde 69

C5

$$\mathbf{a} \quad k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{b} - \vec{a}) \quad \left. \begin{array}{l} \vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \end{array} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\mathbf{b} \quad l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + t(\vec{c} - \vec{a}) \quad \left. \begin{array}{l} \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ en } \vec{c} - \vec{a} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \end{pmatrix} \end{array} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

$$\mathbf{c} \quad \vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix}$$

$$p: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t(\vec{c} - \vec{m}) \quad \left. \begin{array}{l} \vec{c} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \text{ en } \vec{c} - \vec{m} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 4 \\ -4\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 8 \\ -9 \end{pmatrix} \end{array} \right\} p: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ -9 \end{pmatrix}$$

d $\vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ en $\vec{c} - \vec{a} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ (zie vraag b)

Dus een vectorvoorstelling van het lijnstuk AC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ -5 \end{pmatrix} \wedge 0 \leq t \leq 1$.

C6 a $\begin{cases} x = 1 + 2t \\ y = 6 - 3t \end{cases} \Big| \begin{matrix} 3 \\ 2 \end{matrix}$ geeft $\begin{cases} 3x = 3 + 6t \\ 2y = 12 - 6t \\ 3x + 2y = 15 \end{cases}$

Dus $k: 3x + 2y = 15$.

b Stel $x = 3t$. Dan is $12t - 3y = 6$, dus $-3y = -12t + 6$ oftewel $y = 4t - 2$.

Dus $l: x = 3t \wedge y = 4t - 2$.

c Stel $x = t$, dan is $y = 5t - 1$ en $m: x = t \wedge y = 5t - 1$.

Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

10.3 Vectoren en hoeken

Bladzijde 70

40 a $A(a_x, a_y)$, dus $\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$ en $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$.

Kwadrateren geeft $|\vec{a}|^2 = a_x^2 + a_y^2$.

$B(b_x, b_y)$, dus $\vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$ en $|\vec{b}| = \sqrt{b_x^2 + b_y^2}$.

Kwadrateren geeft $|\vec{b}|^2 = b_x^2 + b_y^2$.

b $AB = d(A, B) = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$

Kwadrateren geeft $AB^2 = (b_x - a_x)^2 + (b_y - a_y)^2$
 $= b_x^2 - 2a_x b_x + a_x^2 + b_y^2 - 2a_y b_y + a_y^2$
 $= a_x^2 + a_y^2 + b_x^2 + b_y^2 - 2a_x b_x - 2a_y b_y$

Bladzijde 72

41 $\angle(k, l) \approx 180^\circ - 108,4^\circ = 71,6^\circ$

42 a $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -6 \end{pmatrix} = 2 \cdot -1 + 3 \cdot -6 = -20$

b $\vec{c} \cdot \vec{d} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = -1 \cdot 2 + 0 \cdot 4 = -2$

c $\vec{e} \cdot \vec{f} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 2 \cdot 3 + -3 \cdot 2 = 0$

d $\vec{g} \cdot \vec{h} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2\sqrt{2} \\ -\sqrt{3} \end{pmatrix} = \sqrt{2} \cdot 2\sqrt{2} + \sqrt{3} \cdot -\sqrt{3} = 1$

Bladzijde 73

43 a $\cos(\angle(\vec{a}, \vec{b})) = \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{3 \cdot 2 + 4 \cdot 1}{\sqrt{25} \cdot \sqrt{5}} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$

Dus $\angle(\vec{a}, \vec{b}) \approx 26,6^\circ$.

b $\cos(\angle(\vec{c}, \vec{d})) = \frac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right|} = \frac{4 \cdot -1 + -1 \cdot 3}{\sqrt{17} \cdot \sqrt{10}} = \frac{-7}{\sqrt{170}}$

Dus $\angle(\vec{c}, \vec{d}) \approx 122,5^\circ$.

$$\text{c } \cos(\angle(\vec{e}, \vec{f})) = \frac{\begin{pmatrix} 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} 0 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right|} = \frac{0 \cdot 2 + 3 \cdot 5}{3 \cdot \sqrt{29}} = \frac{15}{3\sqrt{29}} = \frac{5}{\sqrt{29}}$$

Dus $\angle(\vec{e}, \vec{f}) \approx 21,8^\circ$.

$$\text{d } \cos(\angle(\vec{g}, \vec{h})) = \frac{\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|} = \frac{2 \cdot 5 + -5 \cdot 2}{\sqrt{29} \cdot \sqrt{29}} = \frac{0}{29} = 0$$

Dus $\angle(\vec{g}, \vec{h}) = 90^\circ$.

$$\text{44 a } \cos(\angle(\vec{r}_k, \vec{r}_l)) = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{1 \cdot 2 + 2 \cdot 1}{\sqrt{5} \cdot \sqrt{5}} = \frac{4}{5} \text{ geeft } \angle(\vec{r}_k, \vec{r}_l) = 36,86...^\circ$$

Dus $\angle(k, l) \approx 36,9^\circ$.

$$\text{b } \cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right|} = \frac{3 \cdot -1 + 1 \cdot 2}{\sqrt{10} \cdot \sqrt{5}} = \frac{-1}{\sqrt{50}} \text{ geeft } \angle(\vec{r}_m, \vec{r}_n) = 98,13...^\circ$$

Dus $\angle(m, n) = 180^\circ - 98,13...^\circ \approx 81,9^\circ$.

$$\text{c } \cos(\angle(\vec{r}_p, \vec{r}_q)) = \frac{\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{-1 \cdot 2 + 1 \cdot 1}{\sqrt{2} \cdot \sqrt{5}} = \frac{-1}{\sqrt{10}} \text{ geeft } \angle(\vec{r}_p, \vec{r}_q) = 108,43...^\circ$$

Dus $\angle(p, q) = 180^\circ - 108,43...^\circ \approx 71,6^\circ$.

$$\text{45 a } \cos(\angle(\vec{r}_k, \vec{r}_l)) = \frac{\begin{pmatrix} 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right|} = \frac{3 \cdot 3 + -1 \cdot 5}{\sqrt{10} \cdot \sqrt{34}} = \frac{4}{\sqrt{340}} \text{ geeft } \angle(\vec{r}_k, \vec{r}_l) = 77,47...^\circ$$

Dus $\angle(k, l) \approx 77,5^\circ$.

$$\text{b } n: x = 3t - 2 \wedge y = 10t + 3 \text{ oftewel } n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + u \cdot \begin{pmatrix} 3 \\ 10 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 10 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 10 \end{pmatrix} \right|} = \frac{2 \cdot 3 + -1 \cdot 10}{\sqrt{5} \cdot \sqrt{109}} = \frac{-4}{\sqrt{545}} \text{ geeft } \angle(\vec{r}_m, \vec{r}_n) = 99,86...^\circ$$

Dus $\angle(m, n) = 180^\circ - 99,86...^\circ \approx 80,1^\circ$.

$$\text{46 a } \vec{r}_{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{AC}, \vec{r}_{BC})) = \frac{\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right|} = \frac{2 \cdot 1 + -5 \cdot 4}{\sqrt{29} \cdot \sqrt{17}} = \frac{-18}{\sqrt{493}} \text{ geeft } \angle(\vec{r}_{AC}, \vec{r}_{BC}) = 144,16...^\circ$$

Dus $\angle(AC, BC) = 180^\circ - 144,16...^\circ \approx 35,8^\circ$.

$$\begin{aligned} \mathbf{b} \quad \overrightarrow{BA} &= \vec{a} - \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \\ \overrightarrow{BC} &= \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \\ \cos(\angle(\overrightarrow{BA}, \overrightarrow{BC})) &= \frac{\begin{pmatrix} -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix}}{\left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|} = \frac{-3 \cdot -1 + 1 \cdot -4}{\sqrt{10} \cdot \sqrt{17}} = \frac{-1}{\sqrt{170}} \text{ geeft } \angle(\overrightarrow{BA}, \overrightarrow{BC}) = 94,39\dots^\circ \\ \text{Dus } \angle ABC &\approx 94,4^\circ. \end{aligned}$$

47 $\angle POQ = \angle(\overrightarrow{OP}, \overrightarrow{OQ})$

$$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 1\frac{1}{2} \end{pmatrix} \text{ en } Q(4t, 3 - 3t), \text{ dus } \overrightarrow{OQ} = \begin{pmatrix} 4t \\ 3 - 3t \end{pmatrix}.$$

$$\cos(\angle(\overrightarrow{OP}, \overrightarrow{OQ})) = \frac{\begin{pmatrix} 2 \\ 1\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 4t \\ 3 - 3t \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 1\frac{1}{2} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 4t \\ 3 - 3t \end{pmatrix} \right|} = \frac{2 \cdot 4t + 1\frac{1}{2} \cdot (3 - 3t)}{\sqrt{6\frac{1}{4}} \cdot \sqrt{(4t)^2 + (3 - 3t)^2}} = \frac{8t + 4\frac{1}{2} - 4\frac{1}{2}t}{2\frac{1}{2}\sqrt{16t^2 + 9 - 18t + 9t^2}} = \frac{3\frac{1}{2}t + 4\frac{1}{2}}{2\frac{1}{2}\sqrt{25t^2 - 18t + 9}}$$

$$\text{Er moet gelden } \angle(\overrightarrow{OP}, \overrightarrow{OQ}) = 60^\circ, \text{ dus los op } \cos(60^\circ) = \frac{3\frac{1}{2}t + 4\frac{1}{2}}{2\frac{1}{2}\sqrt{25t^2 - 18t + 9}}.$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = -0,09\dots$ en $t = 2,32\dots$

$t = -0,09\dots$ geeft het punt $Q_1(-0,4; 3,3)$.

$t = 2,32\dots$ geeft het punt $Q_2(9,3; -4,0)$.

48 $\vec{a} \cdot \vec{b} = \begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} q \\ -p \end{pmatrix} = pq - pq = 0$

$$\vec{a} \cdot \vec{b} = 0, \text{ dus } \cos(\angle(\vec{a}, \vec{b})) = 0 \text{ en hieruit volgt } \angle(\vec{a}, \vec{b}) = 90^\circ.$$

Bladzijde 74

49 a Een richtingsvector gaat van het punt $(0, 2)$ naar het punt $(3, 0)$.

$$\text{Dit is de vector } \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

$$\text{Dus een richtingsvector van } k \text{ is } \vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

b De vector $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op $\vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

$$\text{Dus } \vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ staat loodrecht op } k.$$

c De vector $\vec{n}_l = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ staat loodrecht op l .

Bladzijde 76

50 a $\begin{cases} x = 2 + 4t \\ y = 3 + 5t \end{cases} \left| \begin{matrix} 5 \\ 4 \end{matrix} \right|$ geeft $\begin{cases} 5x = 10 + 20t \\ 4y = 12 + 20t \end{cases} \quad \underline{5x - 4y = -2}$

$$\text{Dus } k: 5x - 4y = -2.$$

*

b Neem $x = 7u$. Dan is $21u - 7y = 8$, dus $-7y = -21u + 8$ oftewel $y = 3u - 1\frac{1}{7}$.

$$\text{Dit geeft } l: x = 7u \wedge y = 3u - 1\frac{1}{7}.$$

$$\text{Dus } l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -1\frac{1}{7} \end{pmatrix} + u \cdot \begin{pmatrix} 7 \\ 3 \end{pmatrix}.$$

*

51 a $\vec{r}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.
 $k: 4x - 3y = c$
 door $(2, -1)$ $\left. \vphantom{\begin{matrix} k: 4x - 3y = c \\ \text{door } (2, -1) \end{matrix}} \right\} c = 4 \cdot 2 - 3 \cdot -1 = 11$
 Dus $k: 4x - 3y = 11$.
 $\vec{r}_l = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.
 $l: 7x + 4y = c$
 door $(0, 0)$ $\left. \vphantom{\begin{matrix} l: 7x + 4y = c \\ \text{door } (0, 0) \end{matrix}} \right\} c = 0$
 Dus $l: 7x + 4y = 0$.

b $\vec{n}_m = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, dus $\vec{r}_m = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
 m door $(-3, 0)$, dus $\vec{s}_m = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.
 Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + v \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
 $\vec{n}_n = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\vec{r}_n = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.
 n door $(0, 0)$, dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
 Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = w \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

52 a $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
 $l: 5x + 2y = c$
 door $A(4, 1)$ $\left. \vphantom{\begin{matrix} l: 5x + 2y = c \\ \text{door } A(4, 1) \end{matrix}} \right\} c = 5 \cdot 4 + 2 \cdot 1 = 22$
 Dus $l: 5x + 2y = 22$.

b $n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 $n: 3x + 2y = c$
 door $B(5, -1)$ $\left. \vphantom{\begin{matrix} n: 3x + 2y = c \\ \text{door } B(5, -1) \end{matrix}} \right\} c = 3 \cdot 5 + 2 \cdot -1 = 13$
 Dus $n: 3x + 2y = 13$.

53 a $\vec{n}_k = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 l door $A(5, 2)$, dus $\vec{s}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
 Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

b $n \perp m$, dus $\vec{r}_n = \vec{n}_m = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.
 n door $B(1, -3)$, dus $\vec{s}_n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.
 Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + u \cdot \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

54 a $\begin{cases} 3 + t = -1 + 3u \\ 3 + 2t = 5 - 4u \end{cases}$ oftewel $\begin{cases} t - 3u = -4 \\ 2t + 4u = 2 \end{cases} \begin{array}{l} |2| \\ |1| \end{array}$ geeft $\begin{cases} 2t - 6u = -8 \\ 2t + 4u = 2 \end{cases}$
 $\frac{-10u = -10}{u = 1}$

$u = 1$ geeft $x = -1 + 3 \cdot 1 = 2 \wedge y = 5 - 4 \cdot 1 = 1$
 Dus $S(2, 1)$.

b $m: y = ax + b$

$$a = \frac{\Delta y}{\Delta x} = \frac{-1 - 5}{3 - -3} = -1$$

$$\left. \begin{array}{l} y = -x + b \\ \text{door } B(3, -1) \end{array} \right\} \begin{array}{l} -3 + b = -1 \\ b = 2 \end{array}$$

Dus $m: y = -x + 2$.

$$\left. \begin{array}{l} n \perp p, \text{ dus } n: 5x + y = c. \\ \text{door } C(-4, -2) \end{array} \right\} c = 5 \cdot -4 - 2 = -22$$

Dus $n: 5x + y = -22$.

Substitutie van $y = -x + 2$ in $5x + y = -22$ geeft $5x - x + 2 = -22$

$$4x = -24$$

$$x = -6$$

$x = -6$ geeft $y = 6 + 2 = 8$

Dus $T(-6, 8)$.

c $\left\{ \begin{array}{l} 5 + 4t = 2w - 4 \\ 3 - t = w - 6 \end{array} \right.$ oftewel $\left\{ \begin{array}{l} 4t - 2w = -9 \\ -t - w = -9 \end{array} \right. \left| \begin{array}{l} 1 \\ 2 \end{array} \right|$ geeft $\left\{ \begin{array}{l} 4t - 2w = -9 \\ -2t - 2w = -18 \end{array} \right.$

$$6t = 9$$

$$t = 1\frac{1}{2}$$

$t = 1\frac{1}{2}$ geeft $x = 5 + 4 \cdot 1\frac{1}{2} = 11$ en $y = 3 - 1\frac{1}{2} = 1\frac{1}{2}$

Dus $U(11, 1\frac{1}{2})$.

Bladzijde 77

55 a k snijden met de x -as geeft $y = 0$

$$3 + t = 0$$

$$t = -3$$

$t = -3$ geeft $x = -2 - 3 \cdot 5 = -17$, dus $A(-17, 0)$.

$l \perp k$, dus $\vec{n}_l = \vec{r}_k = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$.

$$\left. \begin{array}{l} l: 5x + y = c \\ \text{door } A(-17, 0) \end{array} \right\} c = 5 \cdot -17 + 0 = -85$$

Dus $l: 5x + y = -85$.

b $n \parallel m$, dus $\vec{r}_n = \vec{r}_m = \vec{b} - \vec{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

n door $(0, 0)$, dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = u \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

c $\vec{r}_q = \vec{e} - \vec{o} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$p \perp q$, dus $\vec{n}_p = \vec{r}_q = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

$$\left. \begin{array}{l} p: x - 4y = c \\ \text{door } D(-4, 7) \end{array} \right\} c = -4 - 4 \cdot 7 = -32$$

Dus $p: x - 4y = -32$.

56 a $\vec{n}_l = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

$$\cos(\angle(\vec{r}_k, \vec{r}_l)) = \frac{\begin{pmatrix} -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} -4 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|} = \frac{-4 \cdot 5 + 1 \cdot 2}{\sqrt{17} \cdot \sqrt{29}} = \frac{-18}{\sqrt{493}} \text{ geeft } \angle(\vec{r}_k, \vec{r}_l) = 144,16\dots^\circ$$

Dus $\angle(k, l) = 180^\circ - 144,16\dots^\circ \approx 35,8^\circ$.

$$\mathbf{b} \quad \vec{n}_m = \begin{pmatrix} 3 \\ -7 \end{pmatrix} \text{ en } \vec{n}_n = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

$$\cos(\angle(\vec{n}_m, \vec{n}_n)) = \frac{\begin{pmatrix} 3 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ -7 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right|} = \frac{3 \cdot 1 + (-7) \cdot 2}{\sqrt{58} \cdot \sqrt{5}} = \frac{-11}{\sqrt{290}} \text{ geeft } \angle(\vec{n}_m, \vec{n}_n) = 130,23\dots^\circ$$

Dus $\angle(m, n) = 180^\circ - 130,23\dots^\circ \approx 49,8^\circ$.

$$\mathbf{c} \quad \tan(\alpha) = \text{rc}_p = 2 \text{ geeft } \alpha = 63,43\dots^\circ$$

$$\tan(\beta) = \text{rc}_q = -1 \text{ geeft } \beta = -45^\circ$$

$$\alpha - \beta = 63,43\dots^\circ - (-45^\circ) = 108,43\dots^\circ$$

Dus $\angle(p, q) = 180 - 108,43\dots^\circ \approx 71,6^\circ$.

57

$$\mathbf{a} \quad \vec{r}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}, \text{ dus } \vec{n}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

$$\left. \begin{array}{l} k: 3x + 4y = c \\ \text{door } (5, 0) \end{array} \right\} c = 3 \cdot 5 + 4 \cdot 0 = 15$$

Dus $k: 3x + 4y = 15$.

$$d(A, k) = \frac{|3 \cdot 2 + 4 \cdot 1 - 15|}{\sqrt{3^2 + 4^2}} = \frac{|-5|}{\sqrt{25}} = \frac{5}{5} = 1$$

$$\mathbf{b} \quad \vec{r}_l = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \text{ dus } \vec{n}_l = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

$$\left. \begin{array}{l} l: 2x - y = c \\ \text{door } (2, 3) \end{array} \right\} c = 2 \cdot 2 - 3 = 1$$

Dus $l: 2x - y = 1$.

$$d(B, l) = \frac{|2 \cdot 3 - 1 - 1|}{\sqrt{2^2 + (-1)^2}} = \frac{|4|}{\sqrt{5}} = \frac{4}{\sqrt{5}} = \frac{4}{5}\sqrt{5}$$

58

$$\mathbf{a} \quad \vec{r}_{BD} = \vec{d} - \vec{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ en } \vec{s}_{BD} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \text{ dus } BD: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

$$P \text{ op } BD, \text{ dus } \vec{p} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix}.$$

$$\vec{CP} = \vec{p} - \vec{c} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix}$$

$$\mathbf{b} \quad CP \perp BD, \text{ dus } \vec{CP} \cdot \vec{r}_{BD} = 0$$

$$\begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0$$

$$(t-1) \cdot 1 + (3t+4) \cdot 3 = 0$$

$$t-1+9t+12=0$$

$$10t = -11$$

$$t = -1\frac{1}{10}$$

$$t = -1\frac{1}{10} \text{ geeft } P(2 - 1\frac{1}{10}, 5 + 3 \cdot -1\frac{1}{10}) = P(\frac{9}{10}, 1\frac{7}{10})$$

$$\mathbf{c} \quad CP = OP, \text{ dus } |\vec{CP}| = |\vec{OP}|$$

$$\left| \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \right| = \left| \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} \right|$$

$$\sqrt{(t-1)^2 + (3t+4)^2} = \sqrt{(2+t)^2 + (5+3t)^2}$$

kwadrateren geeft

$$(t-1)^2 + (3t+4)^2 = (2+t)^2 + (5+3t)^2$$

$$t^2 - 2t + 1 + 9t^2 + 24t + 16 = 4 + 4t + t^2 + 25 + 30t + 9t^2$$

$$-12t = 12$$

$$t = -1$$

$$t = -1 \text{ geeft } P(2 + (-1), 5 + 3 \cdot (-1)) = P(1, 2)$$

a $\vec{r}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

$$\left. \begin{array}{l} k: 3x + 4y = c \\ \text{door } (4, 0) \end{array} \right\} c = 3 \cdot 4 + 4 \cdot 0 = 12$$

Dus $k: 3x + 4y = 12$.

$P(p, 0)$ is een punt op de x -as.

$$d(P, k) = \frac{|3 \cdot p + 4 \cdot 0 - 12|}{\sqrt{3^2 + 4^2}} = \frac{|3p - 12|}{5}$$

$$d(P, k) = 3 \text{ geeft } \frac{|3p - 12|}{5} = 3$$

$$|3p - 12| = 15$$

$$3p - 12 = 15 \vee 3p - 12 = -15$$

$$3p = 27 \vee 3p = -3$$

$$p = 9 \vee p = -1$$

Dus $P_1(9, 0)$ en $P_2(-1, 0)$.

b $l \parallel k$, dus $l: 3x + 4y = c$.

$(4, 0)$ is een punt op k , dus er moet gelden $d((4, 0), l) = 2$.

$$\text{Dit geeft } \frac{|3 \cdot 4 + 4 \cdot 0 - c|}{\sqrt{3^2 + 4^2}} = 2$$

$$\frac{|12 - c|}{5} = 2$$

$$|12 - c| = 10$$

$$12 - c = 10 \vee 12 - c = -10$$

$$c = 2 \vee c = 22$$

Dus $l_1: 3x + 4y = 2$ en $l_2: 3x + 4y = 22$.

$$l \parallel k, \text{ dus } \vec{r}_l = \vec{r}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}.$$

$$l_1 \text{ door } (2, -1), \text{ dus } \vec{s}_{l_1} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ en } l_2 \text{ door } (6, 1), \text{ dus } \vec{s}_{l_2} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}.$$

$$\text{Dus } l_1: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + u \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix} \text{ en } l_2: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} + v \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix}.$$

Bladzijde 78

a $A(3, 0)$ en $B(0, 4)$

$$P \text{ op } l, \text{ dus } \vec{p} = \begin{pmatrix} 6 + 2t \\ -1 - 3t \end{pmatrix}.$$

$$\vec{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 6 + 2t \\ -1 - 3t \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2t + 3 \\ -3t - 1 \end{pmatrix}$$

$$\vec{BP} = \vec{p} - \vec{b} = \begin{pmatrix} 6 + 2t \\ -1 - 3t \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2t + 6 \\ -3t - 5 \end{pmatrix}$$

$$AP \perp BP, \text{ dus } \vec{AP} \cdot \vec{BP} = 0$$

$$\begin{pmatrix} 2t + 3 \\ -3t - 1 \end{pmatrix} \cdot \begin{pmatrix} 2t + 6 \\ -3t - 5 \end{pmatrix} = 0$$

$$(2t + 3)(2t + 6) + (-3t - 1)(-3t - 5) = 0$$

$$4t^2 + 12t + 6t + 18 + 9t^2 + 15t + 3t + 5 = 0$$

$$13t^2 + 36t + 23 = 0$$

$$D = 36^2 - 4 \cdot 13 \cdot 23 = 100, \text{ dus } \sqrt{D} = 10$$

$$t = \frac{-36 \pm 10}{26} = -1 \vee t = \frac{-36 - 10}{26} = -1\frac{10}{13}$$

$$t = -1 \text{ geeft } P(6 + 2 \cdot -1, -1 - 3 \cdot -1) = P(4, 2)$$

$$t = -1\frac{10}{13} \text{ geeft } P(6 + 2 \cdot -1\frac{10}{13}, -1 - 3 \cdot -1\frac{10}{13}) = P(2\frac{6}{13}, 4\frac{4}{13})$$

$$\text{Dus } P(4, 2) \text{ of } P(2\frac{6}{13}, 4\frac{4}{13}).$$

b $AP = BP$, dus $|\overrightarrow{AP}| = |\overrightarrow{BP}|$

$$\left| \begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix} \right| = \left| \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix} \right|$$

$$\sqrt{(2t+3)^2 + (-3t-1)^2} = \sqrt{(2t+6)^2 + (-3t-5)^2}$$

kwadrateren geeft

$$(2t+3)^2 + (-3t-1)^2 = (2t+6)^2 + (-3t-5)^2$$

$$4t^2 + 12t + 9 + 9t^2 + 6t + 1 = 4t^2 + 24t + 36 + 9t^2 + 30t + 25$$

$$-36t = 51$$

$$t = -1\frac{5}{12}$$

$t = -1\frac{5}{12}$ geeft $P(6 + 2 \cdot -1\frac{5}{12}, -1 - 3 \cdot -1\frac{5}{12}) = P(3\frac{1}{6}, 3\frac{1}{4})$.

61 $l \perp k$, dus $l: 2x + y = c$.
 door $C(4\frac{1}{2}, 0)$ } $c = 2 \cdot 4\frac{1}{2} + 0 = 9$

Dus $l: 2x + y = 9$.

$$\begin{cases} x - 2y = -3 \\ 2x + y = 9 \end{cases} \begin{matrix} |2| \\ |1| \end{matrix} \text{ geeft } \begin{cases} 2x - 4y = -6 \\ 2x + y = 9 \end{cases}$$

$$\begin{matrix} -5y = -15 \\ y = 3 \end{matrix}$$

$y = 3$ geeft $2x + 3 = 9$

$$2x = 6$$

$x = 3$, dus $P(3, 3)$.

$$\overrightarrow{PA} = \vec{a} - \vec{p} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$$\overrightarrow{PB} = \vec{b} - \vec{p} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{PA}, \overrightarrow{PB})) = \frac{\begin{pmatrix} -2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right|} = \frac{-2 \cdot 3 + -3 \cdot -1}{\sqrt{13} \cdot \sqrt{10}} = \frac{-3}{\sqrt{130}} \text{ geeft } \angle(\overrightarrow{PA}, \overrightarrow{PB}) = 105,25\dots^\circ$$

Dus $\angle APB \approx 105,3^\circ$.

Bladzijde 79

C7 a $\cos(\angle(\vec{a}, \vec{b})) = \frac{\begin{pmatrix} 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|} = \frac{4 \cdot 2 + 1 \cdot -3}{\sqrt{17} \cdot \sqrt{13}} = \frac{5}{\sqrt{221}}$

Dus $\angle(\vec{a}, \vec{b}) \approx 70,3^\circ$.

b $\cos(\angle(\vec{c}, \vec{d})) = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right|} = \frac{1 \cdot -3 + 2 \cdot 1}{\sqrt{5} \cdot \sqrt{10}} = \frac{-1}{\sqrt{50}}$

Dus $\angle(\vec{c}, \vec{d}) \approx 98,1^\circ$.

C8 $\cos(\angle(\vec{r}_k, \vec{r}_l)) = \frac{\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -4 \end{pmatrix} \right|} = \frac{2 \cdot 1 + 1 \cdot -4}{\sqrt{5} \cdot \sqrt{17}} = \frac{-2}{\sqrt{85}} \text{ geeft } \angle(\vec{r}_k, \vec{r}_l) = 102,52\dots^\circ$

Dus $\angle(k, l) = 180^\circ - 102,52\dots^\circ \approx 77,5^\circ$.

C9 a $\vec{r}_k = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

$k: 2x - 5y = c$
 door $(6, 1)$ } $c = 2 \cdot 6 - 5 \cdot 1 = 7$

Dus $k: 2x - 5y = 7$.

b $\vec{n}_l = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$.
 l door $(5, 0)$, dus $\vec{s}_l = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$.
Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + u \cdot \begin{pmatrix} 7 \\ 3 \end{pmatrix}$.

C10 $l \perp k$, dus $\vec{r}_l = \vec{n}_k = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.
 l door $A(4, 6)$, dus $\vec{s}_l = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$.
Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

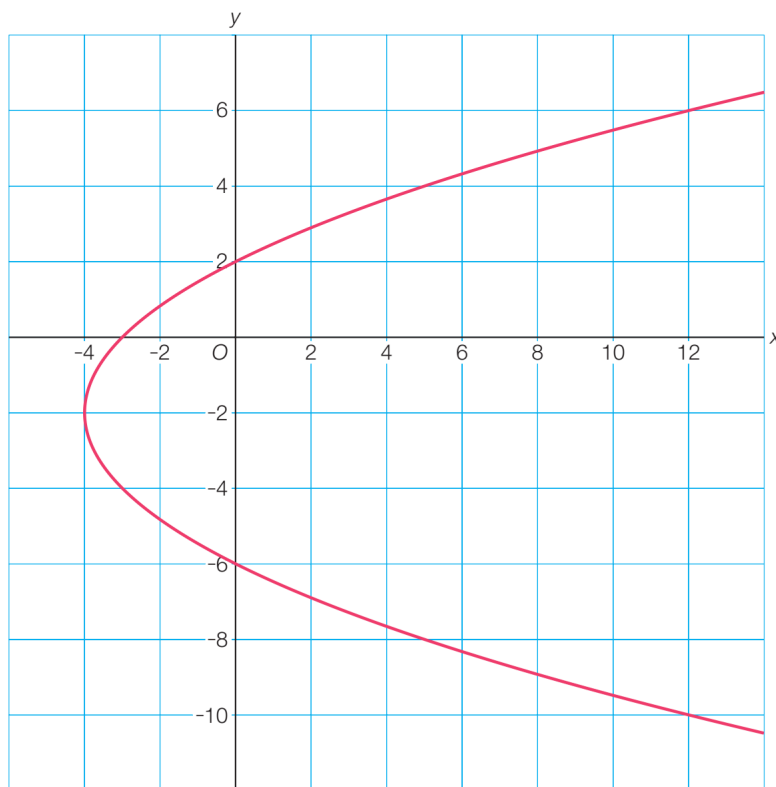
10.4 Vectoren bij snelheid en versnelling

Bladzijde 80

62 a $x(0) = 0 - 0 = 0$ en $y(0) = 0 - 6 = -6$, dus op $t = 0$ is P in het punt $(0, -6)$.

b

t	-2	-1	0	1	2	3	4	5	6
x	12	5	0	-3	-4	-3	0	5	12
y	-10	-8	-6	-4	-2	0	2	4	6



c De verplaatsing van P op $[2, 3]$ wordt gegeven door $\begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Bladzijde 83

63 a Evenwijdig met de y -as, dus $x'(t) = 0$
 $t^2 - 4 = 0$
 $t^2 = 4$
 $t = 2 \vee t = -2$
 $y'(2) = 2 \neq 0$ en $y'(-2) = -6 \neq 0$
 $x(2) = -5\frac{1}{3}$ en $y(2) = 0$, dus $(-5\frac{1}{3}, 0)$.
 $x(-2) = 5\frac{1}{3}$ en $y(-2) = 8$, dus $(5\frac{1}{3}, 8)$.

b Door de oorsprong, dus $x(t) = 0 \wedge y(t) = 0$

$$\begin{aligned}\frac{1}{3}t^3 - 4t &= 0 \wedge t^2 - 2t = 0 \\ \frac{1}{3}t(t^2 - 12) &= 0 \wedge t(t - 2) = 0 \\ (t = 0 \vee t^2 = 12) &\wedge (t = 0 \vee t = 2) \\ t &= 0\end{aligned}$$

$l: ax + by = 0$

$$\vec{r}_l = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \text{ dus } \vec{n}_l = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Dus $l: x - 2y = 0$.

c Naar links bewegen betekent $x'(t) < 0$ en omhoog bewegen betekent $y'(t) > 0$, dus $x'(t) < 0 \wedge y'(t) > 0$ en dit geeft $t^2 - 4 < 0 \wedge 2t - 2 > 0$.

d $t^2 - 4 < 0 \wedge 2t - 2 > 0$

$$\begin{aligned}t^2 &< 4 \wedge 2t > 2 \\ -2 &< t < 2 \wedge t > 1 \\ 1 &< t < 2\end{aligned}$$

Bladzijde 84

64 a $x(t) = t^2 - 2$ geeft $x'(t) = 2t$

$y(t) = t^3 - 4t$ geeft $y'(t) = 3t^2 - 4$

Evenwijdig met de x -as, dus $y'(t) = 0$

$$3t^2 - 4 = 0$$

$$3t^2 = 4$$

$$t^2 = \frac{4}{3}$$

$$t = \sqrt{\frac{4}{3}} \vee t = -\sqrt{\frac{4}{3}}$$

$$t = \frac{2}{3}\sqrt{3} \vee t = -\frac{2}{3}\sqrt{3}$$

$$x'(\frac{2}{3}\sqrt{3}) = 1\frac{1}{3}\sqrt{3} \neq 0 \text{ en } x'(-\frac{2}{3}\sqrt{3}) = -1\frac{1}{3}\sqrt{3} \neq 0$$

$$x(\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3} \text{ en } y(\frac{2}{3}\sqrt{3}) = \frac{4}{3} \cdot \frac{2}{3}\sqrt{3} - 4 \cdot \frac{2}{3}\sqrt{3} = \frac{8}{9}\sqrt{3} - \frac{8}{3}\sqrt{3} = -1\frac{7}{9}\sqrt{3}$$

$$x(-\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3} \text{ en } y(-\frac{2}{3}\sqrt{3}) = \frac{4}{3} \cdot -\frac{2}{3}\sqrt{3} - 4 \cdot -\frac{2}{3}\sqrt{3} = -\frac{8}{9}\sqrt{3} + \frac{8}{3}\sqrt{3} = 1\frac{7}{9}\sqrt{3}$$

Dus $(-\frac{2}{3}, -1\frac{7}{9}\sqrt{3})$ en $(-\frac{2}{3}, 1\frac{7}{9}\sqrt{3})$.

b $x'(-1) = -2$ en $y'(-1) = -1$

Dus de baansnelheid op $t = -1$ is $v(-1) = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

c $x'(t) > 0 \wedge y'(t) < 0$ geeft $2t > 0 \wedge 3t^2 - 4 < 0$

$$t > 0 \wedge 3t^2 < 4$$

$$t > 0 \wedge t^2 < \frac{4}{3}$$

$$t > 0 \wedge -\frac{2}{3}\sqrt{3} < t < \frac{2}{3}\sqrt{3}$$

$$0 < t < \frac{2}{3}\sqrt{3}$$

d $x = 2$ geeft $t^2 - 2 = 2$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$y(2) = 0$ en $y(-2) = 0$, dus de baan snijdt zichzelf in $(2, 0)$.

$$\vec{v}(2) = \begin{pmatrix} x'(2) \\ y'(2) \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

$$\vec{v}(-2) = \begin{pmatrix} x'(-2) \\ y'(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} 4 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 8 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ 8 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -4 \\ 8 \end{pmatrix} \right|} = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right|} = \frac{1 \cdot -1 + 2 \cdot 2}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5} \text{ geeft } \varphi = 53,13\dots^\circ$$

Dus de gevraagde hoek is $53,1^\circ$.

- a** $x(t) = 0$ geeft $2t - \frac{1}{6}t^3 = 0$
 $\frac{1}{6}t(12 - t^2) = 0$
 $t = 0 \vee t^2 = 12$
 $t = 0 \vee t = \sqrt{12} \vee t = -\sqrt{12}$
 $t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$
 $t = 0$ geeft $(0, 0)$
 $y(2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot 2\sqrt{3} = 3 - 4\sqrt{3}$, dus $B(0, 3 - 4\sqrt{3})$.
 $y(-2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot -2\sqrt{3} = 3 + 4\sqrt{3}$, dus $A(0, 3 + 4\sqrt{3})$.
 $d(A, B) = 3 + 4\sqrt{3} - (3 - 4\sqrt{3}) = 8\sqrt{3}$

- b** $x(t) = 2t - \frac{1}{6}t^3$ geeft $x'(t) = 2 - \frac{1}{2}t^2$
 $y(t) = \frac{1}{4}t^2 - 2t$ geeft $y'(t) = \frac{1}{2}t - 2$
Evenwijdig met de x -as, dus $y'(t) = 0$
 $\frac{1}{2}t - 2 = 0$
 $\frac{1}{2}t = 2$
 $t = 4$
 $x'(4) = -6 \neq 0$
 $x(4) = -2\frac{2}{3}$ en $y(4) = -4$
Dus evenwijdig met de x -as in $(-2\frac{2}{3}, -4)$.
Evenwijdig met de y -as, dus $x'(t) = 0$
 $2 - \frac{1}{2}t^2 = 0$
 $\frac{1}{2}t^2 = 2$
 $t^2 = 4$
 $t = 2 \vee t = -2$
 $y'(2) = -1 \neq 0$ en $y'(-2) = -3 \neq 0$
 $x(2) = 2\frac{2}{3}$ en $y(2) = -3$
 $x(-2) = -2\frac{2}{3}$ en $y(-2) = 5$
Dus evenwijdig met de y -as in $(2\frac{2}{3}, -3)$ en $(-2\frac{2}{3}, 5)$.
- c** Bij de oorsprong hoort $t = 0$ (zie vraag a).
 $x'(0) = 2$ en $y'(0) = -2$
Dus de baansnelheid in de oorsprong is $v(0) = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$.
- d** $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(2 - \frac{1}{2}t^2)^2 + (\frac{1}{2}t - 2)^2}$
Invullen van de formules en gebruiken van de optie minimum.
De minimale baansnelheid is ongeveer 0,97 voor $t \approx 2,11$.

- a** Als de bal in het hoogste punt is, geldt $y'(t) = 0$.
 $y(t) = 12t - 5t^2$ geeft $y'(t) = 12 - 10t$
 $y'(t) = 0$ geeft $12 - 10t = 0$
 $10t = 12$
 $t = 1,2$
- b** $x(t) = 20t$ geeft $x'(t) = 20$
 $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{20^2 + (12 - 10t)^2} = \sqrt{400 + 144 - 240t + 100t^2} = \sqrt{100t^2 - 240t + 544}$
 $v'(t) = \frac{1}{2\sqrt{100t^2 - 240t + 544}} \cdot (200t - 240) = \frac{100t - 120}{\sqrt{100t^2 - 240t + 544}}$
 $v'(1,2) = \frac{100 \cdot 1,2 - 120}{\sqrt{100 \cdot 1,2^2 - 240 \cdot 1,2 + 544}} = 0$
- c** $x'(t) = 20$ geeft $x''(t) = 0$
 $y'(t) = 12 - 10t$ geeft $y''(t) = -10$
 $\vec{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$
In een hoogste punt is de raaklijn horizontaal en is er dus geen verticale verplaatsing, dus in een hoogste punt geldt $v'(t) = x''(t)$.
Dus in A geldt $v'(1,2) = x''(1,2) = 0$.

67 a De baanversnelling op $t = 2$ is $a(2) = \frac{10 - 7}{\sqrt{20 - 28 + 17}} = \frac{3}{\sqrt{9}} = 1$.

b $\vec{v}(2) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ en $\vec{a}(2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
 $v(2) = \sqrt{9} = 3$

Dus $a(2) = \frac{\vec{v}(2) \cdot \vec{a}(2)}{v(2)} = \frac{\begin{pmatrix} 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{3} = \frac{0 \cdot 2 + 3 \cdot 1}{3} = \frac{3}{3} = 1$.

c $\vec{v}(t) \cdot \vec{a}(t) = 5t - 7$ en $v(t) = \sqrt{5t^2 - 14t + 17}$ geeft $a(t) = \frac{5t - 7}{\sqrt{5t^2 - 14t + 17}}$

68 a Snijden met de x -as, dus $y(t) = 0$ geeft $t^3 - 3t = 0$

$$t(t^2 - 3) = 0$$

$$t = 0 \vee t^2 = 3$$

$$t = 0 \vee t = \sqrt{3} \vee t = -\sqrt{3}$$

De baan passeert D twee keer, dat is dus op $t = -\sqrt{3}$ en $t = \sqrt{3}$. Dus op $t = 0$ is P in C .

Snijden met de y -as, dus $x(t) = 0$ geeft $t^2 - 4 = 0$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$y(2) = 2$, dus de baan passeert A op $t = 2$ en B op $t = -2$.

Dus de punten worden in de volgorde B, D, C, D, A doorlopen.

b $\vec{r}(t) = \begin{pmatrix} t^2 - 4 \\ t^3 - 3t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 2t \\ 3t^2 - 3 \end{pmatrix}$ en

$$v(t) = \sqrt{(2t)^2 + (3t^2 - 3)^2} = \sqrt{4t^2 + 9t^4 - 18t^2 + 9} = \sqrt{9t^4 - 14t^2 + 9}.$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{9t^4 - 14t^2 + 9}} \cdot (36t^3 - 28t) = \frac{18t^3 - 14t}{\sqrt{9t^4 - 14t^2 + 9}}$$

c De baansnelheid in A is $v(2) = \sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9} = \sqrt{97}$.

De baanversnelling in A is $a(2) = \frac{18 \cdot 2^3 - 14 \cdot 2}{\sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9}} = \frac{116}{\sqrt{97}} = 1\frac{19}{97}\sqrt{97}$.

d $a(t) = 0$ geeft $18t^3 - 14t = 0$

$$2t(9t^2 - 7) = 0$$

$$t = 0 \vee t^2 = \frac{7}{9}$$

$$t = 0 \vee t = \sqrt{\frac{7}{9}} \vee t = -\sqrt{\frac{7}{9}}$$

$$t = 0 \vee t = \frac{1}{3}\sqrt{7} \vee t = -\frac{1}{3}\sqrt{7}$$

$$v(0) = \sqrt{0 + 9} = 3$$

$$v(\frac{1}{3}\sqrt{7}) = \sqrt{9 \cdot (\frac{7}{9})^2 - 14 \cdot \frac{7}{9} + 9} = \sqrt{\frac{32}{9}} = 1\frac{1}{3}\sqrt{2}$$

$$v(-\frac{1}{3}\sqrt{7}) = \sqrt{9 \cdot (\frac{7}{9})^2 - 14 \cdot \frac{7}{9} + 9} = 1\frac{1}{3}\sqrt{2}$$

e $v(t) = 2$ geeft $\sqrt{9t^4 - 14t^2 + 9} = 2$

$$9t^4 - 14t^2 + 9 = 4$$

$$9t^4 - 14t^2 + 5 = 0$$

$$\text{Stel } t^2 = u.$$

$$9u^2 - 14u + 5 = 0$$

$$D = (-14)^2 - 4 \cdot 9 \cdot 5 = 16, \text{ dus } \sqrt{D} = 4$$

$$u = \frac{14 \pm 4}{18} = 1 \vee \frac{14 - 4}{18} = \frac{5}{9}$$

$$t^2 = 1 \vee t^2 = \frac{5}{9}$$

$$t = 1 \vee t = -1 \vee t = \sqrt{\frac{5}{9}} \vee t = -\sqrt{\frac{5}{9}}$$

$$t = 1 \vee t = -1 \vee t = \frac{1}{3}\sqrt{5} \vee t = -\frac{1}{3}\sqrt{5}$$

- f** De baan passeert D op $t = \sqrt{3}$ en $t = -\sqrt{3}$ (zie vraag a).
 $x(\sqrt{3}) = -1$ en $x(-\sqrt{3}) = -1$, dus de baan snijdt zichzelf in $D(-1, 0)$.

$$\vec{v}(\sqrt{3}) = \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix}$$

$$\vec{v}(-\sqrt{3}) = \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix}}{\left| \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix} \right|} = \frac{\begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} \sqrt{3} \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix} \right|} = \frac{\sqrt{3} \cdot -\sqrt{3} + 3 \cdot 3}{\sqrt{12} \cdot \sqrt{12}} = \frac{6}{12} = \frac{1}{2} \text{ geeft } \varphi = 60^\circ$$

Dus de gevraagde hoek is 60° .

- 69 a** Snijden met de x -as, dus $y(t) = 0$ geeft $\frac{1}{2}t^2 - 2t = 0$

$$\frac{1}{2}t(t - 4) = 0$$

$$t = 0 \vee t = 4$$

$x(0) = -2$, dus de baan passeert A op $t = 0$ en B op $t = 4$.

Dus de punten worden in de volgorde C, A, D, B doorlopen.

- b** $\vec{r}(t) = \begin{pmatrix} \frac{1}{2}t^2 - 2 \\ \frac{1}{2}t^2 - 2t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} t \\ t - 2 \end{pmatrix}$ en $v(t) = \sqrt{t^2 + (t - 2)^2} = \sqrt{t^2 + t^2 - 4t + 4} = \sqrt{2t^2 - 4t + 4}$.

$$a(t) = v'(t) = \frac{1}{2\sqrt{2t^2 - 4t + 4}} \cdot (4t - 4) = \frac{2t - 2}{\sqrt{2t^2 - 4t + 4}}$$

- c** $\vec{v}(t) = \begin{pmatrix} t \\ t - 2 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\vec{v}(t) \cdot \vec{a}(t) = t + t - 2 = 2t - 2$$

$$\vec{v}(t) \cdot \vec{a}(t) = 0 \text{ geeft } 2t - 2 = 0 \text{ oftewel } t = 1$$

$x(1) = -1\frac{1}{2}$ en $y(1) = -1\frac{1}{2}$, dus in $(-1\frac{1}{2}, -1\frac{1}{2})$ staat de versnellingsvector loodrecht op de snelheidsvector.

- d** $a(t) = 0$ geeft $2t - 2 = 0$ oftewel $t = 1$

$$v(1) = \sqrt{2 \cdot 1^2 - 4 \cdot 1 + 4} = \sqrt{2}$$

Dus de minimale baansnelheid is $\sqrt{2}$ voor $t = 1$.

- e** Bij B hoort $t = 4$ (zie vraag a), dus de baansnelheid in B is $v(4) = \sqrt{2 \cdot 4^2 - 4 \cdot 4 + 4} = \sqrt{20}$.

Snijden met de y -as, dus $x(t) = 0$ geeft $\frac{1}{2}t^2 - 2 = 0$

$$\frac{1}{2}t^2 = 2$$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

Bij C hoort $t = -2$ (zie vraag a), dus de baansnelheid in C is

$$v(-2) = \sqrt{2 \cdot (-2)^2 - 4 \cdot (-2) + 4} = \sqrt{20}.$$

Dus de baansnelheid in B is gelijk aan die in C .

- f** $x(t) = -1\frac{1}{2}$ en $y(t) = 2\frac{1}{2}$ geeft $\begin{cases} \frac{1}{2}t^2 - 2 = -1\frac{1}{2} \\ \frac{1}{2}t^2 - 2t = 2\frac{1}{2} \end{cases}$

$$2t - 2 = -4$$

$$2t = -2$$

$$t = -1$$

$$\vec{r}_k = \vec{v}(-1) = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \text{ geeft } \vec{n}_k = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$k: 3x - y = c$$

$$\text{door } E(-1\frac{1}{2}, 2\frac{1}{2}) \left. \vphantom{\begin{matrix} k: 3x - y = c \\ \text{door } E(-1\frac{1}{2}, 2\frac{1}{2}) \end{matrix}} \right\} c = 3 \cdot -1\frac{1}{2} - 2\frac{1}{2} = -7$$

Dus $k: 3x - y = -7$.

70 $p = x_Q = x_R$ geeft $\frac{1}{2}t_Q^2 - 2 = \frac{1}{2}t_R^2 - 2$, dus $t_Q^2 = t_R^2$ oftewel $t_Q = -t_R$.
 $y_R - y_Q = 12$ geeft $\frac{1}{2}t_R^2 - 2t_R - (\frac{1}{2}t_Q^2 - 2t_Q) = 12$
 Substitutie van $t_Q = -t_R$ in $\frac{1}{2}t_R^2 - 2t_R - (\frac{1}{2}t_Q^2 - 2t_Q) = 12$ geeft $\frac{1}{2}t_R^2 - 2t_R - (\frac{1}{2}t_R^2 + 2t_R) = 12$
 $\frac{1}{2}t_R^2 - 2t_R - \frac{1}{2}t_R^2 - 2t_R = 12$
 $-4t_R = 12$
 $t_R = -3$

Dus $t_Q = 3$.
 $p = x_Q = x_R = x(-3) = 2\frac{1}{2}$
 $y_Q = y(3) = -1\frac{1}{2}$ en $y_R = y(-3) = 10\frac{1}{2}$
 Dus $Q(2\frac{1}{2}, -1\frac{1}{2})$ en $R(2\frac{1}{2}, 10\frac{1}{2})$.

Bladzijde 87

71 a $\vec{r}(t) = \begin{pmatrix} 25t \\ -5t^2 + 15t + 3 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$
 $v(0) = \sqrt{25^2 + 15^2} = 29,15...$
 De baansnelheid waarmee de speer wordt weggegooid, is ongeveer 29,2 m/s.

b Een richtingsvector van een horizontale lijn is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$\vec{v}(0) = \begin{pmatrix} 25 \\ 15 \end{pmatrix}$
 $\cos(\varphi) = \frac{\begin{pmatrix} 25 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\left| \begin{pmatrix} 25 \\ 15 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{25 \cdot 1 + 15 \cdot 0}{\sqrt{850} \cdot 1} = \frac{25}{\sqrt{850}}$ geeft $\varphi = 30,9...^\circ$

De gevraagde hoek is 31° .

c $y'(t) = 0$ geeft $-10t + 15 = 0$
 $-10t = -15$
 $t = 1,5$

$y(1,5) = 14,25$, dus de maximale hoogte is 14,25 meter.

d $y(t) = 0$ geeft $-5t^2 + 15t + 3 = 0$
 $D = 15^2 - 4 \cdot -5 \cdot 3 = 285$
 $t = \frac{-15 \pm \sqrt{285}}{-10} = -0,18... \vee t = \frac{-15 \pm \sqrt{285}}{-10} = 3,18...$

$t = -0,18...$ voldoet niet, dus $t = 3,18...$

$x(3,18...) = 79,70...$

Dus de speer komt ongeveer 79,7 meter van de afworpleijn neer.

e $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$

De versnellingsvector is de valversnelling van -10 m/s^2 .

Bladzijde 88

72 a Snijden met de x -as, dus $y(t) = 0$ geeft $t^2 - 4 = 0$
 $t^2 = 4$
 $t = 2 \vee t = -2$

$\vec{r}(t) = \begin{pmatrix} \frac{1}{3}t^3 - 4t \\ t^2 - 4 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix}$

$\vec{v}(2) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ en $\vec{v}(-2) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

De snelheidsvectoren in A en B zijn verticaal, dus de raaklijnen aan de baan in A en B zijn verticaal.

b $y = 5$ geeft $t^2 - 4 = 5$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$x(3) = -3$, dus $t = 3$ hoort bij C .

$$\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix} \text{ geeft } v(t) = \sqrt{(t^2 - 4)^2 + (2t)^2} = \sqrt{t^4 - 8t^2 + 16 + 4t^2} = \sqrt{t^4 - 4t^2 + 16}$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{t^4 - 4t^2 + 16}} \cdot (4t^3 - 8t) = \frac{2t^3 - 4t}{\sqrt{t^4 - 4t^2 + 16}}$$

De baansnelheid in C is $v(3) = \sqrt{3^4 - 4 \cdot 3^2 + 16} = \sqrt{61}$.

De baanversnelling in C is $a(3) = \frac{2 \cdot 3^3 - 4 \cdot 3}{\sqrt{3^4 - 4 \cdot 3^2 + 16}} = \frac{42}{\sqrt{61}} = \frac{42}{61}\sqrt{61}$.

c $v(t) = 6\frac{1}{2}$ geeft $\sqrt{t^4 - 4t^2 + 16} = 6\frac{1}{2}$

$$t^4 - 4t^2 + 16 = 42\frac{1}{4}$$

$$t^4 - 4t^2 - 26\frac{1}{4} = 0$$

Stel $t^2 = u$.

$$u^2 - 4u - 26\frac{1}{4} = 0$$

$$D = (-4)^2 - 4 \cdot 1 \cdot -26\frac{1}{4} = 121, \text{ dus } \sqrt{D} = 11$$

$$u = \frac{4 + 11}{2} = 7\frac{1}{2} \vee u = \frac{4 - 11}{2} = -3\frac{1}{2}$$

$$t^2 = 7\frac{1}{2} \vee t^2 = -3\frac{1}{2}$$

$$t = \sqrt{7\frac{1}{2}} \vee t = -\sqrt{7\frac{1}{2}}$$

$$t = \frac{1}{2}\sqrt{30} \vee t = -\frac{1}{2}\sqrt{30}$$

$$x(\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot \frac{1}{2}\sqrt{30} - 4 \cdot \frac{1}{2}\sqrt{30} = 1\frac{1}{4}\sqrt{30} - 2\sqrt{30} = -\frac{3}{4}\sqrt{30} \text{ en } y(\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}$$

$$x(-\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot -\frac{1}{2}\sqrt{30} - 4 \cdot -\frac{1}{2}\sqrt{30} = -1\frac{1}{4}\sqrt{30} + 2\sqrt{30} = \frac{3}{4}\sqrt{30} \text{ en } y(-\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}$$

Dus $(-\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$ en $(\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$.

d Invoeren van de formule van de baanversnelling en gebruiken van de optie minimum.

De baanversnelling heeft voor $t \approx 0,851$ een minimum van ongeveer $-0,588$.

e Snijden met de y -as, dus $x(t) = 0$ geeft $\frac{1}{3}t^3 - 4t = 0$

$$\frac{1}{3}t(t^2 - 12) = 0$$

$$t = 0 \vee t^2 = 12$$

$$t = 0 \vee t = \sqrt{12} \vee t = -\sqrt{12}$$

$$t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$$

$$y(2\sqrt{3}) = 8 \text{ en } y(-2\sqrt{3}) = 8$$

Dus de baan snijdt zichzelf in $S(0, 8)$.

$$\vec{v}(2\sqrt{3}) = \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \text{ en } \vec{v}(-2\sqrt{3}) = \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}}{\left| \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix} \right|} = \frac{\begin{pmatrix} 2 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -\sqrt{3} \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ \sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -\sqrt{3} \end{pmatrix} \right|} = \frac{2 \cdot 2 + \sqrt{3} \cdot -\sqrt{3}}{\sqrt{7} \cdot \sqrt{7}} = \frac{1}{7} \text{ geeft } \varphi = 81,7...^\circ$$

Dus de gevraagde hoek is 82° .

C11

a $x(t) = -\frac{1}{3}t^3 + 3t^2$ geeft $x'(t) = -t^2 + 6t$

$$y(t) = \frac{1}{3}t^3 - 9t \text{ geeft } y'(t) = t^2 - 9$$

Evenwijdig met de x -as, dus $y'(t) = 0$

$$t^2 - 9 = 0$$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$$x'(3) = 9 \neq 0 \text{ en } x'(-3) = -27 \neq 0$$

$$x(3) = 18 \text{ en } y(3) = -18$$

$$x(-3) = 36 \text{ en } y(-3) = 18$$

Dus $(18, -18)$ en $(36, 18)$.

b $l: ax + by = c$

$$\vec{r}_l = \begin{pmatrix} x'(1) \\ y'(1) \end{pmatrix} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}, \text{ dus } \vec{n}_l = \begin{pmatrix} 8 \\ 5 \end{pmatrix}.$$

$$\left. \begin{array}{l} l: 8x + 5y = c \\ \text{door } (x(1), y(1)) = (2\frac{2}{3}, -8\frac{2}{3}) \end{array} \right\} c = 8 \cdot 2\frac{2}{3} + 5 \cdot -8\frac{2}{3} = -22$$

Dus $l: 8x + 5y = -22$.

c De baansnelheid op $t = 1$ is $v(1) = \sqrt{5^2 + (-8)^2} = \sqrt{89}$.

C12 a $v(t) = \sqrt{(-t^2 + 6t)^2 + (t^2 - 9)^2} = \sqrt{t^4 - 12t^3 + 36t^2 + t^4 - 18t^2 + 81} = \sqrt{2t^4 - 12t^3 + 18t^2 + 81}$

$$a(t) = v'(t) = \frac{1}{2\sqrt{2t^4 - 12t^3 + 18t^2 + 81}} \cdot (8t^3 - 36t^2 + 36t) = \frac{4t^3 - 18t^2 + 18t}{\sqrt{2t^4 - 12t^3 + 18t^2 + 81}}$$

De baanversnelling op $t = 1$ is $a(1) = \frac{4 - 18 + 18}{\sqrt{2 - 12 + 18 + 81}} = \frac{4}{\sqrt{89}} = \frac{4}{89}\sqrt{89}$.

b $\vec{v}(t) = \begin{pmatrix} -t^2 + 6t \\ t^2 - 9 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} -2t + 6 \\ 2t \end{pmatrix}$

$$\begin{aligned} \vec{v}(t) \cdot \vec{a}(t) &= (-t^2 + 6t)(-2t + 6) + (t^2 - 9) \cdot 2t \\ &= 2t^3 - 6t^2 - 12t^2 + 36t + 2t^3 - 18t \\ &= 4t^3 - 18t^2 + 18t \\ \vec{v}(t) \cdot \vec{a}(t) &= 0 \text{ geeft } 4t^3 - 18t^2 + 18t = 0 \\ &\quad 2t(2t^2 - 9t + 9) = 0 \\ &\quad t = 0 \vee 2t^2 - 9t + 9 = 0 \\ &\quad D = (-9)^2 - 4 \cdot 2 \cdot 9 = 9, \text{ dus } \sqrt{D} = 3 \\ &\quad t = \frac{9 \pm 3}{4} = 3 \vee t = \frac{9 - 3}{4} = 1\frac{1}{2} \end{aligned}$$

Dus voor $t = 0$, $t = 1\frac{1}{2}$ en $t = 3$ staat de versnellingsvector loodrecht op de snelheidsvector.

Kennis- en vaardigheidentest

Bladzijde 89

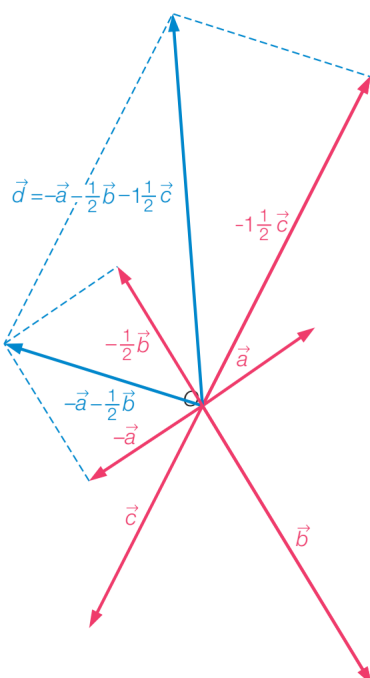
1 a $|\vec{a}| = \left| \begin{pmatrix} 4 \\ 2 \end{pmatrix} \right| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

$$|\vec{b}| = \left| \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$$

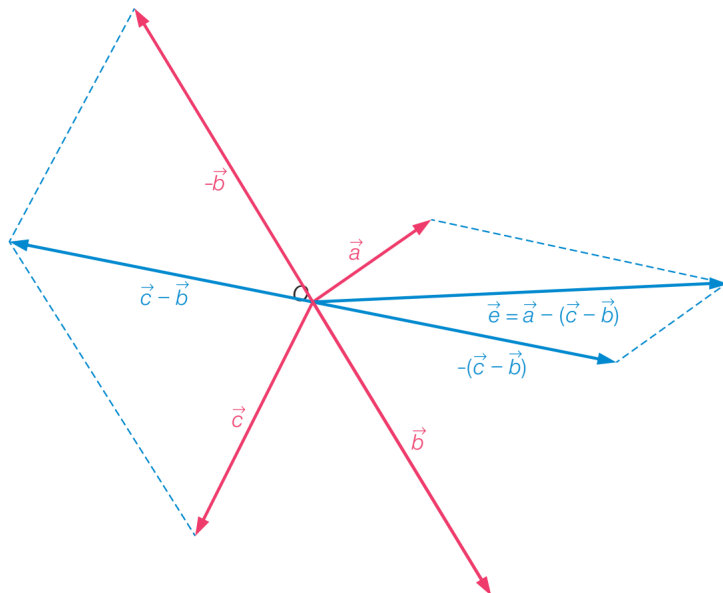
b $\vec{c} = \vec{a} + 2\vec{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

$$\vec{d} = \frac{1}{2}\vec{a} - 3\vec{b} = \frac{1}{2} \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} - 3 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \end{pmatrix} = \begin{pmatrix} -7 \\ 4 \end{pmatrix}$$

2 a



b

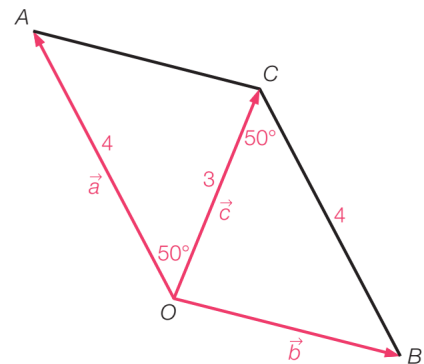


3 a Zie de figuur hiernaast.

b In $\triangle OBC$ is $BC = |\vec{a}| = 4$ en $\angle OCB = \angle AOC = 50^\circ$ (Z-hoeken).De cosinusregel in $\triangle OBC$ geeft

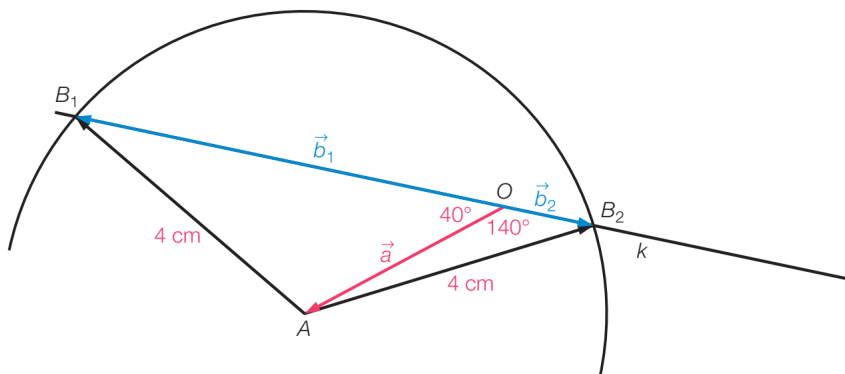
$$|\vec{b}|^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos(50^\circ) = 9,573\dots$$

$$|\vec{b}| \approx 3,09$$



Bladzijde 90

4 Zie de figuur hieronder.



$$\begin{aligned} \text{De sinusregel in } \triangle OAB_1 \text{ geeft } \frac{3}{\sin(\angle B_1)} &= \frac{4}{\sin(40^\circ)} \\ \sin(\angle B_1) &= \frac{3 \sin(40^\circ)}{4} \end{aligned}$$

$$\angle B_1 = 28,82\dots^\circ$$

$$\angle OAB_1 = 180^\circ - 40^\circ - 28,82\dots^\circ = 111,17\dots^\circ$$

$$\text{De sinusregel in } \triangle OAB_1 \text{ geeft } \frac{OB_1}{\sin(111,17\dots^\circ)} = \frac{4}{\sin(40^\circ)}$$

$$\text{Dus } |\vec{b}_1| = OB_1 = \frac{4 \sin(111,17\dots^\circ)}{\sin(40^\circ)} \approx 5,80 \text{ cm.}$$

$$\angle B_2 = \angle B_1 = 28,82\dots^\circ$$

$$\angle OAB_2 = 180^\circ - 140^\circ - 28,82\dots^\circ = 11,17\dots^\circ$$

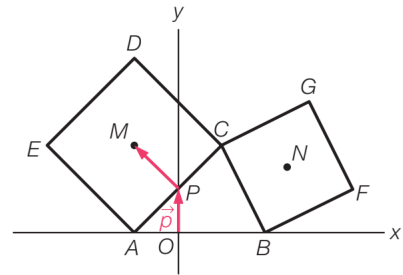
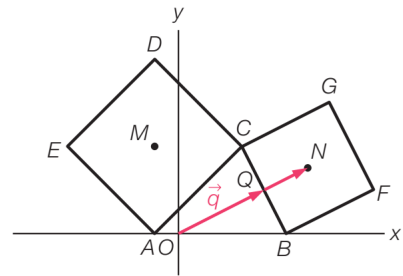
De sinusregel in $\triangle OAB_2$ geeft $\frac{OB_2}{\sin(11,17\dots^\circ)} = \frac{4}{\sin(140^\circ)}$.
 Dus $|\vec{b}_2| = OB_2 = \frac{4 \sin(11,17\dots^\circ)}{\sin(140^\circ)} \approx 1,21 \text{ cm}$.

5 P is het midden van zijde AC .

$$\begin{aligned}\vec{OM} &= \vec{OP} + \vec{PM} = \vec{OP} + \vec{PA}_R \\ \vec{OP} &= \vec{p} = \frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ \vec{PA} &= \vec{a} - \vec{p} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \text{ dus } \vec{PA}_R = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\ \text{Dit geeft } \vec{OM} &= \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \text{ dus } M(-2, 4).\end{aligned}$$

Q is het midden van zijde BC .

$$\begin{aligned}\vec{ON} &= \vec{OQ} + \vec{QN} = \vec{OQ} + \vec{QB}_L \\ \vec{OQ} &= \vec{q} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \\ \vec{QB} &= \vec{b} - \vec{q} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \text{ dus } \vec{QB}_L = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ \text{Dit geeft } \vec{ON} &= \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \text{ dus } N(5, 3).\end{aligned}$$



6 a $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + t(\vec{a} - \vec{c})$
 $\left. \begin{aligned} \vec{b} &= \begin{pmatrix} -4 \\ 5 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \end{aligned} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

b $\vec{m} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

$l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{a} - \vec{m})$
 $\left. \begin{aligned} \vec{a} &= \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ en } \vec{a} - \vec{m} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} -1\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 7\frac{1}{2} \\ 7\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

c $\vec{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ en } \vec{c} - \vec{a} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} -5 \\ -12 \end{pmatrix}$

Dus een vectorvoorstelling van het lijnstuk AC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} -5 \\ -12 \end{pmatrix} \wedge 0 \leq t \leq 1$.

7 Substitutie van $x = -2 + 2t$ en $y = 3 - t$ in $2x - 3y = 22$ geeft $2(-2 + 2t) - 3(3 - t) = 22$
 $-4 + 4t - 9 + 3t = 22$
 $7t = 35$
 $t = 5$

$t = 5$ geeft $x = -2 + 2 \cdot 5 = 8$ en $y = 3 - 5 = -2$, dus $S(8, -2)$.

8 a $\begin{cases} x = 1 + 3t \\ y = 2 + 4t \end{cases} \Big| \begin{cases} 4 \\ 3 \end{cases}$ geeft $\begin{cases} 4x = 4 + 12t \\ 3y = 6 + 12t \\ 4x - 3y = -2 \end{cases}$

Dus $k: 4x - 3y = -2$.

b Stel $x = t$. Dan is $3t - y = 6$, dus $y = 3t - 6$.

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

9 Stel $y = t$. Dan is $x - 3t = -6$, dus $x = 3t - 6$.

Dus $P(3t - 6, t)$ en $M(1\frac{1}{2}t - 3, \frac{1}{2}t)$.

$$\overrightarrow{PM} = \vec{m} - \vec{p} = \begin{pmatrix} 1\frac{1}{2}t - 3 \\ \frac{1}{2}t \end{pmatrix} - \begin{pmatrix} 3t - 6 \\ t \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2}t + 3 \\ -\frac{1}{2}t \end{pmatrix}$$

$$\overrightarrow{PQ} = \overrightarrow{PM}_L = \begin{pmatrix} \frac{1}{2}t \\ -1\frac{1}{2}t + 3 \end{pmatrix}$$

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix} 3t - 6 \\ t \end{pmatrix} + \begin{pmatrix} \frac{1}{2}t \\ -1\frac{1}{2}t + 3 \end{pmatrix} = \begin{pmatrix} 3\frac{1}{2}t - 6 \\ -\frac{1}{2}t + 3 \end{pmatrix}$$

$$\begin{cases} x = 3\frac{1}{2}t - 6 \\ y = -\frac{1}{2}t + 3 \end{cases} \Bigg| \cdot 7 \text{ geeft } \begin{cases} x = 3\frac{1}{2}t - 6 \\ 7y = -3\frac{1}{2}t + 21 \end{cases} +$$

$$x + 7y = 15$$

Dus $l: x + 7y = 15$.

10 a $\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ en $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$$\cos(\angle(\overrightarrow{AB}, \overrightarrow{AC})) = \frac{\begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix}}{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|} = \frac{6 \cdot -1 + 1 \cdot -4}{\sqrt{37} \cdot \sqrt{17}} = \frac{-10}{\sqrt{629}} \text{ geeft } \angle(\overrightarrow{AB}, \overrightarrow{AC}) = 113,49\dots^\circ$$

Dus $\angle BAC \approx 113,5^\circ$.

b $\vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\vec{n} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\vec{r}_{MN} = \vec{m} - \vec{n} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_{MN}, \vec{r}_k)) = \frac{\begin{pmatrix} 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right|} = \frac{1 \cdot 3 + 4 \cdot 1}{\sqrt{17} \cdot \sqrt{10}} = \frac{7}{\sqrt{170}} \text{ geeft } \angle(\vec{r}_{MN}, \vec{r}_k) = 57,52\dots^\circ$$

Dus $\angle(MN, k) \approx 57,5^\circ$.

11 a $\vec{r}_l = \vec{r}_k = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$.

$$l: 7x + 2y = c \left\{ \begin{array}{l} \text{door } A(1, -3) \end{array} \right\} c = 7 \cdot 1 + 2 \cdot -3 = 1$$

Dus $l: 7x + 2y = 1$.

b $\vec{n}_m = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$, dus $\vec{r}_n = \vec{r}_m = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

$$n \text{ door } B(-4, -1), \text{ dus } \vec{s}_n = \begin{pmatrix} -4 \\ -1 \end{pmatrix}.$$

$$\text{Dus } n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} + u \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

c $q \perp p$, dus $\vec{r}_q = \vec{n}_p = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

$$q \text{ door } C(-2, 6), \text{ dus } \vec{s}_q = \begin{pmatrix} -2 \\ 6 \end{pmatrix}.$$

$$\text{Dus } q: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix} + v \cdot \begin{pmatrix} 3 \\ 5 \end{pmatrix}.$$

12

$$\mathbf{a} \quad \begin{cases} 1+t=3+2u \\ 9+3t=1-u \end{cases} \text{ oftewel } \begin{cases} t-2u=2 \\ 3t+u=-8 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} t-2u=2 \\ 6t+2u=-16 \end{cases} +$$

$$\frac{7t}{7t} = -14 \quad t = -2$$

$$t = -2 \text{ geeft } x = 1 - 2 = -1 \text{ en } y = 9 + 3 \cdot -2 = 3$$

Dus $S(-1, 3)$.

$$\mathbf{b} \quad m \perp k, \text{ dus } \vec{n}_m = \vec{r}_k = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

$$\left. \begin{array}{l} m: x+3y=c \\ \text{door } A(3, -5) \end{array} \right\} c = 3 + 3 \cdot -5 = -12$$

$$\text{Dus } m: x+3y = -12.$$

$$y = 0 \text{ geeft } x = -12, \text{ dus } B(-12, 0).$$

$$x = 0 \text{ geeft } 3y = -12 \text{ oftewel } y = -4, \text{ dus } C(0, -4).$$

$$|\overrightarrow{BC}| = d(B, C) = \sqrt{(0 - (-12))^2 + (-4 - 0)^2} = \sqrt{160} = 4\sqrt{10}$$

13

$$\mathbf{a} \quad x(t) = 4t - t^2 \text{ geeft } x'(t) = 4 - 2t$$

$$y(t) = \frac{2}{3}t^3 - t^2 \text{ geeft } y'(t) = 2t^2 - 2t$$

$$\text{Evenwijdig met de } x\text{-as, dus } y'(t) = 0$$

$$2t^2 - 2t = 0$$

$$2t(t-1) = 0$$

$$t = 0 \vee t = 1$$

$$x'(0) = 4 \neq 0 \text{ en } x'(1) = 2 \neq 0$$

$$x(0) = 0 \text{ en } y(0) = 0$$

$$x(1) = 3 \text{ en } y(1) = -\frac{1}{3}$$

Dus evenwijdig met de x -as in $(0, 0)$ en $(3, -\frac{1}{3})$.

$$\text{Evenwijdig met de } y\text{-as, dus } x'(t) = 0$$

$$4 - 2t = 0$$

$$2t = 4$$

$$t = 2$$

$$y'(2) = 4 \neq 0$$

$$x(2) = 4 \text{ en } y(2) = 1\frac{1}{3}$$

Dus evenwijdig met de y -as in $(4, 1\frac{1}{3})$.

$$\mathbf{b} \quad \text{Snijden met de } x\text{-as, dus } y(t) = 0 \text{ geeft } \frac{2}{3}t^3 - t^2 = 0$$

$$t^2(\frac{2}{3}t - 1) = 0$$

$$t = 0 \vee \frac{2}{3}t = 1$$

$$t = 0 \vee t = 1\frac{1}{2}$$

Bij de oorsprong hoort $t = 0$ (zie vraag a), dus bij het snijpunt met de positieve x -as hoort $t = 1\frac{1}{2}$.

$$\vec{v}(1\frac{1}{2}) = \begin{pmatrix} x'(1\frac{1}{2}) \\ y'(1\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix}$$

Bij de x -as hoort richtingsvector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\cos(\varphi) = \frac{\begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{2 \cdot 1 + 3 \cdot 0}{\sqrt{13} \cdot 1} = \frac{2}{\sqrt{13}} \text{ geeft } \varphi = 56,30\dots^\circ$$

Dus de gevraagde hoek is $56,3^\circ$.

$$\mathbf{c} \quad v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(4-2t)^2 + (2t^2-2t)^2}$$

Invoeren van de formule en gebruiken van de optie minimum.

De minimale baansnelheid van P is ongeveer 1,60 voor $t \approx 1,32$.

$$\begin{aligned} \mathbf{d} \quad v(t) &= \sqrt{(4-2t)^2 + (2t^2-2t)^2} = \sqrt{16-16t+4t^2+4t^4-8t^3+4t^2} = \sqrt{4t^4-8t^3+8t^2-16t+16} \\ a(t) &= v'(t) = \frac{1}{2\sqrt{4t^4-8t^3+8t^2-16t+16}} \cdot (16t^3-24t^2+16t-16) = \frac{8t^3-12t^2+8t-8}{\sqrt{4t^4-8t^3+8t^2-16t+16}} \\ \text{De baanversnelling op } t=3 \text{ is } a(3) &= \frac{216-108+24-8}{\sqrt{324-216+72-48+16}} = \frac{124}{\sqrt{148}} = \frac{124}{2\sqrt{37}} = \frac{1}{37}\sqrt{37}. \end{aligned}$$

Gemengde opgaven

Bladzijde 92

1

$$\mathbf{a} \quad \vec{n}_l = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \text{ dus } \vec{r}_l = \begin{pmatrix} 4 \\ -3 \end{pmatrix}.$$

$$\cos(\angle(\vec{r}_k, \vec{r}_l)) = \frac{\begin{pmatrix} 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix}}{\left| \begin{pmatrix} 5 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 4 \\ -3 \end{pmatrix} \right|} = \frac{5 \cdot 4 + (-1) \cdot (-3)}{\sqrt{26} \cdot \sqrt{25}} = \frac{23}{5\sqrt{26}} \text{ geeft } \angle(\vec{r}_k, \vec{r}_l) = 25,55\dots^\circ$$

Dus $\angle(k, l) \approx 25,6^\circ$.

$$\mathbf{b} \quad m \perp k, \text{ dus } \vec{n}_m = \vec{r}_k = \begin{pmatrix} 5 \\ -1 \end{pmatrix}.$$

$$\left. \begin{array}{l} m: 5x - y = c \\ \text{door } A(1, -3) \end{array} \right\} c = 5 \cdot 1 + 3 = 8$$

Dus $m: 5x - y = 8$.

Substitutie van $x = -1 + 5t$ en $y = -t$ in $5x - y = 8$ geeft $5(-1 + 5t) + t = 8$

$$-5 + 25t + t = 8$$

$$26t = 13$$

$$t = \frac{1}{2}$$

$t = \frac{1}{2}$ geeft $x = -1 + 5 \cdot \frac{1}{2} = 1\frac{1}{2}$ en $y = -\frac{1}{2}$, dus $B(1\frac{1}{2}, -\frac{1}{2})$.

$$\mathbf{c} \quad P \text{ op } k, \text{ dus } \vec{p} = \begin{pmatrix} -1 + 5t \\ -t \end{pmatrix}.$$

$$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} -1 + 5t \\ -t \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5t - 2 \\ -t + 3 \end{pmatrix}$$

De lijnen AP en l zijn evenwijdig en $\vec{n}_l = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, dus $\begin{pmatrix} 5t - 2 \\ -t + 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 0$

$$(5t - 2) \cdot 3 + (-t + 3) \cdot 4 = 0$$

$$15t - 6 - 4t + 12 = 0$$

$$11t = -6$$

$$t = -\frac{6}{11}$$

$t = -\frac{6}{11}$ geeft $P(-1 + 5 \cdot -\frac{6}{11}, \frac{6}{11}) = P(-3\frac{8}{11}, \frac{6}{11})$

$$\mathbf{d} \quad \text{Substitutie van } x = -1 + 5t \text{ en } y = -t \text{ in } 3x + 4y = 8 \text{ geeft } 3(-1 + 5t) + 4 \cdot -t = 8$$

$$-3 + 15t - 4t = 8$$

$$11t = 11$$

$$t = 1$$

$t = 1$ geeft $x = -1 + 5 = 4$ en $y = -1$, dus $S(4, -1)$.

$$\left. \begin{array}{l} n \perp l, \text{ dus } n: 4x - 3y = c. \\ \text{door } S(4, -1) \end{array} \right\} c = 4 \cdot 4 - 3 \cdot (-1) = 19$$

Dus $n: 4x - 3y = 19$.

2

$$\mathbf{a} \quad \vec{r}_m = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$\cos(\angle(\vec{r}_k, \vec{r}_m)) = \frac{\begin{pmatrix} -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right|} = \frac{-2 \cdot 2 + 3 \cdot (-5)}{\sqrt{13} \cdot \sqrt{29}} = \frac{-19}{\sqrt{377}} \text{ geeft } \angle(\vec{r}_k, \vec{r}_m) = 168,11\dots^\circ$$

Dus $\angle(k, m) = 180^\circ - 168,11\dots^\circ \approx 11,9^\circ$.

$$\text{b } \begin{cases} 1 - 2t = 2u - 6 \\ 2 + 3t = -5u + 4 \end{cases} \text{ oftewel } \begin{cases} -2t - 2u = -7 \\ 3t + 5u = 2 \end{cases} \left| \begin{array}{r} -3 \\ 2 \end{array} \right| \text{ geeft } \begin{cases} 6t + 6u = 21 \\ 6t + 10u = 4 \end{cases}$$

$$\begin{array}{r} -4u = 17 \\ u = -4\frac{1}{4} \end{array}$$

$$u = -4\frac{1}{4} \text{ geeft } x = 2 \cdot -4\frac{1}{4} - 6 = -14\frac{1}{2} \text{ en } y = -5 \cdot -4\frac{1}{4} + 4 = 25\frac{1}{4}$$

$$\text{Dus } S(-14\frac{1}{2}, 25\frac{1}{4}).$$

$$\text{c } n \perp m, \text{ dus } \vec{n}_n = \vec{r}_m = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

$$\left. \begin{array}{l} n: 2x - 5y = c \\ \text{door } O(0, 0) \end{array} \right\} n: 2x - 5y = 0$$

$$\text{d } p \perp l, \text{ dus } p: 3x - y = c. \left. \begin{array}{l} \text{door } A(6, 8) \end{array} \right\} c = 3 \cdot 6 - 8 = 10$$

$$\text{Dus } p: 3x - y = 10.$$

$$\text{Substitutie van } x = 1 - 2t \text{ en } y = 2 + 3t \text{ in } 3x - y = 10 \text{ geeft } 3(1 - 2t) - (2 + 3t) = 10$$

$$3 - 6t - 2 - 3t = 10$$

$$-9t = 9$$

$$t = -1$$

$$t = -1 \text{ geeft } x = 1 - 2 \cdot -1 = 3 \text{ en } y = 2 + 3 \cdot -1 = -1$$

$$\text{Dus } T(3, -1).$$

$$\text{3 a } a = 3, \text{ dus } A(3, 0) \text{ en } D(0, 6).$$

$$\vec{DA} = \vec{a} - \vec{d} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$\vec{OC} = \vec{OD} + \vec{DA}_L = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

$$\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$\vec{OP} = \vec{OA} + \vec{AP} = \vec{OA} + \vec{AC}_R = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 9 \\ -3 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$$

$$\text{Dus } P(12, -3).$$

$$\text{b } \vec{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 2a \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ 2a \end{pmatrix}$$

$$\vec{OB} = \vec{OA} + \vec{AB} = \vec{OA} + \vec{AD}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} = \begin{pmatrix} 3a \\ a \end{pmatrix}$$

$$B(3a, a) \text{ op } y = \frac{1}{3}x^2 - 2 \text{ geeft } \frac{1}{3} \cdot (3a)^2 - 2 = a$$

$$\frac{1}{3} \cdot 9a^2 - 2 = a$$

$$3a^2 - a - 2 = 0$$

$$D = (-1)^2 - 4 \cdot 3 \cdot -2 = 25, \text{ dus } \sqrt{D} = 5$$

$$a = \frac{1+5}{6} = 1 \vee a = \frac{1-5}{6} = -\frac{2}{3}$$

$$a > 0, \text{ dus } a = 1.$$

$$\text{4 a } \left. \begin{array}{l} AB: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{b} - \vec{a}) \\ \vec{a} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix} \end{array} \right\} AB: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$x = 99 \text{ oftewel } -1 + t = 99 \text{ geeft } t = 100$$

$$t = 100 \text{ geeft } y = 5 + 100 \cdot -2 = -195$$

$$\text{Dus } D(99, -195) \text{ ligt op de lijn } AB.$$

$$\mathbf{b} \quad \left. \begin{aligned} BC: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + t(\vec{c} - \vec{b}) \\ \vec{b} &= \begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ en } \vec{c} - \vec{b} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{aligned} \right\} BC: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$k \perp BC, \text{ dus } \vec{n}_k = \vec{r}_{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

$$\left. \begin{aligned} k: x + 3y &= c \\ \text{door } A(-1, 5) \end{aligned} \right\} c = -1 + 3 \cdot 5 = 14$$

$$\text{Dus } k: x + 3y = 14.$$

$$\text{Substitutie van } x = 2 + t \text{ en } y = -1 + 3t \text{ in } x + 3y = 14 \text{ geeft } 2 + t + 3(-1 + 3t) = 14$$

$$2 + t - 3 + 9t = 14$$

$$10t = 15$$

$$t = 1\frac{1}{2}$$

$$t = 1\frac{1}{2} \text{ geeft } x = 2 + 1\frac{1}{2} = 3\frac{1}{2} \text{ en } y = -1 + 3 \cdot 1\frac{1}{2} = 3\frac{1}{2}, \text{ dus } S(3\frac{1}{2}, 3\frac{1}{2}).$$

$$\mathbf{c} \quad \text{Stel } P(a, b).$$

$$\vec{AP} = \vec{p} - \vec{a} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} a+1 \\ b-5 \end{pmatrix} \text{ en } \vec{BP} = \vec{p} - \vec{b} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} a-2 \\ b+1 \end{pmatrix}$$

$$AP = BP, \text{ dus } |\vec{AP}| = |\vec{BP}|$$

$$\left| \begin{pmatrix} a+1 \\ b-5 \end{pmatrix} \right| = \left| \begin{pmatrix} a-2 \\ b+1 \end{pmatrix} \right|$$

$$\sqrt{(a+1)^2 + (b-5)^2} = \sqrt{(a-2)^2 + (b+1)^2}$$

kwadrateren geeft

$$(a+1)^2 + (b-5)^2 = (a-2)^2 + (b+1)^2$$

$$a^2 + 2a + 1 + b^2 - 10b + 25 = a^2 - 4a + 4 + b^2 + 2b + 1$$

$$6a - 12b = -21$$

$$a = 2b - 3\frac{1}{2}$$

$$\angle APB = 90^\circ, \text{ dus } \vec{AP} \cdot \vec{BP} = 0$$

$$\begin{pmatrix} a+1 \\ b-5 \end{pmatrix} \cdot \begin{pmatrix} a-2 \\ b+1 \end{pmatrix} = 0$$

$$(a+1)(a-2) + (b-5)(b+1) = 0$$

$$a^2 - a - 2 + b^2 - 4b - 5 = 0$$

$$\text{Substitutie van } a = 2b - 3\frac{1}{2} \text{ in } a^2 - a - 2 + b^2 - 4b - 5 = 0 \text{ geeft}$$

$$(2b - 3\frac{1}{2})^2 - (2b - 3\frac{1}{2}) - 2 + b^2 - 4b - 5 = 0$$

$$4b^2 - 14b + 12\frac{1}{4} - 2b + 3\frac{1}{2} - 2 + b^2 - 4b - 5 = 0$$

$$5b^2 - 20b + 8\frac{3}{4} = 0$$

$$D = (-20)^2 - 4 \cdot 5 \cdot 8\frac{3}{4} = 225, \text{ dus } \sqrt{D} = 15$$

$$b = \frac{20 + 15}{10} = 3\frac{1}{2} \vee b = \frac{20 - 15}{10} = \frac{1}{2}$$

$$b = 3\frac{1}{2} \text{ geeft } a = 2 \cdot 3\frac{1}{2} - 3\frac{1}{2} = 3\frac{1}{2}$$

$$b = \frac{1}{2} \text{ geeft } a = 2 \cdot \frac{1}{2} - 3\frac{1}{2} = -2\frac{1}{2}$$

$$\text{Dus } P_1(3\frac{1}{2}, 3\frac{1}{2}) \text{ en } P_2(-2\frac{1}{2}, \frac{1}{2}).$$

$$\mathbf{d} \quad \left. \begin{aligned} BC: \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ (zie vraag b)} \\ BC: \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} p \\ 38 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3 \end{pmatrix} \end{aligned} \right\} q = 1$$

$$BC \text{ door } B(2, -1), \text{ dus } 38 + 3u = -1$$

$$3u = -39$$

$$u = -13$$

$$q = 1 \text{ en } u = -13 \text{ geeft } p - 13 = 2$$

$$p = 15$$

$$\text{Dus voor } p = 15 \text{ en } q = 1.$$

5

$$\mathbf{a} \quad \overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix}$$

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{BA}_R = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} 16 \\ 8 \end{pmatrix} = \begin{pmatrix} 24 \\ 8 \end{pmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OC} + \overrightarrow{BA} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 16 \end{pmatrix} = \begin{pmatrix} 16 \\ 24 \end{pmatrix}$$

$$\left. \begin{aligned} CD: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{c} + t(\vec{d} - \vec{c}) \\ \vec{c} &= \begin{pmatrix} 24 \\ 8 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} 16 \\ 24 \end{pmatrix} - \begin{pmatrix} 24 \\ 8 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -2 \end{pmatrix} \end{aligned} \right\} CD: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 24 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$x = 22$ geeft $24 + t = 22$, dus $t = -2$.

$t = -2$ geeft $y = 8 + -2 \cdot -2 = 12$

Verder geldt $x_D \leq x_P \leq x_C$.

Dus P ligt op de zijde CD .

$$\mathbf{b} \quad AP: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t(\vec{p} - \vec{a}) \left. \begin{aligned} \vec{a} &= \begin{pmatrix} 0 \\ 16 \end{pmatrix} \text{ en } \vec{p} - \vec{a} = \begin{pmatrix} 22 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 16 \end{pmatrix} = \begin{pmatrix} 22 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 11 \\ -2 \end{pmatrix} \end{aligned} \right\} AP: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 16 \end{pmatrix} + t \cdot \begin{pmatrix} 11 \\ -2 \end{pmatrix}$$

$$Q \text{ op } AP, \text{ dus } \vec{q} = \begin{pmatrix} 11t \\ 16 - 2t \end{pmatrix}.$$

$$\overrightarrow{DQ} = \vec{q} - \vec{d} = \begin{pmatrix} 11t \\ 16 - 2t \end{pmatrix} - \begin{pmatrix} 16 \\ 24 \end{pmatrix} = \begin{pmatrix} 11t - 16 \\ -2t - 8 \end{pmatrix}$$

$$DQ \perp AP, \text{ dus } \overrightarrow{DQ} \cdot \overrightarrow{AP} = 0$$

$$\begin{pmatrix} 11t - 16 \\ -2t - 8 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ -2 \end{pmatrix} = 0$$

$$(11t - 16) \cdot 11 + (-2t - 8) \cdot -2 = 0$$

$$121t - 176 + 4t + 16 = 0$$

$$125t = 160$$

$$t = 1\frac{7}{25}$$

$$t = 1\frac{7}{25} \text{ geeft } Q(11 \cdot 1\frac{7}{25}, 16 - 2 \cdot 1\frac{7}{25}) = Q(14\frac{2}{25}, 13\frac{11}{25})$$

6

$$\angle OPQ = \angle(\overrightarrow{PO}, \overrightarrow{PQ})$$

Q op k , dus $Q(q, -2q)$.

$$\overrightarrow{PO} = -\vec{p} = \begin{pmatrix} -2 \\ -7 \end{pmatrix} \text{ en } \overrightarrow{PQ} = \vec{q} - \vec{p} = \begin{pmatrix} q \\ -2q \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} q - 2 \\ -2q - 7 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{PO}, \overrightarrow{PQ})) = \frac{\begin{pmatrix} -2 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} q - 2 \\ -2q - 7 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -7 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} q - 2 \\ -2q - 7 \end{pmatrix} \right|} = \frac{-2q + 4 + 14q + 49}{\sqrt{53} \cdot \sqrt{(q - 2)^2 + (-2q - 7)^2}} = \frac{12q + 53}{\sqrt{53} \cdot \sqrt{(q - 2)^2 + (-2q - 7)^2}}$$

$$\text{Los op } \frac{12q + 53}{\sqrt{53} \cdot \sqrt{(q - 2)^2 + (-2q - 7)^2}} = \cos(20^\circ).$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $q = -1,255\dots$ en $q = 2,908\dots$

$q = -1,255\dots$ geeft $Q_1(-1,255\dots; -2 \cdot -1,255\dots) \approx Q_1(-1,26; 2,51)$

$q = 2,908\dots$ geeft $Q_2(2,908\dots; -2 \cdot 2,908\dots) \approx Q_2(2,91; -5,82)$

7

a $p = 4$, dus $B(4, 6)$ en $C(1, 6)$.

$$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \text{ dus } OB: \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

 D op OB , dus $D(2t, 3t)$.

$$\vec{r}_{OD} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ en } \overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2t \\ 3t \end{pmatrix} - \begin{pmatrix} 8 \\ 0 \end{pmatrix} = \begin{pmatrix} 2t-8 \\ 3t \end{pmatrix}$$

 $OD \perp AD$, dus $\vec{r}_{OD} \cdot \overrightarrow{AD} = 0$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2t-8 \\ 3t \end{pmatrix} = 0$$

$$2(2t-8) + 3 \cdot 3t = 0$$

$$4t - 16 + 9t = 0$$

$$13t = 16$$

$$t = 1\frac{3}{13}$$

$$t = 1\frac{3}{13} \text{ geeft } D(2 \cdot 1\frac{3}{13}, 3 \cdot 1\frac{3}{13}) = D(2\frac{6}{13}, 3\frac{9}{13})$$

b $OA = 8$, $BC = 3$ en $OB = \sqrt{p^2 + 36}$ Stel $OS = x$, dan is $BS = \sqrt{p^2 + 36} - x$.Uit $\triangle OAS \sim \triangle BCS$ volgt $\frac{OA}{BC} = \frac{OS}{BS}$

$$\frac{8}{3} = \frac{x}{\sqrt{p^2 + 36} - x}$$

$$3x = 8(\sqrt{p^2 + 36} - x)$$

$$3x = 8\sqrt{p^2 + 36} - 8x$$

$$11x = 8\sqrt{p^2 + 36}$$

$$x = \frac{8}{11}\sqrt{p^2 + 36}$$

$$|\overrightarrow{OS}| = |\overrightarrow{OA}| \text{ geeft } \frac{8}{11}\sqrt{p^2 + 36} = 8$$

$$\sqrt{p^2 + 36} = 11$$

$$p^2 + 36 = 121$$

$$p^2 = 85$$

$$p = \sqrt{85} \vee p = -\sqrt{85}$$

 $p > 3$, dus $p = \sqrt{85}$.

8

a Zie de figuur hiernaast.

 E is het snijpunt van de diagonalen DH en CI van het vierkant $DCHI$ en is dus het midden van diagonaal DH .

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{AB}_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\overrightarrow{OH} = \overrightarrow{OB} + 2\overrightarrow{BC} = \overrightarrow{OB} + 2\overrightarrow{AD} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$\overrightarrow{OE} = \vec{e} = \frac{1}{2}(\vec{d} + \vec{h}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \text{ dus } E(3, 5).$$

b $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, dus $C(4, 3)$.

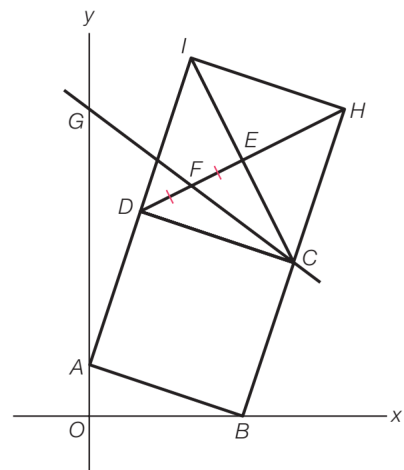
$$\overrightarrow{OF} = \vec{f} = \frac{1}{2}(\vec{d} + \vec{e}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 4 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 4\frac{1}{2} \end{pmatrix}$$

$$CF: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{c} + t(\vec{f} - \vec{c})$$

$$\vec{c} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ en } \vec{f} - \vec{c} = \begin{pmatrix} 2 \\ 4\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1\frac{1}{2} \end{pmatrix} \triangleq \begin{pmatrix} -4 \\ 3 \end{pmatrix} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} CF: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

Snijden met de y -as, dus $x = 0$ geeft $4 - 4t = 0$ oftewel $t = 1$. $t = 1$ geeft $y = 3 + 3 = 6$, dus $G(0, 6)$.

$$CG = \sqrt{(0-4)^2 + (6-3)^2} = \sqrt{25} = 5$$



a $x(t) = 2t$ geeft $x'(t) = 2$

$$y(t) = 4e^{\frac{1}{2}t} - 4 \text{ geeft } y'(t) = 4e^{\frac{1}{2}t} \cdot \frac{1}{2} = 2e^{\frac{1}{2}t}$$

$$\text{rc} = 3, \text{ dus } \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$3 \cdot x'(t) = 1 \cdot y'(t)$$

$$3 \cdot 2 = 2e^{\frac{1}{2}t}$$

$$e^{\frac{1}{2}t} = 3$$

$$\frac{1}{2}t = \ln(3)$$

$$t = 2 \ln(3)$$

$$x'(2 \ln(3)) = 2 \text{ en } y'(2 \ln(3)) = 2e^{\ln(3)} = 2 \cdot 3 = 6$$

$$\text{De baansnelheid in } A \text{ is } v(2 \ln(3)) = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}.$$

b $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{2^2 + (2e^{\frac{1}{2}t})^2} = \sqrt{4 + 4e^t}$

$$a(t) = v'(t) = \frac{1}{2\sqrt{4 + 4e^t}} \cdot 4e^t = \frac{4e^t}{2\sqrt{4(1 + e^t)}} = \frac{4e^t}{4\sqrt{1 + e^t}} = \frac{e^t}{\sqrt{1 + e^t}}$$

$$a(t) = 1 \text{ geeft } \frac{e^t}{\sqrt{1 + e^t}} = 1$$

$$\sqrt{1 + e^t} = e^t$$

$$1 + e^t = (e^t)^2$$

$$(e^t)^2 - e^t - 1 = 0$$

$$\text{Stel } e^t = u.$$

$$u^2 - u - 1 = 0$$

$$D = (-1)^2 - 4 \cdot 1 \cdot -1 = 5$$

$$u = \frac{1 + \sqrt{5}}{2} \vee u = \frac{1 - \sqrt{5}}{2}$$

$$u = \frac{1}{2} + \frac{1}{2}\sqrt{5} \vee u = \frac{1}{2} - \frac{1}{2}\sqrt{5}$$

$$e^t = \frac{1}{2} + \frac{1}{2}\sqrt{5} \vee e^t = \frac{1}{2} - \frac{1}{2}\sqrt{5}$$

$$t = \ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$$

$$x\left(\ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)\right) = 2 \ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right) \approx 0,96$$

$$y\left(\ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)\right) = 4\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^{\frac{1}{2}} - 4 \approx 1,09$$

$$\text{Dus } B(0,96; 1,09).$$

c $P(2t, 4e^{\frac{1}{2}t} - 4)$ en $Q(2, 0)$ geeft $\overrightarrow{PQ} = \vec{q} - \vec{p} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2t \\ 4e^{\frac{1}{2}t} - 4 \end{pmatrix} = \begin{pmatrix} 2 - 2t \\ 4 - 4e^{\frac{1}{2}t} \end{pmatrix}$

$$\overrightarrow{PR} = \overrightarrow{PQ}_L = \begin{pmatrix} 4e^{\frac{1}{2}t} - 4 \\ 2 - 2t \end{pmatrix}$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = \begin{pmatrix} 2t \\ 4e^{\frac{1}{2}t} - 4 \end{pmatrix} + \begin{pmatrix} 4e^{\frac{1}{2}t} - 4 \\ 2 - 2t \end{pmatrix} = \begin{pmatrix} 4e^{\frac{1}{2}t} + 2t - 4 \\ 4e^{\frac{1}{2}t} - 2t - 2 \end{pmatrix}$$

$$y_R = 4e^{\frac{1}{2}t} - 2t - 2 \text{ geeft } \frac{dy_R}{dt} = 4e^{\frac{1}{2}t} \cdot \frac{1}{2} - 2 = 2e^{\frac{1}{2}t} - 2$$

$$\frac{dy_R}{dt} = 0 \text{ geeft } 2e^{\frac{1}{2}t} - 2 = 0$$

$$e^{\frac{1}{2}t} = 1$$

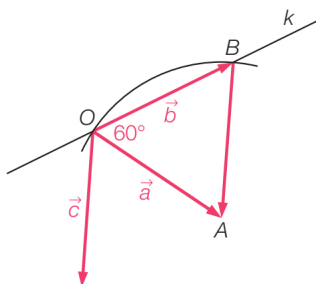
$$\frac{1}{2}t = 0$$

$$t = 0$$

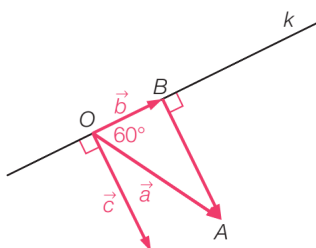
$$t = 0 \text{ geeft } y_R = 4e^0 - 2 \cdot 0 - 2 = 2$$

Dus de minimale y -coördinaat van R is 2.

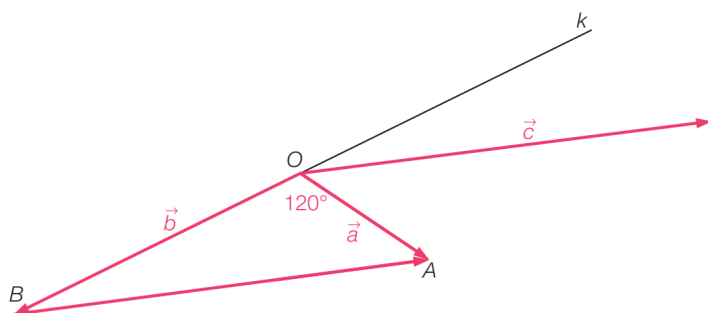
- 10 a** A is het eindpunt van $\vec{a}$ en B is het eindpunt van $\vec{b}$.
 Teken een cirkelboog van de cirkel met middelpunt A en straal OA .
 Deze cirkelboog snijdt k in O en B . Je hebt nu $\vec{b}$ en $\vec{BA} = \vec{a} - \vec{b}$.
 Teken $\vec{c}$ evenwijdig met $\vec{BA}$ en even lang als $\vec{BA}$.



- b** A is het eindpunt van $\vec{a}$ en B is het eindpunt van $\vec{b}$.
 $|\vec{c}| = \sqrt{3} \cdot |\vec{b}|$ betekent dat driehoek OBA een bijzondere rechthoekige driehoek is met zijden waarvan de lengten zich verhouden als $1 : 2 : \sqrt{3}$.
 Teken lijnstuk AB loodrecht op k . Je hebt nu $\vec{b}$ en $\vec{BA} = \vec{a} - \vec{b}$.
 Teken $\vec{c}$ evenwijdig met $\vec{BA}$ en even lang als $\vec{BA}$.



- c** Zie de figuur hieronder.
 A is het eindpunt van $\vec{a}$, B is het eindpunt van $\vec{b}$, $|\vec{b}| = 2 \cdot |\vec{a}|$ oftewel $OB = 2 \cdot OA$, $\vec{BA} = \vec{a} - \vec{b}$ en $\vec{c} = \vec{a} - \vec{b}$.



De cosinusregel in $\triangle OAB$ geeft $AB^2 = OA^2 + (2 \cdot OA)^2 - 2 \cdot OA \cdot 2 \cdot OA \cdot \cos(120^\circ)$

$$AB^2 = OA^2 + 4 \cdot OA^2 - 4 \cdot OA^2 \cdot -\frac{1}{2}$$

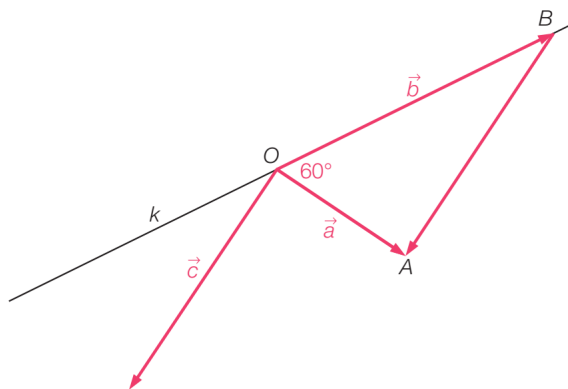
$$AB^2 = 7 \cdot OA^2$$

$$AB = \sqrt{7} \cdot OA$$

$$AB = |\vec{BA}| = |\vec{c}| \text{ en } OA = |\vec{a}|, \text{ dus } |\vec{c}| = \sqrt{7} \cdot |\vec{a}|.$$

Zie de figuur bovenaan de volgende bladzijde.

A is het eindpunt van $\vec{a}$, B is het eindpunt van $\vec{b}$, $|\vec{b}| = 2 \cdot |\vec{a}|$ oftewel $OB = 2 \cdot OA$, $\vec{BA} = \vec{a} - \vec{b}$ en $\vec{c} = \vec{a} - \vec{b}$.



De cosinusregel in $\triangle OAB$ geeft $AB^2 = OA^2 + (2 \cdot OA)^2 - 2 \cdot OA \cdot 2 \cdot OA \cdot \cos(60^\circ)$

$$AB^2 = OA^2 + 4 \cdot OA^2 - 4 \cdot OA^2 \cdot \frac{1}{2}$$

$$AB^2 = 3 \cdot OA^2$$

$$AB = \sqrt{3} \cdot OA$$

$$AB = |\vec{BA}| = |\vec{c}| \text{ en } OA = |\vec{a}|, \text{ dus } |\vec{c}| = \sqrt{3} \cdot |\vec{a}|.$$

Bladzijde 95

- 11 a** $a = 2$ geeft $k: y = 2x - 4$ en $A(t, 2t - 4)$, dus $\vec{OA} = \begin{pmatrix} t \\ 2t - 4 \end{pmatrix}$.

Het punt M is het midden van OA geeft $\vec{OM} = \frac{1}{2} \cdot \begin{pmatrix} t \\ 2t - 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t \\ t - 2 \end{pmatrix}$.

$$\vec{MB} = \vec{OM}_L = \begin{pmatrix} -t + 2 \\ \frac{1}{2}t \end{pmatrix}$$

$$\vec{OB} = \vec{OM} + \vec{MB} = \begin{pmatrix} \frac{1}{2}t \\ t - 2 \end{pmatrix} + \begin{pmatrix} -t + 2 \\ \frac{1}{2}t \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}t + 2 \\ 1\frac{1}{2}t - 2 \end{pmatrix}$$

$$\begin{cases} x = -\frac{1}{2}t + 2 \\ y = 1\frac{1}{2}t - 2 \end{cases} \Bigg| \begin{matrix} 3 \\ 1 \end{matrix} \text{ geeft } \begin{cases} 3x = -\frac{1}{2}t + 6 \\ y = 1\frac{1}{2}t - 2 \end{cases} +$$

$$3x + y = 4$$

Dus $l: 3x + y = 4$.

- b** $\vec{OA} = \begin{pmatrix} t \\ at - 4 \end{pmatrix}$

Het punt M is het midden van OA geeft $\vec{OM} = \frac{1}{2} \cdot \begin{pmatrix} t \\ at - 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t \\ \frac{1}{2}at - 2 \end{pmatrix}$.

$$\vec{MB} = \vec{OM}_L = \begin{pmatrix} -\frac{1}{2}at + 2 \\ \frac{1}{2}t \end{pmatrix}$$

$$\vec{OB} = \vec{OM} + \vec{MB} = \begin{pmatrix} \frac{1}{2}t \\ \frac{1}{2}at - 2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}at + 2 \\ \frac{1}{2}t \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}at + \frac{1}{2}t + 2 \\ \frac{1}{2}at + \frac{1}{2}t - 2 \end{pmatrix}$$

B beweegt over een horizontale lijn l als $y_B = \frac{1}{2}at + \frac{1}{2}t - 2$ constant is, dus als $a = -1$.

- c** $a = 3$ geeft $\vec{OA} = \begin{pmatrix} t \\ 3t - 4 \end{pmatrix}$

Het punt M is het midden van OA geeft $\vec{OM} = \frac{1}{2} \cdot \begin{pmatrix} t \\ 3t - 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t \\ 1\frac{1}{2}t - 2 \end{pmatrix}$.

$$\vec{MB} = \vec{OM}_L = \begin{pmatrix} -1\frac{1}{2}t + 2 \\ \frac{1}{2}t \end{pmatrix}$$

$$\vec{OB} = \vec{OM} + \vec{MB} = \begin{pmatrix} \frac{1}{2}t \\ 1\frac{1}{2}t - 2 \end{pmatrix} + \begin{pmatrix} -1\frac{1}{2}t + 2 \\ \frac{1}{2}t \end{pmatrix} = \begin{pmatrix} -t + 2 \\ 2t - 2 \end{pmatrix}$$

$B(-t + 2, 2t - 2)$ op k als $3(-t + 2) - 4 = 2t - 2$

$$-3t + 6 - 4 = 2t - 2$$

$$-5t = -4$$

$$t = \frac{4}{5}$$

$$t = \frac{4}{5} \text{ geeft } B(-\frac{4}{5} + 2, 2 \cdot \frac{4}{5} - 2) = B(1\frac{1}{5}, -\frac{2}{5})$$

12 a $y(t) = 0$ geeft $t^2 - 1 = 0$

$$t^2 = 1$$

$$t = 1 \vee t = -1$$

$$x(1) = 0 \text{ en } x(-1) = 6$$

Dus de snijpunten met de x -as zijn $(0, 0)$ en $(6, 0)$.

$$x(t) = 0 \text{ geeft } -t^3 + 3t^2 - 2t = 0$$

$$-t(t^2 - 3t + 2) = 0$$

$$-t(t-1)(t-2) = 0$$

$$t = 0 \vee t = 1 \vee t = 2$$

$$y(0) = -1 \text{ en } y(2) = 3$$

Dus de snijpunten met de y -as zijn $(0, -1)$, $(0, 0)$ en $(0, 3)$.

b $x(t) = -t^3 + 3t^2 - 2t$ geeft $x'(t) = -3t^2 + 6t - 2$

$$y(t) = t^2 - 1 \text{ geeft } y'(t) = 2t$$

Evenwijdig met de x -as als $y'(t) = 0$ en evenwijdig met de y -as als $x'(t) = 0$.

$$x'(t) = 0 \text{ geeft } -3t^2 + 6t - 2 = 0$$

$$D = 6^2 - 4 \cdot (-3) \cdot (-2) = 12, \text{ dus } \sqrt{D} = 2\sqrt{3}$$

$$t = \frac{-6 + 2\sqrt{3}}{-6} \vee t = \frac{-6 - 2\sqrt{3}}{-6}$$

$$t = 1 - \frac{1}{3}\sqrt{3} \vee t = 1 + \frac{1}{3}\sqrt{3}$$

$$y'(t) = 0 \text{ geeft } 2t = 0$$

$$t = 0$$

Dus raaklijn evenwijdig met de x -as voor $t = 0$ en evenwijdig met de y -as voor $t = 1 - \frac{1}{3}\sqrt{3}$ en $t = 1 + \frac{1}{3}\sqrt{3}$.

c Evenwijdig met k en $rc_k = 2$, dus $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$2 \cdot x'(t) = 1 \cdot y'(t)$$

$$2(-3t^2 + 6t - 2) = 2t$$

$$-6t^2 + 12t - 4 = 2t$$

$$-6t^2 + 10t - 4 = 0$$

$$3t^2 - 5t + 2 = 0$$

$$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1, \text{ dus } \sqrt{D} = 1$$

$$t = \frac{5+1}{6} = 1 \vee t = \frac{5-1}{6} = \frac{2}{3}$$

$$t = 1 \text{ geeft } (0, 0) \text{ (zie vraag a)}$$

$$x(\frac{2}{3}) = -\frac{8}{27} \text{ en } y(\frac{2}{3}) = -\frac{5}{9}$$

Dus de gevraagde punten zijn $(0, 0)$ en $(-\frac{8}{27}, -\frac{5}{9})$.

d $y = 8$ geeft $t^2 - 1 = 8$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$$x(3) = -6 \text{ en } x(-3) = 60, \text{ dus bij } (-6, 8) \text{ hoort } t = 3.$$

$$x'(3) = -11 \text{ en } y'(3) = 6$$

De baansnelheid in $(-6, 8)$ is $v(3) = \sqrt{(-11)^2 + 6^2} = \sqrt{157}$.

e $y = -\frac{3}{4}$ geeft $t^2 - 1 = -\frac{3}{4}$

$$t^2 = \frac{1}{4}$$

$$t = \frac{1}{2} \vee t = -\frac{1}{2}$$

$$x(\frac{1}{2}) = -\frac{3}{8} \text{ en } x(-\frac{1}{2}) = \frac{7}{8}, \text{ dus bij } (-\frac{3}{8}, -\frac{3}{4}) \text{ hoort } t = \frac{1}{2}.$$

$$v(t) = \sqrt{(-3t^2 + 6t - 2)^2 + (2t)^2} = \sqrt{9t^4 - 18t^3 + 6t^2 - 18t^3 + 36t^2 - 12t + 6t^2 - 12t + 4 + 4t^2}$$

$$= \sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}} \cdot (36t^3 - 108t^2 + 104t - 24) = \frac{18t^3 - 54t^2 + 52t - 12}{\sqrt{9t^4 - 36t^3 + 52t^2 - 24t + 4}}$$

$$\text{De baanversnelling in } (-\frac{3}{8}, -\frac{3}{4}) \text{ is } a(\frac{1}{2}) = \frac{18 \cdot \frac{1}{8} - 54 \cdot \frac{1}{4} + 52 \cdot \frac{1}{2} - 12}{\sqrt{9 \cdot \frac{1}{16} - 36 \cdot \frac{1}{8} + 52 \cdot \frac{1}{4} - 24 \cdot \frac{1}{2} + 4}} = \frac{2\frac{3}{4}}{\sqrt{1\frac{1}{16}}} = \frac{11}{17}\sqrt{17}.$$

11 Integraalrekening

11.1 Primitieven en integralen

Bladzijde 98

- V1** a $f(x) = (2x - 1)^6$ geeft $f'(x) = 6(2x - 1)^5 \cdot 2 = 12(2x - 1)^5$
- b $f(x) = \frac{10 \cdot 3^x}{\ln(3)} = \frac{10}{\ln(3)} \cdot 3^x$ geeft $f'(x) = \frac{10}{\ln(3)} \cdot 3^x \ln(3) = 10 \cdot 3^x$
- c $f(x) = x \ln(x) + x$ geeft $f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} + 1 = \ln(x) + 1 + 1 = \ln(x) + 2$
- d $f(x) = 6x^2 \sqrt{x^2 + 1}$ geeft

$$f'(x) = 12x \cdot \sqrt{x^2 + 1} + 6x^2 \cdot \frac{1}{2\sqrt{x^2 + 1}} \cdot 2x = 12x\sqrt{x^2 + 1} + \frac{6x^3}{\sqrt{x^2 + 1}} = \frac{12x(x^2 + 1) + 6x^3}{\sqrt{x^2 + 1}} = \frac{18x^3 + 12x}{\sqrt{x^2 + 1}}$$
- e $f(x) = \frac{e^{4x} - 5}{2e^{3x}}$ geeft

$$f'(x) = \frac{2e^{3x} \cdot e^{4x} \cdot 4 - (e^{4x} - 5) \cdot 2e^{3x} \cdot 3}{(2e^{3x})^2} = \frac{8e^{7x} - 6e^{7x} + 30e^{3x}}{4e^{6x}} = \frac{2e^{7x} + 30e^{3x}}{4e^{6x}} = \frac{e^{4x} + 15}{2e^{3x}}$$
- f $f(x) = \frac{1}{4}\cos(2x) + \frac{1}{2}x\sin(2x)$ geeft

$$f'(x) = -\frac{1}{4}\sin(2x) \cdot 2 + \frac{1}{2} \cdot \sin(2x) + \frac{1}{2}x \cdot \cos(2x) \cdot 2 = -\frac{1}{2}\sin(2x) + \frac{1}{2}\sin(2x) + x\cos(2x) = x\cos(2x)$$

Bladzijde 99

- V2** a $f(x) = x + \frac{1}{1 + e^x} - \ln(1 + e^x) = x + (1 + e^x)^{-1} - \ln(1 + e^x)$ geeft

$$f'(x) = 1 - (1 + e^x)^{-2} \cdot e^x - \frac{1}{1 + e^x} \cdot e^x = 1 - \frac{e^x}{(1 + e^x)^2} - \frac{e^x}{1 + e^x}$$

$$= \frac{(1 + e^x)^2 - e^x - e^x(1 + e^x)}{(1 + e^x)^2} = \frac{1 + 2e^x + e^{2x} - e^x - e^x - e^{2x}}{(1 + e^x)^2} = \frac{1}{(1 + e^x)^2}$$
- b $f(x) = \sin(x) - \frac{1}{3}\sin^3(x)$ geeft

$$f'(x) = \cos(x) - \sin^2(x) \cdot \cos(x) = \cos(x) - (1 - \cos^2(x)) \cdot \cos(x) = \cos(x) - \cos(x) + \cos^3(x) = \cos^3(x)$$
- c $f(x) = x + 2 - \frac{4}{x + 2} - 4\ln(x + 2) = x + 2 - 4(x + 2)^{-1} - 4\ln(x + 2)$ geeft

$$f'(x) = 1 + 4(x + 2)^{-2} - 4 \cdot \frac{1}{x + 2} = 1 + \frac{4}{(x + 2)^2} - \frac{4}{x + 2} = \frac{(x + 2)^2 + 4 - 4(x + 2)}{(x + 2)^2}$$

$$= \frac{x^2 + 4x + 4 + 4 - 4x - 8}{(x + 2)^2} = \frac{x^2}{(x + 2)^2}$$
- d $f(x) = (0,2x^2 - 0,4x + 1,2)\sqrt{2x + 3}$ geeft

$$f'(x) = (0,4x - 0,4) \cdot \sqrt{2x + 3} + (0,2x^2 - 0,4x + 1,2) \cdot \frac{1}{2\sqrt{2x + 3}} \cdot 2$$

$$= (0,4x - 0,4)\sqrt{2x + 3} + \frac{0,2x^2 - 0,4x + 1,2}{\sqrt{2x + 3}} = \frac{(0,4x - 0,4)(2x + 3)}{\sqrt{2x + 3}} + \frac{0,2x^2 - 0,4x + 1,2}{\sqrt{2x + 3}}$$

$$= \frac{0,8x^2 + 1,2x - 0,8x - 1,2 + 0,2x^2 - 0,4x + 1,2}{\sqrt{2x + 3}} = \frac{x^2}{\sqrt{2x + 3}}$$

- 1** $g(x) = x^3 + c$ geeft $g'(x) = 3x^2$ en dit is $f(x)$.

$$\left. \begin{array}{l} g(x) = x^3 + c \\ g(2) = 15 \end{array} \right\} \begin{array}{l} 2^3 + c = 15 \\ 8 + c = 15 \\ c = 7 \end{array}$$

Bladzijde 100

- 2 a** $F(x) = \frac{1}{9}(3x+4)^3$ geeft $F'(x) = \frac{1}{3}(3x+4)^2 \cdot 3 = (3x+4)^2$
 $F'(x) = f(x)$, dus F is een primitieve van f .
- b** $F(x) = x - \ln(x+2)$ geeft $F'(x) = 1 - \frac{1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+1}{x+2}$
 $F'(x) = f(x)$, dus F is een primitieve van f .
- c** $F(x) = \frac{-1}{x^2+1} = -(x^2+1)^{-1}$ geeft $F'(x) = (x^2+1)^{-2} \cdot 2x = \frac{2x}{(x^2+1)^2}$
 $F'(x) = f(x)$, dus F is een primitieve van f .
- 3 a** $F(x) = (x^2+1)^6 + 1$ geeft $F'(x) = 6(x^2+1)^5 \cdot 2x = 12x(x^2+1)^5$
 $F'(x) = f(x)$, dus F is een primitieve van f .
- b** $G(x) = \frac{5^{3x-1}}{3 \ln(5)} = \frac{1}{3 \ln(5)} \cdot 5^{3x-1}$ geeft $G'(x) = \frac{1}{3 \ln(5)} \cdot 5^{3x-1} \ln(5) \cdot 3 = 5^{3x-1}$
 $G'(x) = g(x)$, dus G is een primitieve van g .
- c** $H(x) = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + 3$ geeft $H'(x) = x \cdot \ln(x) + \frac{1}{2}x^2 \cdot \frac{1}{x} - \frac{1}{2}x = x \ln(x) + \frac{1}{2}x - \frac{1}{2}x = x \ln(x)$
 $H'(x) = h(x)$, dus H is een primitieve van h .
- d** $J(x) = \frac{e^{3x}-10}{2e^x} - 4 = \frac{1}{2}e^{2x} - 5e^{-x} - 4$ geeft $J'(x) = \frac{1}{2}e^{2x} \cdot 2 - 5e^{-x} \cdot -1 = e^{2x} + \frac{5}{e^x} = \frac{e^{3x}+5}{e^x}$
 $J'(x) = j(x)$, dus J is een primitieve van j .
- e** $K(x) = \sin^3(x)$ geeft $K'(x) = 3\sin^2(x) \cdot \cos(x) = 3(1 - \cos^2(x)) \cdot \cos(x) = 3\cos(x) - 3\cos^3(x)$
 $K'(x) = k(x)$, dus K is een primitieve van k .

- 4** $F(x) = \ln(x + \sqrt{x^2-1})$ geeft

$$F'(x) = \frac{1}{x + \sqrt{x^2-1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right) = \frac{1}{x + \sqrt{x^2-1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{1}{x + \sqrt{x^2-1}} \cdot \frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}}$$

$$= \frac{\sqrt{x^2-1} + x}{x + \sqrt{x^2-1}} \cdot \frac{1}{\sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$$
 $F'(x) = f(x)$, dus F is een primitieve van f .

- 5 a** $F(x) = \frac{1}{5}x^5$ geeft $F'(x) = x^4$
Dus $F(x) = \frac{1}{5}x^5$ is een primitieve van $f(x) = x^4$.
- b** $G(x) = 6\sqrt{x}$ geeft $G'(x) = 6 \cdot \frac{1}{2\sqrt{x}} = \frac{3}{\sqrt{x}}$
Dus $G(x) = 6\sqrt{x}$ is een primitieve van $g(x) = \frac{3}{\sqrt{x}}$.
- c** $H(x) = \frac{2^x}{\ln(2)} = \frac{1}{\ln(2)} \cdot 2^x$ geeft $H'(x) = \frac{1}{\ln(2)} \cdot 2^x \ln(2) = 2^x$
Dus $H(x) = \frac{2^x}{\ln(2)}$ is een primitieve van $h(x) = 2^x$.
- d** $K(x) = 2\sin(x) - 3\cos(x)$ geeft $K'(x) = 2\cos(x) + 3\sin(x)$.
Dus $K(x) = 2\sin(x) - 3\cos(x)$ is een primitieve van $k(x) = 2\cos(x) + 3\sin(x)$.

Bladzijde 102

- 6** $F(x) = ax + c$ geeft $F'(x) = a$
Dus $F(x) = ax + c$ zijn de primitieven van $f(x) = a$.
- $F(x) = \frac{1}{n+1}x^{n+1} + c$ geeft $F'(x) = \frac{1}{n+1} \cdot (n+1)x^n = x^n$
Dus $F(x) = \frac{1}{n+1}x^{n+1} + c$ zijn de primitieven van $f(x) = x^n$.
- $F(x) = \frac{g^x}{\ln(g)} + c = \frac{1}{\ln(g)} \cdot g^x + c$ geeft $F'(x) = \frac{1}{\ln(g)} \cdot g^x \cdot \ln(g) = g^x$
Dus $F(x) = \frac{g^x}{\ln(g)} + c$ zijn de primitieven van $f(x) = g^x$.
- $F(x) = e^x + c$ geeft $F'(x) = e^x$
Dus $F(x) = e^x + c$ zijn de primitieven van $f(x) = e^x$.

$$F(x) = \ln|x| + c = \begin{cases} \ln(x) + c & \text{voor } x > 0 \\ \ln(-x) + c & \text{voor } x < 0 \end{cases} \text{ geeft } F'(x) = \begin{cases} \frac{1}{x} & \text{voor } x > 0 \\ \frac{1}{-x} \cdot -1 = \frac{1}{x} & \text{voor } x < 0 \end{cases}$$

Dus $F(x) = \ln|x| + c$ zijn de primitieven van $f(x) = \frac{1}{x}$.

$$F(x) = -\cos(x) + c \text{ geeft } F'(x) = \sin(x)$$

Dus $F(x) = -\cos(x) + c$ zijn de primitieven van $f(x) = \sin(x)$.

$$F(x) = \sin(x) + c \text{ geeft } F'(x) = \cos(x)$$

Dus $F(x) = \sin(x) + c$ zijn de primitieven van $f(x) = \cos(x)$.

7 a $f(x) = x^{-1}$ geeft $F(x) = \frac{1}{-1+1} x^{-1+1} + c$ kan niet kloppen, want $\frac{1}{-1+1} = \frac{1}{0}$ bestaat niet en $x^0 = 1$ voor $x \neq 0$.

b $f(x) = x^{-1} = \frac{1}{x}$ geeft $F(x) = \ln|x| + c$

8 a $f(x) = 2x^3 + 5x^4$ geeft $F(x) = \frac{1}{2}x^4 + x^5 + c$

b $f(x) = \frac{2x^3 + 3x}{x^4} = 2 \cdot \frac{1}{x} + 3x^{-3}$ geeft $F(x) = 2 \ln|x| + \frac{3}{-2}x^{-2} + c = 2 \ln|x| - \frac{3}{2x^2} + c$

c $f(x) = 2e^x$ geeft $F(x) = 2e^x + c$

d $f(x) = 5 \cdot 2^x$ geeft $F(x) = 5 \cdot \frac{2^x}{\ln(2)} + c = \frac{5 \cdot 2^x}{\ln(2)} + c$

e $f(x) = \sin(x) - \cos(x)$ geeft $F(x) = -\cos(x) - \sin(x) + c$

f $f(x) = \frac{x\sqrt{x} - 2\sqrt{x}}{3x} = \frac{1}{3}x^{\frac{1}{2}} - \frac{2}{3}x^{-\frac{1}{2}}$ geeft $F(x) = \frac{1}{3} \cdot \frac{1}{\frac{1}{2}}x^{\frac{1}{2}+1} - \frac{2}{3} \cdot \frac{1}{-\frac{1}{2}}x^{-\frac{1}{2}+1} + c = \frac{2}{9}x\sqrt{x} - \frac{4}{3}\sqrt{x} + c$

9 a $f(x) = x^3 - 3x$ geeft $F(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + c$

b $f(x) = ex + e^2$ geeft $F(x) = e \cdot \frac{1}{2}x^2 + e^2x + c = \frac{1}{2}ex^2 + e^2x + c$

c $f(x) = \frac{x^4 - 6}{2x^3} = \frac{1}{2}x - 3x^{-3}$ geeft $F(x) = \frac{1}{4}x^2 - \frac{3}{-2}x^{-2} + c = \frac{1}{4}x^2 + \frac{3}{2x^2} + c$

d $f(x) = x^e + e^x$ geeft $F(x) = \frac{1}{e+1}x^{e+1} + e^x + c$

e $f(x) = 4^x + x^4$ geeft $F(x) = \frac{4^x}{\ln(4)} + \frac{1}{5}x^5 + c$

f $f(x) = \frac{10}{3x} - \frac{5}{3\sqrt{x}} = \frac{10}{3} \cdot \frac{1}{x} - \frac{5}{3}x^{-\frac{1}{2}}$ geeft $F(x) = \frac{10}{3} \ln|x| - \frac{5}{3} \cdot \frac{1}{-\frac{1}{2}}x^{\frac{1}{2}} + c = \frac{10}{3} \ln|x| - \frac{10}{3}\sqrt{x} + c$

Bladzijde 103

10 a $f(x) = \frac{8}{x^3} - 1 = 8x^{-3} - 1$ geeft $F(x) = -4x^{-2} - x + c = -\frac{4}{x^2} - x + c$

b $f(x) = \frac{-x^3 + 2x + 3}{x^4} = -\frac{1}{x} + 2x^{-3} + 3x^{-4}$ geeft

$$F(x) = -\ln|x| - x^{-2} - x^{-3} + c = -\ln|x| - \frac{1}{x^2} - \frac{1}{x^3} + c$$

c $f(x) = \frac{2}{3}x^3 - 5x\sqrt{x} + 2 \cdot \sqrt[3]{x} = \frac{2}{3}x^3 - 5x^{\frac{3}{2}} + 2x^{\frac{1}{3}}$ geeft

$$F(x) = \frac{1}{6}x^4 - \frac{5}{2 \cdot \frac{1}{2}}x^{2\frac{1}{2}} + \frac{2}{\frac{1}{3}}x^{\frac{1}{3}+1} + c = \frac{1}{6}x^4 - 2x^2 \cdot \sqrt{x} + \frac{1}{2}x \cdot \sqrt[3]{x} + c$$

d $f(x) = ex^2 - 2\sin(x)$ geeft $F(x) = e \cdot \frac{1}{3}x^3 + 2\cos(x) + c = \frac{1}{3}ex^3 + 2\cos(x) + c$

e $f(x) = \frac{x^2 \cdot \sqrt{x} - 3x\sqrt{x} + 1}{2x^2} = \frac{1}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-2}$ geeft

$$F(x) = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}}x^{\frac{1}{2}+1} - \frac{3}{2} \cdot \frac{1}{-\frac{1}{2}}x^{-\frac{1}{2}+1} + \frac{1}{2} \cdot \frac{1}{-2}x^{-2+1} + c = \frac{1}{3}x\sqrt{x} - 3\sqrt{x} - \frac{1}{4x} + c$$

f $f(x) = 3 \cdot 10^x - 5e^x$ geeft $F(x) = 3 \cdot \frac{10^x}{\ln(10)} - 5e^x + c = \frac{3 \cdot 10^x}{\ln(10)} - 5e^x + c$

- 11** $F_q(x) = (3x + q)x\sqrt{x} = 3x^{2\frac{1}{2}} + qx^{1\frac{1}{2}}$ geeft $F'_q(x) = 7\frac{1}{2}x^{\frac{1}{2}} + 1\frac{1}{2}qx^{\frac{1}{2}} = 7\frac{1}{2}x\sqrt{x} + 1\frac{1}{2}q\sqrt{x}$
 Dit moet gelijk zijn aan $f'_p(x) = p\sqrt{x} + 2px\sqrt{x}$, dus $2p = 7\frac{1}{2}$ en $p = 1\frac{1}{2}q$

$$p = 3\frac{3}{4} \text{ en } q = \frac{2}{3}p = \frac{2}{3} \cdot \frac{15}{4} = 2\frac{1}{2}$$

 Dus voor $p = 3\frac{3}{4}$ en $q = 2\frac{1}{2}$.

- 12** **a** $f(x) = 2x - 3$ geeft $F(x) = x^2 - 3x + c$
b $F(1) = 2$ geeft $1^2 - 3 \cdot 1 + c = 2$

$$1 - 3 + c = 2$$

$$c = 4$$

 Dus $F(x) = x^2 - 3x + 4$.
c De grafiek van $F(x) = x^2 - 3x + c$ raakt de x -as, dus

$$\left. \begin{aligned} D &= 0 \\ D &= (-3)^2 - 4 \cdot 1 \cdot c \end{aligned} \right\} \begin{aligned} 9 - 4c &= 0 \\ 4c &= 9 \\ c &= 2\frac{1}{4} \end{aligned}$$

 Dus $F(x) = x^2 - 3x + 2\frac{1}{4}$.

- 13** $f(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1$ geeft $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$
 $F(1) = 7$ geeft $\frac{1}{5} - \frac{2}{3} + 1 + c = 7$

$$c = 6\frac{7}{15}$$

 Dus $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 6\frac{7}{15}$.

- 14** $F(x) = (ax^2 + bx + c)e^{2x}$ geeft
 $F'(x) = (2ax + b) \cdot e^{2x} + (ax^2 + bx + c) \cdot e^{2x} \cdot 2 = (2ax^2 + (2a + 2b)x + b + 2c)e^{2x}$
 Dit moet gelijk zijn aan $f(x) = x^2 e^{2x}$, dus $2a = 1 \wedge 2a + 2b = 0 \wedge b + 2c = 0$.

$$\left. \begin{aligned} 2a &= 1 \text{ geeft } a = \frac{1}{2} \\ 2a + 2b &= 0 \end{aligned} \right\} \begin{aligned} 2 \cdot \frac{1}{2} + 2b &= 0 \\ 2b &= -1 \\ b &= -\frac{1}{2} \end{aligned}$$

$$\left. \begin{aligned} b &= -\frac{1}{2} \\ b + 2c &= 0 \end{aligned} \right\} \begin{aligned} -\frac{1}{2} + 2c &= 0 \\ 2c &= \frac{1}{2} \\ c &= \frac{1}{4} \end{aligned}$$

 Dus $a = \frac{1}{2}$, $b = -\frac{1}{2}$ en $c = \frac{1}{4}$.

- 15** **a** $A(p) = O(\text{rechthoek}) + O(\text{driehoek}) = p \cdot b + \frac{1}{2} \cdot p \cdot (ap + b - b) = bp + \frac{1}{2}p \cdot ap = \frac{1}{2}ap^2 + bp$
b $A'(p) = ap + b$
 Dus $A'(p) = f(p)$.

Bladzijde 105

- 16** **a** $f(x) = 0$ geeft $3x^2 - x^3 = 0$

$$x^2(3 - x) = 0$$

$$x = 0 \vee x = 3$$

$$O(V) = \int_0^3 (3x^2 - x^3) dx = \left[x^3 - \frac{1}{4}x^4 \right]_0^3 = 27 - \frac{1}{4} \cdot 81 - 0 = 6\frac{3}{4}$$

b $\int_0^p (3x^2 - x^3) dx = 3\frac{3}{8}$

$$\left[x^3 - \frac{1}{4}x^4 \right]_0^p = 3\frac{3}{8}$$

$$p^3 - \frac{1}{4}p^4 - 0 = 3\frac{3}{8}$$

$$p^3 - \frac{1}{4}p^4 = 3\frac{3}{8}$$

 Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 1,84$.

17 $O(W) = \frac{1}{2} \cdot O(V)$

$$\int_{\frac{1}{3}\pi}^p \sin(x) dx = \frac{1}{2} \int_0^\pi \sin(x) dx$$

$$[-\cos(x)]_{\frac{1}{3}\pi}^p = \frac{1}{2} [-\cos(x)]_0^\pi$$

$$-\cos(p) + \cos(\frac{1}{3}\pi) = \frac{1}{2} (-\cos(\pi) + \cos(0))$$

$$-\cos(p) + \frac{1}{2} = \frac{1}{2} (1 + 1)$$

$$-\cos(p) + \frac{1}{2} = 1$$

$$-\cos(p) = \frac{1}{2}$$

$$\cos(p) = -\frac{1}{2}$$

$$p = \frac{2}{3}\pi$$

Bladzijde 106

18 a $O(V) = \int_0^{12} \sqrt{x} dx = \int_0^{12} x^{\frac{1}{2}} dx = \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^{12} = \left[\frac{2}{3} x\sqrt{x} \right]_0^{12} = \frac{2}{3} \cdot 12\sqrt{12} - 0 = 8\sqrt{12} = 16\sqrt{3}$

b $O(V) = 18$ geeft $\int_0^p \sqrt{x} dx = 18$

$$\left[\frac{2}{3} x\sqrt{x} \right]_0^p = 18$$

$$\frac{2}{3} p\sqrt{p} = 18$$

$$p^{1\frac{1}{2}} = 27$$

$$p = 27^{\frac{2}{3}}$$

$$p = 9$$

c $O(W_1) = O(W_2)$

$$\int_0^4 \sqrt{x} dx = \int_4^p \sqrt{x} dx$$

$$\left[\frac{2}{3} x\sqrt{x} \right]_0^4 = \left[\frac{2}{3} x\sqrt{x} \right]_4^p$$

$$\frac{2}{3} \cdot 4\sqrt{4} - 0 = \frac{2}{3} p\sqrt{p} - \frac{2}{3} \cdot 4\sqrt{4}$$

$$\frac{2}{3} p\sqrt{p} = 2 \cdot \frac{2}{3} \cdot 4\sqrt{4}$$

$$p^{1\frac{1}{2}} = 16$$

$$p = 16^{\frac{2}{3}}$$

$$p = (2^4)^{\frac{2}{3}}$$

$$p = 2^{2\frac{2}{3}}$$

$$p = 4 \cdot \sqrt[3]{4}$$

19 a $O(V) = 5$

$$\int_0^p e^x dx = 5$$

$$[e^x]_0^p = 5$$

$$e^p - e^0 = 5$$

$$e^p - 1 = 5$$

$$e^p = 6$$

$$p = \ln(6)$$

b $O(W) = 2 \cdot O(V)$

$$\int_p^{2p} e^x dx = 2 \int_0^p e^x dx$$

$$[e^x]_p^{2p} = 2[e^x]_0^p$$

$$e^{2p} - e^p = 2(e^p - e^0)$$

$$e^{2p} - e^p = 2e^p - 2$$

$$(e^p)^2 - 3e^p + 2 = 0$$

$$\text{Stel } e^p = u.$$

$$u^2 - 3u + 2 = 0$$

$$(u - 1)(u - 2) = 0$$

$$u = 1 \vee u = 2$$

$$e^p = 1 \vee e^p = 2$$

$$p = 0 \vee p = \ln(2)$$

$$p = 0 \text{ voldoet niet, dus } p = \ln(2).$$

20 a $F(x) = x \ln(x) - x$ geeft $F'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$

$$F'(x) = f(x), \text{ dus } F \text{ is een primitieve van } f.$$

b $O(V) = \int_1^{e^2} \ln(x) dx = [x \ln(x) - x]_1^{e^2} = e^2 \ln(e^2) - e^2 - (\ln(1) - 1) = 2e^2 - e^2 + 1 = e^2 + 1$

c $O(V) = 10$

$$\int_1^p \ln(x) dx = 10$$

$$[x \ln x - x]_1^p = 10$$

$$p \ln(p) - p - (\ln(1) - 1) = 10$$

$$p \ln(p) - p + 1 = 10$$

$$p \ln(p) - p = 9$$

$$\text{Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft } p \approx 8,174.$$

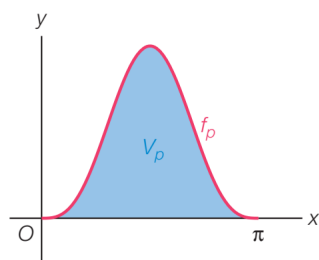
21 a $F_p(x) = \frac{1}{3}p \cos^3(x) - p \cos(x)$ geeft $F_p'(x) = \frac{1}{3}p \cdot 3 \cos^2(x) \cdot -\sin(x) + p \sin(x) = -p \sin(x) \cos^2(x) + p \sin(x)$
 $= p \sin(x)(-\cos^2(x) + 1) = p \sin(x) \sin^2(x) = p \sin^3(x)$

$$F_p'(x) = f_p(x), \text{ dus } F_p \text{ is een primitieve van } f_p.$$

b $f_p(x) = 0$ geeft $p \sin^3(x) = 0$

$$\sin(x) = 0$$

$$x = k \cdot \pi$$



$$O(V_p) = \int_0^\pi f_p(x) dx = \left[\frac{1}{3}p \cos^3(x) - p \cos(x) \right]_0^\pi = \frac{1}{3}p \cdot -1 - p \cdot -1 - \left(\frac{1}{3}p \cdot 1 - p \cdot 1 \right)$$

$$= -\frac{1}{3}p + p - \left(\frac{1}{3}p - p \right) = \frac{2}{3}p + \frac{2}{3}p = \frac{4}{3}p$$

$$O(V_p) = 4 \text{ geeft } \frac{4}{3}p = 4$$

$$p = 3$$

Bladzijde 107**C1**

a $F(x) = 3(2x^3 - 10)^4$ geeft $F'(x) = 12(2x^3 - 10)^3 \cdot 6x^2 = 72x^2(2x^3 - 10)^3$
 $F'(x) = f(x)$, dus F is een primitieve van f .

b $G(x) = x^3 \ln(x)$ geeft $G'(x) = 3x^2 \cdot \ln(x) + x^3 \cdot \frac{1}{x} = 3x^2 \ln(x) + x^2 = x^2(3 \ln(x) + 1)$

$G'(x) = g(x)$, dus G is een primitieve van g .

c $K(x) = \cos^3(x)$ geeft $K'(x) = 3 \cos^2(x) \cdot -\sin(x) = -3(1 - \sin^2(x)) \sin(x) = 3 \sin^3(x) - 3 \sin(x)$
 $K'(x) = k(x)$, dus K is een primitieve van k .

C2

a $f(x) = \frac{3x^3 + 2x^2 - 7x}{x^2} = 3x + 2 - 7 \cdot \frac{1}{x}$ geeft $F(x) = \frac{1}{2}x^2 + 2x - 7 \ln|x| + c$

b $f(x) = \sqrt[3]{x} + \frac{5}{3\sqrt{x}} - 1 = x^{\frac{1}{3}} + \frac{5}{3}x^{-\frac{1}{2}} - 1$ geeft $F(x) = \frac{1}{1\frac{1}{3}}x^{1\frac{1}{3}} + \frac{5}{3} \cdot \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} - x + c = \frac{3}{4}x \cdot \sqrt[3]{x} + 3\frac{1}{3}\sqrt{x} - x + c$

c $f(x) = 5^x - e^x$ geeft $F(x) = \frac{5^x}{\ln(5)} - e^x + c$

C3

a $O(V) = \int_1^{e^2} \frac{1}{x} dx = [\ln|x|]_1^{e^2} = \ln(e^2) - \ln(1) = 2 - 0 = 2$

b $O(V) = 2\frac{1}{2}$ geeft $\int_1^p \frac{1}{x} dx = 2\frac{1}{2}$
 $[\ln|x|]_1^p = 2\frac{1}{2}$
 $\ln(p) - \ln(1) = 2\frac{1}{2}$
 $\ln(p) - 0 = 2\frac{1}{2}$
 $\ln(p) = 2\frac{1}{2}$
 $p = e^{2\frac{1}{2}}$
 $p = e^2 \cdot \sqrt{e}$

11.2 Oppervlakten**Bladzijde 108****22**

a $F(x) = a(3x + 1)^6 + c$ geeft $F'(x) = 6a(3x + 1)^5 \cdot 3 = 18a(3x + 1)^5$

b $F'(x) = 18a(3x + 1)^5$ moet gelijk zijn aan $f(x) = (3x + 1)^5$, dus $18a = 1$ en dit geeft $a = \frac{1}{18}$.

Bladzijde 109**23**

$\left[\frac{1}{a} F(ax + b) + c \right]' = \frac{1}{a} \cdot f(ax + b) \cdot a = f(ax + b)$

Dus $\frac{1}{a} F(ax + b) + c$ zijn de primitieven van $f(ax + b)$.

24

a $f(x) = (ax + b)^n$ geeft $F(x) = \frac{1}{a} \cdot \frac{1}{n+1} (ax + b)^{n+1} + c$

b $f(x) = g^{ax+b}$ geeft $F(x) = \frac{1}{a} \cdot \frac{g^{ax+b}}{\ln(g)} + c$

c $f(x) = \frac{1}{ax+b}$ geeft $F(x) = \frac{1}{a} \ln|ax + b| + c$

d $f(x) = \cos(ax + b)$ geeft $F(x) = \frac{1}{a} \sin(ax + b) + c$

- 25** a $f(x) = (2x - 1)^6$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{7}(2x - 1)^7 + c = \frac{1}{14}(2x - 1)^7 + c$
 b $f(x) = \frac{1}{(3x + 4)^3} = (3x + 4)^{-3}$ geeft $F(x) = \frac{1}{3} \cdot \frac{1}{-2}(3x + 4)^{-2} + c = -\frac{1}{6(3x + 4)^2} + c$
 c $f(x) = 4\sqrt{3 - 2x} = 4(3 - 2x)^{\frac{1}{2}}$ geeft $F(x) = 4 \cdot \frac{1}{-2} \cdot \frac{1}{\frac{1}{2}}(3 - 2x)^{\frac{1}{2} + 1} + c = -1\frac{1}{3}(3 - 2x)\sqrt{3 - 2x} + c$
 d $f(x) = \frac{2}{\sqrt{1 - x}} = 2(1 - x)^{-\frac{1}{2}}$ geeft $F(x) = 2 \cdot \frac{1}{-1} \cdot \frac{1}{\frac{1}{2}}(1 - x)^{\frac{1}{2} + 1} + c = -4\sqrt{1 - x} + c$
 e $f(x) = \sin(2x + \frac{1}{3}\pi)$ geeft $F(x) = -\frac{1}{2}\cos(2x + \frac{1}{3}\pi) + c$
 f $f(x) = 3\cos(\frac{1}{2}x - \frac{1}{6}\pi)$ geeft $F(x) = 3 \cdot 2\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c = 6\sin(\frac{1}{2}x - \frac{1}{6}\pi) + c$

- 26** a $f(x) = \frac{3}{(2x - 1)^4} = 3(2x - 1)^{-4}$ geeft $F(x) = 3 \cdot \frac{1}{2} \cdot \frac{1}{-3}(2x - 1)^{-3} + c = -\frac{1}{2(2x - 1)^3} + c$
 b $f(x) = (4x + 3)\sqrt{4x + 3} = (4x + 3)^{\frac{1}{2}}$ geeft $F(x) = \frac{1}{4} \cdot \frac{1}{\frac{1}{2}}(4x + 3)^{\frac{1}{2} + 1} + c = \frac{1}{10}(4x + 3)^2 \cdot \sqrt{4x + 3} + c$
 c $f(x) = 2\sin(3x + \frac{1}{2}\pi)$ geeft $F(x) = -2 \cdot \frac{1}{3}\cos(3x + \frac{1}{2}\pi) + c = -\frac{2}{3}\cos(3x + \frac{1}{2}\pi) + c$
 d $f(x) = 8 \cdot \sqrt[3]{3 - 4x} = 8(3 - 4x)^{\frac{1}{3}}$ geeft $F(x) = 8 \cdot \frac{1}{-4} \cdot \frac{1}{\frac{1}{3}}(3 - 4x)^{\frac{1}{3} + 1} + c = -1\frac{1}{2}(3 - 4x) \cdot \sqrt[3]{3 - 4x} + c$
 e $f(x) = \frac{6}{3x + 1} = 6 \cdot \frac{1}{3x + 1}$ geeft $F(x) = 6 \cdot \frac{1}{3}\ln|3x + 1| + c = 2\ln|3x + 1| + c$
 f $f(x) = e^{4x + 1} - 4e^{4x}$ geeft $F(x) = \frac{1}{4}e^{4x + 1} - \frac{1}{e} \cdot \frac{4e^{4x}}{\ln(4)} + c = \frac{1}{4}e^{4x + 1} - \frac{4e^{4x}}{e\ln(4)} + c$
 g $f(x) = 3x - \cos(1 - 2x)$ geeft $F(x) = \frac{1}{2}x^2 + \frac{1}{2}\sin(1 - 2x) + c$
 h $f(x) = \frac{5}{4x^2 - 12x + 9} = \frac{5}{(2x - 3)^2}$ geeft
 $F(x) = 5 \cdot \frac{1}{2} \cdot \frac{1}{-1}(2x - 3)^{-1} + c = -\frac{5}{2(2x - 3)} + c = -\frac{5}{4x - 6} + c$

- 27** a $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin(2x - \frac{1}{6}\pi) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{6}\pi)\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = -\frac{1}{2}\cos(\frac{1}{2}\pi) + \frac{1}{2}\cos(\frac{1}{6}\pi) = 0 + \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} = \frac{1}{4}\sqrt{3}$
 b $\int_0^{\frac{1}{2}\pi} (2x + \cos(\frac{1}{2}x)) dx = \left[x^2 + 2\sin(\frac{1}{2}x)\right]_0^{\frac{1}{2}\pi} = \frac{1}{4}\pi^2 + 2\sin(\frac{1}{4}\pi) - (0 + 2\sin(0)) = \frac{1}{4}\pi^2 + 2 \cdot \frac{1}{2}\sqrt{2} - 0 = \frac{1}{4}\pi^2 + \sqrt{2}$

Bladzijde 110

- 28** $f(x) = 0$ geeft $1 + 2\cos(\frac{1}{2}x - \frac{5}{6}\pi) = 0$
 $2\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -1$
 $\cos(\frac{1}{2}x - \frac{5}{6}\pi) = -\frac{1}{2}$
 $\frac{1}{2}x - \frac{5}{6}\pi = \frac{2}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}x - \frac{5}{6}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$
 $\frac{1}{2}x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi$
 $x = 3\pi + k \cdot 4\pi \vee x = \frac{1}{3}\pi + k \cdot 4\pi$
 x in $[0, 4\pi]$ geeft $x = \frac{1}{3}\pi \vee x = 3\pi$
 $O(V) = \int_{\frac{1}{3}\pi}^{3\pi} (1 + 2\cos(\frac{1}{2}x - \frac{5}{6}\pi)) dx = \left[x + 4\sin(\frac{1}{2}x - \frac{5}{6}\pi)\right]_{\frac{1}{3}\pi}^{3\pi} = 3\pi + 4\sin(\frac{2}{3}\pi) - (\frac{1}{3}\pi + 4\sin(-\frac{2}{3}\pi))$
 $= 3\pi + 4 \cdot \frac{1}{2}\sqrt{3} - (\frac{1}{3}\pi + 4 \cdot -\frac{1}{2}\sqrt{3}) = 3\pi + 2\sqrt{3} - \frac{1}{3}\pi + 2\sqrt{3} = 2\frac{2}{3}\pi + 4\sqrt{3}$

29 a $f(x) = \frac{9}{3x+2} = 9 \cdot \frac{1}{3x+2}$ geeft $F(x) = 9 \cdot \frac{1}{3} \cdot \ln|3x+2| + c = 3 \ln|3x+2| + c$

$$O(U) = \int_0^{\frac{1}{3}} \frac{9}{3x+2} dx = [3 \ln|3x+2|]_0^{\frac{1}{3}} = 3 \ln(3) - 3 \ln(2) = 3 \ln(1\frac{1}{2})$$

$$O(V) = \int_{\frac{1}{3}}^p \frac{9}{3x+2} dx = [3 \ln|3x+2|]_{\frac{1}{3}}^p = 3 \ln(3p+2) - 3 \ln(3)$$

$$O(U) = O(V) \text{ geeft } 3 \ln(1\frac{1}{2}) = 3 \ln(3p+2) - 3 \ln(3)$$

$$\ln(3p+2) = \ln(1\frac{1}{2}) + \ln(3)$$

$$\ln(3p+2) = \ln(4\frac{1}{2})$$

$$3p+2 = 4\frac{1}{2}$$

$$3p = 2\frac{1}{2}$$

$$p = \frac{5}{6}$$

b $O(W) = \ln(\frac{27}{8})$

$$\int_q^{2q} \frac{9}{3x+2} dx = \ln(\frac{27}{8})$$

$$[3 \ln|3x+2|]_q^{2q} = \ln(\frac{27}{8})$$

$$3 \ln(6q+2) - 3 \ln(3q+2) = \ln(\frac{27}{8})$$

$$3 \ln\left(\frac{6q+2}{3q+2}\right) = \ln(\frac{27}{8})$$

$$3 \ln\left(\frac{6q+2}{3q+2}\right) = \ln((\frac{3}{2})^3)$$

$$3 \ln\left(\frac{6q+2}{3q+2}\right) = 3 \ln(\frac{3}{2})$$

$$\ln\left(\frac{6q+2}{3q+2}\right) = \ln(\frac{3}{2})$$

$$\frac{6q+2}{3q+2} = \frac{3}{2}$$

$$2(6q+2) = 3(3q+2)$$

$$12q+4 = 9q+6$$

$$3q = 2$$

$$q = \frac{2}{3}$$

30 $f(x) = 2e^{1-\frac{1}{3}x}$ geeft $F(x) = 2 \cdot -3e^{1-\frac{1}{3}x} + c = -6e^{1-\frac{1}{3}x} + c$

$$O(V) = 2 \cdot O(W) \text{ geeft } \int_{-p}^0 2e^{1-\frac{1}{3}x} dx = 2 \int_0^p 2e^{1-\frac{1}{3}x} dx$$

$$[-6e^{1-\frac{1}{3}x}]_{-p}^0 = 2[-6e^{1-\frac{1}{3}x}]_0^p$$

$$-6e^1 + 6e^{1+\frac{1}{3}p} = 2(-6e^{1-\frac{1}{3}p} + 6e^1)$$

$$-6e + 6e \cdot e^{\frac{1}{3}p} = -12e \cdot e^{-\frac{1}{3}p} + 12e$$

$$-1 + e^{\frac{1}{3}p} = -2e^{-\frac{1}{3}p} + 2$$

$$e^{\frac{1}{3}p} - 3 + 2(e^{\frac{1}{3}p})^{-1} = 0$$

$$\text{Stel } e^{\frac{1}{3}p} = u.$$

$$u - 3 + 2u^{-1} = 0$$

$$u^2 - 3u + 2 = 0$$

$$(u-1)(u-2) = 0$$

$$u = 1 \vee u = 2$$

$$e^{\frac{1}{3}p} = 1 \vee e^{\frac{1}{3}p} = 2$$

$$\frac{1}{3}p = 0 \vee \frac{1}{3}p = \ln(2)$$

$$p = 0 \vee p = 3 \ln(2)$$

$p = 0$ voldoet niet, dus $p = 3 \ln(2)$.

31 a De regel gaat over functies van de vorm $f(ax + b)$ en g is niet van deze vorm.

b $G(x) = p(x^3 - 4)^{\frac{1}{2}} + c$ geeft $G'(x) = \frac{1}{2}p(x^3 - 4)^{-\frac{1}{2}} \cdot 3x^2 = \frac{3}{2}px^2 \cdot \sqrt{x^3 - 4}$
Er bestaat geen waarde van p waarvoor $G'(x) = 2\sqrt{x^3 - 4}$.

Bladzijde 111

32 a Stel A is het gebied begrensd door de x -as, de lijnen $x = a$ en $x = b$ en de grafiek van f .

Dit geeft $O(A) = \int_a^b f(x) dx$.

Stel B is het gebied begrensd door de x -as, de lijnen $x = a$ en $x = b$ en de grafiek van g .

Dit geeft $O(B) = \int_a^b g(x) dx$.

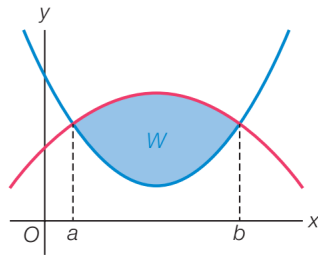
Dus $O(V) = O(A) - O(B) = \int_a^b f(x) dx - \int_a^b g(x) dx$.

b $O(V) = \int_a^b f(x) dx - \int_a^b g(x) dx = [F(x)]_a^b - [G(x)]_a^b = F(b) - F(a) - (G(b) - G(a))$
 $= F(b) - F(a) - G(b) + G(a) = F(b) - G(b) - F(a) + G(a)$
 $= F(b) - G(b) - (F(a) - G(a))$

c $O(V) = F(b) - G(b) - (F(a) - G(a)) = [F(x) - G(x)]_a^b = \int_a^b (f(x) - g(x)) dx$

Bladzijde 112

33 a Schuif de grafieken van f en g beide q omhoog. Zie de figuur.



$O(W) = \int_a^b (f(x) + q) dx - \int_a^b (g(x) + q) dx = \int_a^b (f(x) + q - g(x) - q) dx = \int_a^b (f(x) - g(x)) dx$

b $\int_a^b p \cdot f(x) dx = [p \cdot F(x)]_a^b = p \cdot F(b) - p \cdot F(a) = p(F(b) - F(a)) = p[F(x)]_a^b = p \int_a^b f(x) dx$

Bladzijde 113

34 a $O(V) = \int_0^\pi \sin(x) dx = [-\cos(x)]_0^\pi = -\cos(\pi) + \cos(0) = 1 + 1 = 2$

$O(W) = -\int_\pi^{2\pi} \sin(x) dx = -[-\cos(x)]_\pi^{2\pi} = [\cos(x)]_\pi^{2\pi} = \cos(2\pi) - \cos(\pi) = 1 - (-1) = 2$

b $\int_0^{2\pi} \sin(x) dx = [-\cos(x)]_0^{2\pi} = -\cos(2\pi) + \cos(0) = -1 + 1 = 0$

$\int_0^{2\pi} \sin(x) dx = \int_0^\pi \sin(x) dx + \int_\pi^{2\pi} \sin(x) dx = O(V) + -O(W) = 2 - 2 = 0$

35 $f(x) = g(x)$ geeft $\sin(x) = \cos(2x)$

$$\cos(x - \frac{1}{2}\pi) = \cos(2x)$$

$$x - \frac{1}{2}\pi = 2x + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x + k \cdot 2\pi$$

$$-x = \frac{1}{2}\pi + k \cdot 2\pi \vee 3x = \frac{1}{2}\pi + k \cdot 2\pi$$

$$x = -\frac{1}{2}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$$

$O(V) = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (f(x) - g(x)) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin(x) - \cos(2x)) dx = [-\cos(x) - \frac{1}{2}\sin(2x)]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi}$
 $= -\cos(\frac{5}{6}\pi) - \frac{1}{2}\sin(\frac{5}{3}\pi) - (-\cos(\frac{1}{6}\pi) - \frac{1}{2}\sin(\frac{1}{3}\pi)) = \frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{3})$
 $= \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$

36

a $x^3 = 2x$

$$x = 0 \vee x^2 = 2$$

$$x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2}$$

$$O(V) = \int_0^{\sqrt{2}} (2x - x^3) dx = \left[x^2 - \frac{1}{4}x^4 \right]_0^{\sqrt{2}} = 2 - \frac{1}{4} \cdot 4 - 0 = 1$$

b $\int_0^p (2x - x^3) dx = \frac{1}{2}$ met $0 < p < \sqrt{2}$

$$\left[x^2 - \frac{1}{4}x^4 \right]_0^p = \frac{1}{2}$$

$$p^2 - \frac{1}{4}p^4 - 0 = \frac{1}{2}$$

$$4p^2 - p^4 = 2$$

$$p^4 - 4p^2 + 2 = 0$$

$$\text{Stel } p^2 = u.$$

$$u^2 - 4u + 2 = 0$$

$$D = (-4)^2 - 4 \cdot 1 \cdot 2 = 8, \text{ dus } \sqrt{D} = 2\sqrt{2}$$

$$u = \frac{4 + 2\sqrt{2}}{2} = 2 + \sqrt{2} \vee u = \frac{4 - 2\sqrt{2}}{2} = 2 - \sqrt{2}$$

$$p^2 = 2 + \sqrt{2} \vee p^2 = 2 - \sqrt{2}$$

$$p^2 = 2 + \sqrt{2} \text{ voldoet niet, dus } p^2 = 2 - \sqrt{2}$$

$$p = \sqrt{2 - \sqrt{2}} \vee p = -\sqrt{2 - \sqrt{2}}$$

$$p = -\sqrt{2 - \sqrt{2}} \text{ voldoet niet, dus } p = \sqrt{2 - \sqrt{2}}.$$

37

$f(x) = 1$ geeft $x^3 - 5x^2 + 6x + 1 = 1$

$$x^3 - 5x^2 + 6x = 0$$

$$x(x^2 - 5x + 6) = 0$$

$$x(x - 2)(x - 3) = 0$$

$$x = 0 \vee x = 2 \vee x = 3$$

$$O(V) = \int_0^2 (f(x) - 1) dx = \int_0^2 (x^3 - 5x^2 + 6x) dx = \left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2 \right]_0^2 = \frac{1}{4} \cdot 2^4 - \frac{5}{3} \cdot 2^3 + 3 \cdot 2^2 - 0 = 2\frac{2}{3}$$

$$O(W) = \int_2^3 (1 - f(x)) dx = \int_2^3 (1 - (x^3 - 5x^2 + 6x + 1)) dx = \int_2^3 (1 - x^3 + 5x^2 - 6x - 1) dx$$

$$= \int_2^3 (-x^3 + 5x^2 - 6x) dx = \left[-\frac{1}{4}x^4 + \frac{5}{3}x^3 - 3x^2 \right]_2^3$$

$$= -\frac{1}{4} \cdot 3^4 + \frac{5}{3} \cdot 3^3 - 3 \cdot 3^2 - \left(-\frac{1}{4} \cdot 2^4 + \frac{5}{3} \cdot 2^3 - 3 \cdot 2^2 \right) = \frac{5}{12}$$

$$\frac{O(V)}{O(W)} = \frac{2\frac{2}{3}}{\frac{5}{12}} = \frac{8}{3} \cdot \frac{12}{5} = \frac{32}{5} = 6\frac{2}{5}$$

Dus $O(V)$ is $6\frac{2}{5}$ keer zo groot als $O(W)$.

Bladzijde 114

38

$f(x) = g(x)$ geeft $x - 4\sqrt{3x - 5} = -x$

$$4\sqrt{3x - 5} = 2x$$

$$16(3x - 5) = (2x)^2$$

$$48x - 80 = 4x^2$$

$$4x^2 - 48x + 80 = 0$$

$$x^2 - 12x + 20 = 0$$

$$(x - 2)(x - 10) = 0$$

$$x = 2 \vee x = 10$$

$$O(V) = \int_2^{10} (g(x) - f(x)) dx = \int_2^{10} (-x - x + 4\sqrt{3x - 5}) dx = \int_2^{10} (-2x + 4(3x - 5)^{\frac{1}{2}}) dx$$

$$= \left[-x^2 + 4 \cdot \frac{1}{3} \cdot \frac{1}{\frac{1}{2}} (3x - 5)^{\frac{1}{2}} \right]_2^{10} = \left[-x^2 + \frac{8}{9}(3x - 5)\sqrt{3x - 5} \right]_2^{10} = -100 + 111\frac{1}{9} - (-4 + \frac{8}{9}) = 14\frac{2}{9}$$

39 Knikken als $x^2 - 9 = 0$

$$x^2 = 9$$

$$x = 3 \vee x = -3$$

Dus $x_B = 3$.

$f(3) = 6$, dus $k: y = 6$.

Voor $-3 \leq x \leq 3$ geldt $f(x) = -(x^2 - 9) + 2x = -x^2 + 2x + 9$.

$$-x^2 + 2x + 9 = 6$$

$$-x^2 + 2x + 3 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \vee x = 3$$

$$\begin{aligned} O(V) &= \int_{-1}^3 (f(x) - 6) dx = \int_{-1}^3 (-x^2 + 2x + 9 - 6) dx = \int_{-1}^3 (-x^2 + 2x + 3) dx = \left[-\frac{1}{3}x^3 + x^2 + 3x \right]_{-1}^3 \\ &= -\frac{1}{3} \cdot 3^3 + 3^2 + 3 \cdot 3 - \left(-\frac{1}{3} \cdot (-1)^3 + (-1)^2 + 3 \cdot (-1) \right) = 10\frac{2}{3} \end{aligned}$$

40 $f(x) = q$ geeft $3^x = q$, dus $x = {}^3\log(q)$.

$$\begin{aligned} O(V) &= \int_0^{{}^3\log(q)} (q - f(x)) dx = \int_0^{{}^3\log(q)} (q - 3^x) dx = \left[qx - \frac{3^x}{\ln(3)} \right]_0^{{}^3\log(q)} = q \cdot {}^3\log(q) - \frac{q}{\ln(3)} - \left(0 - \frac{1}{\ln(3)} \right) \\ &= q \cdot {}^3\log(q) + \frac{1 - q}{\ln(3)} \end{aligned}$$

$$\begin{aligned} O(W) &= \int_{{}^3\log(q)}^2 (f(x) - q) dx = \int_{{}^3\log(q)}^2 (3^x - q) dx = \left[\frac{3^x}{\ln(3)} - qx \right]_{{}^3\log(q)}^2 = \frac{9}{\ln(3)} - 2q - \left(\frac{q}{\ln(3)} - q \cdot {}^3\log(q) \right) \\ &= \frac{9 - q}{\ln(3)} - 2q + q \cdot {}^3\log(q) \end{aligned}$$

$$O(V) = O(W) \text{ geeft } q \cdot {}^3\log(q) + \frac{1 - q}{\ln(3)} = \frac{9 - q}{\ln(3)} - 2q + q \cdot {}^3\log(q)$$

$$2q = \frac{8}{\ln(3)}$$

$$q = \frac{4}{\ln(3)}$$

41 a $f(x) = -1\frac{1}{2}$ geeft $\frac{x^2 + x + 1}{x} = -1\frac{1}{2}$

$$x^2 + x + 1 = -1\frac{1}{2}x$$

$$x^2 + 2\frac{1}{2}x + 1 = 0$$

$$(x + \frac{1}{2})(x + 2) = 0$$

$$x = -\frac{1}{2} \vee x = -2$$

$$\begin{aligned} O(V) &= \int_{-2}^{-\frac{1}{2}} (f(x) - (-1\frac{1}{2})) dx = \int_{-2}^{-\frac{1}{2}} \left(\frac{x^2 + x + 1}{x} + 1\frac{1}{2} \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x + 1 + \frac{1}{x} + 1\frac{1}{2} \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x + \frac{1}{x} + 2\frac{1}{2} \right) dx \\ &= \left[\frac{1}{2}x^2 + \ln|x| + 2\frac{1}{2}x \right]_{-2}^{-\frac{1}{2}} = \frac{1}{2} \cdot \left(-\frac{1}{2} \right)^2 + \ln\left(\frac{1}{2}\right) + 2\frac{1}{2} \cdot -\frac{1}{2} - \left(\frac{1}{2} \cdot (-2)^2 + \ln(2) + 2\frac{1}{2} \cdot -2 \right) \\ &= \frac{1}{8} - \ln(2) - 1\frac{1}{4} - 2 - \ln(2) + 5 = 1\frac{7}{8} - 2\ln(2) \end{aligned}$$

b $O(W) = 2$ geeft $\int_1^p (f(x) - (x + 1)) dx = 2$

$$\int_1^p \left(\frac{x^2 + x + 1}{x} - (x + 1) \right) dx = 2$$

$$\int_1^p \left(x + 1 + \frac{1}{x} - x - 1 \right) dx = 2$$

$$\int_1^p \frac{1}{x} dx = 2$$

$$[\ln|x|]_1^p = 2$$

$$\ln(p) - 0 = 2$$

$$\ln(p) = 2$$

$$p = e^2$$

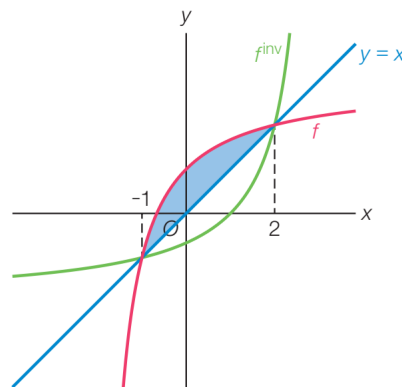
Bladzijde 115

- 42** De snijpunten van de grafiek f en de grafiek van f^{inv} liggen op de lijn $y = x$.

$$\begin{aligned} f(x) = x \text{ geeft } 3 - \frac{4}{x+2} &= x \\ 3(x+2) - 4 &= x(x+2) \\ 3x + 6 - 4 &= x^2 + 2x \\ x^2 - x - 2 &= 0 \\ (x+1)(x-2) &= 0 \\ x &= -1 \vee x = 2 \end{aligned}$$

Zie de figuur hiernaast.

$$\begin{aligned} O(V) &= 2 \int_{-1}^2 (f(x) - x) dx = 2 \int_{-1}^2 \left(3 - \frac{4}{x+2} - x \right) dx \\ &= 2 \left[3x - 4 \ln|x+2| - \frac{1}{2}x^2 \right]_{-1}^2 \\ &= 2 \left(3 \cdot 2 - 4 \ln(4) - \frac{1}{2} \cdot 2^2 - \left(3 \cdot (-1) - 4 \ln(1) - \frac{1}{2} \cdot (-1)^2 \right) \right) \\ &= 2 \left(6 - 4 \ln(4) - 2 - (-3 - 0 - \frac{1}{2}) \right) \\ &= 2 \left(7\frac{1}{2} - 4 \ln(4) \right) = 15 - 8 \ln(4) \end{aligned}$$



- 43** $P(p, 2e^{0,2p})$

$$O(\text{rechthoek}) = p \cdot 2e^{0,2p}$$

$$O(V) = O(W) \text{ geeft } O(W) = \frac{1}{2} \cdot O(\text{rechthoek})$$

$$\int_0^p (2e^{0,2x}) dx = p \cdot e^{0,2p}$$

$$[10e^{0,2x}]_0^p = p \cdot e^{0,2p}$$

$$10e^{0,2p} - 10 = p \cdot e^{0,2p}$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 7,97$.

Bladzijde 116

- 44 a** $f_p(x) = \frac{p}{\sqrt{1-x}} = p(1-x)^{-\frac{1}{2}}$ geeft $F_p(x) = p \cdot \frac{1}{-\frac{1}{2}} \cdot \frac{1}{2} (1-x)^{\frac{1}{2}} + c = -2p\sqrt{1-x} + c$

$$\begin{aligned} \text{b } O(V_q) &= \int_q^0 (f_6(x) - f_2(x)) dx = \int_q^0 \left(\frac{6}{\sqrt{1-x}} - \frac{2}{\sqrt{1-x}} \right) dx = \int_q^0 \left(\frac{4}{\sqrt{1-x}} \right) dx = [-8\sqrt{1-x}]_q^0 \\ &= -8 + 8\sqrt{1-q} \end{aligned}$$

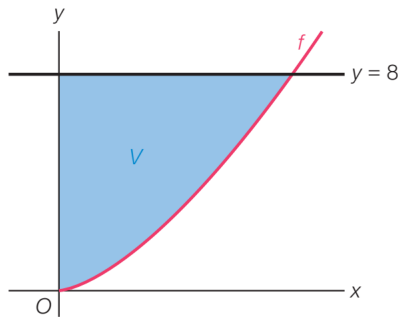
$$O(V_q) = 12 \text{ geeft } -8 + 8\sqrt{1-q} = 12$$

$$8\sqrt{1-q} = 20$$

$$64(1-q) = 400$$

$$1-q = 6\frac{1}{4}$$

$$q = -5\frac{1}{4}$$



$$f(x) = 8 \text{ geeft } x\sqrt{x} = 8 \\ x^3 = 64 \\ x = 4$$

$$O(V) = \int_0^4 (8 - f(x)) dx = \int_0^4 (8 - x\sqrt{x}) dx = \int_0^4 (8 - x^{1\frac{1}{2}}) dx = \left[8x - \frac{2}{5}x^{2\frac{1}{2}} \right]_0^4 = \left[8x - \frac{2}{5}x^2 \cdot \sqrt{x} \right]_0^4 \\ = 8 \cdot 4 - \frac{2}{5} \cdot 4^2 \cdot \sqrt{4} - 0 = 32 - 12\frac{4}{5} - 0 = 19\frac{1}{5}$$

b $f(x) = x\sqrt{x} = x^{1\frac{1}{2}}$ geeft $f'(x) = 1\frac{1}{2}x^{\frac{1}{2}} = 1\frac{1}{2}\sqrt{x}$

$$L = \int_0^4 \sqrt{1 + (f'(x))^2} dx = \int_0^4 \sqrt{1 + (1\frac{1}{2}\sqrt{x})^2} dx = \int_0^4 \sqrt{1 + 2\frac{1}{4}x} dx = \int_0^4 (1 + 2\frac{1}{4}x)^{\frac{1}{2}} dx \\ = \left[\frac{1}{2\frac{1}{4}} \cdot \frac{1}{1\frac{1}{2}} (1 + 2\frac{1}{4}x)^{1\frac{1}{2}} \right]_0^4 = \left[\frac{8}{27} (1 + 2\frac{1}{4}x) \sqrt{1 + 2\frac{1}{4}x} \right]_0^4 = \frac{8}{27} (10\sqrt{10} - 1)$$

$$\text{omtrek } V = \frac{8}{27} (10\sqrt{10} - 1) + 4 + 8 \approx 21,07$$

c $L = \int_0^p \sqrt{1 + (f'(x))^2} dx = \left[\frac{8}{27} (1 + 2\frac{1}{4}x) \sqrt{1 + 2\frac{1}{4}x} \right]_0^p = \frac{8}{27} (1 + 2\frac{1}{4}p) \sqrt{1 + 2\frac{1}{4}p} - \frac{8}{27}$

$$\text{omtrek } W = 10 \text{ geeft } \frac{8}{27} (1 + 2\frac{1}{4}p) \sqrt{1 + 2\frac{1}{4}p} - \frac{8}{27} + p + p\sqrt{p} = 10$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 2,29$.

46 $f_p(x) = p \ln(x)$ geeft $f'_p(x) = \frac{p}{x}$

Stel $A(a, p \ln(a))$.

$$\text{rc}_k = f'_p(a) \text{ geeft } \frac{p \ln(a)}{a} = \frac{p}{a} \\ \ln(a) = 1 \\ a = e$$

Dus $k: y = \frac{p}{e}x$.

$$f_p(x) = 0 \text{ geeft } p \ln(x) = 0 \\ \ln(x) = 0 \\ x = 1$$

$$O(V) = \int_0^e \frac{p}{e}x dx - \int_1^e f_p(x) dx = \left[\frac{p}{2e}x^2 \right]_0^e - [px \ln(x) - px]_1^e = \frac{1}{2}pe - 0 - (pe - pe - (0 - p)) = \frac{1}{2}pe - p \\ = p(\frac{1}{2}e - 1)$$

$$O(W) = \int_1^e f_p(x) dx = [px \ln(x) - px]_1^e = pe - pe - (0 - p) = p$$

Dus $\frac{O(V)}{O(W)} = \frac{p(\frac{1}{2}e - 1)}{p} = \frac{1}{2}e - 1$ en dit is onafhankelijk van p .

Bladzijde 117**C4**

a $f(x) = 4(3x-2)^5$ geeft $F(x) = 4 \cdot \frac{1}{3} \cdot \frac{1}{6}(3x-2)^6 + c = \frac{2}{9}(3x-2)^6 + c$

b $f(x) = \frac{5}{\sqrt{2-3x}} - \frac{3}{2-3x} = 5(2-3x)^{-\frac{1}{2}} - 3 \cdot \frac{1}{2-3x}$ geeft

$$F(x) = 5 \cdot \frac{1}{-\frac{1}{2}} \cdot \frac{1}{2} (2-3x)^{\frac{1}{2}} - 3 \cdot \frac{1}{-\frac{1}{2}} \ln|2-3x| + c = -3\frac{1}{3}\sqrt{2-3x} + \ln|2-3x| + c$$

c $f(x) = 3 \cdot 10^{3x-2}$ geeft $F(x) = 3 \cdot \frac{1}{3} \cdot \frac{10^{3x-2}}{\ln(10)} + c = \frac{10^{3x-2}}{\ln(10)} + c$

d $f(x) = 2 - 3 \sin(\pi x - 2)$ geeft $F(x) = 2x + 3 \cdot \frac{1}{\pi} \cos(\pi x - 2) + c = 2x + \frac{3}{\pi} \cos(\pi x - 2) + c$

C5

a $f(x) = g(x)$ geeft $2\sqrt{x} = x$
 $4x = x^2$
 $x = 0 \vee x = 4$

$$O(V) = \int_0^4 (f(x) - g(x)) dx = \int_0^4 (2\sqrt{x} - x) dx = \int_0^4 (2x^{\frac{1}{2}} - x) dx = \left[2 \cdot \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} - \frac{1}{2} x^2 \right]_0^4 = \left[\frac{4}{3} x\sqrt{x} - \frac{1}{2} x^2 \right]_0^4$$

$$= \frac{4}{3} \cdot 4 \cdot 2 - \frac{1}{2} \cdot 4^2 - 0 = \frac{32}{3} - 8 - 0 = 2\frac{2}{3}$$

b $\int_0^p (f(x) - g(x)) dx = 1\frac{1}{3}$

$$\left[\frac{4}{3} x\sqrt{x} - \frac{1}{2} x^2 \right]_0^p = 1\frac{1}{3}$$

$$\frac{4}{3} p\sqrt{p} - \frac{1}{2} p^2 - 0 = 1\frac{1}{3}$$

$$\frac{4}{3} p\sqrt{p} - \frac{1}{2} p^2 = 1\frac{1}{3}$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 1,51$.

11.3 Inhouden**Bladzijde 118**

- 47** Het groene lichaam is te benaderen door de cilinder met straal $f(x)$ en hoogte h .
 Dus de inhoud is ongeveer $\pi \cdot (f(x))^2 \cdot h$.

Bladzijde 119

48 $f(x) = 2x$ geeft $3 - \sqrt{x} = 2x$
 $\sqrt{x} = 3 - 2x$
 $x = (3 - 2x)^2$

$$x = 9 - 12x + 4x^2$$

$$4x^2 - 13x + 9 = 0$$

$$D = (-13)^2 - 4 \cdot 4 \cdot 9 = 25, \text{ dus } \sqrt{D} = 5$$

$$x = \frac{13 \pm 5}{8} = 2\frac{1}{4} \vee x = \frac{13 - 5}{8} = 1$$

$x = 2\frac{1}{4}$ voldoet niet, dus $x = 1$.

$f(x) = 0$ geeft $3 - \sqrt{x} = 0$

$$\sqrt{x} = 3$$

$$x = 9$$

$$I(L) = \pi \int_0^1 (2x)^2 dx + \pi \int_1^9 (3 - \sqrt{x})^2 dx = \pi \int_0^1 4x^2 dx + \pi \int_1^9 (9 - 6x^{\frac{1}{2}} + x) dx$$

$$= \pi \left[\frac{4}{3} x^3 \right]_0^1 + \pi \left[9x - \frac{6}{\frac{1}{2}} x^{\frac{1}{2}} + \frac{1}{2} x^2 \right]_1^9 = \pi \left[\frac{4}{3} x^3 \right]_0^1 + \pi \left[9x - 4x\sqrt{x} + \frac{1}{2} x^2 \right]_1^9$$

$$= \frac{4}{3}\pi - 0 + \pi(9 \cdot 9 - 4 \cdot 9 \cdot 3 + \frac{1}{2} \cdot 9^2 - (9 \cdot 1 - 4 \cdot 1 \cdot 1 + \frac{1}{2} \cdot 1^2)) = 1\frac{1}{3}\pi + 8\pi = 9\frac{1}{3}\pi$$

49 a $I(K) = \pi \int_0^1 (e^x)^2 dx = \pi \int_0^1 e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_0^1 = \pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)$

$$I(L_2) = \pi \int_1^2 (e^x)^2 dx = \pi \int_1^2 e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_1^2 = \pi \left(\frac{1}{2} e^4 - \frac{1}{2} e^2 \right)$$

$$\frac{I(L_2)}{I(K)} = \frac{\pi \left(\frac{1}{2} e^4 - \frac{1}{2} e^2 \right)}{\pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)} = \frac{\frac{1}{2} \pi e^2 (e^2 - 1)}{\frac{1}{2} \pi (e^2 - 1)} = e^2$$

Dus $I(L_2)$ is e^2 keer zo groot als $I(K)$.

b $I(L_p) = \pi \int_1^p (e^x)^2 dx = \pi \int_1^p e^{2x} dx = \pi \left[\frac{1}{2} e^{2x} \right]_1^p = \pi \left(\frac{1}{2} e^{2p} - \frac{1}{2} e^2 \right)$

$$\frac{I(L_p)}{I(K)} = \frac{\pi \left(\frac{1}{2} e^{2p} - \frac{1}{2} e^2 \right)}{\pi \left(\frac{1}{2} e^2 - \frac{1}{2} \right)} = \frac{\frac{1}{2} \pi (e^{2p} - e^2)}{\frac{1}{2} \pi (e^2 - 1)} = \frac{e^{2p} - e^2}{e^2 - 1}$$

$$\begin{aligned} \frac{I(L_p)}{I(K)} &= 10 \text{ geeft } \frac{e^{2p} - e^2}{e^2 - 1} = 10 \\ e^{2p} - e^2 &= 10e^2 - 10 \\ e^{2p} &= 11e^2 - 10 \\ 2p &= \ln(11e^2 - 10) \\ p &= \frac{1}{2} \ln(11e^2 - 10) = 2,1333... \end{aligned}$$

Dus $p \approx 2,133$.

50 a $g(x) = \frac{3}{x}$

b $I(M) = \pi \int_1^3 \left(\frac{3}{x} \right)^2 dx = \pi \int_1^3 9x^{-2} dx = \pi \left[-9x^{-1} \right]_1^3 = \pi \left[-\frac{9}{x} \right]_1^3 = \pi \left(-\frac{9}{3} + \frac{9}{1} \right) = 6\pi$

Het lichaam L dat ontstaat als V wentelt om de lijn $y = 2$ is even groot als het lichaam M dat ontstaat als W wentelt om de x -as, omdat V en de lijn $y = 2$ beide 2 omlaag zijn geschoven.

51 a $O(V) = \int_1^5 (f(x) - 3) dx = \int_1^5 \left(3 + \frac{4}{2x-1} - 3 \right) dx = \int_1^5 \frac{4}{2x-1} dx = \left[4 \cdot \frac{1}{2} \ln|2x-1| \right]_1^5 = [2 \ln|2x-1|]_1^5$
 $= 2 \ln(9) - 2 \ln(1) = 2 \ln(9) - 0 = 2 \ln(9)$

b Stel U is het vlakdeel dat ontstaat als V over een afstand 3 omlaag wordt geschoven.

Dan wordt U ingesloten door de grafiek van de functie $g(x) = f(x) - 3 = \frac{4}{2x-1}$ en de lijnen $x = 1$ en $x = 5$.

$$\begin{aligned} I(L) &= \pi \int_1^5 \left(\frac{4}{2x-1} \right)^2 dx = \pi \int_1^5 \frac{16}{(2x-1)^2} dx = \pi \int_1^5 16(2x-1)^{-2} dx = \pi \left[16 \cdot \frac{1}{2} \cdot (2x-1)^{-1} \right]_1^5 = \pi \left[-\frac{8}{2x-1} \right]_1^5 \\ &= \pi \left(-\frac{8}{9} + 8 \right) = 7\frac{1}{9}\pi \end{aligned}$$

52 $I(L_1) + I(L_2) = \pi \int_2^8 (\sqrt{x-2})^2 dx = \pi \int_2^8 (x-2) dx = \pi \left[\frac{1}{2} x^2 - 2x \right]_2^8 = \pi \left(\frac{1}{2} \cdot 8^2 - 2 \cdot 8 - \left(\frac{1}{2} \cdot 2^2 - 2 \cdot 2 \right) \right) = 18\pi$

$$I(L_1) = \frac{1}{2} \cdot 18\pi \text{ geeft } \pi \int_2^a (\sqrt{x-2})^2 dx = 9\pi$$

$$\pi \int_2^a (x-2) dx = 9\pi$$

$$\int_2^a (x-2) dx = 9$$

$$\left[\frac{1}{2} x^2 - 2x \right]_2^a = 9$$

$$\frac{1}{2} a^2 - 2a - (2 - 4) = 9$$

$$\frac{1}{2} a^2 - 2a - 7 = 0$$

$$a^2 - 4a - 14 = 0$$

$$D = (-4)^2 - 4 \cdot 1 \cdot (-14) = 72, \text{ dus } \sqrt{D} = 6\sqrt{2}$$

$$a = \frac{4 + 6\sqrt{2}}{2} = 2 + 3\sqrt{2} \vee a = \frac{4 - 6\sqrt{2}}{2} = 2 - 3\sqrt{2}$$

$a = 2 - 3\sqrt{2}$ voldoet niet, dus $a = 2 + 3\sqrt{2}$.

$$53 \quad a \quad O(V) = \int_0^p 3^{\frac{1}{2}x+1} dx = \left[2 \cdot \frac{3^{\frac{1}{2}x+1}}{\ln(3)} \right]_0^p = \left[\frac{2 \cdot 3^{\frac{1}{2}p+1}}{\ln(3)} \right]_0^p = \frac{2 \cdot 3^{\frac{1}{2}p+1}}{\ln(3)} - \frac{6}{\ln(3)} = \frac{2 \cdot 3^{\frac{1}{2}p+1} - 6}{\ln(3)}$$

$$O(V) = \frac{12}{\ln(3)} \text{ geeft } 2 \cdot 3^{\frac{1}{2}p+1} - 6 = 12$$

$$2 \cdot 3^{\frac{1}{2}p+1} = 18$$

$$3^{\frac{1}{2}p+1} = 9$$

$$3^{\frac{1}{2}p+1} = 3^2$$

$$\frac{1}{2}p + 1 = 2$$

$$\frac{1}{2}p = 1$$

$$p = 2$$

$$b \quad I(L_p) = \pi \int_0^p (3^{\frac{1}{2}x+1})^2 dx = \pi \int_0^p 3^{x+2} dx = \pi \left[\frac{3^{x+2}}{\ln(3)} \right]_0^p = \pi \left(\frac{3^{p+2}}{\ln(3)} - \frac{9}{\ln(3)} \right) = \pi \left(\frac{3^{p+2} - 9}{\ln(3)} \right)$$

$$I(L_p) = \frac{18\pi}{\ln(3)} \text{ geeft } 3^{p+2} - 9 = 18$$

$$3^{p+2} = 27$$

$$3^{p+2} = 3^3$$

$$p + 2 = 3$$

$$p = 1$$

$$54 \quad a \quad I(L) = 8\frac{2}{3}\pi - 2\frac{1}{6}\pi = 6\frac{1}{2}\pi$$

$$b \quad \pi \int_1^3 (f(x) - g(x))^2 dx = \pi \int_1^3 (x - \frac{1}{2}x)^2 dx = \pi \int_1^3 (\frac{1}{2}x)^2 dx = \pi \int_1^3 \frac{1}{4}x^2 dx = \pi \left[\frac{1}{12}x^3 \right]_1^3 = \pi \left(\frac{1}{12} \cdot 3^3 - \frac{1}{12} \cdot 1^3 \right) = 2\frac{1}{6}\pi$$

$$\pi \int_1^3 ((f(x))^2 - (g(x))^2) dx = \pi \int_1^3 (x^2 - (\frac{1}{2}x)^2) dx = \pi \int_1^3 (x^2 - \frac{1}{4}x^2) dx = \pi \int_1^3 \frac{3}{4}x^2 dx = \pi \left[\frac{1}{4}x^3 \right]_1^3 = \pi \left(\frac{1}{4} \cdot 3^3 - \frac{1}{4} \cdot 1^3 \right) = 6\frac{1}{2}\pi$$

Dus Irma heeft gelijk.

55 Je kunt $I(L)$ niet berekenen met $\pi \int_a^b ((f(x))^2 - (g(x))^2) dx$ of $\pi \int_a^b ((g(x))^2 - (f(x))^2) dx$, want dan krijg je de inhoud van een uitgehold omwentelingslichaam en daar is bij L geen sprake van. In dit geval bereken je $I(L)$ met $\pi \int_a^b (f(x))^2 dx$.

$$56 \quad f(x) = g(x) \text{ geeft } \sqrt{x} = \frac{1}{2}x$$

$$x = \frac{1}{4}x^2$$

$$x = 0 \vee \frac{1}{4}x = 1$$

$$x = 0 \vee x = 4$$

$$I(L) = \pi \int_0^4 ((f(x))^2 - (g(x))^2) dx = \pi \int_0^4 (x - \frac{1}{4}x^2) dx = \pi \left[\frac{1}{2}x^2 - \frac{1}{12}x^3 \right]_0^4 = \pi \left(\frac{1}{2} \cdot 4^2 - \frac{1}{12} \cdot 4^3 - 0 \right) = 2\frac{2}{3}\pi$$

$$57 \quad f(x) = g(x) \text{ geeft } -x = x^2 - 4x$$

$$x^2 = 3x$$

$$x = 0 \vee x = 3$$

$$I(L) = \pi \int_0^3 ((g(x))^2 - (f(x))^2) dx = \pi \int_0^3 ((x^2 - 4x)^2 - x^2) dx = \pi \int_0^3 (x^4 - 8x^3 + 16x^2 - x^2) dx = \pi \int_0^3 (x^4 - 8x^3 + 15x^2) dx = \pi \left[\frac{1}{5}x^5 - 2x^4 + 5x^3 \right]_0^3 = \pi \left(\frac{1}{5} \cdot 3^5 - 2 \cdot 3^4 + 5 \cdot 3^3 - 0 \right) = 21\frac{3}{5}\pi$$

58 $f(x) = g(x)$ geeft $e^{2x} = 15 - 2e^x$
 $(e^x)^2 = 15 - 2e^x$
 Stel $e^x = u$.
 $u^2 = 15 - 2u$
 $u^2 + 2u - 15 = 0$
 $(u - 3)(u + 5) = 0$
 $u = 3 \vee u = -5$
 $e^x = 3 \vee e^x = -5$
 $x = \ln(3)$

$$\begin{aligned} I(L) &= \pi \int_0^{\ln(3)} ((g(x))^2 - (f(x))^2) dx = \pi \int_0^{\ln(3)} ((15 - 2e^x)^2 - (e^{2x})^2) dx = \pi \int_0^{\ln(3)} (225 - 60e^x + 4e^{2x} - e^{4x}) dx \\ &= \pi \left[225x - 60e^x + 2e^{2x} - \frac{1}{4}e^{4x} \right]_0^{\ln(3)} = \pi \left(225 \ln(3) - 60 \cdot 3 + 2 \cdot 3^2 - \frac{1}{4} \cdot 3^4 - (0 - 60 + 2 - \frac{1}{4}) \right) \\ &= \pi (225 \ln(3) - 180 + 2 \cdot 9 - \frac{1}{4} \cdot 81 + 58\frac{1}{4}) = \pi (225 \ln(3) - 124) \end{aligned}$$

59 a $f(x) = 3$ geeft $\frac{1}{2}x + \frac{5}{2x} = 3$
 $x^2 + 5 = 6x$
 $x^2 - 6x + 5 = 0$
 $(x - 1)(x - 5) = 0$
 $x = 1 \vee x = 5$

$$\begin{aligned} O(V) &= \int_1^5 (3 - f(x)) dx = \int_1^5 \left(3 - \frac{1}{2}x - \frac{5}{2x} \right) dx = \left[3x - \frac{1}{4}x^2 - 2\frac{1}{2}\ln|x| \right]_1^5 \\ &= 3 \cdot 5 - \frac{1}{4} \cdot 5^2 - 2\frac{1}{2}\ln(5) - \left(3 \cdot 1 - \frac{1}{4} \cdot 1^2 - 2\frac{1}{2}\ln(1) \right) = 6 - 2\frac{1}{2}\ln(5) \end{aligned}$$

b $I(L) = \pi \int_1^5 (3^2 - (f(x))^2) dx = \pi \int_1^5 \left(9 - \left(\frac{1}{2}x + \frac{5}{2x} \right)^2 \right) dx = \pi \int_1^5 \left(9 - \left(\frac{1}{2}x + \frac{5}{2x^{-1}} \right)^2 \right) dx =$
 $= \pi \int_1^5 \left(9 - \frac{1}{4}x^2 - \frac{5}{2} - \frac{25}{4}x^{-2} \right) dx = \pi \int_1^5 \left(6\frac{1}{2} - \frac{1}{4}x^2 - \frac{25}{4}x^{-2} \right) dx = \pi \left[6\frac{1}{2}x - \frac{1}{12}x^3 + \frac{25}{4}x^{-1} \right]_1^5$
 $= \pi \left[6\frac{1}{2}x - \frac{1}{12}x^3 + \frac{25}{4x} \right]_1^5 = \pi \left(6\frac{1}{2} \cdot 5 - \frac{1}{12} \cdot 5^3 + \frac{25}{4 \cdot 5} - \left(6\frac{1}{2} \cdot 1 - \frac{1}{12} \cdot 1^3 + \frac{25}{4 \cdot 1} \right) \right) = 10\frac{2}{3}\pi$

60 a $f(x) = 10$ geeft $\frac{10}{x\sqrt{x}} = 10$
 $x\sqrt{x} = 1$
 $x = 1$

$$\begin{aligned} O(V) &= \int_1^4 (10 - f(x)) dx = \int_1^4 \left(10 - \frac{10}{x\sqrt{x}} \right) dx = \int_1^4 (10 - 10x^{-1\frac{1}{2}}) dx = \left[10x - 10 \cdot \frac{1}{-\frac{1}{2}}x^{-\frac{1}{2}} \right]_1^4 \\ &= \left[10x + \frac{20}{\sqrt{x}} \right]_1^4 = 10 \cdot 4 + \frac{20}{2} - \left(10 \cdot 1 + \frac{20}{1} \right) = 20 \end{aligned}$$

b $I(L) = \pi \int_1^4 (10^2 - (f(x))^2) dx = \pi \int_1^4 \left(100 - \frac{100}{x^3} \right) dx = \pi \int_1^4 (100 - 100x^{-3}) dx$
 $= \pi \left[100x + 50x^{-2} \right]_1^4 = \pi \left[100x + \frac{50}{x^2} \right]_1^4 = \pi \left(100 \cdot 4 + \frac{50}{4^2} - \left(100 \cdot 1 + \frac{50}{1^2} \right) \right) = 253\frac{1}{8}\pi$

61 Voor $x < 0$ geldt $f(x) = -\frac{1}{4}x^2 - 2x + \frac{1}{2}x = -\frac{1}{4}x^2 - \frac{1}{2}x$.

$$x_A = \frac{\frac{1}{2}}{-\frac{1}{2}} = -3$$

$$f(-3) = 2\frac{1}{4}, \text{ dus } A(-3, 2\frac{1}{4}).$$

$$\text{Voor } x \geq 0 \text{ geldt } f(x) = -\frac{1}{4}x^2 + 2x + \frac{1}{2}x = -\frac{1}{4}x^2 + 2\frac{1}{2}x.$$

$$-\frac{1}{4}x^2 + 2\frac{1}{2}x = 2\frac{1}{4}$$

$$-\frac{1}{4}x^2 + 2\frac{1}{2}x - 2\frac{1}{4} = 0$$

$$x^2 - 10x + 9 = 0$$

$$(x-1)(x-9) = 0$$

$$x = 1 \vee x = 9$$

$$\begin{aligned} I(L) &= \pi \int_1^9 ((f(x))^2 - (2\frac{1}{4})^2) dx = \pi \int_1^9 ((-\frac{1}{4}x^2 + 2\frac{1}{2}x)^2 - 5\frac{1}{16}) dx = \pi \int_1^9 (\frac{1}{16}x^4 - 1\frac{1}{4}x^3 + 6\frac{1}{4}x^2 - 5\frac{1}{16}) dx \\ &= \pi [\frac{1}{80}x^5 - \frac{5}{16}x^4 + 2\frac{1}{12}x^3 - 5\frac{1}{16}x]_1^9 \\ &= \pi (\frac{1}{80} \cdot 9^5 - \frac{5}{16} \cdot 9^4 + 2\frac{1}{12} \cdot 9^3 - 5\frac{1}{16} \cdot 9 - (\frac{1}{80} \cdot 1^5 - \frac{5}{16} \cdot 1^4 + 2\frac{1}{12} \cdot 1^3 - 5\frac{1}{16} \cdot 1)) = 164\frac{4}{15}\pi \end{aligned}$$

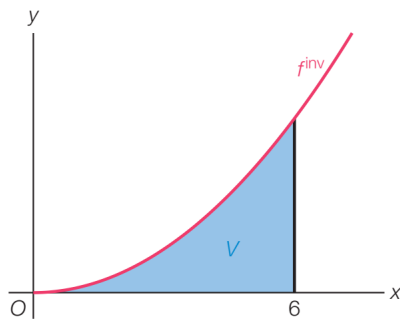
Bladzijde 124

62 a Voor f geldt $y = 3\sqrt{x}$, dus voor f^{inv} geldt $x = 3\sqrt{y}$

$$x^2 = 9y$$

$$y = \frac{1}{9}x^2$$

Dus $f^{\text{inv}}(x) = \frac{1}{9}x^2$ met $x \geq 0$.



$$I(L) = \pi \int_a^b (f^{\text{inv}}(x))^2 dy = \pi \int_0^6 \frac{1}{81}x^4 dx = \pi [\frac{1}{405}x^5]_0^6 = \pi (\frac{1}{405} \cdot 6^5 - 0) = 19\frac{1}{5}\pi$$

De gebruikte grenzen zijn 0 en 6.

b Voor f geldt $y = 3\sqrt{x}$

$$y^2 = 9x$$

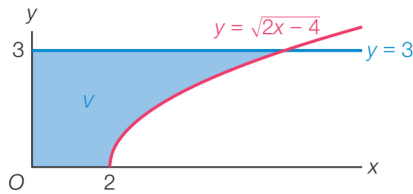
$$x = \frac{1}{9}y^2$$

$$x^2 = \frac{1}{81}y^4$$

$$\mathbf{c} \quad I(L) = \pi \int_0^6 x^2 dy = \pi \int_0^6 \frac{1}{81}y^4 dy = \pi [\frac{1}{405}y^5]_0^6 = \pi (\frac{1}{405} \cdot 6^5 - 0) = 19\frac{1}{5}\pi$$

Bladzijde 125

$$\mathbf{63} \quad I(L) = I(\text{cilinder}) - \pi \int_0^2 x^2 dy = \pi \cdot 4^2 \cdot 2 - \pi \int_0^2 y^4 dy = 32\pi - \pi [\frac{1}{5}y^5]_0^2 = 32\pi - \pi (\frac{32}{5} - 0) = 25\frac{3}{5}\pi$$



$$y = \sqrt{2x-4} \text{ geeft } y^2 = 2x-4$$

$$2x = y^2 + 4$$

$$x = \frac{1}{2}y^2 + 2$$

$$x^2 = \left(\frac{1}{2}y^2 + 2\right)^2$$

$$x^2 = \frac{1}{4}y^4 + 2y^2 + 4$$

$$\begin{aligned} I(L) &= \pi \int_0^3 x^2 dy = \pi \int_0^3 \left(\frac{1}{4}y^4 + 2y^2 + 4\right) dy = \pi \left[\frac{1}{20}y^5 + \frac{2}{3}y^3 + 4y\right]_0^3 = \pi \left(\frac{1}{20} \cdot 3^5 + \frac{2}{3} \cdot 3^3 + 4 \cdot 3 - 0\right) \\ &= 42\frac{3}{20}\pi \end{aligned}$$

b $y = 3$ geeft $\sqrt{2x-4} = 3$

$$2x - 4 = 9$$

$$2x = 13$$

$$x = 6\frac{1}{2}$$

$$\begin{aligned} I(M) &= \pi \int_0^{6\frac{1}{2}} 3^2 dx - \pi \int_2^{6\frac{1}{2}} (\sqrt{2x-4})^2 dx = \pi \int_0^{6\frac{1}{2}} 9 dx - \pi \int_2^{6\frac{1}{2}} (2x-4) dx = \pi [9x]_0^{6\frac{1}{2}} - \pi [x^2 - 4x]_2^{6\frac{1}{2}} \\ &= \pi(9 \cdot 6\frac{1}{2} - 0) - \pi((6\frac{1}{2})^2 - 4 \cdot 6\frac{1}{2} - (2^2 - 4 \cdot 2)) = 58\frac{1}{2}\pi - 20\frac{1}{4}\pi = 38\frac{1}{4}\pi \end{aligned}$$

65 a $I(L) = \pi \int_0^4 x^2 dy = \pi \int_0^4 y dy = \pi \left[\frac{1}{2}y^2\right]_0^4 = \pi\left(\frac{1}{2} \cdot 4^2 - 0\right) = 8\pi$

b $P(p, q)$ ligt op de parabool, dus $q = p^2$.

$$I(L_p) = I(M_q) \text{ geeft } \pi \int_0^p y^2 dx = \pi \int_0^q x^2 dy$$

$$\int_0^p x^4 dx = \int_0^{p^2} y dy$$

$$\left[\frac{1}{5}x^5\right]_0^p = \left[\frac{1}{2}y^2\right]_0^{p^2}$$

$$\frac{1}{5}p^5 - 0 = \frac{1}{2}p^4 - 0$$

$$2p^5 = 5p^4$$

$$p^4 = 0 \vee 2p = 5$$

$$p = 0 \vee p = 2\frac{1}{2}$$

$p = 0$ voldoet niet, dus $p = 2\frac{1}{2}$ en $q = (2\frac{1}{2})^2 = 6\frac{1}{4}$.

Bladzijde 126

66 Voor f geldt $y = x^2$.

$$I(K) = \pi \int_0^a x^2 dy = \pi \int_0^a y dy = \pi \left[\frac{1}{2}y^2\right]_0^a = \pi\left(\frac{1}{2}a^2 - 0\right) = \frac{1}{2}\pi a^2$$

$$I(L) = \pi \int_a^b x^2 dy = \pi \int_a^b y dy = \pi \left[\frac{1}{2}y^2\right]_a^b = \pi\left(\frac{1}{2}b^2 - \frac{1}{2}a^2\right)$$

$$I(K) = I(L) \text{ geeft } \frac{1}{2}\pi a^2 = \pi\left(\frac{1}{2}b^2 - \frac{1}{2}a^2\right)$$

$$\frac{1}{2}a^2 = \frac{1}{2}b^2 - \frac{1}{2}a^2$$

$$\frac{1}{2}b^2 = a^2$$

$$b^2 = 2a^2$$

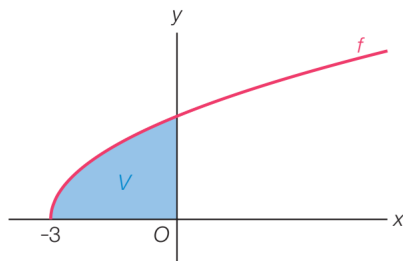
$$b = \sqrt{2} \cdot a$$

Dus $k = \sqrt{2}$.

67 a $f(x) = 0$ geeft $2x + 6 = 0$

$$2x = -6$$

$$x = -3$$



$$I(L) = \pi \int_{-3}^0 (\sqrt{2x+6})^2 dx = \pi \int_{-3}^0 (2x+6) dx = \pi [x^2 + 6x]_{-3}^0 = \pi(0 - ((-3)^2 + 6 \cdot (-3))) = 9\pi$$

b $f(0) = \sqrt{6}$

$$y = \sqrt{2x+6} \text{ geeft } y^2 = 2x+6$$

$$2x = y^2 - 6$$

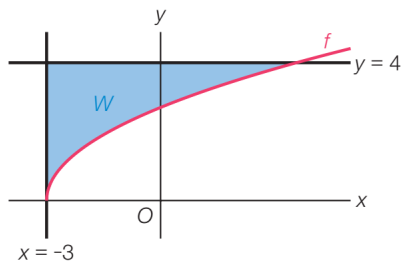
$$x = \frac{1}{2}y^2 - 3$$

$$x^2 = (\frac{1}{2}y^2 - 3)^2$$

$$x^2 = \frac{1}{4}y^4 - 3y^2 + 9$$

$$\begin{aligned} I(M) &= \pi \int_0^{\sqrt{6}} x^2 dy = \pi \int_0^{\sqrt{6}} (\frac{1}{4}y^4 - 3y^2 + 9) dy = \pi [\frac{1}{20}y^5 - y^3 + 9y]_0^{\sqrt{6}} \\ &= \pi(\frac{1}{20} \cdot (\sqrt{6})^5 - (\sqrt{6})^3 + 9\sqrt{6} - 0) = \pi(\frac{1}{20} \cdot 36\sqrt{6} - 6\sqrt{6} + 9\sqrt{6}) = 4\frac{4}{5}\pi\sqrt{6} \end{aligned}$$

c



Stel U is het vlakdeel dat ontstaat als W over een afstand 3 naar rechts wordt geschoven.

Dan wordt U ingesloten door de grafiek van de functie $g(x) = \sqrt{2(x-3)} + 6 = \sqrt{2x}$, de y -as en de lijn $y = 4$.

$$y = \sqrt{2x} \text{ geeft } y^2 = 2x$$

$$x = \frac{1}{2}y^2$$

$$x^2 = \frac{1}{4}y^4$$

$$I(N) = \pi \int_0^4 x^2 dy = \pi \int_0^4 \frac{1}{4}y^4 dy = \pi [\frac{1}{20}y^5]_0^4 = \pi(\frac{1}{20} \cdot 4^5 - 0) = 51\frac{1}{5}\pi$$

68 $y = \ln(x)$ geeft $x = e^y$

$$x^2 = e^{2y}$$

$$I(L_1) = \pi \int_0^a x^2 dy = \pi \int_0^a e^{2y} dy = \pi [\frac{1}{2}e^{2y}]_0^a = \pi(\frac{1}{2}e^{2a} - \frac{1}{2})$$

$$I(L_1) = \frac{1}{2} \cdot \pi \int_0^3 x^2 dy \text{ geeft } \pi(\frac{1}{2}e^{2a} - \frac{1}{2}) = \frac{1}{2} \cdot \pi(\frac{1}{2}e^6 - \frac{1}{2})$$

$$e^{2a} - 1 = \frac{1}{2}e^6 - \frac{1}{2}$$

$$e^{2a} = \frac{1}{2}e^6 + \frac{1}{2}$$

$$2a = \ln(\frac{1}{2}e^6 + \frac{1}{2})$$

$$a = \frac{1}{2}\ln(\frac{1}{2}e^6 + \frac{1}{2})$$

$$a \approx 2,655$$

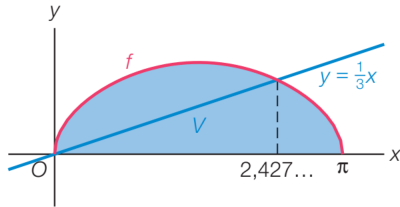
69 a $I(L) = \pi \int_{-4}^4 (x^2 + 4) dx = \pi \left[\frac{1}{3}x^3 + 4x \right]_{-4}^4 = \pi \left(\frac{1}{3} \cdot 4^3 + 4 \cdot 4 - \left(\frac{1}{3} \cdot (-4)^3 + 4 \cdot (-4) \right) \right) = 74\frac{2}{3}\pi$

b $O(V) = \int_{-4}^4 \sqrt{x^2 + 4} dx$ en van $\sqrt{x^2 + 4}$ is geen primitieve te vinden met de regels die tot nu toe zijn geweest.

Bladzijde 127

70 De GR geeft $I(L) = \pi \int_0^{0,852\dots} (2^2 - (xe^x)^2) dx \approx 8,26$.

71



Los op $\sqrt{\sin(x)} = \frac{1}{3}x$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $x = 0$ en $x = 2,427\dots$

De GR geeft $O(V) = \int_0^\pi \sqrt{\sin(x)} dx = 2,396\dots$ en $\int_0^{2,427\dots} (\sqrt{\sin(x)} - \frac{1}{3}x) dx = 1,019\dots$

1,019... is niet gelijk aan de helft van 2,396..., dus de lijn $y = \frac{1}{3}x$ verdeelt het vlakdeel V niet in twee delen met gelijke oppervlakte.

72 a $f(x) = 0$ geeft $2 \ln(x) - \ln^2(x) = 0$
 $\ln(x)(2 - \ln(x)) = 0$
 $\ln(x) = 0 \vee \ln(x) = 2$
 $x = 1 \vee x = e^2$

Dus de nulpunten zijn 1 en e^2 .

b $f(x) = 2 \ln(x) - \ln^2(x)$ geeft $f'(x) = \frac{2}{x} - 2 \ln(x) \cdot \frac{1}{x} = \frac{2 - 2 \ln(x)}{x}$
 $f'(x) = 0$ geeft $2 - 2 \ln(x) = 0$
 $\ln(x) = 1$
 $x = e$

$f(e) = 2 \cdot 1 - 1^2 = 1$, dus de top is $(e, 1)$.

c $F(x) = 4x \ln(x) - x \ln^2(x) - 4x$ geeft
 $F'(x) = 4 \cdot \ln(x) + 4x \cdot \frac{1}{x} - \left(1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} \right) - 4 = 4 \ln(x) + 4 - \ln^2(x) - 2 \ln(x) - 4$
 $= 2 \ln(x) - \ln^2(x)$

$F'(x) = f(x)$, dus F is een primitieve van f .

d $O(V) = \int_1^{e^2} (2 \ln(x) - \ln^2(x)) dx = [4x \ln(x) - x \ln^2(x) - 4x]_1^{e^2}$
 $= 4e^2 \cdot 2 - e^2 \cdot 2^2 - 4e^2 - (0 - 0 - 4) = 8e^2 - 4e^2 - 4e^2 + 4 = 4$

e De GR geeft $I(L) = \pi \int_1^{e^2} (2 \ln(x) - \ln^2(x))^2 dx \approx 9,778$.

Bladzijde 128

C6

a $f(x) = g(x)$ geeft $4\sqrt{2x} = 2x^2$

$$16 \cdot 2x = 4x^4$$

$$32x = 4x^4$$

$$x = 0 \vee 4x^3 = 32$$

$$x = 0 \vee x^3 = 8$$

$$x = 0 \vee x = 2$$

$$I(K) = \pi \int_0^2 ((f(x))^2 - (g(x))^2) dx = \pi \int_0^2 ((4\sqrt{2x})^2 - (2x^2)^2) dx = \pi \int_0^2 (32x - 4x^4) dx = \pi \left[16x^2 - \frac{4}{5}x^5 \right]_0^2$$

$$= \pi(16 \cdot 2^2 - \frac{4}{5} \cdot 2^5 - 0) = 38\frac{2}{5}\pi$$

b $y = 4\sqrt{2x}$ geeft $y^2 = 32x$

$$x = \frac{1}{32}y^2$$

$$x^2 = \frac{1}{1024}y^4$$

$$I(L) = \pi \int_0^8 x^2 dy = \pi \int_0^8 \frac{1}{1024}y^4 dy = \pi \left[\frac{1}{5120}y^5 \right]_0^8 = \pi \left(\frac{1}{5120} \cdot 8^5 - 0 \right) = 6\frac{2}{5}\pi$$

C7

Los op $0,1x^2 e^x = 5$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $x = 2,271\dots$

De GR geeft $O(V) = \int_0^{2,271\dots} (5 - 0,1x^2 e^x) dx \approx 9,02$.

11.4 Toepassingen van integralen

Bladzijde 129

73

$$I(\text{bol}) = 2 \cdot \pi \int_0^r y^2 dx = 2\pi \int_0^r (r^2 - x^2) dx = 2\pi \left[r^2x - \frac{1}{3}x^3 \right]_0^r = 2\pi(r^3 - \frac{1}{3}r^3 - 0) = 2\pi \cdot \frac{2}{3}r^3 = \frac{4}{3}\pi r^3$$

Bladzijde 130

74

$$a \quad I(T) = \pi \int_{\frac{1}{4}r}^{\frac{1}{2}r} y^2 dx = \pi \int_{\frac{1}{4}r}^{\frac{1}{2}r} (r^2 - x^2) dx = \pi \left[r^2x - \frac{1}{3}x^3 \right]_{\frac{1}{4}r}^{\frac{1}{2}r} = \pi(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot (\frac{1}{2}r)^3 - (r^2 \cdot \frac{1}{4}r - \frac{1}{3} \cdot (\frac{1}{4}r)^3))$$

$$= \pi(\frac{1}{2}r^3 - \frac{1}{24}r^3 - \frac{1}{4}r^3 + \frac{1}{192}r^3) = \frac{41}{192}\pi r^3$$

$$\frac{I(T)}{I(S)} = \frac{\frac{41}{192}\pi r^3}{\frac{29}{192}\pi r^3} = \frac{41}{29} = 1\frac{12}{29}$$

Dus de inhoud van bolschijf T is $1\frac{12}{29}$ keer zo groot als de inhoud van bolschijf S .

b $I(K_p) = 24\pi$ geeft $\frac{1}{3}\pi \cdot (\frac{2}{3} \cdot 6)^2 \cdot 6 - \frac{1}{3}\pi \cdot (\frac{2}{3}p)^2 \cdot p = 24\pi$

$$32 - \frac{4}{27}p^3 = 24$$

$$\frac{4}{27}p^3 = 8$$

$$p^3 = 54$$

$$p = \sqrt[3]{54}$$

Bladzijde 131

75

$$I(S) = \pi \int_{\frac{1}{3}r}^r y^2 dx = \pi \int_{\frac{1}{3}r}^r (r^2 - x^2) dx = \pi \left[r^2x - \frac{1}{3}x^3 \right]_{\frac{1}{3}r}^r = \pi(r^3 - \frac{1}{3}r^3 - (r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot (\frac{1}{3}r)^3))$$

$$= \pi(r^3 - \frac{1}{3}r^3 - \frac{1}{3}r^3 + \frac{1}{81}r^3) = \frac{28}{81}\pi r^3$$

76

$$I(S_p) = \pi \int_{-3}^p y^2 dx = \pi \int_{-3}^p (36 - x^2) dx = \pi \left[36x - \frac{1}{3}x^3 \right]_{-3}^p = \pi(36p - \frac{1}{3}p^3 - (36 \cdot -3 - \frac{1}{3} \cdot (-3)^3))$$

$$= \pi(36p - \frac{1}{3}p^3 + 99)$$

$$I(B) = \frac{4}{3}\pi \cdot 6^3 = 288\pi$$

$$I(S_p) = \frac{1}{2} \cdot I(B) \text{ geeft } \pi(36p - \frac{1}{3}p^3 + 99) = \frac{1}{2} \cdot 288\pi$$

$$36p - \frac{1}{3}p^3 + 99 = 144$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 1,27$.

77 a Lijn l door $O(0, 0)$ en A heeft $rc_l = \frac{2-0}{4-0} = \frac{1}{2}$.

$k \perp l$, dus $rc_k = -2$.

$$\left. \begin{array}{l} k: y = -2x + b \\ \text{door } A(4, 2) \end{array} \right\} \begin{array}{l} -2 \cdot 4 + b = 2 \\ -8 + b = 2 \\ b = 10 \end{array}$$

Dus $k: y = -2x + 10$.

$$\begin{aligned} I(K) &= \pi \int_0^4 ((-2x + 10)^2 - (20 - x^2)) dx = \pi \int_0^4 (4x^2 - 40x + 100 - 20 + x^2) dx = \pi \int_0^4 (5x^2 - 40x + 80) dx \\ &= \pi \left[\frac{5}{3}x^3 - 20x^2 + 80x \right]_0^4 = \pi \left(\frac{5}{3} \cdot 4^3 - 20 \cdot 4^2 + 80 \cdot 4 - 0 \right) = 106\frac{2}{3}\pi \end{aligned}$$

b m door B geeft $y = ap$.

B op c geeft $p^2 + y^2 = 20$.

Invullen van $y = ap$ in $p^2 + y^2 = 20$ geeft $p^2 + (ap)^2 = 20$

$$p^2 + a^2p^2 = 20$$

$$a^2p^2 = 20 - p^2$$

$$a^2 = \frac{20 - p^2}{p^2}$$

c $I(L) = \pi \int_0^p (20 - x^2 - (ax)^2) dx = \pi \int_0^p (20 - x^2 - a^2x^2) dx = \pi \left[20x - \frac{1}{3}x^3 - \frac{1}{3}a^2x^3 \right]_0^p$

$$= \pi \left(20p - \frac{1}{3}p^3 - \frac{1}{3}a^2p^3 - 0 \right) = \pi \left(20p - \frac{1}{3}p^3 - \frac{1}{3} \cdot \frac{20 - p^2}{p^2} \cdot p^3 \right)$$

$$= \pi \left(20p - \frac{1}{3}p^3 - 6\frac{2}{3}p + \frac{1}{3}p^3 \right) = 13\frac{1}{3}p\pi$$

$$I(L) = 40\pi \text{ geeft } 13\frac{1}{3}p\pi = 40\pi$$

$$p = 3$$

$$p = 3 \text{ invullen in } a^2 = \frac{20 - p^2}{p^2} \text{ geeft } a^2 = \frac{20 - 3^2}{3^2} = \frac{11}{9}, \text{ dus } a = \sqrt{\frac{11}{9}} = \frac{1}{3}\sqrt{11}.$$

78 a De gemiddelde snelheid gedurende de eerste zes seconden is $\frac{0 + 10}{2} = 5$ m/s.

Er wordt $6 \cdot 5 = 30$ meter afgelegd gedurende deze zes seconden.

b $O(\triangle OAB) = \frac{1}{2} \cdot 6 \cdot 10 = 30$ en dit is de afgelegde afstand gedurende de eerste zes seconden.

c $v(t) = at$ met $a = \frac{10 - 0}{6 - 0} = \frac{5}{3}$, dus $v(t) = \frac{5}{3}t$.

$$s(t) = \frac{1}{2} \cdot t \cdot v(t) = \frac{1}{2}t \cdot \frac{5}{3}t = \frac{5}{6}t^2$$

d $s(t) = \frac{5}{6}t^2$ geeft $s'(t) = \frac{5}{3}t$

$s'(t) = v(t)$, dus s is een primitieve van v .

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79 $v(t) = \frac{1}{2}t^2 + e^t - 1$ geeft $s(t) = \frac{1}{6}t^3 + e^t - t + c$

$$s(0) = 0 \text{ geeft } 0 + e^0 - 0 + c = 0$$

$$1 + c = 0$$

$$c = -1$$

$$\text{Dus } s(t) = \frac{1}{6}t^3 + e^t - t - 1.$$

80 $a(t) = mt$

$$m = \frac{a(20) - a(0)}{20 - 0} = \frac{5 - 0}{20} = \frac{1}{4}$$

$$a(t) = \frac{1}{4}t \text{ en } v(0) = 1 \text{ geeft } v(t) = \frac{1}{8}t^2 + 1$$

De afgelegde afstand gedurende deze 20 seconden is

$$\int_0^{20} \left(\frac{1}{8}t^2 + 1 \right) dt = \left[\frac{1}{24}t^3 + t \right]_0^{20} = \frac{1}{24} \cdot 20^3 + 20 - 0 = 353\frac{1}{3} \text{ meter.}$$

- 81 a** $a(t) = -t^2 + 6t$ en $v(0) = 0$ geeft $v(t) = -\frac{1}{3}t^3 + 3t^2$
 $v(6) = -\frac{1}{3} \cdot 6^3 + 3 \cdot 6^2 = 36$
 Dus de snelheid op $t = 6$ is 36 m/s.
- b** De afgelegde afstand gedurende deze 6 seconden is
 $\int_0^6 (-\frac{1}{3}t^3 + 3t^2) dt = [-\frac{1}{12}t^4 + t^3]_0^6 = -\frac{1}{12} \cdot 6^4 + 6^3 - 0 = 108$ meter.
- c** $s(6) = 108$ en $v(6) = 36$ geeft $s(10) = 108 + 4 \cdot 36 = 252$
 Dus op $t = 10$ is 252 meter afgelegd.
- d** $500 - 108 = 392$
 392 meter met een snelheid van 36 m/s duurt $\frac{392}{36} = 10\frac{8}{9}$ seconden.
 Dus op $t = 6 + 10\frac{8}{9} = 16\frac{8}{9}$ is in totaal 500 meter afgelegd.

Bladzijde 134

- 82 a** $54 \text{ km/u} = 15 \text{ m/s}$
 De constante versnelling is a .
 $v(t) = at + b$ en $v(0) = 15$ geeft $v(t) = at + 15$
 $v(t) = at + 15$ en $s(0) = 0$ geeft $s(t) = \frac{1}{2}at^2 + 15t$
 Noem de remtijd t_r , dan geldt $v(t_r) = 0$ en $s(t_r) = 0,75$.
 $v(t_r) = 0$ geeft $at_r + 15 = 0$
 $at_r = -15$
 $s(t_r) = 0,75$ geeft $\frac{1}{2}at_r^2 + 15t_r = 0,75$ oftewel $\frac{1}{2}at_r \cdot t_r + 15t_r = 0,75$.
 Invullen van $at_r = -15$ in $\frac{1}{2}at_r \cdot t_r + 15t_r = 0,75$ geeft $-7,5t_r + 15t_r = 0,75$
 $7,5t_r = 0,75$
 $t_r = 0,1$

Dus de botsing duurt 0,1 seconde.

Alternatieve uitwerking

Het rekenwerk wordt eenvoudiger als het probleem 'achterstevoren' wordt aangepakt.

Noem de remtijd t_r en neem $v(0) = 0$, $s(0) = 0$, $v(t_r) = 15$ en $s(t_r) = 0,75$.

Dit geeft $v(t) = at$ en $s(t) = \frac{1}{2}at^2$. Dan geldt

$$\left. \begin{array}{l} at_r = 15 \\ \frac{1}{2}at_r^2 = 0,75 \text{ oftewel } \frac{1}{2}at_r \cdot t_r = 0,75 \end{array} \right\} \begin{array}{l} 7,5t_r = 0,75 \\ t_r = 0,1 \end{array}$$

Dus de botsing duurt 0,1 seconde.

- b** $at_r = 15$
 $t_r = 0,1$ } $a \cdot 0,1 = -15$
 $a = -150$

Dus de versnelling tijdens de botsing is -150 m/s^2 .

Dit is 15 keer zo groot als de versnelling van de zwaartekracht.

- 83 a** $v(t) = -0,2t^2 + 4t$ en $s(0) = 0$ geeft $s(t) = -\frac{1}{15}t^3 + 2t^2$
 $s(10) = -\frac{1}{15} \cdot 10^3 + 2 \cdot 10^2 = 133\frac{1}{3}$
 $v(t) = 0,8t^2 - 24t + 180$ geeft $s(t) = \frac{4}{15}t^3 - 12t^2 + 180t + c$
 $s(10) = 133\frac{1}{3}$ geeft $\frac{4}{15} \cdot 10^3 - 12 \cdot 10^2 + 180 \cdot 10 + c = 133\frac{1}{3}$
 $866\frac{2}{3} + c = 133\frac{1}{3}$
 $c = -733\frac{1}{3}$
 Dus $s(t) = \frac{4}{15}t^3 - 12t^2 + 180t - 733\frac{1}{3}$.
 $s(15) = \frac{4}{15} \cdot 15^3 - 12 \cdot 15^2 + 180 \cdot 15 - 733\frac{1}{3} = 166\frac{2}{3}$
 Dus de totale afstand die de auto gedurende deze 15 seconden aflegt is $166\frac{2}{3}$ meter.

b gemiddelde snelheid $= \frac{166\frac{2}{3}}{15} = 11\frac{1}{9}$

Los op $-0,2t^2 + 4t = 11\frac{1}{9}$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 3,333\dots$

Los op $0,8t^2 - 24t + 180 = 11\frac{1}{9}$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 11,273\dots$

Dus op $t \approx 3,33$ en $t \approx 11,27$ is de snelheid gelijk aan de gemiddelde snelheid.

c Los op $-\frac{1}{15}t^3 + 2t^2 = 100$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 8,317\dots$

Dus na ongeveer 8,3 seconden heeft de auto 100 meter afgelegd.

84 a $a(t) = -4e^{-0,1t}$ geeft $v(t) = 40e^{-0,1t} + c$

$v(3) = 32$ geeft $40e^{-0,3} + c = 32$

$29,632\dots + c = 32$

$c = 2,367\dots$

Dus $v(t) = 40e^{-0,1t} + 2,367\dots$

$v(0) = 40 + 2,367\dots = 42,367\dots$

Dus de gevraagde snelheid is ongeveer 42,4 m/s.

b $v(t) = 40e^{-0,1t} + 2,367\dots$ geeft $s(t) = -400e^{-0,1t} + 2,367\dots \cdot t + c$

$s(0) = 0$ geeft $-400 + 0 + c = 0$

$c = 400$

Dus $s(t) = -400e^{-0,1t} + 2,367\dots \cdot t + 400$.

2 minuten = 120 seconden

$s(120) = -400 \cdot e^{-12} + 2,367\dots \cdot 120 + 400 \approx 684$

Dus in de eerste twee minuten na $t = 0$ heeft ze 684 meter afgelegd.

Bladzijde 135

C8 a $I(S_p) = \pi \int_{-2p}^p y^2 dx = \pi \int_{-2p}^p (16 - x^2) dx = \pi \left[16x - \frac{1}{3}x^3 \right]_{-2p}^p = \pi \left(16p - \frac{1}{3}p^3 - (16 \cdot -2p - \frac{1}{3} \cdot (-2p)^3) \right)$

$= \pi \left(16p - \frac{1}{3}p^3 + 32p - 2\frac{2}{3}p^3 \right) = \pi(48p - 3p^3)$

$I(S_p) = 45\pi$ geeft $\pi(48p - 3p^3) = 45\pi$

$48p - 3p^3 = 45$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p = 1$.

b De lijn $y = -x + 4$ snijdt c in de punten $(0, 4)$ en $(4, 0)$.

$I(L) = \pi \int_0^4 (16 - x^2 - (-x + 4)^2) dx = \pi \int_0^4 (16 - x^2 - x^2 + 8x - 16) dx = \pi \int_0^4 (-2x^2 + 8x) dx$

$= \pi \left[-\frac{2}{3}x^3 + 4x^2 \right]_0^4 = \pi \left(-\frac{2}{3} \cdot 4^3 + 4 \cdot 4^2 - 0 \right) = 21\frac{1}{3}\pi$

C9 a $a(t) = 4e^{0,1t} - 4$ geeft $v(t) = 40e^{0,1t} - 4t + c$

$v(0) = 0$ geeft $40 + c = 0$

$c = -40$

Dus $v(t) = 40e^{0,1t} - 4t - 40$.

$v(8) = 40e^{0,8} - 4 \cdot 8 - 40 = 17,021\dots$

Dus de snelheid op $t = 8$ is ongeveer 17,02 m/s.

b De afgelegde afstand gedurende deze 10 seconden is

$\int_0^{10} (40e^{0,1t} - 4t - 40) dt = [400e^{0,1t} - 2t^2 - 40t]_0^{10} = 400e - 2 \cdot 10^2 - 40 \cdot 10 - (400 - 0 - 0) \approx 87$ meter.

Bladzijde 136

- 1 a** $F(x) = x^3 e^x$ geeft $F'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = (x^3 + 3x^2) e^x$
 $F'(x) = f(x)$, dus F is een primitieve van f .
- b** $G(x) = \frac{1}{2}x^2 - x + 2 \ln(x+1)$ geeft $G'(x) = x - 1 + 2 \cdot \frac{1}{x+1} = \frac{(x-1)(x+1)}{x+1} + \frac{2}{x+1} = \frac{x^2+1}{x+1}$
 $G'(x) = g(x)$ dus G is een primitieve van g .
- 2 a** $f(x) = 10x\sqrt{x} = 10x^{1\frac{1}{2}}$ geeft $F(x) = 10 \cdot \frac{1}{2\frac{1}{2}} x^{2\frac{1}{2}} + c = 4x^2 \cdot \sqrt{x} + c$
- b** $f(x) = 2 \cdot 3^x$ geeft $F(x) = 2 \cdot \frac{3^x}{\ln(3)} + c = \frac{2 \cdot 3^x}{\ln(3)} + c$
- c** $f(x) = \frac{x+1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ geeft $F(x) = \frac{1}{\frac{1}{2}} x^{\frac{3}{2}} + \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + c = \frac{2}{3} x\sqrt{x} + 2\sqrt{x} + c$
- d** $f(x) = 6e^x + x^2$ geeft $F(x) = 6e^x + \frac{1}{3}x^3 + c$
- e** $f(x) = 3x - \frac{1}{3x} = 3x - \frac{1}{3} \cdot \frac{1}{x}$ geeft $F(x) = \frac{3}{2}x^2 - \frac{1}{3} \ln|x| + c$
- f** $f(x) = 3 \sin(x) + 2 \cos(x)$ geeft $F(x) = -3 \cos(x) + 2 \sin(x) + c$
- 3** $f(x) = 6x - 3\sqrt{x} = 6x - 3x^{\frac{1}{2}}$ geeft $F(x) = 3x^2 - 3 \cdot \frac{1}{\frac{1}{2}} x^{\frac{3}{2}} + c = 3x^2 - 2x\sqrt{x} + c$
 $F(1) = 5$ geeft $3 - 2 + c = 5$
 $c = 4$
Dus $F(x) = 3x^2 - 2x\sqrt{x} + 4$.
- 4 a** $O(V) = \int_1^2 \frac{2}{x^2} dx = \int_1^2 2x^{-2} dx = [-2x^{-1}]_1^2 = \left[-\frac{2}{x}\right]_1^2 = -1 + 2 = 1$
- b** $\int_1^p \frac{2}{x^2} dx = \left[-\frac{2}{x}\right]_1^p = -\frac{2}{p} + 2$
 $\int_1^p \frac{2}{x^2} dx = \frac{1}{2} \cdot O(V)$ geeft $-\frac{2}{p} + 2 = \frac{1}{2}$
 $\frac{2}{p} = 1\frac{1}{2}$
 $p = 1\frac{1}{3}$
- 5 a** $f(x) = (2x+6)^5$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{6} (2x+6)^6 + c = \frac{1}{12} (2x+6)^6 + c$
- b** $f(x) = (5x+2)^2 \cdot \sqrt{5x+2} = (5x+2)^{2\frac{1}{2}}$ geeft
 $F(x) = \frac{1}{5} \cdot \frac{1}{\frac{3}{2}} (5x+2)^{3\frac{1}{2}} + c = \frac{2}{35} (5x+2)^3 \cdot \sqrt{5x+2} + c$
- c** $f(x) = 4e^{2x+3}$ geeft $F(x) = 4 \cdot \frac{1}{2} e^{2x+3} + c = 2e^{2x+3} + c$
- d** $f(x) = \frac{10}{(3x-1)^2} = 10(3x-1)^{-2}$ geeft
 $F(x) = 10 \cdot \frac{1}{3} \cdot \frac{1}{-1} (3x-1)^{-1} + c = -\frac{10}{3(3x-1)} + c = -\frac{10}{9x-3} + c$
- e** $f(x) = 2 \cos(\pi x + 3)$ geeft $F(x) = 2 \cdot \frac{1}{\pi} \sin(\pi x + 3) + c = \frac{2}{\pi} \sin(\pi x + 3) + c$
- f** $f(x) = \frac{6}{2x+5} = 6 \cdot \frac{1}{2x+5}$ geeft $F(x) = 6 \cdot \frac{1}{2} \cdot \ln|2x+5| + c = 3 \ln|2x+5| + c$

$$\begin{aligned}
6 \quad a \quad f(x) = g(x) \text{ geeft } \frac{x^2 + 4}{x} &= -\frac{1}{3}x + 5\frac{1}{3} \\
x^2 + 4 &= -\frac{1}{3}x^2 + 5\frac{1}{3}x \\
1\frac{1}{3}x^2 - 5\frac{1}{3}x + 4 &= 0 \\
x^2 - 4x + 3 &= 0 \\
(x-1)(x-3) &= 0 \\
x &= 1 \vee x = 3
\end{aligned}$$

$$\begin{aligned}
O(V) &= \int_1^3 (g(x) - f(x)) \, dx = \int_1^3 \left(-\frac{1}{3}x + 5\frac{1}{3} - \frac{x^2 + 4}{x} \right) \, dx = \int_1^3 \left(-\frac{1}{3}x + 5\frac{1}{3} - x - \frac{4}{x} \right) \, dx \\
&= \int_1^3 \left(-1\frac{1}{3}x + 5\frac{1}{3} - 4 \cdot \frac{1}{x} \right) \, dx = \left[-\frac{2}{3}x^2 + 5\frac{1}{3}x - 4 \ln|x| \right]_1^3 \\
&= -\frac{2}{3} \cdot 3^2 + 5\frac{1}{3} \cdot 3 - 4 \ln(3) - \left(-\frac{2}{3} \cdot 1^2 + 5\frac{1}{3} \cdot 1 - 4 \ln(1) \right) = 10 - 4 \ln(3) - (4\frac{2}{3} - 0) = 5\frac{1}{3} - 4 \ln(3)
\end{aligned}$$

$$\begin{aligned}
b \quad O(W) &= \int_1^p \left(\frac{x^2 + 4}{x} - x \right) \, dx = \int_1^p \left(x + \frac{4}{x} - x \right) \, dx = \int_1^p 4 \cdot \frac{1}{x} \, dx = [4 \ln|x|]_1^p = 4 \ln(p) - 4 \ln(1) = 4 \ln(p) \\
O(W) &= 8 \text{ geeft } 4 \ln(p) = 8 \\
\ln(p) &= 2 \\
p &= e^2
\end{aligned}$$

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$$\begin{aligned}
7 \quad a \quad f(x) = 0 \text{ geeft } 2x + 6 &= 0 \\
2x &= -6 \\
x &= -3 \\
g(x) = 0 \text{ geeft } x &= 9 \\
f(x) = g(x) \text{ geeft } \sqrt{2x + 6} &= -x + 9 \\
2x + 6 &= (-x + 9)^2 \\
2x + 6 &= x^2 - 18x + 81 \\
x^2 - 20x + 75 &= 0 \\
(x-5)(x-15) &= 0 \\
x &= 5 \vee x = 15
\end{aligned}$$

$x = 15$ voldoet niet, dus $x = 5$.

$$\begin{aligned}
I(K) &= \pi \int_{-3}^5 (\sqrt{2x+6})^2 \, dx + \pi \int_5^9 (-x+9)^2 \, dx = \pi \int_{-3}^5 (2x+6) \, dx + \pi \int_5^9 (x^2 - 18x + 81) \, dx \\
&= \pi [x^2 + 6x]_{-3}^5 + \pi \left[\frac{1}{3}x^3 - 9x^2 + 81x \right]_5^9 \\
&= \pi(5^2 + 6 \cdot 5 - ((-3)^2 + 6 \cdot (-3))) + \pi \left(\frac{1}{3} \cdot 9^3 - 9 \cdot 9^2 + 81 \cdot 9 - \left(\frac{1}{3} \cdot 5^3 - 9 \cdot 5^2 + 81 \cdot 5 \right) \right) \\
&= 64\pi + 21\frac{1}{3}\pi = 85\frac{1}{3}\pi
\end{aligned}$$

$$\begin{aligned}
b \quad I(L) &= \pi \int_0^5 ((g(x))^2 - (f(x))^2) \, dx = \pi \int_0^5 ((-x+9)^2 - (2x+6)) \, dx = \pi \int_0^5 (x^2 - 18x + 81 - 2x - 6) \, dx \\
&= \pi \int_0^5 (x^2 - 20x + 75) \, dx = \pi \left[\frac{1}{3}x^3 - 10x^2 + 75x \right]_0^5 = \pi \left(\frac{1}{3} \cdot 5^3 - 10 \cdot 5^2 + 75 \cdot 5 - 0 \right) = 166\frac{2}{3}\pi
\end{aligned}$$

$$\begin{aligned}
8 \quad a \quad f(x) = 9 \text{ geeft } 2^x + 1 &= 9 \\
2^x &= 8 \\
x &= 3
\end{aligned}$$

Stel U is het vlakdeel dat ontstaat als V over een afstand 9 omlaag wordt geschoven.

Dan wordt U ingesloten door de grafiek van de functie $g(x) = f(x) - 9 = 2^x - 8$, de x -as en de y -as.

$$\begin{aligned}
I(L) &= \pi \int_0^3 (2^x - 8)^2 \, dx = \pi \int_0^3 (2^{2x} - 16 \cdot 2^x + 64) \, dx = \pi \left[\frac{2^{2x}}{2 \ln(2)} - \frac{16 \cdot 2^x}{\ln(2)} + 64x \right]_0^3 \\
&= \pi \left(\frac{2^6}{2 \ln(2)} - \frac{16 \cdot 2^3}{\ln(2)} + 64 \cdot 3 - \left(\frac{1}{2 \ln(2)} - \frac{16}{\ln(2)} + 0 \right) \right) \approx 238,3
\end{aligned}$$

$$\begin{aligned}
b \quad y = 2^x + 1 \text{ geeft } 2^x &= y - 1 \\
x &= {}^2\log(y-1) \\
x^2 &= {}^2\log^2(y-1)
\end{aligned}$$

$$f(0) = 2$$

$$\text{De GR geeft } I(M) = \pi \int_2^9 x^2 \, dy = \pi \int_2^9 ({}^2\log^2(y-1)) \, dy \approx 100,2.$$

$$9 \quad \frac{1}{2} \cdot I(S_p) = \pi \int_0^p y^2 dx = \pi \int_0^p (64 - x^2) dx = \pi \left[64x - \frac{1}{3}x^3 \right]_0^p = \pi(64p - \frac{1}{3}p^3 - 0) = \pi(64p - \frac{1}{3}p^3)$$

$$\text{Dus } I(S_p) = 2\pi(64p - \frac{1}{3}p^3).$$

$$I(B) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 8^3 = 682\frac{2}{3}\pi$$

$$I(S_p) = \frac{1}{2} \cdot I(B) \text{ geeft } 2\pi(64p - \frac{1}{3}p^3) = \frac{1}{2} \cdot 682\frac{2}{3}\pi$$

$$64p - \frac{1}{3}p^3 = 170\frac{2}{3}$$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 2,78$.

$$10 \quad a \quad a(t) = mt + n$$

$$m = \frac{a(60) - a(0)}{60} = \frac{68 - 8}{60} = 1$$

$$a(t) = t + n \text{ en } a(0) = 8 \text{ geeft } a(t) = t + 8$$

$$a(t) = t + 8 \text{ en } v(0) = 0 \text{ geeft } v(t) = \frac{1}{2}t^2 + 8t$$

$$v(t) = \frac{1}{2}t^2 + 8t \text{ en } s(0) = 0 \text{ geeft } s(t) = \frac{1}{6}t^3 + 4t^2$$

$$s(60) = \frac{1}{6} \cdot 60^3 + 4 \cdot 60^2 = 50400$$

Dus in deze 60 seconden bereikt de raket een hoogte van 50,4 km.

$$b \quad \text{Vanaf } t = 60 \text{ is } a(t) = -10, \text{ dus } v(t) = -10t + c.$$

$$v(60) = \frac{1}{2} \cdot 60^2 + 8 \cdot 60 = 2280 \text{ geeft } -10 \cdot 60 + c = 2280$$

$$-600 + c = 2280$$

$$c = 2880$$

$$\text{Dus } v(t) = -10t + 2880.$$

$$v(t) = -10t + 2880 \text{ geeft } s(t) = -5t^2 + 2880t + d$$

$$s(60) = 50400 \text{ geeft } -5 \cdot 60^2 + 2880 \cdot 60 + d = 50400$$

$$154800 + d = 50400$$

$$d = -104400$$

$$\text{Dus } s(t) = -5t^2 + 2880t - 104400.$$

De hoogte is maximaal als de snelheid 0 m/s is.

$$v(t) = 0 \text{ geeft } -10t + 2880 = 0$$

$$-10t = -2880$$

$$t = 288$$

$$s(288) = -5 \cdot 288^2 + 2880 \cdot 288 - 104400 = 310320$$

Dus de maximale hoogte is 310,32 km.

Gemengde opgaven

Bladzijde 138

$$1 \quad F(x) = (ax^2 + bx + c)e^x + d \text{ geeft}$$

$$F'(x) = (2ax + b) \cdot e^x + (ax^2 + bx + c) \cdot e^x = (ax^2 + (2a + b)x + b + c)e^x$$

Uit $F'(x) = f(x)$ volgt $a = 1$ en $2a + b = 0$ en $b + c = 0$.

Dit geeft $b = -2$ en $c = 2$.

$$\text{Dus } F(x) = (x^2 - 2x + 2)e^x + d.$$

$$F(0) = 0 \text{ geeft } (0^2 - 2 \cdot 0 + 2)e^0 + d = 0$$

$$2 + d = 0$$

$$d = -2$$

$$\text{Dus } F(x) = (x^2 - 2x + 2)e^x - 2.$$

$$F'(x) = f(x) = x^2 e^x \text{ geeft } F''(x) = f'(x) = 2x \cdot e^x + x^2 \cdot e^x = (x^2 + 2x)e^x$$

$$F''(x) = 0 \text{ geeft } x^2 + 2x = 0$$

$$x(x + 2) = 0$$

$$x = 0 \vee x = -2$$

$$F(-2) = ((-2)^2 - 2 \cdot -2 + 2)e^{-2} - 2 = \frac{10}{e^2} - 2, \text{ dus het andere buigpunt is } \left(-2, \frac{10}{e^2} - 2\right)$$

2 a $f(x) = ax$ geeft $x^3 - 3x = ax$
 $x^3 = (a+3)x$
 $x = 0 \vee x^2 = a+3$
 $x = 0 \vee x = \sqrt{a+3} \vee x = -\sqrt{a+3}$
 $x = -\sqrt{a+3}$ voldoet niet, dus $x = 0$ of $x = \sqrt{a+3}$.

$$\begin{aligned} O(V) &= \int_0^{\sqrt{a+3}} (ax - f(x)) dx = \int_0^{\sqrt{a+3}} (ax - (x^3 - 3x)) dx = \int_0^{\sqrt{a+3}} (ax - x^3 + 3x) dx \\ &= \left[\frac{1}{2}ax^2 - \frac{1}{4}x^4 + \frac{3}{2}x^2 \right]_0^{\sqrt{a+3}} = \frac{1}{2}a(\sqrt{a+3})^2 - \frac{1}{4}(\sqrt{a+3})^4 + \frac{3}{2}(\sqrt{a+3})^2 - 0 \\ &= \frac{1}{2}a(a+3) - \frac{1}{4}(a+3)^2 + \frac{3}{2}(a+3) = \frac{1}{2}a^2 + \frac{1}{2}a - \frac{1}{4}a^2 - \frac{1}{2}a - 2\frac{1}{4} + 1\frac{1}{2}a + 4\frac{1}{2} \\ &= \frac{1}{4}a^2 + 1\frac{1}{2}a + 2\frac{1}{4} \end{aligned}$$

$$\begin{aligned} O(V) = 9 \text{ geeft } \frac{1}{4}a^2 + 1\frac{1}{2}a + 2\frac{1}{4} &= 9 \\ a^2 + 6a + 9 &= 36 \\ a^2 + 6a - 27 &= 0 \\ (a-3)(a+9) &= 0 \\ a = 3 \vee a = -9 \end{aligned}$$

$a = -9$ voldoet niet, dus $a = 3$.

b $f(x) = x^3 - 3x$ geeft $f'(x) = 3x^2 - 3$
 $f'(x) = 0$ geeft $3x^2 - 3 = 0$
 $3x^2 = 3$
 $x^2 = 1$
 $x = 1 \vee x = -1$

$x = -1$ voldoet niet, dus $x = 1$.

$f(1) = -2$, dus het laagste punt van de grafiek van f is $(1, -2)$.

k door $(1, -2)$ geeft $a = -2$, dus $k: y = -2x$.

$$\begin{aligned} I(L) &= \pi \int_0^1 ((f(x))^2 - (-2x)^2) dx = \pi \int_0^1 ((x^3 - 3x)^2 - 4x^2) dx = \pi \int_0^1 (x^6 - 6x^4 + 9x^2 - 4x^2) dx \\ &= \pi \int_0^1 (x^6 - 6x^4 + 5x^2) dx = \pi \left[\frac{1}{7}x^7 - \frac{6}{5}x^5 + \frac{5}{3}x^3 \right]_0^1 = \pi \left(\frac{1}{7} \cdot 1^7 - \frac{6}{5} \cdot 1^5 + \frac{5}{3} \cdot 1^3 - 0 \right) = \frac{64}{105}\pi \end{aligned}$$

3 $O(ABCD) = 5 \cdot (2 - p) = 10 - 5p$

V is het vlakdeel dat wordt ingesloten door de grafiek van f en het lijnstuk CD .

$$\begin{aligned} O(V) &= \int_0^5 (2 - f(x)) dx = \int_0^5 \left(2 - \frac{1}{2}x + 1 - \frac{3}{x+1} \right) dx = \int_0^5 \left(3 - \frac{1}{2}x - \frac{3}{x+1} \right) dx = \left[3x - \frac{1}{4}x^2 - 3 \ln|x+1| \right]_0^5 \\ &= 3 \cdot 5 - \frac{1}{4} \cdot 5^2 - 3 \ln(6) - (0 - 0 - 3 \ln(1)) = 8\frac{3}{4} - 3 \ln(6) \end{aligned}$$

$$O(V) = \frac{1}{2} \cdot O(ABCD) \text{ geeft } 8\frac{3}{4} - 3 \ln(6) = 5 - 2\frac{1}{2}p$$

$$2\frac{1}{2}p = 3 \ln(6) - 3\frac{3}{4}$$

$$p = 1\frac{1}{5} \ln(6) - 1\frac{1}{2}$$

4 a $54 \text{ km/u} = 15 \text{ m/s}$

De constante versnelling is a .

$$v_w(t) = at + b \text{ en } v_w(0) = 15 \text{ geeft } v_w(t) = at + 15$$

$$v_w(t) = at + 15 \text{ en } s_w(0) = 0 \text{ geeft } s_w(t) = \frac{1}{2}at^2 + 15t$$

$$v_w(t) = 0 \text{ geeft } at + 15 = 0$$

$$at = -15$$

$$t = -\frac{15}{a}$$

$$\begin{aligned}
s_w\left(-\frac{15}{a}\right) &= 40 \text{ geeft } \frac{1}{2}a \cdot \left(-\frac{15}{a}\right)^2 + 15 \cdot -\frac{15}{a} = 40 \\
\frac{1}{2}a \cdot \frac{225}{a^2} - \frac{225}{a} &= 40 \\
\frac{112\frac{1}{2}}{a} - \frac{225}{a} &= 40 \\
-\frac{112\frac{1}{2}}{a} &= 40 \\
a &= -\frac{112\frac{1}{2}}{40} = -2,81...
\end{aligned}$$

Dus de versnelling van de wielrenner is ongeveer $-2,8 \text{ m/s}^2$.

b $a = -2,5$, $v_w(0) = 15$ en $s_w(0) = 0$ geeft $v_w(t) = -2,5t + 15$ en $s_w(t) = -1,25t^2 + 15t$

$6 \text{ km/u} = 1\frac{2}{3} \text{ m/s}$

Voor de auto geldt $v_a(t) = 1\frac{2}{3}$.

$v_a(t) = 1\frac{2}{3}$ en $s_a(0) = 40$ geeft $s_a(t) = 1\frac{2}{3}t + 40$

$v_w(t) = v_a(t)$ geeft $-2,5t + 15 = 1\frac{2}{3}$

$-2,5t = -13\frac{1}{3}$

$t = 5\frac{1}{3}$

$s_w(5\frac{1}{3}) = -1,25 \cdot (5\frac{1}{3})^2 + 15 \cdot 5\frac{1}{3} = 44\frac{4}{9}$

$s_a(5\frac{1}{3}) = 1\frac{2}{3} \cdot 5\frac{1}{3} + 40 = 48\frac{8}{9}$

Dus op het moment dat de wielrenner en de auto dezelfde snelheid hebben, rijdt de auto $48\frac{8}{9} - 44\frac{4}{9} = 4\frac{4}{9}$ meter voor de wielrenner.

De wielrenner kan dus op tijd stoppen.

5 $\frac{1}{2} \cdot I(S_p) = \pi \int_0^{pr} y^2 dx = \pi \int_0^{pr} (r^2 - x^2) dx = \pi \left[r^2x - \frac{1}{3}x^3 \right]_0^{pr} = \pi(pr^3 - \frac{1}{3}(pr)^3 - 0) = \pi(pr^3 - \frac{1}{3}p^3r^3)$

Dus $I(S_p) = 2\pi(pr^3 - \frac{1}{3}p^3r^3)$.

$I(S_p) = \frac{1}{2} \cdot I(B)$ geeft $2\pi(pr^3 - \frac{1}{3}p^3r^3) = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$

$p - \frac{1}{3}p^3 = \frac{1}{3}$

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $p \approx 0,35$.

Bladzijde 139

6 a $f(x) = 0$ geeft $2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$f(x) = (2x + 1)\sqrt{2x + 1} = (2x + 1)^{1\frac{1}{2}}$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{2\frac{1}{2}}(2x + 1)^{2\frac{1}{2}} + c = \frac{1}{5}(2x + 1)^2 \cdot \sqrt{2x + 1} + c$

$O(V) = \int_{-\frac{1}{2}}^0 f(x) dx = \left[\frac{1}{5}(2x + 1)^2 \cdot \sqrt{2x + 1} \right]_{-\frac{1}{2}}^0 = \frac{1}{5} \cdot 1^2 \cdot \sqrt{1} - 0 = \frac{1}{5}$

$O(W_p) = \int_0^p f(x) dx = \left[\frac{1}{5}(2x + 1)^2 \cdot \sqrt{2x + 1} \right]_0^p = \frac{1}{5}(2p + 1)^2 \cdot \sqrt{2p + 1} - \frac{1}{5} \cdot 1^2 \cdot \sqrt{1}$
 $= \frac{1}{5}(2p + 1)^2 \cdot \sqrt{2p + 1} - \frac{1}{5}$

$O(W_p) = 31 \cdot O(V)$ geeft $\frac{1}{5}(2p + 1)^2 \cdot \sqrt{2p + 1} - \frac{1}{5} = 6\frac{1}{5}$

$\frac{1}{5}(2p + 1)^2 \cdot \sqrt{2p + 1} = 6\frac{2}{5}$

$(2p + 1)^2 \cdot \sqrt{2p + 1} = 32$

$(2p + 1)^5 = 1024$

$2p + 1 = 4$

$2p = 3$

$p = 1\frac{1}{2}$

$$\mathbf{b} \quad I(K) = \pi \int_{-\frac{1}{2}}^0 (f(x))^2 dx = \pi \int_{-\frac{1}{2}}^0 (2x+1)^3 dx = \pi \left[\frac{1}{2} \cdot \frac{1}{4} (2x+1)^4 \right]_{-\frac{1}{2}}^0 = \pi \left[\frac{1}{8} (2x+1)^4 \right]_{-\frac{1}{2}}^0 = \pi \left(\frac{1}{8} \cdot 1^4 - 0 \right) = \frac{1}{8} \pi$$

$$I(L_p) = \pi \int_0^p (f(x))^2 dx = \pi \left[\frac{1}{8} (2x+1)^4 \right]_0^p = \frac{1}{8} \pi [(2x+1)^4]_0^p = \frac{1}{8} \pi ((2p+1)^4 - 1)$$

$$I(L_p) = 624 \cdot I(K) \text{ geeft } \frac{1}{8} \pi ((2p+1)^4 - 1) = 624 \cdot \frac{1}{8} \pi$$

$$(2p+1)^4 - 1 = 624$$

$$(2p+1)^4 = 625$$

$$2p+1 = 5 \vee 2p+1 = -5$$

$$2p = 4 \vee 2p = -6$$

$$p = 2 \vee p = -3$$

$p = -3$ voldoet niet, dus $p = 2$.

$$\mathbf{c} \quad y = (2x+1)\sqrt{2x+1} \text{ geeft } (2x+1)^3 = y^2$$

$$2x+1 = y^{\frac{2}{3}}$$

$$2x = y^{\frac{2}{3}} - 1$$

$$x = \frac{1}{2} y^{\frac{2}{3}} - \frac{1}{2}$$

$$x^2 = \left(\frac{1}{2} y^{\frac{2}{3}} - \frac{1}{2} \right)^2$$

$$f(0) = 1$$

$$I(M) = \pi \int_0^1 x^2 dy = \pi \int_0^1 \left(\frac{1}{2} y^{\frac{2}{3}} - \frac{1}{2} \right)^2 dy = \pi \int_0^1 \left(\frac{1}{4} y^{\frac{4}{3}} - \frac{1}{2} y^{\frac{2}{3}} + \frac{1}{4} \right) dy = \pi \left[\frac{1}{4} \cdot \frac{1}{2\frac{4}{3}} y^{\frac{4}{3}+1} - \frac{1}{2} \cdot \frac{1}{1\frac{2}{3}} y^{\frac{2}{3}+1} + \frac{1}{4} y \right]_0^1$$

$$= \pi \left[\frac{3}{28} y^{\frac{7}{3}} - \frac{3}{10} y^{\frac{5}{3}} + \frac{1}{4} y \right]_0^1 = \pi \left(\frac{3}{28} - \frac{3}{10} + \frac{1}{4} - 0 \right) = \frac{2}{35} \pi$$

$\mathbf{d}$ Stel U is het vlakdeel dat ontstaat als W_3 over een afstand 3 naar links wordt geschoven. Dan wordt U ingesloten door de grafiek van de functie

$$g(x) = (2(x+3)+1)\sqrt{2(x+3)+1} = (2x+7)(\sqrt{2x+7}), \text{ de } x\text{-as, de } y\text{-as en de lijn } x = -3.$$

$$y = (2x+7)\sqrt{2x+7} \text{ geeft } (2x+7)^3 = y^2$$

$$2x+7 = y^{\frac{2}{3}}$$

$$2x = y^{\frac{2}{3}} - 7$$

$$x = \frac{1}{2} y^{\frac{2}{3}} - 3\frac{1}{2}$$

$$x^2 = \left(\frac{1}{2} y^{\frac{2}{3}} - 3\frac{1}{2} \right)^2$$

$$g(0) = 7\sqrt{7} \text{ en } g(-3) = 1$$

$$\text{De GR geeft } I(N) = \pi \int_0^1 3^2 dy + \pi \int_1^{7\sqrt{7}} x^2 dy = \pi \int_0^1 9 dy + \pi \int_1^{7\sqrt{7}} \left(\frac{1}{2} y^{\frac{2}{3}} - 3\frac{1}{2} \right)^2 dy \approx 159,0.$$

7 a $f(x) = e$ geeft $e^{2x} = e$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$g(x) = e \text{ geeft } e^{-x} = e$$

$$-x = 1$$

$$x = -1$$

$$f(x) = g(x) \text{ geeft } e^{2x} = e^{-x}$$

$$2x = -x$$

$$x = 0$$

Stel U is het vlakdeel dat ontstaat als V over een afstand e omlaag wordt geschoven.

Dan wordt U ingesloten door de grafieken van de functies $h(x) = f(x) - e = e^{2x} - e$ en

$k(x) = g(x) - e = e^{-x} - e$ en de x -as.

$$I(L) = \pi \int_{-1}^0 (e^{-x} - e)^2 dx + \pi \int_0^{\frac{1}{2}} (e^{2x} - e)^2 dx = \pi \int_{-1}^0 (e^{-2x} - 2e^{-x+1} + e^2) dx + \pi \int_0^{\frac{1}{2}} (e^{4x} - 2e^{2x+1} + e^2) dx$$

$$= \pi \left[-\frac{1}{2} e^{-2x} + 2e^{-x+1} + e^2 x \right]_{-1}^0 + \pi \left[\frac{1}{4} e^{4x} - e^{2x+1} + e^2 x \right]_0^{\frac{1}{2}}$$

$$= \pi \left(-\frac{1}{2} + 2e + 0 - \left(-\frac{1}{2} e^2 + 2e^2 - e^2 \right) \right) + \pi \left(\frac{1}{4} e^2 - e^2 + \frac{1}{2} e^2 - \left(\frac{1}{4} - e + 0 \right) \right) \approx 5,85$$

$$\begin{aligned}
\mathbf{b} \quad O(W) &= \int_{-1}^0 g(x) \, dx + \int_0^{\frac{1}{2}} f(x) \, dx = \int_{-1}^0 e^{-x} \, dx + \int_0^{\frac{1}{2}} e^{2x} \, dx = [-e^{-x}]_{-1}^0 + \left[\frac{1}{2}e^{2x}\right]_0^{\frac{1}{2}} = -1 + e + \frac{1}{2}e - \frac{1}{2} = 1\frac{1}{2}e - 1\frac{1}{2} \\
\int_{-1}^p g(x) \, dx &= \int_{-1}^p e^{-x} \, dx = [-e^{-x}]_{-1}^p = -e^{-p} + e \\
\int_{-1}^p g(x) \, dx &= \frac{1}{2} \cdot O(W) \text{ geeft } -e^{-p} + e = \frac{3}{4}e - \frac{3}{4} \\
e^{-p} &= \frac{1}{4}e + \frac{3}{4} \\
-p &= \ln\left(\frac{1}{4}e + \frac{3}{4}\right) \\
p &= -\ln\left(\frac{1}{4}e + \frac{3}{4}\right) = \ln\left(\left(\frac{1}{4}e + \frac{3}{4}\right)^{-1}\right) = \ln\left(\frac{1}{\frac{1}{4}e + \frac{3}{4}}\right) = \ln\left(\frac{4}{e+3}\right)
\end{aligned}$$

8 a Van c is het middelpunt $M(5, 0)$ en de straal $r = 5$.

$k: y = ax + 10$ oftewel $k: ax - y + 10 = 0$

$$\begin{aligned}
d(M, k) = r \text{ geeft } \frac{|5a - 0 + 10|}{\sqrt{a^2 + (-1)^2}} &= 5 \\
|5a + 10| &= 5\sqrt{a^2 + 1} \\
25a^2 + 100a + 100 &= 25a^2 + 25 \\
100a &= -75 \\
a &= -\frac{3}{4}
\end{aligned}$$

Dus $k: y = -\frac{3}{4}x + 10$.

$$\begin{aligned}
y = 0 \text{ geeft } -\frac{3}{4}x + 10 &= 0 \\
-\frac{3}{4}x &= -10 \\
x &= 13\frac{1}{3}
\end{aligned}$$

Dus k snijdt de x -as in het punt $Q(13\frac{1}{3}, 0)$.

$$O(V) + O(W) = O(\triangle OQP) - \frac{1}{2} \cdot \pi r^2 = \frac{1}{2} \cdot 13\frac{1}{3} \cdot 10 - \frac{1}{2} \cdot \pi \cdot 5^2 \approx 27,4$$

b Lijn l door M en A heeft $rc_l = 1\frac{1}{3}$.

$$\begin{aligned}
l: y = \frac{1}{3}x + b \quad \left. \begin{array}{l} \text{door } M(5, 0) \end{array} \right\} \begin{array}{l} \frac{1}{3} \cdot 5 + b = 0 \\ 6\frac{2}{3} + b = 0 \\ b = -6\frac{2}{3} \end{array}
\end{aligned}$$

Dus $l: y = \frac{1}{3}x - 6\frac{2}{3}$.

$$\begin{aligned}
k \text{ snijden met } l \text{ geeft } -\frac{3}{4}x + 10 &= \frac{1}{3}x - 6\frac{2}{3} \\
-9x + 120 &= 16x - 80 \\
25x &= 200 \\
x &= 8, \text{ dus } x_A = 8.
\end{aligned}$$

$$\begin{aligned}
I(K) &= \pi \int_0^8 ((-\frac{3}{4}x + 10)^2 - (25 - (x - 5)^2)) \, dx = \pi \int_0^8 (\frac{9}{16}x^2 - 15x + 100 - 25 + x^2 - 10x + 25) \, dx \\
&= \pi \int_0^8 (\frac{9}{16}x^2 - 25x + 100) \, dx = \pi [\frac{25}{48}x^3 - 12\frac{1}{2}x^2 + 100x]_0^8 = \pi(\frac{25}{48} \cdot 8^3 - 12\frac{1}{2} \cdot 8^2 + 100 \cdot 8 - 0) \\
&= 266\frac{2}{3}\pi
\end{aligned}$$

$$\begin{aligned}
I(L) &= \pi \int_8^{13\frac{1}{3}} (-\frac{3}{4}x + 10)^2 \, dx - \pi \int_8^{10} (25 - (x - 5)^2) \, dx \\
&= \pi \int_8^{13\frac{1}{3}} (\frac{9}{16}x^2 - 15x + 100) \, dx - \pi \int_8^{10} (25 - x^2 + 10x - 25) \, dx \\
&= \pi \int_8^{13\frac{1}{3}} (\frac{9}{16}x^2 - 15x + 100) \, dx - \pi \int_8^{10} (-x^2 + 10x) \, dx \\
&= \pi [\frac{3}{16}x^3 - 7\frac{1}{2}x^2 + 100x]_8^{13\frac{1}{3}} - \pi [-\frac{1}{3}x^3 + 5x^2]_8^{10} \\
&= \pi(\frac{3}{16} \cdot (13\frac{1}{3})^3 - 7\frac{1}{2} \cdot (13\frac{1}{3})^2 + 100 \cdot 13\frac{1}{3} - (\frac{3}{16} \cdot 8^3 - 7\frac{1}{2} \cdot 8^2 + 100 \cdot 8)) \\
&\quad - \pi(-\frac{1}{3} \cdot 10^3 + 5 \cdot 10^2 - (-\frac{1}{3} \cdot 8^3 + 5 \cdot 8^2)) = 28\frac{4}{9}\pi - 17\frac{1}{3}\pi = 11\frac{1}{9}\pi \\
p &= \frac{266\frac{2}{3}\pi}{11\frac{1}{9}\pi} = 24
\end{aligned}$$

12 Goniometrische formules

12.1 Goniometrische formules bij vergelijkingen en herleidingen

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- V1** **a** $y = \sin(2x - \frac{1}{3}\pi) = \cos(2x - \frac{1}{3}\pi - \frac{1}{2}\pi) = \cos(2x - \frac{5}{6}\pi)$
b $y = -\sin(3x - \frac{1}{4}\pi) = \sin(3x - \frac{1}{4}\pi + \pi) = \sin(3x + \frac{3}{4}\pi) = \cos(3x + \frac{3}{4}\pi - \frac{1}{2}\pi) = \cos(3x + \frac{1}{4}\pi)$
c $y = \cos(\frac{1}{2}x + \frac{1}{4}\pi) = \sin(\frac{1}{2}x + \frac{1}{4}\pi + \frac{1}{2}\pi) = \sin(\frac{1}{2}x + \frac{3}{4}\pi)$
d $y = -\cos(2(x - \frac{1}{6}\pi)) = \cos(2x - \frac{1}{3}\pi + \pi) = \cos(2x + \frac{2}{3}\pi) = \sin(2x + \frac{2}{3}\pi + \frac{1}{2}\pi) = \sin(2x + 1\frac{1}{6}\pi)$

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- V2** **a** $\sin(2x - \frac{1}{3}\pi) = 0$
 $2x - \frac{1}{3}\pi = k \cdot \pi$
 $2x = \frac{1}{3}\pi + k \cdot \pi$
 $x = \frac{1}{6}\pi + k \cdot \frac{1}{2}\pi$
b $\cos(\frac{1}{2}x + \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$
 $\frac{1}{2}x + \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x + \frac{1}{6}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$
 $\frac{1}{2}x = k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x = k \cdot 4\pi \vee x = -\frac{2}{3}\pi + k \cdot 4\pi$
c $\sin(2x) = \sin(x - \frac{1}{4}\pi)$
 $2x = x - \frac{1}{4}\pi + k \cdot 2\pi \vee 2x = \pi - (x - \frac{1}{4}\pi) + k \cdot 2\pi$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee 2x = \pi - x + \frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee 3x = 1\frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = \frac{5}{12}\pi + k \cdot \frac{2}{3}\pi$
d $\cos(2x + \frac{1}{3}\pi) = \cos(x - \frac{1}{6}\pi)$
 $2x + \frac{1}{3}\pi = x - \frac{1}{6}\pi + k \cdot 2\pi \vee 2x + \frac{1}{3}\pi = -(x - \frac{1}{6}\pi) + k \cdot 2\pi$
 $x = -\frac{1}{2}\pi + k \cdot 2\pi \vee 2x + \frac{1}{3}\pi = -x + \frac{1}{6}\pi + k \cdot 2\pi$
 $x = -\frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{6}\pi + k \cdot 2\pi$
 $x = -\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

- V3** **a** $\sin(2x - \frac{1}{4}\pi) = -1$
 $2x - \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$
 $2x = 1\frac{3}{4}\pi + k \cdot 2\pi$
 $x = \frac{7}{8}\pi + k \cdot \pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{7}{8}\pi \vee x = 1\frac{7}{8}\pi$
b $\sin(x - \frac{1}{3}\pi) = \frac{1}{2}$
 $x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi$
c $\sin(\frac{1}{2}x) = \sin(2x - \frac{1}{6}\pi)$
 $\frac{1}{2}x = 2x - \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \pi - (2x - \frac{1}{6}\pi) + k \cdot 2\pi$
 $-1\frac{1}{2}x = -\frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \pi - 2x + \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{9}\pi + k \cdot -1\frac{1}{3}\pi \vee 2\frac{1}{2}x = 1\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{9}\pi + k \cdot 1\frac{1}{3}\pi \vee x = \frac{7}{15}\pi + k \cdot \frac{4}{5}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{9}\pi \vee x = 1\frac{4}{9}\pi \vee x = \frac{7}{15}\pi \vee x = 1\frac{4}{15}\pi$

d $\cos(x + \frac{1}{4}\pi) = \cos(\frac{1}{2}x + \pi)$
 $x + \frac{1}{4}\pi = \frac{1}{2}x + \pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -(\frac{1}{2}x + \pi) + k \cdot 2\pi$
 $\frac{1}{2}x = \frac{3}{4}\pi + k \cdot 2\pi \vee x + \frac{1}{4}\pi = -\frac{1}{2}x - \pi + k \cdot 2\pi$
 $x = 1\frac{1}{2}\pi + k \cdot 4\pi \vee 1\frac{1}{2}x = -1\frac{1}{4}\pi + k \cdot 2\pi$
 $x = 1\frac{1}{2}\pi + k \cdot 4\pi \vee x = -\frac{5}{6}\pi + k \cdot 1\frac{1}{3}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = 1\frac{1}{2}\pi \vee x = \frac{1}{2}\pi \vee x = 1\frac{5}{6}\pi$

1 a $\sin(2x) + \sin(x) = 0$
 $\sin(2x) = -\sin(x)$
 $\sin(2x) = \sin(x + \pi)$
b $2\sin^2(x) = 3\cos(x)$
 $2(1 - \cos^2(x)) = 3\cos(x)$
 $2 - 2\cos^2(x) = 3\cos(x)$
 $2\cos^2(x) + 3\cos(x) - 2 = 0$

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2 a $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$
 $\sin(2x - \frac{1}{3}\pi) = \cos(x + 1\frac{1}{3}\pi)$
 $\sin(2x - \frac{1}{3}\pi) = \sin(x + 1\frac{5}{6}\pi)$
 $2x - \frac{1}{3}\pi = x + 1\frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \pi - x - 1\frac{5}{6}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
b $\sin(2x - \frac{1}{3}\pi) = -\cos(x + \frac{1}{3}\pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + 1\frac{1}{3}\pi)$
 $2x - \frac{5}{6}\pi = x + 1\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - 1\frac{1}{3}\pi + k \cdot 2\pi$
 $x = 2\frac{1}{6}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

3 a $\sin(x + \frac{1}{2}\pi) = \cos(2x)$
 $\cos(x) = \cos(2x)$
 $x = 2x + k \cdot 2\pi \vee x = -2x + k \cdot 2\pi$
 $-x = k \cdot 2\pi \vee 3x = k \cdot 2\pi$
 $x = k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = 2\pi \vee x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi$
b $\sin(3x) = -\cos(x)$
 $\cos(3x - \frac{1}{2}\pi) = \cos(x + \pi)$
 $3x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \vee 3x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$
 $2x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 4x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot \pi \vee x = -\frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{3}{4}\pi \vee x = 1\frac{3}{4}\pi \vee x = \frac{3}{8}\pi \vee x = \frac{7}{8}\pi \vee x = 1\frac{3}{8}\pi \vee x = 1\frac{7}{8}\pi$
c $\sin^2(x) + \frac{1}{2}\cos(x) = 1$
 $1 - \cos^2(x) + \frac{1}{2}\cos(x) = 1$
 $\cos^2(x) - \frac{1}{2}\cos(x) = 0$
 $\cos(x)(\cos(x) - \frac{1}{2}) = 0$
 $\cos(x) = 0 \vee \cos(x) = \frac{1}{2}$
 $x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi \vee x = \frac{1}{3}\pi \vee x = 1\frac{2}{3}\pi$

$$\begin{aligned} \text{d } \sin\left(\frac{1}{2}x\right) + \sqrt{3} \cdot \cos\left(\frac{1}{2}x\right) &= 0 \\ \sin\left(\frac{1}{2}x\right) &= -\sqrt{3} \cdot \cos\left(\frac{1}{2}x\right) \\ \frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} &= -\sqrt{3} \\ \tan\left(\frac{1}{2}x\right) &= -\sqrt{3} \\ \frac{1}{2}x &= -\frac{1}{3}\pi + k \cdot \pi \\ x &= -\frac{2}{3}\pi + k \cdot 2\pi \\ x \text{ in } [0, 2\pi] \text{ geeft } x &= \frac{1}{3}\pi \end{aligned}$$

$$\begin{aligned} 4 \text{ a } \cos(2\pi t) &= \sin\left(\frac{1}{2}\pi t\right) \\ \cos(2\pi t) &= \cos\left(\frac{1}{2}\pi t - \frac{1}{2}\pi\right) \\ 2\pi t &= \frac{1}{2}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \vee 2\pi t = -\frac{1}{2}\pi t + \frac{1}{2}\pi + k \cdot 2\pi \\ \frac{1}{2}\pi t &= -\frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{2}\pi t = \frac{1}{2}\pi + k \cdot 2\pi \\ t &= -\frac{1}{3} + k \cdot \frac{1}{3} \vee t = \frac{1}{5} + k \cdot \frac{4}{5} \\ t \text{ in } [0, 3] \text{ geeft } t &= 1 \vee t = 2\frac{1}{3} \vee t = \frac{1}{5} \vee t = 1\frac{4}{5} \vee t = 2\frac{3}{5} \\ \text{b } \sin\left(\frac{1}{6}\pi t\right) &= -\cos(\pi t) \\ \cos\left(\frac{1}{6}\pi t - \frac{1}{2}\pi\right) &= \cos(\pi t + \pi) \\ \frac{1}{6}\pi t - \frac{1}{2}\pi &= \pi t + \pi + k \cdot 2\pi \vee \frac{1}{6}\pi t - \frac{1}{2}\pi = -\pi t - \pi + k \cdot 2\pi \\ -\frac{5}{6}\pi t &= \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{1}{6}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \\ t &= -1\frac{4}{5} + k \cdot 2\frac{2}{5} \vee t = -\frac{3}{7} + k \cdot 1\frac{5}{7} \\ t \text{ in } [0, 3] \text{ geeft } t &= \frac{3}{5} \vee t = 3 \vee t = 1\frac{2}{7} \end{aligned}$$

$$\begin{aligned} 5 \text{ a } 2\sin^2(x) + \cos^2(x) + \cos(x) &= 0 \\ 2(1 - \cos^2(x)) + \cos^2(x) + \cos(x) &= 0 \\ 2 - 2\cos^2(x) + \cos^2(x) + \cos(x) &= 0 \\ \cos^2(x) - \cos(x) - 2 &= 0 \\ (\cos(x) + 1)(\cos(x) - 2) &= 0 \\ \cos(x) &= -1 \vee \cos(x) = 2 \\ x &= \pi + k \cdot 2\pi \\ x \text{ in } [0, 2\pi] \text{ geeft } x &= \pi \\ \text{b } \sin\left(2x - \frac{1}{3}\pi\right) &= -\cos\left(\frac{1}{2}x\right) \\ \cos\left(2x - \frac{5}{6}\pi\right) &= \cos\left(\frac{1}{2}x + \pi\right) \\ 2x - \frac{5}{6}\pi &= \frac{1}{2}x + \pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -\frac{1}{2}x - \pi + k \cdot 2\pi \\ \frac{1}{2}x &= 1\frac{5}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = -\frac{1}{6}\pi + k \cdot 2\pi \\ x &= 1\frac{2}{3}\pi + k \cdot 1\frac{1}{3}\pi \vee x = -\frac{1}{15}\pi + k \cdot \frac{4}{5}\pi \\ x \text{ in } [0, 2\pi] \text{ geeft } x &= 1\frac{2}{3}\pi \vee x = \frac{11}{15}\pi \vee x = 1\frac{8}{15}\pi \\ \text{c } \cos\left(\frac{1}{4}\pi t\right) &= -\sin\left(\frac{2}{5}\pi t\right) \\ \sin\left(\frac{1}{4}\pi t + \frac{1}{2}\pi\right) &= \sin\left(\frac{2}{5}\pi t + \pi\right) \\ \frac{1}{4}\pi t + \frac{1}{2}\pi &= \frac{2}{5}\pi t + \pi + k \cdot 2\pi \vee \frac{1}{4}\pi t + \frac{1}{2}\pi = \pi - \frac{2}{5}\pi t - \pi + k \cdot 2\pi \\ -\frac{3}{20}\pi t &= \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{13}{20}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \\ t &= -3\frac{1}{3} + k \cdot 13\frac{1}{3} \vee t = -\frac{10}{13} + k \cdot 3\frac{1}{13} \\ t \text{ in } [0, 10] \text{ geeft } t &= 10 \vee t = 2\frac{4}{13} \vee t = 5\frac{5}{13} \vee t = 8\frac{6}{13} \\ \text{d } \cos\left(\frac{1}{3}\pi x\right) &= \sqrt{3} \cdot \sin\left(\frac{1}{3}\pi x\right) \\ \frac{\sin\left(\frac{1}{3}\pi x\right)}{\cos\left(\frac{1}{3}\pi x\right)} &= \frac{1}{\sqrt{3}} \\ \tan\left(\frac{1}{3}\pi x\right) &= \frac{1}{3}\sqrt{3} \\ \frac{1}{3}\pi x &= \frac{1}{6}\pi + k \cdot \pi \\ x &= \frac{1}{2} + k \cdot 3 \\ x \text{ in } [0, 10] \text{ geeft } x &= \frac{1}{2} \vee x = 3\frac{1}{2} \vee x = 6\frac{1}{2} \vee x = 9\frac{1}{2} \end{aligned}$$

- 6** **a** $2 \sin(x) = \sin(x)$
 $\sin(x) = 0$
b $\sin(2x) = \sin(x)$
 $2x = x + k \cdot 2\pi \vee 2x = \pi - x + k \cdot 2\pi$
c $2 \sin(x) = \cos(x)$ is niet exact op te lossen.
d $2 \sin(x) = \sin(x + \frac{1}{3}\pi)$ is niet exact op te lossen.
e $\sin(2x) = \sin(x + \frac{1}{3}\pi)$
 $2x = x + \frac{1}{3}\pi + k \cdot 2\pi \vee 2x = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$
f $5 \sin(x) = \sin(5x)$ is niet exact op te lossen.

- 7** **a** $f(x) = -\frac{1}{2}\sqrt{2}$ geeft $\sin(x) = -\frac{1}{2}\sqrt{2}$
 $x = \frac{1}{4}\pi + k \cdot 2\pi \vee x = -\frac{1}{4}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{4}\pi \vee x = \frac{3}{4}\pi$
b $g(x) = \frac{1}{2}$ geeft $-\cos(2x) = \frac{1}{2}$
 $\cos(2x) = -\frac{1}{2}$
 $2x = \frac{2}{3}\pi + k \cdot 2\pi \vee 2x = -\frac{2}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \pi \vee x = -\frac{1}{3}\pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = \frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = \frac{4}{3}\pi$
c $f(x) = g(x)$ geeft $\sin(x) = -\cos(2x)$
 $\cos(x - \frac{1}{2}\pi) = \cos(2x + \pi)$
 $x - \frac{1}{2}\pi = 2x + \pi + k \cdot 2\pi \vee x - \frac{1}{2}\pi = -2x - \pi + k \cdot 2\pi$
 $-x = \frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = -\frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$
 $f(x) \leq g(x)$ geeft $x = \frac{1}{2}\pi \vee \frac{1}{6}\pi \leq x \leq \frac{5}{6}\pi$

- 8** $f(x) = g(x)$ geeft $\sin(2x - \frac{1}{3}\pi) = -\cos(x - \frac{1}{6}\pi)$
 $\cos(2x - \frac{5}{6}\pi) = \cos(x + \frac{5}{6}\pi)$
 $2x - \frac{5}{6}\pi = x + \frac{5}{6}\pi + k \cdot 2\pi \vee 2x - \frac{5}{6}\pi = -x - \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{2}{3}\pi + k \cdot 2\pi \vee 3x = k \cdot 2\pi$
 $x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot \frac{2}{3}\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi \vee x = 0 \vee x = \frac{2}{3}\pi \vee x = \frac{1}{3}\pi \vee x = 2\pi$
 $O(V) = \int_0^{\frac{2}{3}\pi} (f(x) - g(x)) dx = \int_0^{\frac{2}{3}\pi} ((\sin(2x - \frac{1}{3}\pi) + \cos(x - \frac{1}{6}\pi))) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) + \sin(x - \frac{1}{6}\pi)\right]_0^{\frac{2}{3}\pi}$
 $= -\frac{1}{2}\cos(\pi) + \sin(\frac{1}{2}\pi) - (-\frac{1}{2}\cos(-\frac{1}{3}\pi) + \sin(-\frac{1}{6}\pi)) = \frac{1}{2} + 1 - (-\frac{1}{4} - \frac{1}{2}) = 2\frac{1}{4}$
 $O(W) = \int_{\frac{1}{3}\pi}^{\frac{2}{3}\pi} (f(x) - g(x)) dx = \int_{\frac{1}{3}\pi}^{\frac{2}{3}\pi} ((\sin(2x - \frac{1}{3}\pi) + \cos(x - \frac{1}{6}\pi))) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) + \sin(x - \frac{1}{6}\pi)\right]_{\frac{1}{3}\pi}^{\frac{2}{3}\pi}$
 $= -\frac{1}{2}\cos(\pi) + \sin(\frac{1}{2}\pi) - (-\frac{1}{2}\cos(\frac{2}{3}\pi) + \sin(\frac{1}{6}\pi)) = \frac{1}{2} - 1 - (-\frac{1}{4} - \frac{1}{2}) = \frac{1}{4}$
De oppervlakte van V is $\frac{2\frac{1}{4}}{\frac{1}{4}} = 9$ keer zo groot als de oppervlakte van W .

9 $f(x) = \frac{1}{2}$ geeft $\sin(2x - \frac{1}{3}\pi) = \frac{1}{2}$

$$2x - \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$2x = \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = \frac{1}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{4}\pi + k \cdot \pi \vee x = \frac{7}{12}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{4}\pi \vee x = \frac{1}{4}\pi + \pi \vee x = \frac{7}{12}\pi \vee x = \frac{7}{12}\pi + \pi$$

Uit de figuur volgt dat het deel boven de lijn $y = \frac{1}{2}$ het kleinste deel is.

$$\begin{aligned} \text{Dus } O &= \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (f(x) - \frac{1}{2}) dx = \int_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} (\sin(2x - \frac{1}{3}\pi) - \frac{1}{2}) dx = \left[-\frac{1}{2}\cos(2x - \frac{1}{3}\pi) - \frac{1}{2}x \right]_{\frac{1}{4}\pi}^{\frac{7}{12}\pi} \\ &= -\frac{1}{2}\cos(\frac{5}{6}\pi) - \frac{7}{24}\pi - (-\frac{1}{2}\cos(\frac{1}{6}\pi) - \frac{1}{8}\pi) = \frac{1}{4}\sqrt{3} - \frac{7}{24}\pi + \frac{1}{4}\sqrt{3} + \frac{1}{8}\pi = \frac{1}{2}\sqrt{3} - \frac{1}{6}\pi. \end{aligned}$$

10 Er geldt $\vec{a} = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$, $\vec{b} = \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}$ en $\angle(\vec{a}, \vec{b}) = \alpha - \beta$.

$$\cos(\alpha - \beta) = \cos(\angle(\vec{a}, \vec{b})) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{\begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \cdot \begin{pmatrix} \cos(\beta) \\ \sin(\beta) \end{pmatrix}}{1 \cdot 1} = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

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11 a $\cos(t - u) = \cos(t)\cos(u) + \sin(t)\sin(u)$

u vervangen door $-u$ geeft

$$\cos(t + u) = \cos(t)\cos(-u) + \sin(t)\sin(-u)$$

$$\cos(t + u) = \cos(t)\cos(u) + \sin(t) \cdot -\sin(u)$$

$$\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$$

b $\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$

u vervangen door $u - \frac{1}{2}\pi$ geeft

$$\cos(t + u - \frac{1}{2}\pi) = \cos(t)\cos(u - \frac{1}{2}\pi) + \sin(t) \cdot -\sin(u - \frac{1}{2}\pi)$$

$$\sin(t + u) = \cos(t)\sin(u) + \sin(t)\sin(u + \frac{1}{2}\pi)$$

$$\sin(t + u) = \cos(t)\sin(u) + \sin(t)\cos(u)$$

$$\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$$

c $\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$

u vervangen door $-u$ geeft

$$\sin(t - u) = \sin(t)\cos(-u) + \cos(t)\sin(-u)$$

$$\sin(t - u) = \sin(t)\cos(u) + \cos(t) \cdot -\sin(u)$$

$$\sin(t - u) = \sin(t)\cos(u) - \cos(t)\sin(u)$$

12 a $\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$

t en u beide vervangen door A geeft

$$\sin(A + A) = \sin(A)\cos(A) + \cos(A)\sin(A)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

b $\cos(2A) = \cos^2(A) - \sin^2(A)$

$$\cos(2A) = \cos^2(A) - (1 - \cos^2(A))$$

$$\cos(2A) = \cos^2(A) - 1 + \cos^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1$$

$$\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$$

t en u beide vervangen door A geeft

$$\cos(A + A) = \cos(A)\cos(A) - \sin(A)\sin(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\cos(2A) = 1 - \sin^2(A) - \sin^2(A)$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

13 a $\sin(x - \frac{1}{4}\pi) = \sin(x)\cos(\frac{1}{4}\pi) - \cos(x)\sin(\frac{1}{4}\pi) = \sin(x) \cdot \frac{1}{2}\sqrt{2} - \cos(x) \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}(\sin(x) - \cos(x))$

b $\sin(\frac{1}{12}\pi) = \sin(\frac{1}{3}\pi - \frac{1}{4}\pi) = \sin(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi) - \cos(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

c $\sin(x + \frac{1}{4}\pi) = \sin(x)\cos(\frac{1}{4}\pi) + \cos(x)\sin(\frac{1}{4}\pi) = \sin(x) \cdot \frac{1}{2}\sqrt{2} + \cos(x) \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}(\sin(x) + \cos(x))$

d $\sin(\frac{7}{12}\pi) = \sin(\frac{1}{3}\pi + \frac{1}{4}\pi) = \sin(\frac{1}{3}\pi)\cos(\frac{1}{4}\pi) + \cos(\frac{1}{3}\pi)\sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2} \cdot \frac{1}{2}\sqrt{2}$
 $= \frac{1}{4}\sqrt{6} + \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$

$$14 \quad \mathbf{a} \quad \cos\left(\frac{1}{12}\pi\right) = \cos\left(\frac{1}{3}\pi - \frac{1}{4}\pi\right) = \cos\left(\frac{1}{3}\pi\right)\cos\left(\frac{1}{4}\pi\right) + \sin\left(\frac{1}{3}\pi\right)\sin\left(\frac{1}{4}\pi\right) = \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{2} \\ = \frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$$

$$\mathbf{b} \quad \cos\left(2 \cdot \frac{1}{8}\pi\right) = 1 - 2\sin^2\left(\frac{1}{8}\pi\right)$$

$$2\sin^2\left(\frac{1}{8}\pi\right) = 1 - \cos\left(\frac{1}{4}\pi\right)$$

$$\sin^2\left(\frac{1}{8}\pi\right) = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{1}{4}\pi\right) = \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2} - \frac{1}{4}\sqrt{2}$$

$$\sin\left(\frac{1}{8}\pi\right) = \sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}} \vee \sin\left(\frac{1}{8}\pi\right) = -\sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}}$$

$$\sin\left(\frac{1}{8}\pi\right) = -\sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}} \text{ voldoet niet, want } \sin\left(\frac{1}{8}\pi\right) \text{ is positief.}$$

$$\text{Dus } \sin\left(\frac{1}{8}\pi\right) = \sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2}} = \sqrt{\frac{1}{4}(2 - \sqrt{2})} = \frac{1}{2}\sqrt{2 - \sqrt{2}}.$$

$$15 \quad \mathbf{a} \quad y = \cos(4x)\cos(x) + \sin(4x)\sin(x) = \cos(4x - x) = \cos(3x)$$

$$\mathbf{b} \quad y = \sin\left(\frac{1}{2}x\right)\cos(x) - \cos\left(\frac{1}{2}x\right)\sin(x) = \sin\left(\frac{1}{2}x - x\right) = \sin\left(-\frac{1}{2}x\right)$$

$$\mathbf{c} \quad y = \cos\left(\frac{3}{4}x\right)\cos\left(\frac{1}{4}x\right) - \sin\left(\frac{3}{4}x\right)\sin\left(\frac{1}{4}x\right) = \cos\left(\frac{3}{4}x + \frac{1}{4}x\right) = \cos(x)$$

$$\mathbf{d} \quad y = \sin\left(\frac{1}{3}x\right)\cos\left(\frac{1}{3}x\right) + \cos\left(\frac{1}{3}x\right)\sin\left(\frac{1}{3}x\right) = \sin\left(\frac{1}{3}x + \frac{1}{3}x\right) = \sin\left(\frac{2}{3}x\right)$$

$$16 \quad y = \sin(x)(\sin(4x) - \sin(2x)) + \cos(x)(\cos(4x) + \cos(2x)) \\ = \sin(x)\sin(4x) - \sin(x)\sin(2x) + \cos(x)\cos(4x) + \cos(x)\cos(2x) \\ = \cos(4x)\cos(x) + \sin(4x)\sin(x) + \cos(x)\cos(2x) - \sin(x)\sin(2x) \\ = \cos(4x - x) + \cos(x + 2x) \\ = \cos(3x) + \cos(3x) \\ = 2\cos(3x)$$

$$17 \quad \mathbf{a} \quad \cos(2A) = 2\cos^2(A) - 1 \\ 2\cos^2(A) = 1 + \cos(2A) \\ \cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$$

$$\mathbf{b} \quad \cos(2A) = 1 - 2\sin^2(A) \\ 2\sin^2(A) = 1 - \cos(2A) \\ \sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$$

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$$18 \quad \mathbf{a} \quad 2\sin(x)\cos(x) = \sin(2x) \\ \sin(2x) = \sin(x - 1) \\ 2x = x - 1 + k \cdot 2\pi \vee 2x = \pi - x + 1 + k \cdot 2\pi \\ x = -1 + k \cdot 2\pi \vee 3x = \pi + 1 + k \cdot 2\pi \\ x = -1 + k \cdot 2\pi \vee x = \frac{1}{3}\pi + \frac{1}{3} + k \cdot \frac{2}{3}\pi$$

$$\mathbf{b} \quad \cos^2(2x) = \cos(4x) + \frac{1}{2} \\ \frac{1}{2} + \frac{1}{2}\cos(4x) = \cos(4x) + \frac{1}{2} \\ \frac{1}{2}\cos(4x) = 0 \\ \cos(4x) = 0$$

$$4x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = \frac{1}{8}\pi + k \cdot \frac{1}{4}\pi$$

$$\mathbf{c} \quad \sin^2\left(\frac{1}{2}x\right) = \cos(x) + \frac{1}{4}$$

$$\frac{1}{2} - \frac{1}{2}\cos(x) = \cos(x) + \frac{1}{4}$$

$$-1\frac{1}{2}\cos(x) = \frac{3}{4}$$

$$\cos(x) = -\frac{1}{2}$$

$$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$$

$$\mathbf{d} \quad (\sin(x) + \cos(x))^2 = 1\frac{1}{2}$$

$$\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) = 1\frac{1}{2}$$

$$\sin^2(x) + \cos^2(x) + \sin(2x) = 1\frac{1}{2}$$

$$1 + \sin(2x) = 1\frac{1}{2}$$

$$\sin(2x) = \frac{1}{2}$$

$$2x = \frac{1}{6}\pi + k \cdot 2\pi \vee 2x = \frac{5}{6}\pi + k \cdot 2\pi$$

$$x = \frac{1}{12}\pi + k \cdot \pi \vee x = \frac{5}{12}\pi + k \cdot \pi$$

19 a De evenwichtsstand is $\frac{1+0}{2} = \frac{1}{2}$, dus $a = \frac{1}{2}$.

De amplitude is $1 - \frac{1}{2} = \frac{1}{2}$, dus $b = \frac{1}{2}$.

De periode is π , dus $c = \frac{2\pi}{\pi} = 2$.

Dus $y = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

b $y = \sin^2(x) + \cos(2x) = \frac{1}{2} - \frac{1}{2}\cos(2x) + \cos(2x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

20 $\sin(3x) = \sin(2x + x)$
 $= \sin(2x)\cos(x) + \cos(2x)\sin(x)$
 $= 2\sin(x)\cos(x)\cos(x) + (1 - 2\sin^2(x)) \cdot \sin(x)$
 $= 2\sin(x)\cos^2(x) + \sin(x) - 2\sin^3(x)$
 $= 2\sin(x)(1 - \sin^2(x)) + \sin(x) - 2\sin^3(x)$
 $= 2\sin(x) - 2\sin^3(x) + \sin(x) - 2\sin^3(x)$
 $= 3\sin(x) - 4\sin^3(x)$

21 a $y = 1 - \cos(x) - \sin^2(\frac{1}{2}x)$
 $= 1 - \cos(x) - (\frac{1}{2} - \frac{1}{2}\cos(x))$
 $= 1 - \cos(x) - \frac{1}{2} + \frac{1}{2}\cos(x)$
 $= \frac{1}{2} - \frac{1}{2}\cos(x)$

b $\cos(3x) = \cos(2x + x)$
 $= \cos(2x)\cos(x) - \sin(2x)\sin(x)$
 $= (2\cos^2(x) - 1) \cdot \cos(x) - 2\sin(x)\cos(x)\sin(x)$
 $= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$
 $= 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x)) \cdot \cos(x)$
 $= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x)$
 $= 4\cos^3(x) - 3\cos(x)$

22 a $-\cos(A) = \cos(A + \pi)$

b $\sin(A) = \cos(A - \frac{1}{2}\pi)$

c $\cos(2A) = 1 - 2\sin^2(A)$ oftewel $\sin^2(A) = \frac{1}{2} - \frac{1}{2}\cos(2A)$

d $\cos(2A) = 2\cos^2(A) - 1$ oftewel $\cos^2(A) = \frac{1}{2} + \frac{1}{2}\cos(2A)$

e $\cos(2A) = 1 - 2\sin^2(A)$

Bladzijde 150

C1 a $\sin(4x) = -\cos(x)$
 $\cos(4x - \frac{1}{2}\pi) = \cos(x + \pi)$
 $4x - \frac{1}{2}\pi = x + \pi + k \cdot 2\pi \vee 4x - \frac{1}{2}\pi = -x - \pi + k \cdot 2\pi$

$3x = 1\frac{1}{2}\pi + k \cdot 2\pi \vee 5x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi \vee x = -\frac{1}{10}\pi + k \cdot \frac{2}{5}\pi$

b $2\sin^2(x) + 7\cos(x) + 2 = 0$
 $2(1 - \cos^2(x)) + 7\cos(x) + 2 = 0$
 $2 - 2\cos^2(x) + 7\cos(x) + 2 = 0$
 $-2\cos^2(x) + 7\cos(x) + 4 = 0$

Stel $\cos(x) = u$.

$-2u^2 + 7u + 4 = 0$

$D = 7^2 - 4 \cdot -2 \cdot 4 = 81$, dus $\sqrt{D} = 9$

$u = \frac{-7 + 9}{-4} = -\frac{1}{2} \vee u = \frac{-7 - 9}{-4} = 4$

$\cos(x) = -\frac{1}{2} \vee \cos(x) = 4$

$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{2}{3}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi \vee x = 1\frac{1}{3}\pi$

a $y = \cos(2x) - 3 \sin^2(x)$
 $= \cos(2x) - 3\left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)$
 $= \cos(2x) - \frac{1}{2} + \frac{1}{2}\cos(2x)$
 $= -\frac{1}{2} + 2\frac{1}{2}\cos(2x)$

b $\sin(2x) = 2 \cos^2(x)$
 $2 \sin(x) \cos(x) = 2 \cos^2(x)$
 $\cos(x) = 0 \vee \sin(x) = \cos(x)$
 $\cos(x) = 0 \vee \frac{\sin(x)}{\cos(x)} = 1$
 $\cos(x) = 0 \vee \tan(x) = 1$
 $x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{4}\pi + k \cdot \pi$

12.2 Goniometrische formules bij symmetrie en primitiveren

Bladzijde 151

23 $B(-x_A, y_A)$ en $C(-x_A, -y_A)$

Bladzijde 153

24 a $f(x) = x \cos(x)$

$$f(-x) = -x \cos(-x) = -x \cos(x)$$

Er geldt $f(-x) = -f(x)$, dus de grafiek van f is symmetrisch in de oorsprong.

b $g(x) = x \sin(x)$

$$g(-x) = -x \sin(-x) = -x \cdot -\sin(x) = x \sin(x)$$

Er geldt $g(-x) = g(x)$, dus de grafiek van g is symmetrisch in de y -as.

25 a $f(x) = \cos^2(x) \sin(x)$

$$f(-x) = \cos^2(-x) \sin(-x) = \cos^2(x) \cdot -\sin(x) = -\cos^2(x) \sin(x)$$

Er geldt $f(-x) = -f(x)$, dus de grafiek van f is symmetrisch in de oorsprong.

b Translatie van de grafiek van f over $(-\frac{1}{2}\pi, 0)$ geeft de grafiek van $g(x) = \cos^2(x + \frac{1}{2}\pi) \sin(x + \frac{1}{2}\pi)$.

$$g(x) = \sin^2(x + \pi) \cos(x) = (-\sin(x))^2 \cos(x) = \sin^2(x) \cos(x)$$

$$g(-x) = \sin^2(-x) \cos(-x) = (-\sin(x))^2 \cos(x) = \sin^2(x) \cos(x)$$

Er geldt $g(-x) = g(x)$, dus de grafiek van g is symmetrisch in de y -as en hieruit volgt dat de grafiek van f symmetrisch is in de lijn $x = \frac{1}{2}\pi$.

26 a Translatie van de grafiek van f over $(-\frac{1}{4}\pi, 0)$ geeft de grafiek van

$$g(x) = \cos(x + \frac{1}{4}\pi) + \sin(x + \frac{1}{4}\pi) + 1.$$

$$g(x) = \cos(x) \cos(\frac{1}{4}\pi) - \sin(x) \sin(\frac{1}{4}\pi) + \sin(x) \cos(\frac{1}{4}\pi) + \cos(x) \sin(\frac{1}{4}\pi) + 1$$

$$= \frac{1}{2}\sqrt{2} \cos(x) - \frac{1}{2}\sqrt{2} \sin(x) + \frac{1}{2}\sqrt{2} \sin(x) + \frac{1}{2}\sqrt{2} \cos(x) + 1$$

$$= \sqrt{2} \cos(x) + 1$$

$$g(-x) = \sqrt{2} \cos(-x) + 1 = \sqrt{2} \cos(x) + 1$$

Er geldt $g(-x) = g(x)$, dus de grafiek van g is symmetrisch in de y -as en hieruit volgt dat de grafiek van f symmetrisch is in de lijn $x = \frac{1}{4}\pi$.

b Translatie van de grafiek van f over $(-\frac{3}{4}\pi, -1)$ geeft de grafiek van

$$h(x) = \cos(x + \frac{3}{4}\pi) + \sin(x + \frac{3}{4}\pi).$$

$$h(x) = \cos(x) \cos(\frac{3}{4}\pi) - \sin(x) \sin(\frac{3}{4}\pi) + \sin(x) \cos(\frac{3}{4}\pi) + \cos(x) \sin(\frac{3}{4}\pi)$$

$$= -\frac{1}{2}\sqrt{2} \cos(x) - \frac{1}{2}\sqrt{2} \sin(x) - \frac{1}{2}\sqrt{2} \sin(x) + \frac{1}{2}\sqrt{2} \cos(x)$$

$$= -\sqrt{2} \sin(x)$$

$$h(-x) = -\sqrt{2} \sin(-x) = \sqrt{2} \sin(x)$$

Er geldt $h(-x) = -h(x)$, dus de grafiek van h is symmetrisch in de oorsprong en hieruit volgt dat de grafiek van f symmetrisch is in het punt $(\frac{3}{4}\pi, 1)$.

27 Translatie van de grafiek van f over $(-\frac{1}{6}\pi, 0)$ geeft de grafiek van

$$g(x) = \sin(x + \frac{1}{6}\pi) + \sqrt{3} \cdot \cos(x + \frac{1}{6}\pi).$$

$$\begin{aligned} g(x) &= \sin(x)\cos(\frac{1}{6}\pi) + \cos(x)\sin(\frac{1}{6}\pi) + \sqrt{3}(\cos(x)\cos(\frac{1}{6}\pi) - \sin(x)\sin(\frac{1}{6}\pi)) \\ &= \frac{1}{2}\sqrt{3}\sin(x) + \frac{1}{2}\cos(x) + \sqrt{3}(\frac{1}{2}\sqrt{3}\cos(x) - \frac{1}{2}\sin(x)) \\ &= \frac{1}{2}\sqrt{3}\sin(x) + \frac{1}{2}\cos(x) + \frac{3}{2}\cos(x) - \frac{1}{2}\sqrt{3}\sin(x) \\ &= 2\cos(x) \end{aligned}$$

$$g(-x) = 2\cos(-x) = 2\cos(x)$$

Er geldt $g(-x) = g(x)$, dus de grafiek van g is symmetrisch in de y -as en hieruit volgt dat de grafiek van f symmetrisch is in de lijn $x = \frac{1}{6}\pi$.

28 a $y = \frac{1}{3}\sin^3(x)$ geeft $\frac{dy}{dx} = \sin^2(x) \cdot \cos(x)$ en dit is niet $f(x)$,
dus $y = \frac{1}{3}\sin^3(x)$ is geen primitieve van f .

b $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

c $f(x) = \sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x - \frac{1}{4}\sin(2x) + c$

Bladzijde 154

29 a $f(x) = \cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$ geeft $F(x) = \frac{1}{2}x + \frac{1}{4}\sin(2x) + c$

b $g(x) = \sin^2(3x) = \frac{1}{2} - \frac{1}{2}\cos(6x)$ geeft $G(x) = \frac{1}{2}x - \frac{1}{12}\sin(6x) + c$

c $h(x) = \sin(\frac{1}{2}x)\cos(\frac{1}{2}x) = \frac{1}{2}\sin(x)$ geeft $H(x) = -\frac{1}{2}\cos(x) + c$

30 a $\int_0^{\frac{1}{6}\pi} \sin(2x)\cos(2x) dx = \int_0^{\frac{1}{6}\pi} \frac{1}{2}\sin(4x) dx = \left[-\frac{1}{8}\cos(4x)\right]_0^{\frac{1}{6}\pi}$
 $= -\frac{1}{8}\cos(\frac{2}{3}\pi) + \frac{1}{8}\cos(0) = -\frac{1}{8} \cdot -\frac{1}{2} + \frac{1}{8} \cdot 1 = \frac{3}{16}$

b $\int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{2}\sin^2(x)) dx = \int_{\frac{1}{3}\pi}^{\pi} (2 - \frac{1}{4} + \frac{1}{4}\cos(2x)) dx = \int_{\frac{1}{3}\pi}^{\pi} (1\frac{3}{4} + \frac{1}{4}\cos(2x)) dx$
 $= \left[1\frac{3}{4}x + \frac{1}{8}\sin(2x)\right]_{\frac{1}{3}\pi}^{\pi} = 1\frac{3}{4}\pi + \frac{1}{8}\sin(2\pi) - (1\frac{3}{4} \cdot \frac{1}{3}\pi + \frac{1}{8}\sin(\frac{2}{3}\pi))$
 $= 1\frac{3}{4}\pi + 0 - (\frac{7}{12}\pi + \frac{1}{8} \cdot \frac{1}{2}\sqrt{3}) = 1\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$

31 $f(x) = 0$ geeft $\sin(2x) = 0$

$$2x = k \cdot \pi$$

$$x = k \cdot \frac{1}{2}\pi$$

$$x \text{ in } [0, \frac{1}{2}\pi] \text{ geeft } x = 0 \vee x = \frac{1}{2}\pi$$

$$\begin{aligned} I(L) &= \pi \int_0^{\frac{1}{2}\pi} (f(x))^2 dx = \pi \int_0^{\frac{1}{2}\pi} \sin^2(2x) dx = \pi \int_0^{\frac{1}{2}\pi} (\frac{1}{2} - \frac{1}{2}\cos(4x)) dx = \pi \left[\frac{1}{2}x - \frac{1}{8}\sin(4x)\right]_0^{\frac{1}{2}\pi} \\ &= \pi(\frac{1}{4}\pi - \frac{1}{8}\sin(2\pi) - (0 - \frac{1}{8}\sin(0))) = \frac{1}{4}\pi^2 \end{aligned}$$

32 $f(x) = 0$ geeft $\sin^2(x) + \sin(x) + \frac{1}{4} = 0$

$$(\sin(x) + \frac{1}{2})^2 = 0$$

$$\sin(x) = -\frac{1}{2}$$

$$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$$

$$\begin{aligned} O(V) &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\sin^2(x) + \sin(x) + \frac{1}{4}) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\frac{1}{2} - \frac{1}{2}\cos(2x) + \sin(x) + \frac{1}{4}) dx \\ &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (\frac{3}{4} - \frac{1}{2}\cos(2x) + \sin(x)) dx = [\frac{3}{4}x - \frac{1}{4}\sin(2x) - \cos(x)]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \\ &= \frac{11}{8}\pi - \frac{1}{4}\sin(\frac{5}{3}\pi) - \cos(\frac{5}{6}\pi) - (\frac{7}{8}\pi - \frac{1}{4}\sin(\frac{1}{3}\pi) - \cos(\frac{1}{6}\pi)) \\ &= \frac{11}{8}\pi - \frac{1}{4} \cdot -\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3} - (\frac{7}{8}\pi - \frac{1}{4} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}) \\ &= \frac{11}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} - \frac{7}{8}\pi + \frac{1}{8}\sqrt{3} - \frac{1}{2}\sqrt{3} \\ &= \frac{1}{2}\pi - \frac{3}{4}\sqrt{3} \end{aligned}$$

33 a $f(x) = 2\sin^2(x) + \sin(x) - 1$ geeft $f'(x) = 4\sin(x)\cos(x) + \cos(x)$

$$f'(x) = 0 \text{ geeft } 4\sin(x)\cos(x) + \cos(x) = 0$$

$$\cos(x)(4\sin(x) + 1) = 0$$

$$\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$$

$$\cos(x) = 0 \text{ geeft } x = \frac{1}{2}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi$$

$$f(\frac{1}{2}\pi) = 2\sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) - 1 = 2 \cdot 1^2 + 1 - 1 = 2$$

$$f(\frac{3}{2}\pi) = 2\sin^2(\frac{3}{2}\pi) + \sin(\frac{3}{2}\pi) - 1 = 2 \cdot (-1)^2 + -1 - 1 = 0$$

$$\sin(x) = -\frac{1}{4} \text{ geeft } f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} - 1 = -\frac{1}{8}$$

$$\text{Dus } B_f = [-\frac{1}{8}, 2].$$

b $f(x) = 0$ geeft $2\sin^2(x) + \sin(x) - 1 = 0$

$$\text{Stel } \sin(x) = u.$$

$$2u^2 + u - 1 = 0$$

$$D = 1^2 - 4 \cdot 2 \cdot -1 = 9, \text{ dus } \sqrt{D} = 3$$

$$u = \frac{-1+3}{4} = \frac{1}{2} \vee u = \frac{-1-3}{4} = -1$$

$$\sin(x) = \frac{1}{2} \vee \sin(x) = -1$$

$$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = \frac{3}{2}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = \frac{3}{2}\pi$$

$$\begin{aligned} O(V) &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (2\sin^2(x) + \sin(x) - 1) dx = \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (1 - \cos(2x) + \sin(x) - 1) dx \\ &= \int_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} (-\cos(2x) + \sin(x)) dx = [-\frac{1}{2}\sin(2x) - \cos(x)]_{\frac{1}{6}\pi}^{\frac{5}{6}\pi} \\ &= -\frac{1}{2}\sin(\frac{5}{3}\pi) - \cos(\frac{5}{6}\pi) - (-\frac{1}{2}\sin(\frac{1}{3}\pi) - \cos(\frac{1}{6}\pi)) \\ &= -\frac{1}{2} \cdot -\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3} - (-\frac{1}{2} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}) \\ &= \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} = \frac{3}{2}\sqrt{3} \end{aligned}$$

34 a $F(x) = \frac{1}{3}\cos^3(x) - \cos(x)$ geeft $F'(x) = \cos^2(x) \cdot -\sin(x) + \sin(x) = -\sin(x)(1 - \sin^2(x)) + \sin(x)$
 $= -\sin(x) + \sin^3(x) + \sin(x) = \sin^3(x)$

Er geldt $F'(x) = f(x)$, dus F is een primitieve van f .

b $f(x) = 0$ geeft $\sin^3(x) = 0$

$$\sin(x) = 0$$

$$x = k \cdot \pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = 0 \vee x = \pi$$

$$O(V) = \int_0^\pi \sin^3(x) dx = \left[\frac{1}{3}\cos^3(x) - \cos(x) \right]_0^\pi = \frac{1}{3}\cos^3(\pi) - \cos(\pi) - \left(\frac{1}{3}\cos^3(0) - \cos(0) \right)$$

$$= \frac{1}{3} \cdot (-1)^3 - (-1) - \left(\frac{1}{3} \cdot 1^3 - 1 \right) = -\frac{1}{3} + 1 - \frac{1}{3} + 1 = 1\frac{1}{3}$$

c Raken in A met $x_A = \frac{1}{3}\pi$, dus $f(\frac{1}{3}\pi) = g(\frac{1}{3}\pi) \wedge f'(\frac{1}{3}\pi) = g'(\frac{1}{3}\pi)$.

$$f(\frac{1}{3}\pi) = \sin^3(\frac{1}{3}\pi) = \left(\frac{1}{2}\sqrt{3}\right)^3 = \frac{3}{8}\sqrt{3}$$

$$g(\frac{1}{3}\pi) = a + b\cos(\frac{1}{3}\pi) = a + \frac{1}{2}b$$

$$f(x) = \sin^3(x) \text{ geeft } f'(x) = 3\sin^2(x)\cos(x)$$

$$g(x) = a + b\cos(x) \text{ geeft } g'(x) = -b\sin(x)$$

$$f'(\frac{1}{3}\pi) = 3\sin^2(\frac{1}{3}\pi)\cos(\frac{1}{3}\pi) = 3 \cdot \left(\frac{1}{2}\sqrt{3}\right)^2 \cdot \frac{1}{2} = 1\frac{1}{8}$$

$$g'(\frac{1}{3}\pi) = -b\sin(\frac{1}{3}\pi) = -b \cdot \frac{1}{2}\sqrt{3}$$

$$f'(\frac{1}{3}\pi) = g'(\frac{1}{3}\pi) \text{ geeft } -b \cdot \frac{1}{2}\sqrt{3} = 1\frac{1}{8}$$

$$b = -\frac{1\frac{1}{8}}{\frac{1}{2}\sqrt{3}} = -\frac{2\frac{1}{4}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\frac{1}{4}\sqrt{3}}{3} = -\frac{3}{4}\sqrt{3}$$

$$f(\frac{1}{3}\pi) = g(\frac{1}{3}\pi) \text{ geeft } a + \frac{1}{2}b = \frac{3}{8}\sqrt{3}$$

$$\text{Samen met } b = -\frac{3}{4}\sqrt{3} \text{ geeft dit } a + \frac{1}{2} \cdot -\frac{3}{4}\sqrt{3} = \frac{3}{8}\sqrt{3}$$

$$a - \frac{3}{8}\sqrt{3} = \frac{3}{8}\sqrt{3}$$

$$a = \frac{3}{4}\sqrt{3}$$

$$\text{Dus } a = \frac{3}{4}\sqrt{3} \text{ en } b = -\frac{3}{4}\sqrt{3}.$$

Bladzijde 155

35 $f_p(x) = 0$ geeft $p\sin(\frac{1}{3}x)\cos(\frac{1}{3}x) = 0$

$$\sin(\frac{1}{3}x) = 0 \vee \cos(\frac{1}{3}x) = 0$$

$$\frac{1}{3}x = k \cdot \pi \vee \frac{1}{3}x = \frac{1}{2}\pi + k \cdot \pi$$

$$x = k \cdot 3\pi \vee x = 1\frac{1}{2}\pi + k \cdot 3\pi$$

$$x \text{ in } [0, 1\frac{1}{2}\pi] \text{ geeft } x = 0 \vee x = 1\frac{1}{2}\pi$$

Dus op $[0, 1\frac{1}{2}\pi]$ geldt voor $p > 0$ dat $f_p(x) \geq 0$.

$$O(V_p) = \int_0^{1\frac{1}{2}\pi} p\sin(\frac{1}{3}x)\cos(\frac{1}{3}x) dx = \int_0^{1\frac{1}{2}\pi} \frac{1}{2}p\sin(\frac{2}{3}x) dx = \left[\frac{1}{2}p \cdot -\frac{1}{2}\cos(\frac{2}{3}x) \right]_0^{1\frac{1}{2}\pi} = -\frac{3}{4}p\cos(\pi) + \frac{3}{4}p\cos(0)$$

$$= \frac{3}{4}p + \frac{3}{4}p = 1\frac{1}{2}p$$

$$O(V_p) = 10 \text{ geeft } 1\frac{1}{2}p = 10$$

$$p = 6\frac{2}{3}$$

C3 a Translatie van de grafiek van f over $(-\frac{3}{4}\pi, 0)$ geeft de grafiek van

$$g(x) = \cos(x + \frac{3}{4}\pi) - \sin(x + \frac{3}{4}\pi).$$

$$\begin{aligned} g(x) &= \cos(x) \cos(\frac{3}{4}\pi) - \sin(x) \sin(\frac{3}{4}\pi) - (\sin(x) \cos(\frac{3}{4}\pi) + \cos(x) \sin(\frac{3}{4}\pi)) \\ &= -\frac{1}{2}\sqrt{2} \cos(x) - \frac{1}{2}\sqrt{2} \sin(x) - (-\frac{1}{2}\sqrt{2} \sin(x) + \frac{1}{2}\sqrt{2} \cos(x)) \\ &= -\frac{1}{2}\sqrt{2} \cos(x) - \frac{1}{2}\sqrt{2} \sin(x) + \frac{1}{2}\sqrt{2} \sin(x) - \frac{1}{2}\sqrt{2} \cos(x) \\ &= -\sqrt{2} \cos(x) \end{aligned}$$

$$g(-x) = -\sqrt{2} \cos(-x) = -\sqrt{2} \cos(x)$$

Er geldt $g(-x) = g(x)$, dus de grafiek van g is symmetrisch in de y -as en hieruit volgt dat de grafiek van f symmetrisch is in de lijn $x = \frac{3}{4}\pi$.

b Translatie van de grafiek van f over $(-\frac{1}{4}\pi, 0)$ geeft de grafiek van

$$h(x) = \cos(x + \frac{1}{4}\pi) - \sin(x + \frac{1}{4}\pi).$$

$$\begin{aligned} h(x) &= \cos(x) \cos(\frac{1}{4}\pi) - \sin(x) \sin(\frac{1}{4}\pi) - (\sin(x) \cos(\frac{1}{4}\pi) + \cos(x) \sin(\frac{1}{4}\pi)) \\ &= \frac{1}{2}\sqrt{2} \cos(x) - \frac{1}{2}\sqrt{2} \sin(x) - (\frac{1}{2}\sqrt{2} \sin(x) + \frac{1}{2}\sqrt{2} \cos(x)) \\ &= \frac{1}{2}\sqrt{2} \cos(x) - \frac{1}{2}\sqrt{2} \sin(x) - \frac{1}{2}\sqrt{2} \sin(x) - \frac{1}{2}\sqrt{2} \cos(x) \\ &= -\sqrt{2} \sin(x) \end{aligned}$$

$$h(-x) = -\sqrt{2} \sin(-x) = \sqrt{2} \sin(x)$$

Er geldt $h(-x) = -h(x)$, dus de grafiek van h is symmetrisch in de oorsprong en hieruit volgt dat de grafiek van f symmetrisch is in het punt $(\frac{1}{4}\pi, 0)$.

C4 $f(0) = 4 \sin^2(0) = 0$ en $f(\frac{2}{3}\pi) = 4 \sin^2(\pi) = 0$

$$\begin{aligned} O(V) &= \int_0^{\frac{2}{3}\pi} 4 \sin^2(\frac{1}{2}x) dx = \int_0^{\frac{2}{3}\pi} (4(\frac{1}{2} - \frac{1}{2} \cos(3x))) dx = \int_0^{\frac{2}{3}\pi} (2 - 2 \cos(3x)) dx \\ &= [2x - \frac{2}{3} \sin(3x)]_0^{\frac{2}{3}\pi} = 2 \cdot \frac{2}{3}\pi - \frac{2}{3} \sin(2\pi) - (0 - \frac{2}{3} \sin(0)) \\ &= 1\frac{1}{3}\pi \end{aligned}$$

12.3 Eenparige cirkelbewegingen en harmonische trillingen

Bladzijde 156

36 a P bevindt zich voor het eerst weer in $(1, 0)$ als t gelijk is aan 2π .

Dus na 2π seconden.

b $A(\cos(\frac{1}{3}\pi), \sin(\frac{1}{3}\pi)) = A(\frac{1}{2}, \frac{1}{2}\sqrt{3})$, $B(\cos(\frac{5}{6}\pi), \sin(\frac{5}{6}\pi)) = B(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$ en

$$AB = \sqrt{(-\frac{1}{2}\sqrt{3} - \frac{1}{2})^2 + (\frac{1}{2} - \frac{1}{2}\sqrt{3})^2} = \sqrt{\frac{3}{4} + \frac{1}{2}\sqrt{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{2}\sqrt{3} + \frac{3}{4}} = \sqrt{2}.$$

c Uit $\angle AOB = \frac{1}{2}\pi$ volgt dat $\triangle AOB$ een gelijkbenige rechthoekige driehoek is, dus de zijden van $\triangle AOB$ verhouden zich als $1 : 1 : \sqrt{2}$. Van de zijden OA en OB is de lengte gelijk aan de straal van de eenheidscirkel oftewel 1, en dus heeft zijde AB lengte $\sqrt{2}$.

Bladzijde 157

37 De formules $\sin^2(A) + \cos^2(A) = 1$ met $A = \frac{1}{2}t$ en met $A = t$

en $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ met $A = \frac{1}{2}t$ en $B = t$.

38 a $PQ = 1$ geeft $\sqrt{2 - 2\cos(\frac{1}{2}t)} = 1$

$$2 - 2\cos(\frac{1}{2}t) = 1$$

$$2\cos(\frac{1}{2}t) = 1$$

$$\cos(\frac{1}{2}t) = \frac{1}{2}$$

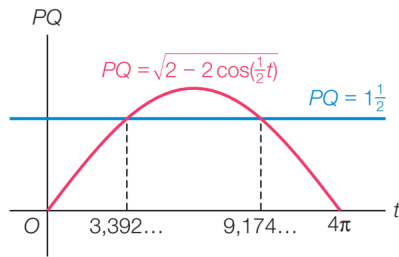
$$\frac{1}{2}t = \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{1}{2}t = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$t = \frac{2}{3}\pi + k \cdot 4\pi \vee t = -\frac{2}{3}\pi + k \cdot 4\pi$$

$$t \text{ in } [0, 4\pi] \text{ geeft } t = \frac{2}{3}\pi \vee t = 3\frac{1}{3}\pi$$

b Los op $\sqrt{2 - 2\cos(\frac{1}{2}t)} = 1\frac{1}{2}$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 3,392\dots$ en $t = 9,174\dots$



$PQ > 1\frac{1}{2}$ geeft $3,39 < t < 9,17$

c $PQ = \sqrt{2 - 2\cos(\frac{1}{2}t)} = \sqrt{2 - 2(1 - 2\sin^2(\frac{1}{4}t))} = \sqrt{2 - 2 + 4\sin^2(\frac{1}{4}t)} = \sqrt{4\sin^2(\frac{1}{4}t)} = 2|\sin(\frac{1}{4}t)|$

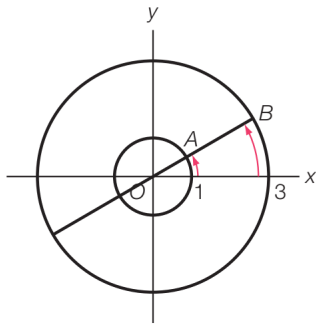
d Op $t = \frac{1}{6}\pi$ is $PQ = 2|\sin(\frac{1}{4} \cdot \frac{1}{6}\pi)| \approx 0,26$.

Bladzijde 158

39 a $t = \frac{1}{4}\pi$ geeft $A(\cos(\frac{1}{2}\pi), \sin(\frac{1}{2}\pi)) = A(0, 1)$ en $B(3\cos(\frac{1}{4}\pi), 3\sin(\frac{1}{4}\pi)) = B(1\frac{1}{2}\sqrt{2}, 1\frac{1}{2}\sqrt{2})$

$$AB = \sqrt{(1\frac{1}{2}\sqrt{2} - 0)^2 + (1\frac{1}{2}\sqrt{2} - 1)^2} = \sqrt{4\frac{1}{2} + 4\frac{1}{2} - 3\sqrt{2} + 1} = \sqrt{10 - 3\sqrt{2}}$$

b



De minimale afstand tussen A en B is $3 - 1 = 2$ en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B.

Dit geeft $2t = t + k \cdot 2\pi$

$$t = k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = 0 \vee t = 2\pi$$

De maximale afstand tussen A en B is $3 + 1 = 4$ en die ontstaat als de draaiingshoek van A gelijk is aan de draaiingshoek van B plus π .

Dit geeft $2t = t + \pi + k \cdot 2\pi$

$$t = \pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \pi$$

c $AB^2 = (3\cos(t) - \cos(2t))^2 + (3\sin(t) - \sin(2t))^2$
 $= 9\cos^2(t) - 6\cos(t)\cos(2t) + \cos^2(2t) + 9\sin^2(t) - 6\sin(t)\sin(2t) + \sin^2(2t)$
 $= 9 + 1 - 6(\cos(2t)\cos(t) + \sin(2t)\sin(t))$
 $= 10 - 6\cos(2t - t)$
 $= 10 - 6\cos(t)$

Dus $AB = \sqrt{10 - 6\cos(t)}$.

d $AB = 7$ geeft $\sqrt{10 - 6\cos(t)} = \sqrt{7}$

$$10 - 6\cos(t) = 7$$

$$6\cos(t) = 3$$

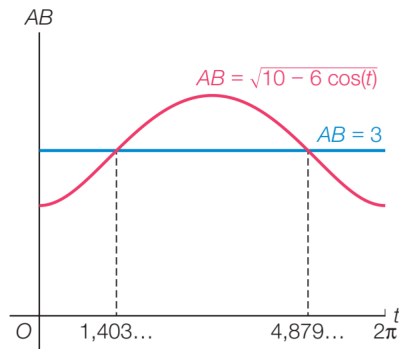
$$\cos(t) = \frac{1}{2}$$

$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{3}\pi \vee t = \frac{5}{3}\pi$$

e Los op $\sqrt{10 - 6 \cos(t)} = 3$.

Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t = 1,403\dots$ en $t = 4,879\dots$



$AB < 3$ geeft $0 \leq t < 1,40 \vee 4,88 < t \leq 2\pi$

40 a $y = \frac{1}{2}\sqrt{2}$ geeft op de eenheidscirkel punten met draaiingshoek $\frac{1}{4}\pi$ en draaiingshoek $\frac{3}{4}\pi$. Tussen deze twee punten ligt een kwart van de eenheidscirkel. De lengte van dit deel van de baan is dus $\frac{1}{4} \cdot 2\pi \cdot 1 = \frac{1}{2}\pi$.

Verder hebben deze punten x -coördinaten $\frac{1}{2}\sqrt{2}$ en $-\frac{1}{2}\sqrt{2}$, dus $PQ = \sqrt{2}$.

b A op $y = \frac{2}{3}x^2$ geeft $\sin(2t) = \frac{2}{3}\cos^2(2t)$

$$\sin(2t) = \frac{2}{3}(1 - \sin^2(2t))$$

$$\sin(2t) = \frac{2}{3} - \frac{2}{3}\sin^2(2t)$$

$$1\frac{1}{2}\sin(2t) = 1 - \sin^2(2t)$$

$$\sin^2(2t) + 1\frac{1}{2}\sin(2t) - 1 = 0$$

$$(\sin(2t) - \frac{1}{2})(\sin(2t) + 2) = 0$$

$$\sin(2t) = \frac{1}{2} \vee \sin(2t) = -2$$

$$2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{12}\pi \vee t = 1\frac{1}{12}\pi \vee t = \frac{5}{12}\pi \vee t = 1\frac{5}{12}\pi$$

$$t = \frac{1}{12}\pi \text{ en } t = 1\frac{1}{12}\pi \text{ geeft } R(\cos(\frac{1}{6}\pi), \sin(\frac{1}{6}\pi)) = R(\frac{1}{2}\sqrt{3}, \frac{1}{2})$$

$$t = \frac{5}{12}\pi \text{ en } t = 1\frac{5}{12}\pi \text{ geeft } S(\cos(\frac{5}{6}\pi), \sin(\frac{5}{6}\pi)) = S(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$$

$$\text{Dus } RS = \frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3}) = \sqrt{3}.$$

41 a Substitutie van $x = 2 \cos(t)$ en $y = 2 \sin(t)$ in $y = \frac{1}{3}x\sqrt{3}$ geeft $2 \sin(t) = \frac{1}{3} \cdot 2 \cos(t) \cdot \sqrt{3}$

$$\frac{\sin(t)}{\cos(t)} = \frac{1}{3}\sqrt{3}$$

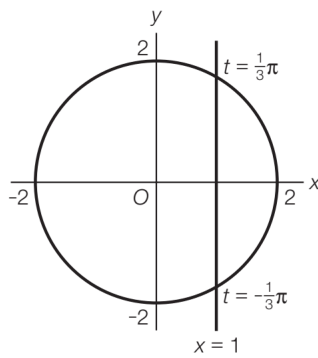
$$\tan(t) = \frac{1}{3}\sqrt{3}$$

$$t = \frac{1}{6}\pi + k \cdot \pi$$

$$t = \frac{1}{6}\pi \text{ geeft het snijpunt } (2 \cos(\frac{1}{6}\pi), 2 \sin(\frac{1}{6}\pi)) = (\sqrt{3}, 1).$$

$$t = 1\frac{1}{6}\pi \text{ geeft het snijpunt } (2 \cos(1\frac{1}{6}\pi), 2 \sin(1\frac{1}{6}\pi)) = (-\sqrt{3}, -1).$$

- b** $x = 1$ geeft $2 \cos(t) = 1$
 $\cos(t) = \frac{1}{2}$
 $t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$



De baan ligt rechts van de lijn $x = 1$ als $2 \cos(t) > 1$.

Dit geeft $-\frac{1}{3}\pi < t < \frac{1}{3}\pi$.

De baan is de cirkel met middelpunt $(0, 0)$ en straal 2.

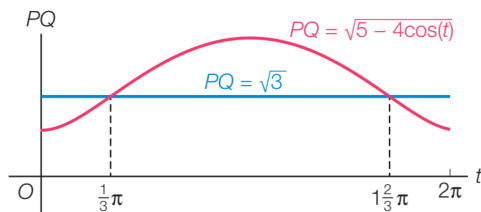
$\frac{2}{3}\pi = \frac{1}{3}$ deel van de cirkel ligt rechts van de lijn $x = 1$.

De lengte van dit deel is $\frac{1}{3} \cdot 2\pi \cdot 2 = \frac{4}{3}\pi$.

- c** $PQ^2 = (2 \cos(t) - \cos(2t))^2 + (2 \sin(t) - \sin(2t))^2$
 $= 4 \cos^2(t) - 4 \cos(t) \cos(2t) + \cos^2(2t) + 4 \sin^2(t) - 4 \sin(t) \sin(2t) + \sin^2(2t)$
 $= 4 + 1 - 4(\cos(2t) \cos(t) + \sin(2t) \sin(t))$
 $= 5 - 4 \cos(2t - t)$
 $= 5 - 4 \cos(t)$

Dus $PQ = \sqrt{5 - 4 \cos(t)}$.

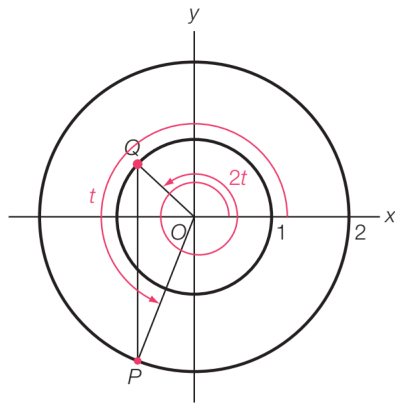
- d** $PQ = \sqrt{3}$ geeft $\sqrt{5 - 4 \cos(t)} = \sqrt{3}$
 $5 - 4 \cos(t) = 3$
 $4 \cos(t) = 2$
 $\cos(t) = \frac{1}{2}$
 $t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$
 t in $[0, 2\pi]$ geeft $t = \frac{1}{3}\pi \vee t = \frac{5}{3}\pi$



Dus gedurende $\frac{5}{3}\pi - \frac{1}{3}\pi = \frac{4}{3}\pi$ seconde per rondgang.

- e** $x_P = x_Q$ geeft $2 \cos(t) = \cos(2t)$
 Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft
 $t = 1,9455\dots$ en $t = 4,3376\dots$
 Voor de getekende situatie geldt $t \approx 1,946$.

- f Voor de andere situatie geldt $t \approx 4,338$.



Bladzijde 159

42
$$\left. \begin{array}{l} y_{P'} = y_P \\ y_P = 3 \sin(\frac{2}{5}\pi t) \end{array} \right\} y_{P'} = 3 \sin(\frac{2}{5}\pi t)$$

Bladzijde 160

43 a De frequentie is $\frac{50\pi}{2\pi} = 25$ Hz.

b Per trilling is de afgelegde afstand $4 \cdot 5 = 20$ cm.

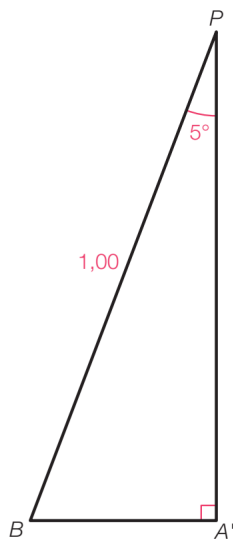
Er zijn 25 trillingen per seconde, dus per kwartier zijn er $15 \cdot 60 \cdot 25 = 22\,500$ trillingen.

De afgelegde afstand in één kwartier is $22\,500 \cdot 20 = 450\,000$ cm = 4,5 km.

44 amplitude = 10 mm geeft $b = 10$
frequentie = 3 Hz geeft $c = 2\pi \cdot 3 = 6\pi$
Dus $u = 10 \sin(6\pi t)$.

45 a De omtrek van de cirkel met middelpunt P en straal 1,00 m is $2\pi \cdot 1 = 2\pi$ m.
De lengte van de boog BC is $\frac{10}{360} \cdot 2\pi \approx 0,1745$ m.

b



$$\sin(5^\circ) = \frac{A'B}{1}, \text{ dus } A'B = \sin(5^\circ)$$

$$BC = 2A'B = 2 \sin(5^\circ) \approx 0,1743 \text{ m}$$

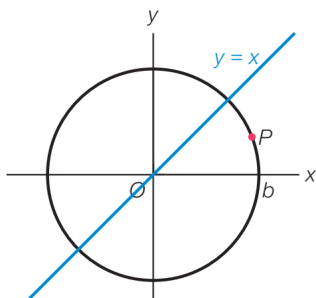
c $b = A'B = \sin(5^\circ) \approx 0,09$

$$c = \frac{2\pi}{T} = \frac{2\pi}{2\pi \cdot \sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}} = \sqrt{\frac{9,81}{1}} \approx 3,13$$

d $T = 2\pi \cdot \sqrt{\frac{l}{g}} = 2\pi \cdot \sqrt{\frac{1}{9,81}} \approx 2,01$ sec, dus de klok geeft 1 tik per seconde.

- 46 a Uit de bewegingsvergelijkingen volgt dat het punt P met constante snelheid een cirkel doorloopt. Er is dus sprake van een eenparige cirkelbeweging. Bij de projectie P'' van P op de x -as voert P'' dus een harmonische trilling uit waarbij de formule $x_{P''} = b \cos(ct)$ hoort.

b



Door de figuur een achttste slag tegen de klok in te draaien, zie je dat de projectie van P op de lijn $y = x$ op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y -as. Dus de projectie van P op de lijn $y = x$ voert een harmonische trilling uit.

C5 a $PQ^2 = (\cos(3t) - 4\cos(\frac{2}{3}t))^2 + (\sin(3t) - 4\sin(\frac{2}{3}t))^2$
 $= \cos^2(3t) - 8\cos(3t)\cos(\frac{2}{3}t) + 16\cos^2(\frac{2}{3}t) + \sin^2(3t) - 8\sin(3t)\sin(\frac{2}{3}t) + 16\sin^2(\frac{2}{3}t)$
 $= 1 + 16 - 8(\cos(3t)\cos(\frac{2}{3}t) + \sin(3t)\sin(\frac{2}{3}t))$
 $= 17 - 8\cos(3t - \frac{2}{3}t)$
 $= 17 - 8\cos(2\frac{1}{3}t)$

Dus $PQ = \sqrt{17 - 8\cos(2\frac{1}{3}t)}$.

b $PQ = \sqrt{13}$ geeft $\sqrt{17 - 8\cos(2\frac{1}{3}t)} = \sqrt{13}$
 $17 - 8\cos(2\frac{1}{3}t) = 13$
 $8\cos(2\frac{1}{3}t) = 4$
 $\cos(2\frac{1}{3}t) = \frac{1}{2}$
 $2\frac{1}{3}t = \frac{1}{3}\pi + k \cdot 2\pi \vee 2\frac{1}{3}t = -\frac{1}{3}\pi + k \cdot 2\pi$
 $t = \frac{1}{7}\pi + k \cdot \frac{6}{7}\pi \vee t = -\frac{1}{7}\pi + k \cdot \frac{6}{7}\pi$
 t in $[0, \pi]$ geeft $t = \frac{1}{7}\pi \vee t = \pi \vee t = \frac{5}{7}\pi$

C6 a amplitude = 10 cm geeft $b = 10$
 frequentie = 75 Hz geeft $c = 2\pi \cdot 75 = 150\pi$
 Dus $u = 10 \sin(150\pi t)$.

- b Per trilling is de afgelegde afstand $4 \cdot 10 = 40$ cm.
 Er zijn 75 trillingen per seconde, dus per 10 minuten zijn er $10 \cdot 60 \cdot 75 = 45\,000$ trillingen.
 De afgelegde afstand in 10 minuten is $45\,000 \cdot 40 = 1\,800\,000$ cm = 18 km.

12.4 Bewegingsvergelijkingen met goniometrische formules

Bladzijde 162

- 47 a** Snijden met de x -as, dus $y = 0$ geeft

$$\sin(4t) = 0$$

$$4t = k \cdot \pi$$

$$t = k \cdot \frac{1}{4}\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = 0 \vee t = \frac{1}{4}\pi \vee t = \frac{1}{2}\pi \vee t = \frac{3}{4}\pi \vee t = \pi \vee t = 1\frac{1}{4}\pi \vee t = 1\frac{1}{2}\pi \vee t = 1\frac{3}{4}\pi \vee t = 2\pi$$

Het punt P gaat door de oorsprong voor $t = 0$, $t = \pi$ en $t = 2\pi$

en snijdt de x -as in $(-1, 0)$ voor $t = 1\frac{1}{2}\pi$ en in $(1, 0)$ voor $t = \frac{1}{2}\pi$.

$$t = \frac{1}{4}\pi \text{ en } t = \frac{3}{4}\pi \text{ geven } x = \sin(\frac{1}{4}\pi) = \sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}, \text{ dus } x_B = \frac{1}{2}\sqrt{2}.$$

$$t = 1\frac{1}{4}\pi \text{ en } t = 1\frac{3}{4}\pi \text{ geven } x = \sin(1\frac{1}{4}\pi) = \sin(1\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}, \text{ dus } x_A = -\frac{1}{2}\sqrt{2}.$$

- b** C is een top, dus $y_C = 1$.

$$y = 1 \text{ geeft } \sin(4t) = 1$$

$$4t = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{8}\pi \vee t = \frac{5}{8}\pi \vee t = 1\frac{1}{8}\pi \vee t = 1\frac{5}{8}\pi$$

$$x(\frac{1}{8}\pi) = \sin(\frac{1}{8}\pi) = 0,3826\dots$$

$$x(\frac{5}{8}\pi) = \sin(\frac{5}{8}\pi) = 0,9238\dots$$

$$x(1\frac{1}{8}\pi) = \sin(1\frac{1}{8}\pi) = -0,3826\dots$$

$$x(1\frac{5}{8}\pi) = \sin(1\frac{5}{8}\pi) = -0,9238\dots$$

$$\text{Dus } x_C \approx 0,383.$$

Bladzijde 164

- 48 a** De baansnelheid in B is $v(\frac{1}{3}\pi) = \sqrt{(2\cos(\frac{2}{3}\pi))^2 + \cos^2(\frac{1}{3}\pi)} = \sqrt{(2 \cdot -\frac{1}{2})^2 + (\frac{1}{2})^2} = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$.

- b** De baansnelheid in O is $v(0) = \sqrt{(2\cos(0))^2 + \cos^2(0)} = \sqrt{(2 \cdot 1)^2 + 1^2} = \sqrt{5}$.

Voor $t = \pi$ en $t = 2\pi$ krijg je dezelfde baansnelheid.

$$\mathbf{c} \quad \vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2\cos(0) \\ \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\tan(\varphi) = \frac{1}{2} \text{ geeft } \varphi \approx 26,6^\circ$$

De gevraagde hoek is $26,6^\circ$.

Voor $t = \pi$ en $t = 2\pi$ krijg je dezelfde hoek.

- 49 a** $y = \frac{1}{2}$ geeft $\sin(t) = \frac{1}{2}$

$$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$$

$$x(\frac{1}{6}\pi) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}, \text{ dus } x_D = \frac{1}{2}\sqrt{3}.$$

$$x(\frac{5}{6}\pi) = \sin(1\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}, \text{ dus } x_C = -\frac{1}{2}\sqrt{3}.$$

$$\text{Dus } CD = \frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3}) = \sqrt{3}.$$

$$\mathbf{b} \quad \vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} 2\cos(\frac{1}{3}\pi) \\ \cos(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2}\sqrt{3} \end{pmatrix}$$

$$\tan(\varphi) = \frac{1}{2}\sqrt{3} \text{ geeft } \varphi = 40,89\dots^\circ$$

De gevraagde hoek is $40,9^\circ$.

- c De baan snijdt zichzelf in de oorsprong voor $t = 0$ en $t = \pi$.

$$\vec{v}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ (zie vraag 48c)}$$

$$\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(2\pi) \\ \cos(\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right|} = \frac{2 \cdot 2 + 1 \cdot -1}{\sqrt{5} \cdot \sqrt{5}} = \frac{3}{5} \text{ geeft } \varphi = 53,13\dots^\circ$$

De gevraagde hoek is $53,1^\circ$.

- d $v(t) = \sqrt{(2 \cos(2t))^2 + \cos^2(t)} = \sqrt{4 \cos^2(2t) + \cos^2(t)}$ geeft

$$a(t) = v'(t) = \frac{1}{2\sqrt{4 \cos^2(2t) + \cos^2(t)}} \cdot (8 \cos(2t) \cdot -\sin(2t) \cdot 2 + 2 \cos(t) \cdot -\sin(t))$$

$$= \frac{-8 \sin(2t) \cos(2t) - \sin(t) \cos(t)}{\sqrt{4 \cos^2(2t) + \cos^2(t)}}$$

$$\text{De baanversnelling van } P \text{ in } D \text{ is } a\left(\frac{1}{6}\pi\right) = \frac{-8 \sin\left(\frac{1}{3}\pi\right) \cos\left(\frac{1}{3}\pi\right) - \sin\left(\frac{1}{6}\pi\right) \cos\left(\frac{1}{6}\pi\right)}{\sqrt{4 \cos^2\left(\frac{1}{3}\pi\right) + \cos^2\left(\frac{1}{6}\pi\right)}} \approx -2,95.$$

- 50 a $y = 1$ geeft $\sin(3t) = 1$

$$3t = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi \vee t = 1\frac{1}{2}\pi$$

$$t = \frac{5}{6}\pi \text{ geeft } A(\sin(1\frac{2}{3}\pi), \sin(2\frac{1}{2}\pi)) = A(-\frac{1}{2}\sqrt{3}, 1)$$

$$x = -1 \text{ geeft } \sin(2t) = -1$$

$$2t = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{3}{4}\pi + k \cdot \pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{3}{4}\pi \vee t = 1\frac{3}{4}\pi$$

$$t = \frac{3}{4}\pi \text{ geeft } B(\sin(1\frac{1}{2}\pi), \sin(2\frac{1}{4}\pi)) = B(-1, \frac{1}{2}\sqrt{2})$$

- b $x(t) = \sin(2t)$ en $y(t) = \sin(3t)$ substitueren in $y = x$ geeft

$$\sin(3t) = \sin(2t)$$

$$3t = 2t + k \cdot 2\pi \vee 3t = \pi - 2t + k \cdot 2\pi$$

$$t = k \cdot 2\pi \vee 5t = \pi + k \cdot 2\pi$$

$$t = k \cdot 2\pi \vee t = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = 0 \vee t = 2\pi \vee t = \frac{1}{5}\pi \vee t = \frac{3}{5}\pi \vee t = \pi \vee t = 1\frac{2}{5}\pi \vee t = 1\frac{4}{5}\pi$$

Bij de oorsprong horen $t = 0$, $t = \pi$ en $t = 2\pi$.

Dus bij de vier andere punten horen $t = \frac{1}{5}\pi$, $t = \frac{3}{5}\pi$, $t = 1\frac{2}{5}\pi$ en $t = 1\frac{4}{5}\pi$.

- c $x(t) = \frac{1}{2}$ geeft $\sin(2t) = \frac{1}{2}$

$$2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{1}{12}\pi + k \cdot \pi \vee t = \frac{5}{12}\pi + k \cdot \pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{1}{12}\pi \vee t = 1\frac{1}{12}\pi \vee t = \frac{5}{12}\pi \vee t = 1\frac{5}{12}\pi$$

$$y(t) = \frac{1}{2}\sqrt{2} \text{ geeft } \sin(3t) = \frac{1}{2}\sqrt{2}$$

$$3t = \frac{1}{4}\pi + k \cdot 2\pi \vee 3t = \frac{3}{4}\pi + k \cdot 2\pi$$

$$t = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi \vee t = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{1}{12}\pi \vee t = \frac{3}{4}\pi \vee t = 1\frac{5}{12}\pi \vee t = \frac{1}{4}\pi \vee t = 1\frac{7}{12}\pi$$

Voor $t = \frac{1}{12}\pi$ en $t = 1\frac{5}{12}\pi$ passeert P het punt $(\frac{1}{2}, \frac{1}{2}\sqrt{2})$, dus de baan snijdt zichzelf in $C(\frac{1}{2}, \frac{1}{2}\sqrt{2})$.

d $x(t) = \sin(2t)$ geeft $x'(t) = 2 \cos(2t)$

$y(t) = \sin(3t)$ geeft $y'(t) = 3 \cos(3t)$

De baansnelheid in C is

$$v(\frac{1}{12}\pi) = \sqrt{(2 \cos(\frac{1}{6}\pi))^2 + (3 \cos(\frac{1}{4}\pi))^2} = \sqrt{(2 \cdot \frac{1}{2}\sqrt{3})^2 + (3 \cdot \frac{1}{2}\sqrt{2})^2} = \sqrt{3 + 4\frac{1}{2}} = \sqrt{\frac{15}{2}} = \frac{1}{2}\sqrt{30}.$$

Voor $t = 1\frac{5}{12}\pi$ krijg je dezelfde baansnelheid.

e $\vec{v}(0) = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 2 \cos(0) \\ 3 \cos(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ en $\vec{v}(\pi) = \begin{pmatrix} x'(\pi) \\ y'(\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(2\pi) \\ 3 \cos(3\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$$\cos(\varphi) = \frac{\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right|} = \frac{2 \cdot 2 + 3 \cdot -3}{\sqrt{13} \cdot \sqrt{13}} = \frac{-5}{13} \text{ geeft } \varphi = 112,6\dots^\circ$$

De gevraagde hoek is $180^\circ - 112,6\dots^\circ \approx 67^\circ$.

Bladzijde 165

51 a $x = 1$ geeft $\sin(t - \frac{1}{6}\pi) = 1$

$$t - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{2}{3}\pi + k \cdot 2\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{2}{3}\pi$$

$$y(\frac{2}{3}\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}, \text{ dus } A(1, -\frac{1}{2}\sqrt{3}).$$

b $x = \frac{1}{2}$ geeft $\sin(t - \frac{1}{6}\pi) = \frac{1}{2}$

$$t - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee t - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = \pi + k \cdot 2\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = \frac{1}{3}\pi \vee t = \pi$$

$$y(\frac{1}{3}\pi) = \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3} \text{ geeft } B(\frac{1}{2}, \frac{1}{2}\sqrt{3})$$

$$y(\pi) = \sin(2\pi) = 0 \text{ geeft } C(\frac{1}{2}, 0)$$

$$\text{Dus } BC = \frac{1}{2}\sqrt{3}.$$

c $x(t) = \sin(t - \frac{1}{6}\pi)$ en $y(t) = \sin(2t)$ substitueren in $y = x$ geeft

$$\sin(2t) = \sin(t - \frac{1}{6}\pi)$$

$$2t = t - \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = \pi - t + \frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee 3t = 1\frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$$

$$t \text{ in } [0, 2\pi] \text{ geeft } t = 1\frac{5}{6}\pi \vee t = \frac{7}{18}\pi \vee t = 1\frac{13}{18}\pi \vee t = 1\frac{13}{18}\pi$$

d $x(t) = 0$ geeft $\sin(t - \frac{1}{6}\pi) = 0$

$$t - \frac{1}{6}\pi = k \cdot \pi$$

$$t = \frac{1}{6}\pi + k \cdot \pi$$

Dus de baan snijdt zichzelf voor $t = \frac{1}{6}\pi$ en voor $t = 1\frac{1}{6}\pi$ in $(0, \frac{1}{2}\sqrt{3})$.

$$x(t) = \sin(t - \frac{1}{6}\pi) \text{ geeft } x'(t) = \cos(t - \frac{1}{6}\pi)$$

$$y(t) = \sin(2t) \text{ geeft } y'(t) = 2 \cos(2t)$$

$$\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} x'(\frac{1}{6}\pi) \\ y'(\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(0) \\ 2 \cos(\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ en } \vec{v}(1\frac{1}{6}\pi) = \begin{pmatrix} x'(1\frac{1}{6}\pi) \\ y'(1\frac{1}{6}\pi) \end{pmatrix} = \begin{pmatrix} \cos(\pi) \\ 2 \cos(2\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0, \text{ dus de hoek waaronder de baan zichzelf snijdt is } 90^\circ.$$

e De baansnelheid op $t = \frac{1}{3}\pi$ is

$$v(\frac{1}{3}\pi) = \sqrt{\cos^2(\frac{1}{6}\pi) + (2 \cos(\frac{2}{3}\pi))^2} = \sqrt{(\frac{1}{2}\sqrt{3})^2 + (2 \cdot -\frac{1}{2})^2} = \sqrt{\frac{3}{4} + 1} = \sqrt{\frac{7}{4}} = \frac{1}{2}\sqrt{7}.$$

a $x(t) = \sin^2(t) + \cos(t)$ geeft $x'(t) = 2 \sin(t) \cos(t) - \sin(t)$
 $x'(t) = 0$ geeft $2 \sin(t) \cos(t) - \sin(t) = 0$
 $\sin(t)(2 \cos(t) - 1) = 0$
 $\sin(t) = 0 \vee \cos(t) = \frac{1}{2}$
 $t = k \cdot \pi \vee t = \frac{1}{3}\pi + k \cdot 2\pi \vee t = -\frac{1}{3}\pi + k \cdot 2\pi$
 $0 \leq t \leq 2\pi$ geeft $t = 0 \vee t = \pi \vee t = 2\pi \vee t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi$

Bij $t = 0$, $t = \pi$ en $t = 2\pi$ geldt $y = 0$.

$$x(\frac{1}{3}\pi) = \sin^2(\frac{1}{3}\pi) + \cos(\frac{1}{3}\pi) = (\frac{1}{2}\sqrt{3})^2 + \frac{1}{2} = 1\frac{1}{4} \text{ en } y(\frac{1}{3}\pi) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}, \text{ dus } A(1\frac{1}{4}, \frac{1}{2}\sqrt{3}).$$

$$x(\frac{2}{3}\pi) = \sin^2(\frac{2}{3}\pi) + \cos(\frac{2}{3}\pi) = (-\frac{1}{2}\sqrt{3})^2 + \frac{1}{2} = 1\frac{1}{4} \text{ en } y(\frac{2}{3}\pi) = \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3}, \text{ dus } B(1\frac{1}{4}, \frac{1}{2}\sqrt{3}).$$

$$AB = \frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3}) = \sqrt{3}$$

$$y = \frac{1}{2} \text{ geeft } \sin(t) = \frac{1}{2}$$

$$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$$

$$x(\frac{1}{6}\pi) = \sin^2(\frac{1}{6}\pi) + \cos(\frac{1}{6}\pi) = (\frac{1}{2})^2 + \frac{1}{2}\sqrt{3} = \frac{1}{4} + \frac{1}{2}\sqrt{3}, \text{ dus } D(\frac{1}{4} + \frac{1}{2}\sqrt{3}, \frac{1}{2}).$$

$$x(\frac{5}{6}\pi) = \sin^2(\frac{5}{6}\pi) + \cos(\frac{5}{6}\pi) = (\frac{1}{2})^2 - \frac{1}{2}\sqrt{3} = \frac{1}{4} - \frac{1}{2}\sqrt{3}, \text{ dus } C(\frac{1}{4} - \frac{1}{2}\sqrt{3}, \frac{1}{2}).$$

$$CD = \frac{1}{4} + \frac{1}{2}\sqrt{3} - (\frac{1}{4} - \frac{1}{2}\sqrt{3}) = \sqrt{3}$$

Dus $AB = CD$.

b $y(t) = \sin(t)$ geeft $y'(t) = \cos(t)$

$$x'(\frac{1}{6}\pi) = 2 \sin(\frac{1}{6}\pi) \cos(\frac{1}{6}\pi) - \sin(\frac{1}{6}\pi) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2} = \frac{1}{2}\sqrt{3} - \frac{1}{2}$$

$$y'(\frac{1}{6}\pi) = \cos(\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$$

$$\text{De baansnelheid in } D \text{ is } v(\frac{1}{6}\pi) = \sqrt{(\frac{1}{2}\sqrt{3} - \frac{1}{2})^2 + (\frac{1}{2}\sqrt{3})^2} = \sqrt{\frac{3}{4} - \frac{1}{2}\sqrt{3} + \frac{1}{4} + \frac{3}{4}} = \sqrt{1\frac{3}{4} - \frac{1}{2}\sqrt{3}}.$$

c $\vec{v}(\frac{5}{6}\pi) = \begin{pmatrix} x'(\frac{5}{6}\pi) \\ y'(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} 2 \sin(\frac{5}{6}\pi) \cos(\frac{5}{6}\pi) - \sin(\frac{5}{6}\pi) \\ \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\sqrt{3} - \frac{1}{2} \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \triangleq \begin{pmatrix} \sqrt{3} + 1 \\ \sqrt{3} \end{pmatrix}$

$$\tan(\varphi) = \frac{\sqrt{3}}{\sqrt{3} + 1} \text{ geeft } \varphi = 32,37\dots^\circ$$

De gevraagde hoek is $32,4^\circ$.

a $x(t) = 0$ geeft $\sin(2t) = 0$
 $2t = k \cdot \pi$
 $t = k \cdot \frac{1}{2}\pi$
 $0 \leq t \leq 2\pi$ geeft $t = 0 \vee t = \frac{1}{2}\pi \vee t = \pi \vee t = \frac{3}{2}\pi \vee t = 2\pi$

$$y(0) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$$

$$y(\frac{1}{2}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$$

$$y(\pi) = \sin(\frac{2}{3}\pi) = -\frac{1}{2}\sqrt{3}$$

$$y(\frac{3}{2}\pi) = \sin(\frac{7}{6}\pi) = -\frac{1}{2}$$

$$y(2\pi) = \sin(\frac{4}{3}\pi) = \frac{1}{2}\sqrt{3}$$

Dus $A(0, \frac{1}{2}\sqrt{3})$, $B(0, \frac{1}{2})$, $C(0, -\frac{1}{2}\sqrt{3})$ en $D(0, -\frac{1}{2})$.

b $x = -\frac{1}{2}$ geeft $\sin(2t) = -\frac{1}{2}$

$$2t = \frac{7}{6}\pi + k \cdot 2\pi \vee 2t = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$t = \frac{7}{12}\pi + k \cdot \pi \vee t = -\frac{5}{12}\pi + k \cdot \pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{7}{12}\pi \vee t = \frac{11}{12}\pi \vee t = \frac{5}{12}\pi \vee t = \frac{19}{12}\pi$$

$$y(\frac{7}{12}\pi) = \sin(\frac{11}{12}\pi) \approx 0,26$$

$$y(\frac{11}{12}\pi) = \sin(\frac{1}{12}\pi) = -\frac{1}{2}\sqrt{2} \approx -0,71$$

$$y(\frac{5}{12}\pi) = \sin(\frac{7}{12}\pi) \approx -0,26$$

$$y(\frac{19}{12}\pi) = \sin(\frac{5}{12}\pi) = \frac{1}{2}\sqrt{2} \approx 0,71$$

Dus $E(-\frac{1}{2}, \frac{1}{2}\sqrt{2})$ en $H(-\frac{1}{2}, -\frac{1}{2}\sqrt{2})$ en dit geeft $EH = \frac{1}{2}\sqrt{2} - (-\frac{1}{2}\sqrt{2}) = \sqrt{2}$.

c $x(t) = \sin(2t)$ geeft $x'(t) = 2 \cos(2t)$

$y(t) = \sin(t + \frac{1}{3}\pi)$ geeft $y'(t) = \cos(t + \frac{1}{3}\pi)$

$$\vec{v}(\frac{1}{2}\pi) = \begin{pmatrix} x'(\frac{1}{2}\pi) \\ y'(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 2 \cos(\pi) \\ \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \text{ en } \vec{r}_{y\text{-as}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|} = \frac{-2 \cdot 0 + -\frac{1}{2}\sqrt{3} \cdot 1}{\sqrt{4\frac{3}{4}} \cdot 1} = \frac{-\frac{1}{2}\sqrt{3}}{\sqrt{4\frac{3}{4}}} \text{ geeft } \varphi = 113,4\dots^\circ$$

De gevraagde hoek is $180^\circ - 113,4\dots^\circ \approx 67^\circ$.

d De baansnelheid in E is

$$v(1\frac{11}{12}\pi) = \sqrt{(2 \cos(3\frac{5}{6}\pi))^2 + \cos^2(2\frac{1}{4}\pi)} = \sqrt{(2 \cdot \frac{1}{2}\sqrt{3})^2 + (\frac{1}{2}\sqrt{2})^2} = \sqrt{3 + \frac{1}{2}} = \sqrt{\frac{7}{2}} = \frac{1}{2}\sqrt{14}.$$

54 a $x(a) = \sin(2a)$ en $y(a) = \sin(a + \frac{1}{3}\pi)$, dus $T(\sin(2a), \sin(a + \frac{1}{3}\pi))$.

$x(a + \pi) = \sin(2(a + \pi)) = \sin(2a + 2\pi) = \sin(2a)$ en $y(a + \pi) = \sin(a + \pi + \frac{1}{3}\pi) = \sin(a + 1\frac{1}{3}\pi)$,
dus $U(\sin(2a), \sin(a + 1\frac{1}{3}\pi))$.

$$TU = |y_T - y_U| = |\sin(a + \frac{1}{3}\pi) - \sin(a + 1\frac{1}{3}\pi)| = |\sin(a + \frac{1}{3}\pi) - (-\sin(a + \frac{1}{3}\pi))| = |2 \sin(a + \frac{1}{3}\pi)|$$

b $TU = 1$ geeft

$$|2 \sin(a + \frac{1}{3}\pi)| = 1$$

$$2 \sin(a + \frac{1}{3}\pi) = 1 \vee 2 \sin(a + \frac{1}{3}\pi) = -1$$

$$\sin(a + \frac{1}{3}\pi) = \frac{1}{2} \vee \sin(a + \frac{1}{3}\pi) = -\frac{1}{2}$$

$$a + \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = \frac{1}{6}\pi + k \cdot 2\pi \vee a + \frac{1}{3}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$a = -\frac{1}{6}\pi + k \cdot 2\pi \vee a = \frac{1}{2}\pi + k \cdot 2\pi \vee a = \frac{5}{6}\pi + k \cdot 2\pi \vee a = -\frac{1}{2}\pi + k \cdot 2\pi$$

$$0 \leq a \leq 2\pi \text{ en } 0 \leq a + \pi \leq 2\pi \text{ oftewel } 0 \leq a \leq \pi \text{ geeft } a = \frac{1}{2}\pi \vee a = \frac{5}{6}\pi$$

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55 a Algemeen geldt $-1 \leq \sin(A) \leq 1$.

Dus zowel $-1 \leq \sin(t + \frac{1}{4}\pi) \leq 1$ als $-1 \leq \sin(2t) \leq 1$ oftewel $-1 \leq x(t) \leq 1$ en $-1 \leq y(t) \leq 1$.

Dus bij de getekende baan geldt $-1 \leq x \leq 1$ en $-1 \leq y \leq 1$.

b $x(t) = -1$ geeft $\sin(t + \frac{1}{4}\pi) = -1$

$$t + \frac{1}{4}\pi = 1\frac{1}{2}\pi + k \cdot 2\pi$$

$$t = 1\frac{1}{4}\pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = 1\frac{1}{4}\pi$$

Dus voor $t = 1\frac{1}{4}\pi$ bevindt P zich in $(-1, 1)$.

$x(t) = 1$ geeft $\sin(t + \frac{1}{4}\pi) = 1$

$$t + \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{1}{4}\pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{4}\pi$$

Dus voor $t = \frac{1}{4}\pi$ bevindt P zich in $(1, 1)$.

$x(t) = 0$ geeft $\sin(t + \frac{1}{4}\pi) = 0$

$$t + \frac{1}{4}\pi = k \cdot \pi$$

$$t = -\frac{1}{4}\pi + k \cdot \pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{3}{4}\pi \vee t = 1\frac{3}{4}\pi$$

Dus voor $t = \frac{3}{4}\pi$ en voor $t = 1\frac{3}{4}\pi$ bevindt P zich in $(0, -1)$.

c $y = px^2 + q$ } $p \cdot 0^2 + q = -1$
door $(0, -1)$ } $q = -1$

$y = px^2 - 1$ } $p \cdot 1^2 - 1 = 1$
door $(1, 1)$ } $p = 2$

Dus $p = 2$ en $q = -1$.

- 56** **a** Voor $1\frac{1}{4}\pi \leq t \leq 2\frac{1}{4}\pi$ wordt de kromme van links naar rechts één keer doorlopen.
b De formules $\cos(2A) = 1 - 2\sin^2(A)$, $-\cos(A) = \cos(A + \pi)$, $\cos(A) = \sin(A + \frac{1}{2}\pi)$ en $\sin(A + 2\pi) = \sin(A)$ zijn gebruikt.

- 57** Substitutie van $x = \sin(t - \frac{1}{4}\pi)$ in $-2x^2 + 1$ geeft

$$\begin{aligned} -2x^2 + 1 &= -2\sin^2(t - \frac{1}{4}\pi) + 1 = 1 - 2\sin^2(t - \frac{1}{4}\pi) \\ &= \cos(2(t - \frac{1}{4}\pi)) \\ &= \cos(2t - \frac{1}{2}\pi) \\ &= \sin(2t) \text{ en dit is gelijk aan } y. \end{aligned}$$

Omdat $x = \sin(t - \frac{1}{4}\pi)$, geldt $-1 \leq x \leq 1$.

Dus bij de grafiek hoort de formule $y = -2x^2 + 1$ met $-1 \leq x \leq 1$.

- 58** **a** $-2 \leq 2\sin(t) \leq 2$, dus $-2 \leq x \leq 2$.
 $-1 \leq \sin(2t - \frac{1}{2}\pi) \leq 1$, dus $-1 \leq y \leq 1$.
b $K: y = ax^2 + b$ door $(0, -1)$ geeft $K: y = ax^2 - 1$
 $K: y = ax^2 - 1$ door $(2, 1)$ geeft $4a - 1 = 1$ oftewel $4a = 2$, dus $a = \frac{1}{2}$.
 Vermoedelijk hoort bij K de formule $y = \frac{1}{2}x^2 - 1$.
 Substitutie van $x = 2\sin(t)$ in $\frac{1}{2}x^2 - 1$ geeft

$$\begin{aligned} \frac{1}{2}x^2 - 1 &= \frac{1}{2}(2\sin(t))^2 - 1 = \frac{1}{2} \cdot 4\sin^2(t) - 1 \\ &= -(1 - 2\sin^2(t)) \\ &= -\cos(2t) \\ &= \cos(2t + \pi) \\ &= \sin(2t + 1\frac{1}{2}\pi) \\ &= \sin(2t - \frac{1}{2}\pi) \text{ en dit is gelijk aan } y. \end{aligned}$$

Dus bij K hoort inderdaad de formule $y = \frac{1}{2}x^2 - 1$.

- 59** Substitutie van $x = \sin(t)$ in $4x^2 - 4x^4$ geeft

$$\begin{aligned} 4x^2 - 4x^4 &= 4\sin^2(t) - 4\sin^4(t) = 4\sin^2(t)(1 - \sin^2(t)) \\ &= 4\sin^2(t)\cos^2(t) \\ &= (2\sin(t)\cos(t))^2 \\ &= \sin^2(2t) \text{ en dit is gelijk aan } y^2. \end{aligned}$$

Omdat $x = \sin(t)$, geldt $-1 \leq x \leq 1$.

Dus bij de baan van P hoort de formule $y^2 = 4x^2 - 4x^4$ met $-1 \leq x \leq 1$.

- 60** $y = \sin(3t) = \sin(2t + t)$

$$\begin{aligned} &= \sin(2t)\cos(t) + \cos(2t)\sin(t) \\ &= 2\sin(t)\cos(t)\cos(t) + (1 - 2\sin^2(t))\sin(t) \\ &= 2\sin(t)\cos^2(t) + \sin(t) - 2\sin^3(t) \\ &= 2\sin(t)(1 - \sin^2(t)) + \sin(t) - 2\sin^3(t) \\ &= 2\sin(t) - 2\sin^3(t) + \sin(t) - 2\sin^3(t) \\ &= 3\sin(t) - 4\sin^3(t) \end{aligned}$$

Omdat $x = \sin(t)$, geldt $y = 3x - 4x^3$.

Dus alle punten van K liggen op de grafiek van $y = 3x - 4x^3$.

- 61** $y = \cos(3t) = \cos(2t + t)$

$$\begin{aligned} &= \cos(2t)\cos(t) - \sin(2t)\sin(t) \\ &= (2\cos^2(t) - 1)\cos(t) - 2\sin(t)\cos(t)\sin(t) \\ &= 2\cos^3(t) - \cos(t) - 2\sin^2(t)\cos(t) \\ &= 2\cos^3(t) - \cos(t) - 2(1 - \cos^2(t))\cos(t) \\ &= 2\cos^3(t) - \cos(t) - 2\cos(t) + 2\cos^3(t) \\ &= 4\cos^3(t) - 3\cos(t) \end{aligned}$$

$x = 2\cos(t)$, dus $\cos(t) = \frac{1}{2}x$.

Dit geeft $y = 4 \cdot (\frac{1}{2}x)^3 - 3 \cdot \frac{1}{2}x = \frac{1}{2}x^3 - 1\frac{1}{2}x$.

Omdat $x(t) = 2\cos(t)$, geldt $-2 \leq x \leq 2$.

Dus bij de baan van P hoort de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \leq x \leq 2$.

- 62** Het midden van OP noemen we M .
 Er geldt $MS \perp OP$ en $MS = OM = \frac{1}{2}OP$.
 Dus $\vec{OS} = \vec{OM} + \vec{MS} = \frac{1}{2}\vec{OP} + \frac{1}{2}\vec{OP}_L$.

63 $\vec{OT} = \frac{1}{2}\vec{OP} + \frac{1}{2}\vec{OP}_R = \frac{1}{2} \cdot \begin{pmatrix} 2\sin(t) \\ 2\sin(2t) \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} 2\sin(2t) \\ -2\sin(t) \end{pmatrix} = \begin{pmatrix} \sin(t) + \sin(2t) \\ \sin(2t) - \sin(t) \end{pmatrix}$

De bewegingsvergelijkingen van T zijn $\begin{cases} x_T(t) = \sin(t) + \sin(2t) \\ y_T(t) = \sin(2t) - \sin(t) \end{cases}$

$$\begin{aligned} y_T(t) = 0 &\text{ geeft } \sin(2t) - \sin(t) = 0 \\ \sin(2t) &= \sin(t) \\ 2t &= t + k \cdot 2\pi \vee 2t = \pi - t + k \cdot 2\pi \\ t &= k \cdot 2\pi \vee 3t = \pi + k \cdot 2\pi \\ t &= k \cdot 2\pi \vee t = \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi \\ 0 \leq t \leq \pi &\text{ geeft } t = 0 \vee t = \frac{1}{3}\pi \vee t = \pi \end{aligned}$$

Voor $t = 0$ en $t = \pi$ valt P samen met O .

$$x_T(\frac{1}{3}\pi) = \sin(\frac{1}{3}\pi) + \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3} = \sqrt{3}$$

Dus het snijpunt van de baan van T met de x -as is $(\sqrt{3}, 0)$.

64 a $\vec{OQ} = \vec{OA} + \vec{AQ} = \vec{OA} + \frac{1}{2}\vec{AP} + \frac{1}{2}\vec{AP}_R$

$$\vec{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 2\sin(t) \\ 2\sin(2t) \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\sin(t) - 2 \\ 2\sin(2t) \end{pmatrix} \text{ geeft } \vec{AP}_R = \begin{pmatrix} 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix}$$

$$\vec{OQ} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} 2\sin(t) - 2 \\ 2\sin(2t) \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix} = \begin{pmatrix} 2 + \sin(t) - 1 + \sin(2t) \\ \sin(2t) + 1 - \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(2t) + \sin(t) + 1 \\ \sin(2t) - \sin(t) + 1 \end{pmatrix}$$

De bewegingsvergelijkingen van Q zijn $\begin{cases} x_Q(t) = \sin(2t) + \sin(t) + 1 \\ y_Q(t) = \sin(2t) - \sin(t) + 1 \end{cases}$

b $\vec{OR} = \vec{OA} + \vec{AR} = \vec{OA} + \vec{AP}_R = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix} = \begin{pmatrix} 2 + 2\sin(2t) \\ 2 - 2\sin(t) \end{pmatrix}$

$$\begin{aligned} y_R(t) = 1 &\text{ geeft } 2 - 2\sin(t) = 1 \\ 2\sin(t) &= 1 \\ \sin(t) &= \frac{1}{2} \\ t &= \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi \\ 0 \leq t \leq \pi &\text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi \end{aligned}$$

$$x_R(\frac{1}{6}\pi) = 2 + 2\sin(\frac{1}{3}\pi) = 2 + 2 \cdot \frac{1}{2}\sqrt{3} = 2 + \sqrt{3}$$

$$x_R(\frac{5}{6}\pi) = 2 + 2\sin(\frac{5}{3}\pi) = 2 + 2 \cdot -\frac{1}{2}\sqrt{3} = 2 - \sqrt{3}$$

Dus de snijpunten zijn $(2 + \sqrt{3}, 1)$ en $(2 - \sqrt{3}, 1)$.

65 a $\vec{OQ} = \vec{OP} + \vec{PQ} = \vec{OP} + \vec{PA}_L$

$$\vec{PA} = \vec{a} - \vec{p} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2\sin(t) \\ 2\sin(t - \frac{1}{4}\pi) \end{pmatrix} = \begin{pmatrix} 4 - 2\sin(t) \\ -2\sin(t - \frac{1}{4}\pi) \end{pmatrix} \text{ geeft } \vec{PA}_L = \begin{pmatrix} 2\sin(t - \frac{1}{4}\pi) \\ 4 - 2\sin(t) \end{pmatrix}$$

$$\vec{OQ} = \begin{pmatrix} 2\sin(t) \\ 2\sin(t - \frac{1}{4}\pi) \end{pmatrix} + \begin{pmatrix} 2\sin(t - \frac{1}{4}\pi) \\ 4 - 2\sin(t) \end{pmatrix} = \begin{pmatrix} 2\sin(t) + 2\sin(t - \frac{1}{4}\pi) \\ 2\sin(t - \frac{1}{4}\pi) + 4 - 2\sin(t) \end{pmatrix}$$

Dus de bewegingsvergelijkingen van Q zijn $\begin{cases} x_Q(t) = 2\sin(t) + 2\sin(t - \frac{1}{4}\pi) \\ y_Q(t) = 2\sin(t - \frac{1}{4}\pi) - 2\sin(t) + 4 \end{cases}$

67 $x_P(t) = 0$ geeft $2 \sin(2t) = 0$

$$\sin(2t) = 0$$

$$2t = k \cdot \pi$$

$$t = k \cdot \frac{1}{2}\pi$$

$$t \text{ in } [\frac{1}{4}\pi, \frac{1}{4}\pi] \text{ geeft } t = \frac{1}{2}\pi \vee t = \pi$$

$$\begin{cases} x_P(t) = 2 \sin(2t) \\ y_P(t) = 2 \sin(t + \frac{1}{4}\pi) \end{cases} \text{ geeft } \begin{cases} x'_P(t) = 4 \cos(2t) \\ y'_P(t) = 2 \cos(t + \frac{1}{4}\pi) \end{cases}$$

$$\begin{cases} x'_P(\frac{1}{2}\pi) = 4 \cos(\pi) = 4 \cdot -1 = -4 \\ y'_P(\frac{1}{2}\pi) = 2 \cos(\frac{3}{4}\pi) = 2 \cdot -\frac{1}{2}\sqrt{2} = -\sqrt{2} \end{cases}$$

Dus de baansnelheid van P in A is $\sqrt{(-4)^2 + (-\sqrt{2})^2} = \sqrt{18} = 3\sqrt{2}$

$$\overrightarrow{OP} = \begin{pmatrix} 2 \sin(2t) \\ 2 \sin(t + \frac{1}{4}\pi) \end{pmatrix} \text{ en } \overrightarrow{OP}_R = \begin{pmatrix} 2 \sin(t + \frac{1}{4}\pi) \\ -2 \sin(2t) \end{pmatrix} \text{ geeft}$$

$$\begin{aligned} \overrightarrow{OQ} &= \frac{1}{2}(\overrightarrow{OP} + \overrightarrow{OP}_R) = \frac{1}{2} \cdot \left(\begin{pmatrix} 2 \sin(2t) \\ 2 \sin(t + \frac{1}{4}\pi) \end{pmatrix} + \begin{pmatrix} 2 \sin(t + \frac{1}{4}\pi) \\ -2 \sin(2t) \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 2 \sin(2t) + 2 \sin(t + \frac{1}{4}\pi) \\ 2 \sin(t + \frac{1}{4}\pi) - 2 \sin(2t) \end{pmatrix} \\ &= \begin{pmatrix} \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{pmatrix}, \text{ dus } \begin{cases} x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{cases} \end{aligned}$$

$x_Q(t) = 0$ geeft $\sin(2t) + \sin(t + \frac{1}{4}\pi) = 0$

$$\sin(2t) = -\sin(t + \frac{1}{4}\pi)$$

$$\sin(2t) = \sin(t + \frac{1}{4}\pi)$$

$$2t = t + \frac{1}{4}\pi + k \cdot 2\pi \vee 2t = \pi - t - \frac{1}{4}\pi + k \cdot 2\pi$$

$$t = \frac{1}{4}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{4}\pi + k \cdot 2\pi$$

$$t = \frac{1}{4}\pi + k \cdot 2\pi \vee t = -\frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$$

$$t \text{ in } [\frac{1}{4}\pi, \frac{1}{4}\pi] \text{ geeft } t = \frac{1}{4}\pi \vee t = \frac{7}{12}\pi$$

$$\begin{cases} x_Q(t) = \sin(2t) + \sin(t + \frac{1}{4}\pi) \\ y_Q(t) = \sin(t + \frac{1}{4}\pi) - \sin(2t) \end{cases} \text{ geeft } \begin{cases} x'_Q(t) = 2 \cos(2t) + \cos(t + \frac{1}{4}\pi) \\ y'_Q(t) = \cos(t + \frac{1}{4}\pi) - 2 \cos(2t) \end{cases}$$

$$\begin{cases} x'_Q(\frac{7}{12}\pi) = 2 \cos(\frac{7}{6}\pi) + \cos(\frac{5}{6}\pi) = 2 \cdot -\frac{1}{2}\sqrt{3} + -\frac{1}{2}\sqrt{3} = -1\frac{1}{2}\sqrt{3} \\ y'_Q(\frac{7}{12}\pi) = \cos(\frac{5}{6}\pi) - 2 \cos(\frac{7}{6}\pi) = -\frac{1}{2}\sqrt{3} - 2 \cdot -\frac{1}{2}\sqrt{3} = \frac{1}{2}\sqrt{3} \end{cases}$$

Dus de baansnelheid van Q in B is $\sqrt{(-1\frac{1}{2}\sqrt{3})^2 + (\frac{1}{2}\sqrt{3})^2} = \sqrt{\frac{15}{2}} = \frac{1}{2}\sqrt{30}$.

Dus de baansnelheid van P in A is $\frac{3\sqrt{2}}{\frac{1}{2}\sqrt{30}} = 6\sqrt{\frac{1}{15}} = 6 \cdot \frac{1}{\sqrt{15}}\sqrt{15} = \frac{2}{5}\sqrt{15}$ keer

zo groot als de baansnelheid van Q in B .

Bladzijde 171

C7 $y = \frac{1}{2}$ geeft $\sin(t) = \frac{1}{2}$

$$t = \frac{1}{6}\pi + k \cdot 2\pi \vee t = \frac{5}{6}\pi + k \cdot 2\pi$$

$$0 \leq t \leq 2\pi \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{5}{6}\pi$$

$$x(\frac{1}{6}\pi) = \sin(\frac{2}{3}\pi) = \frac{1}{2}\sqrt{3} \text{ en } x(\frac{5}{6}\pi) = \sin(\frac{5}{3}\pi) = -\frac{1}{2}\sqrt{3}$$

Dus $A(-\frac{1}{2}\sqrt{3}, \frac{1}{2})$, $B(\frac{1}{2}\sqrt{3}, \frac{1}{2})$ en $AB = \frac{1}{2}\sqrt{3} - (-\frac{1}{2}\sqrt{3}) = \sqrt{3}$.

b $x(t) = \sin(4t)$ geeft $x'(t) = 4 \cos(4t)$

$y(t) = \sin(t)$ geeft $y'(t) = \cos(t)$

$$\vec{v}_{\left(\frac{5}{6}\pi\right)} = \begin{pmatrix} x'(\frac{5}{6}\pi) \\ y'(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} 4 \cos(3\frac{1}{3}\pi) \\ \cos(\frac{5}{6}\pi) \end{pmatrix} = \begin{pmatrix} -2 \\ -\frac{1}{2}\sqrt{3} \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ \sqrt{3} \end{pmatrix}$$

$\tan(\varphi) = \frac{\sqrt{3}}{4}$ geeft $\varphi = 23,41\dots^\circ$

De gevraagde hoek is $23,4^\circ$.

c De baansnelheid in A is $v_{\left(\frac{5}{6}\pi\right)} = \sqrt{(-2)^2 + (-\frac{1}{2}\sqrt{3})^2} = \sqrt{4 + \frac{3}{4}} = \sqrt{\frac{19}{4}} = \frac{1}{2}\sqrt{19}$.

C8 Substitutie van $x = \sin(t)$ in $2x^2 - 1$ geeft

$$2x^2 - 1 = 2 \sin^2(t) - 1 = -(1 - 2 \sin^2(t))$$

$$= -\cos(2t)$$

$$= \cos(2t + \pi) \text{ en dit is gelijk aan } y.$$

Omdat $x = \sin(t)$, geldt $-1 \leq x \leq 1$.

Dus bij de grafiek hoort de formule $y = 2x^2 - 1$ met $-1 \leq x \leq 1$.

C9 $\vec{OQ} = \vec{OP} + \vec{PQ} = \vec{OP} + \frac{1}{2}\vec{PA} + \frac{1}{2}\vec{PA}_R$

$$\vec{PA} = \vec{a} - \vec{p} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} - \begin{pmatrix} \sin(t) \\ \cos(2t) \end{pmatrix} = \begin{pmatrix} -\sin(t) \\ 2 - \cos(2t) \end{pmatrix} \text{ geeft } \vec{PA}_R = \begin{pmatrix} 2 - \cos(2t) \\ \sin(t) \end{pmatrix}$$

$$\begin{aligned} \vec{OQ} &= \begin{pmatrix} \sin(t) \\ \cos(2t) \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} -\sin(t) \\ 2 - \cos(2t) \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} 2 - \cos(2t) \\ \sin(t) \end{pmatrix} = \begin{pmatrix} \sin(t) - \frac{1}{2}\sin(t) + 1 - \frac{1}{2}\cos(2t) \\ \cos(2t) + 1 - \frac{1}{2}\cos(2t) + \frac{1}{2}\sin(t) \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}\sin(t) + 1 - \frac{1}{2}\cos(2t) \\ \frac{1}{2}\cos(2t) + 1 + \frac{1}{2}\sin(t) \end{pmatrix} \end{aligned}$$

De bewegingsvergelijkingen van Q zijn $\begin{cases} x_Q(t) = 1 + \frac{1}{2}\sin(t) - \frac{1}{2}\cos(2t) \\ y_Q(t) = 1 + \frac{1}{2}\sin(t) + \frac{1}{2}\cos(2t) \end{cases}$

Kennis- en vaardigheidentest

Bladzijde 172

1 a $\sin(3x - \frac{1}{4}\pi) = \cos(2x)$
 $\cos(3x - \frac{3}{4}\pi) = \cos(2x)$
 $3x - \frac{3}{4}\pi = 2x + k \cdot 2\pi \vee 3x - \frac{3}{4}\pi = -2x + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot 2\pi \vee 5x = \frac{3}{4}\pi + k \cdot 2\pi$
 $x = \frac{3}{4}\pi + k \cdot 2\pi \vee x = \frac{3}{20}\pi + k \cdot \frac{2}{5}\pi$
 x in $[0, \pi]$ geeft $x = \frac{3}{4}\pi \vee x = \frac{3}{20}\pi \vee x = \frac{11}{20}\pi \vee x = \frac{19}{20}\pi$

b $2 \cos^2(x) + \sin(x) = 1$
 $2(1 - \sin^2(x)) + \sin(x) = 1$
 $2 - 2 \sin^2(x) + \sin(x) = 1$
 $2 \sin^2(x) - \sin(x) - 1 = 0$
 Stel $\sin(x) = u$.
 $2u^2 - u - 1 = 0$
 $D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$, dus $\sqrt{D} = 3$
 $u = \frac{1+3}{4} = 1 \vee u = \frac{1-3}{4} = -\frac{1}{2}$
 $\sin(x) = 1 \vee \sin(x) = -\frac{1}{2}$
 $x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$

$$\begin{aligned}
\text{c } \cos\left(\frac{2}{5}\pi t\right) &= -\sin\left(\frac{1}{6}\pi t\right) \\
\cos\left(\frac{2}{5}\pi t\right) &= \sin\left(\frac{1}{6}\pi t + \pi\right) \\
\cos\left(\frac{2}{5}\pi t\right) &= \cos\left(\frac{1}{6}\pi t + \frac{1}{2}\pi\right) \\
\frac{2}{5}\pi t &= \frac{1}{6}\pi t + \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{2}{5}\pi t = -\frac{1}{6}\pi t - \frac{1}{2}\pi + k \cdot 2\pi \\
\frac{7}{30}\pi t &= \frac{1}{2}\pi + k \cdot 2\pi \vee \frac{17}{30}\pi t = -\frac{1}{2}\pi + k \cdot 2\pi \\
t &= 2\frac{1}{7} + k \cdot 8\frac{4}{7} \vee t = -\frac{15}{17} + k \cdot 3\frac{9}{17} \\
t \text{ in } [0, 10] &\text{ geeft } t = 2\frac{1}{7} \vee t = 2\frac{11}{17} \vee t = 6\frac{3}{17} \vee t = 9\frac{12}{17}
\end{aligned}$$

$$\begin{aligned}
2 \text{ a } y &= \sin\left(\frac{1}{2}x\right)\cos\left(1\frac{1}{2}x\right) + \cos\left(\frac{1}{2}x\right)\sin\left(1\frac{1}{2}x\right) = \sin\left(\frac{1}{2}x + 1\frac{1}{2}x\right) = \sin(2x) \\
\text{b } y &= \sin(3x)\sin(5x) + \cos(3x)\cos(5x) = \cos(5x)\cos(3x) + \sin(5x)\sin(3x) = \cos(5x - 3x) = \cos(2x)
\end{aligned}$$

$$\begin{aligned}
3 \text{ a } \sin\left(x + \frac{1}{3}\pi\right) &= 2\sin(2x)\cos(2x) \\
\sin\left(x + \frac{1}{3}\pi\right) &= \sin(4x) \\
x + \frac{1}{3}\pi &= 4x + k \cdot 2\pi \vee x + \frac{1}{3}\pi = \pi - 4x + k \cdot 2\pi \\
-3x &= -\frac{1}{3}\pi + k \cdot 2\pi \vee 5x = \frac{2}{3}\pi + k \cdot 2\pi \\
x &= \frac{1}{9}\pi + k \cdot \frac{2}{5}\pi \vee x = \frac{2}{15}\pi + k \cdot \frac{2}{5}\pi \\
\text{b } \sin^2(2x) + \frac{1}{4} &= \cos(4x) \\
\frac{1}{2} - \frac{1}{2}\cos(4x) + \frac{1}{4} &= \cos(4x) \\
-1\frac{1}{2}\cos(4x) &= -\frac{3}{4} \\
\cos(4x) &= \frac{1}{2} \\
4x &= \frac{1}{3}\pi + k \cdot 2\pi \vee 4x = -\frac{1}{3}\pi + k \cdot 2\pi \\
x &= \frac{1}{12}\pi + k \cdot \frac{1}{2}\pi \vee x = -\frac{1}{12}\pi + k \cdot \frac{1}{2}\pi
\end{aligned}$$

$$\begin{aligned}
4 \text{ a } y &= \sin^2(2x) + 2\cos^2(2x) + 3\cos(4x) \\
&= \sin^2(2x) + \cos^2(2x) + \cos^2(2x) + 3\cos(4x) \\
&= 1 + \frac{1}{2} + \frac{1}{2}\cos(4x) + 3\cos(4x) \\
&= 1\frac{1}{2} + 3\frac{1}{2}\cos(4x) \\
\text{b } y &= \sin(2x)\cos(2x) + 4\sin(4x) - 1 \\
&= \frac{1}{2}\sin(4x) + 4\sin(4x) - 1 \\
&= 4\frac{1}{2}\sin(4x) - 1
\end{aligned}$$

$$\begin{aligned}
5 \text{ a } &\text{Translatie van de grafiek van } f \text{ over } (-1\frac{1}{2}\pi, 0) \text{ geeft de grafiek van} \\
&h(x) = \sin(2(x + 1\frac{1}{2}\pi)) + \cos(x + 1\frac{1}{2}\pi) = \sin(2x + 3\pi) + \cos(x + 1\frac{1}{2}\pi). \\
&h(x) = \sin(2x + \pi) + \cos(x - \frac{1}{2}\pi) = -\sin(2x) + \sin(x) \\
&h(-x) = -\sin(-2x) + \sin(-x) = \sin(2x) - \sin(x) \\
&\text{Er geldt } h(-x) = -h(x), \text{ dus de grafiek van } h \text{ is symmetrisch in de oorsprong} \\
&\text{en hieruit volgt dat de grafiek van } f \text{ symmetrisch is in het punt } (1\frac{1}{2}\pi, 0). \\
\text{b } &\text{Translatie van de grafiek van } g \text{ over } (-\pi, 0) \text{ geeft de grafiek van } k(x) = 2\sin^2(x + \pi)\cos(x + \pi). \\
&k(x) = 2 \cdot (-\sin(x))^2 \cdot -\cos(x) = -2\sin^2(x)\cos(x) \\
&k(-x) = -2\sin^2(-x)\cos(-x) = -2 \cdot (-\sin(x))^2 \cdot \cos(x) = -2\sin^2(x)\cos(x) \\
&\text{Er geldt } k(-x) = k(x), \text{ dus de grafiek van } k \text{ is symmetrisch in de } y\text{-as} \\
&\text{en hieruit volgt dat de grafiek van } g \text{ symmetrisch is in de lijn } x = \pi.
\end{aligned}$$

6 $f(x) = 0$ geeft $2 \sin(x) = 0$

$$\sin(x) = 0$$

$$x = k \cdot \pi$$

$$x \text{ in } [0, \pi] \text{ geeft } x = 0 \vee x = \pi$$

$$\begin{aligned} I(L) &= \pi \int_0^\pi (2 \sin(x))^2 dx = \pi \int_0^\pi 4 \sin^2(x) dx = \pi \int_0^\pi 4 \left(\frac{1}{2} - \frac{1}{2} \cos(2x) \right) dx = \pi \int_0^\pi (2 - 2 \cos(2x)) dx \\ &= \pi [2x - \sin(2x)]_0^\pi = \pi(2\pi - \sin(2\pi) - (0 - \sin(0))) = 2\pi^2 \end{aligned}$$

7 $O(V) = \int_{\frac{1}{3}\pi}^\pi (1 - 2 \cos^2(x) - \cos(x)) dx = \int_{\frac{1}{3}\pi}^\pi (-2 \cos^2(x) - 1 - \cos(x)) dx = \int_{\frac{1}{3}\pi}^\pi (-\cos(2x) - \cos(x)) dx$

$$\begin{aligned} &= \left[-\frac{1}{2} \sin(2x) - \sin(x) \right]_{\frac{1}{3}\pi}^\pi = -\frac{1}{2} \sin(2\pi) - \sin(\pi) - \left(-\frac{1}{2} \sin\left(\frac{2}{3}\pi\right) - \sin\left(\frac{1}{3}\pi\right) \right) \\ &= 0 - 0 + \frac{1}{2} \cdot \frac{1}{2} \sqrt{3} + \frac{1}{2} \sqrt{3} = \frac{3}{4} \sqrt{3} \end{aligned}$$

Bladzijde 173

8 a $AB^2 = (3 \cos(2t) - 2 \cos(\frac{1}{2}t))^2 + (3 \sin(2t) - 2 \sin(\frac{1}{2}t))^2$

$$\begin{aligned} &= 9 \cos^2(2t) - 12 \cos(2t) \cos(\frac{1}{2}t) + 4 \cos^2(\frac{1}{2}t) + 9 \sin^2(2t) - 12 \sin(2t) \sin(\frac{1}{2}t) + 4 \sin^2(\frac{1}{2}t) \\ &= 9 + 4 - 12(\cos(2t) \cos(\frac{1}{2}t) + \sin(2t) \sin(\frac{1}{2}t)) \\ &= 13 - 12 \cos(2t - \frac{1}{2}t) \\ &= 13 - 12 \cos(\frac{3}{2}t) \end{aligned}$$

$$\text{Dus } AB = \sqrt{13 - 12 \cos(\frac{3}{2}t)}.$$

b $AB = \sqrt{7}$ geeft $\sqrt{13 - 12 \cos(\frac{3}{2}t)} = \sqrt{7}$

$$\begin{aligned} 13 - 12 \cos(\frac{3}{2}t) &= 7 \\ 12 \cos(\frac{3}{2}t) &= 6 \\ \cos(\frac{3}{2}t) &= \frac{1}{2} \\ \frac{3}{2}t &= \frac{1}{3}\pi + k \cdot 2\pi \vee \frac{3}{2}t = -\frac{1}{3}\pi + k \cdot 2\pi \\ t &= \frac{2}{9}\pi + k \cdot \frac{1}{3}\pi \vee t = -\frac{2}{9}\pi + k \cdot \frac{1}{3}\pi \\ t \text{ in } [0, 4\pi] \text{ geeft } t &= \frac{2}{9}\pi \vee t = \frac{5}{9}\pi \vee t = \frac{8}{9}\pi \vee t = \frac{11}{9}\pi \vee t = \frac{14}{9}\pi \vee t = \frac{17}{9}\pi \vee t = \frac{20}{9}\pi \end{aligned}$$

9 a De frequentie is $\frac{8\pi}{2\pi} = 4$ Hz.

b Per trilling is de afgelegde afstand $4 \cdot 5 = 20$ dm = 2 m.

Er zijn 4 trillingen per seconde, dus per uur zijn er $3600 \cdot 4 = 14400$ trillingen.

De afgelegde afstand in één uur is $14400 \cdot 2 = 28800$ m = 28,8 km.

10 $x = 2$ geeft $2 \cos(t) = 2$

$$\cos(t) = 1$$

$$t = k \cdot 2\pi$$

$$y(0) = \cos(-\pi) = -1$$

$$y = ax^2 + 1 \text{ door } (2, -1) \text{ geeft } 4a + 1 = -1 \text{ oftewel } 4a = -2, \text{ dus } a = -\frac{1}{2}.$$

Dit geeft $K: y = -\frac{1}{2}x^2 + 1$.

Substitutie van $x = 2 \cos(t)$ in $-\frac{1}{2}x^2 + 1$ geeft $-\frac{1}{2}x^2 + 1 = -\frac{1}{2} \cdot (2 \cos(t))^2 + 1$

$$= -\frac{1}{2} \cdot 4 \cos^2(t) + 1$$

$$= -2 \cos^2(t) + 1$$

$$= -(2 \cos^2(t) - 1)$$

$$= -\cos(2t)$$

$$= \cos(2t + \pi)$$

$$= \cos(2t - \pi) \text{ en dit is gelijk aan } y.$$

Omdat $x = 2 \cos(t)$, geldt $-2 \leq x \leq 2$.

Dus bij de kromme K hoort de formule $y = -\frac{1}{2}x^2 + 1$ met $-2 \leq x \leq 2$.

a $y_P(t) = -\frac{1}{2}\sqrt{3}$ geeft $\cos(2t - \frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$

$$2t - \frac{1}{3}\pi = \frac{5}{6}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$$

$$2t = \frac{1}{6}\pi + k \cdot 2\pi \vee 2t = -\frac{1}{2}\pi + k \cdot 2\pi$$

$$t = \frac{7}{12}\pi + k \cdot \pi \vee t = -\frac{1}{4}\pi + k \cdot \pi$$

$$t \text{ in } [0, \frac{1}{2}\pi] \text{ geeft } t = \frac{7}{12}\pi \vee t = \frac{3}{4}\pi$$

$$x_P(\frac{7}{12}\pi) = \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

$$x_P(\frac{3}{4}\pi) = \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

$$\text{Dus } AB = \frac{1}{2}\sqrt{2} - (-\frac{1}{2}\sqrt{2}) = \sqrt{2}.$$

b $x_P(t) = 0$ geeft $\sin(3t) = 0$

$$3t = k \cdot \pi$$

$$t = k \cdot \frac{1}{3}\pi$$

$$t \text{ in } [0, \frac{1}{2}\pi] \text{ geeft } t = 0 \vee t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi \vee t = \pi \vee t = \frac{4}{3}\pi$$

$$y_P(0) = \cos(-\frac{1}{3}\pi) = \frac{1}{2}$$

$$y_P(\frac{1}{3}\pi) = \cos(\frac{1}{3}\pi) = \frac{1}{2}$$

$$y_P(\frac{2}{3}\pi) = \cos(\pi) = -1$$

$$y_P(\pi) = \cos(\frac{4}{3}\pi) = \frac{1}{2}$$

$$y_P(\frac{4}{3}\pi) = \cos(\frac{1}{3}\pi) = \frac{1}{2}$$

$$x_P(t) = \sin(3t) \text{ geeft } x'_P(t) = 3 \cos(3t)$$

$$y_P(t) = \cos(2t - \frac{1}{3}\pi) \text{ geeft } y'_P(t) = -2 \sin(2t - \frac{1}{3}\pi)$$

$$\vec{v}_P(0) = \begin{pmatrix} x'_P(0) \\ y'_P(0) \end{pmatrix} = \begin{pmatrix} 3 \cos(0) \\ -2 \sin(-\frac{1}{3}\pi) \end{pmatrix} = \begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix}$$

$$\vec{v}_P(\pi) = \begin{pmatrix} x'_P(\pi) \\ y'_P(\pi) \end{pmatrix} = \begin{pmatrix} 3 \cos(3\pi) \\ -2 \sin(\frac{2}{3}\pi) \end{pmatrix} = \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ \sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ \sqrt{3} \end{pmatrix} \right|} = \frac{3 \cdot -3 + \sqrt{3} \cdot \sqrt{3}}{\sqrt{12} \cdot \sqrt{12}} = -\frac{1}{2} \text{ geeft } \varphi = 120^\circ$$

De gevraagde hoek is $180^\circ - 120^\circ = 60^\circ$.

c De baansnelheid is

$$v_P(\frac{2}{3}\pi) = \sqrt{(x'_P(\frac{2}{3}\pi))^2 + (y'_P(\frac{2}{3}\pi))^2} = \sqrt{(3 \cos(2\pi))^2 + (-2 \sin(\pi))^2} = \sqrt{(3 \cdot 1)^2 + (-2 \cdot 0)^2} = 3.$$

d $\vec{OQ} = \vec{OP} + \vec{PQ} = \vec{OP} + \vec{PO}_R$

$$\vec{PO} = \begin{pmatrix} -\sin(3t) \\ -\cos(2t - \frac{1}{3}\pi) \end{pmatrix}, \text{ dus } \vec{PO}_R = \begin{pmatrix} -\cos(2t - \frac{1}{3}\pi) \\ \sin(3t) \end{pmatrix}.$$

$$\vec{OQ} = \begin{pmatrix} \sin(3t) \\ \cos(2t - \frac{1}{3}\pi) \end{pmatrix} + \begin{pmatrix} -\cos(2t - \frac{1}{3}\pi) \\ \sin(3t) \end{pmatrix} = \begin{pmatrix} \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{pmatrix}$$

$$\text{De bewegingsvergelijkingen van } Q \text{ zijn } \begin{cases} x_Q(t) = \sin(3t) - \cos(2t - \frac{1}{3}\pi) \\ y_Q(t) = \cos(2t - \frac{1}{3}\pi) + \sin(3t) \end{cases}$$

$$y_Q(t) = 0 \text{ geeft } \cos(2t - \frac{1}{3}\pi) + \sin(3t) = 0$$

$$\cos(2t - \frac{1}{3}\pi) = -\sin(3t)$$

$$\cos(2t - \frac{1}{3}\pi) = -\cos(3t - \frac{1}{2}\pi)$$

$$\cos(2t - \frac{1}{3}\pi) = \cos(3t + \frac{1}{2}\pi)$$

$$2t - \frac{1}{3}\pi = 3t + \frac{1}{2}\pi + k \cdot 2\pi \vee 2t - \frac{1}{3}\pi = -3t - \frac{1}{2}\pi + k \cdot 2\pi$$

$$-t = \frac{5}{6}\pi + k \cdot 2\pi \vee 5t = -\frac{1}{6}\pi + k \cdot 2\pi$$

$$t = -\frac{5}{6}\pi + k \cdot 2\pi \vee t = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$$

$$t \text{ in } [0, \frac{1}{2}\pi] \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{11}{30}\pi \vee t = \frac{23}{30}\pi$$

$$x_Q(\frac{1}{6}\pi) = -2, x_Q(\frac{11}{30}\pi) \approx -0,618 \text{ en } x_Q(\frac{23}{30}\pi) \approx 1,618$$

$$\text{Dus voor } t = \frac{23}{30}\pi.$$

Bladzijde 174

- 1 a** $f(x) = 0$ geeft $2 \sin^2(x) + \sin(x) = 0$
 $\sin(x)(2 \sin(x) + 1) = 0$
 $\sin(x) = 0 \vee \sin(x) = -\frac{1}{2}$
 $x = k \cdot \pi \vee x = 1\frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi$
 x in $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 2\pi \vee x = 1\frac{1}{6}\pi \vee x = 1\frac{5}{6}\pi$
 $O = \int_0^\pi f(x) dx = \int_0^\pi (2 \sin^2(x) + \sin(x)) dx = \int_0^\pi (1 - \cos(2x) + \sin(x)) dx$
 $= \left[x - \frac{1}{2} \sin(2x) - \cos(x) \right]_0^\pi = \pi - 0 - (-1) - (0 - 0 - 1) = \pi + 2$
- b** $f(x) = 2 \sin^2(x) + \sin(x)$ geeft $f'(x) = 4 \sin(x) \cos(x) + \cos(x)$
 $f'(x) = 0$ geeft $4 \sin(x) \cos(x) + \cos(x) = 0$
 $\cos(x)(4 \sin(x) + 1) = 0$
 $\cos(x) = 0 \vee \sin(x) = -\frac{1}{4}$
 $x = \frac{1}{2}\pi + k \cdot \pi \vee \sin(x) = -\frac{1}{4}$
 $x = \frac{1}{2}\pi + k \cdot \pi$ met x in $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$
 $f(\frac{1}{2}\pi) = 2 \sin^2(\frac{1}{2}\pi) + \sin(\frac{1}{2}\pi) = 2 \cdot 1^2 + 1 = 3$
 $f(1\frac{1}{2}\pi) = 2 \sin^2(1\frac{1}{2}\pi) + \sin(1\frac{1}{2}\pi) = 2 \cdot (-1)^2 - 1 = 1$
 $\sin(x) = -\frac{1}{4}$ geeft $f(x) = 2 \cdot (-\frac{1}{4})^2 - \frac{1}{4} = -\frac{1}{8}$
 $f(x) = p$ heeft precies vier oplossingen voor $-\frac{1}{8} < p < 0 \vee 0 < p < 1$.
- c** De grafiek van f is symmetrisch op het interval $[0, \pi]$ betekent symmetrie in de lijn $x = \frac{1}{2}\pi$.
 Translatie van de grafiek van f over $(-\frac{1}{2}\pi, 0)$ geeft de grafiek van
 $g(x) = 2 \sin^2(x + \frac{1}{2}\pi) + \sin(x + \frac{1}{2}\pi)$ oftewel van $g(x) = 2 \cos^2(x) + \cos(x)$.
 $g(-x) = 2 \cos^2(-x) + \cos(-x) = 2 \cos^2(x) + \cos(x)$
 Er geldt $g(-x) = g(x)$, dus de grafiek van g is symmetrisch in de y -as en hieruit volgt
 dat de grafiek van f symmetrisch is in de lijn $x = \frac{1}{2}\pi$ oftewel symmetrisch op $[0, \pi]$.
- d** $q = f(\frac{1}{2}\pi - \frac{1}{3}\pi) = f(\frac{1}{6}\pi) = 2 \sin^2(\frac{1}{6}\pi) + \sin(\frac{1}{6}\pi) = 2 \cdot (\frac{1}{2})^2 + \frac{1}{2} = 1$
- 2 a** $f(x) = \sin(2x) - 2 \cos(x)$ geeft $f'(x) = 2 \cos(2x) + 2 \sin(x)$
 $f'(x) = 0$ geeft $2 \cos(2x) + 2 \sin(x) = 0$
 $\cos(2x) = -\sin(x)$
 $\cos(2x) = \sin(x + \pi)$
 $\cos(2x) = \cos(x + \frac{1}{2}\pi)$
 $2x = x + \frac{1}{2}\pi + k \cdot 2\pi \vee 2x = -x - \frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi \vee 3x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$
 Bij A hoort $x = -\frac{1}{6}\pi$.
 $f(-\frac{1}{6}\pi) = \sin(-\frac{1}{3}\pi) - 2 \cos(-\frac{1}{6}\pi) = -\frac{1}{2}\sqrt{3} - 2 \cdot \frac{1}{2}\sqrt{3} = -1\frac{1}{2}\sqrt{3}$, dus $A(-\frac{1}{6}\pi, -1\frac{1}{2}\sqrt{3})$.
 $-\frac{1}{6}\pi + \frac{2}{3}\pi = \frac{1}{2}\pi$ en $\frac{1}{2}\pi + \frac{2}{3}\pi = 1\frac{1}{6}\pi$, dus bij B hoort $x = 1\frac{1}{6}\pi$.
 $f(1\frac{1}{6}\pi) = \sin(2\frac{1}{3}\pi) - 2 \cos(1\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3} - 2 \cdot -\frac{1}{2}\sqrt{3} = 1\frac{1}{2}\sqrt{3}$, dus $B(1\frac{1}{6}\pi, 1\frac{1}{2}\sqrt{3})$.
 $a = \text{amplitude} = 1\frac{1}{2}\sqrt{3}$
 $\frac{1}{2}$ periode $= 1\frac{1}{6}\pi - (-\frac{1}{6}\pi) = 1\frac{1}{3}\pi$, dus periode $= 2\frac{2}{3}\pi$ en $b = \frac{2\pi}{2\frac{2}{3}\pi} = \frac{3}{4}$.
 Een hoogste punt voor $t = 1\frac{1}{6}\pi$, dus $c = 1\frac{1}{6}\pi$.

$$\begin{aligned}
\text{b } f(x) = 0 & \text{ geeft } \sin(2x) - 2\cos(x) = 0 \\
& 2\sin(x)\cos(x) - 2\cos(x) = 0 \\
& 2\cos(x)(\sin(x) - 1) = 0 \\
& \cos(x) = 0 \vee \sin(x) = 1 \\
& x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi \\
& x = \frac{1}{2}\pi + k \cdot \pi
\end{aligned}$$

Dus $C(\frac{1}{2}\pi, 0)$.

Translatie van de grafiek van f over $(-\frac{1}{2}\pi, 0)$ geeft de grafiek van

$$g(x) = \sin(2(x + \frac{1}{2}\pi)) - 2\cos(x + \frac{1}{2}\pi).$$

$$g(x) = \sin(2x + \pi) - 2\cos(x + \frac{1}{2}\pi) = -\sin(2x) - 2\sin(x + \pi) = -\sin(2x) + 2\sin(x)$$

$$g(-x) = -\sin(-2x) + 2\sin(-x) = \sin(2x) - 2\sin(x)$$

Er geldt $g(-x) = -g(x)$, dus de grafiek van g is symmetrisch in de oorsprong en hieruit volgt dat de grafiek van f symmetrisch is in het punt $C(\frac{1}{2}\pi, 0)$.

3

$$\begin{aligned}
\text{a } \begin{cases} x_P(t) = 2\cos(\frac{1}{4}\pi t) \\ y_P(t) = 2\sin(\frac{1}{4}\pi t) \end{cases} & \text{ geeft } \begin{cases} x'_P(t) = -\frac{1}{2}\pi\sin(\frac{1}{4}\pi t) \\ y'_P(t) = \frac{1}{2}\pi\cos(\frac{1}{4}\pi t) \end{cases}, \text{ dus } \vec{v}_P(t) = \begin{pmatrix} -\frac{1}{2}\pi\sin(\frac{1}{4}\pi t) \\ \frac{1}{2}\pi\cos(\frac{1}{4}\pi t) \end{pmatrix}. \\
\begin{cases} x'_P(t) = -\frac{1}{2}\pi\sin(\frac{1}{4}\pi t) \\ y'_P(t) = \frac{1}{2}\pi\cos(\frac{1}{4}\pi t) \end{cases} & \text{ geeft } \begin{cases} x''_P(t) = -\frac{1}{8}\pi^2\cos(\frac{1}{4}\pi t) \\ y''_P(t) = -\frac{1}{8}\pi^2\sin(\frac{1}{4}\pi t) \end{cases}, \text{ dus } \vec{a}_P(t) = \begin{pmatrix} -\frac{1}{8}\pi^2\cos(\frac{1}{4}\pi t) \\ -\frac{1}{8}\pi^2\sin(\frac{1}{4}\pi t) \end{pmatrix}. \\
\vec{v}_P(t) \cdot \vec{a}_P(t) &= \begin{pmatrix} -\frac{1}{2}\pi\sin(\frac{1}{4}\pi t) \\ \frac{1}{2}\pi\cos(\frac{1}{4}\pi t) \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{8}\pi^2\cos(\frac{1}{4}\pi t) \\ -\frac{1}{8}\pi^2\sin(\frac{1}{4}\pi t) \end{pmatrix} \\
&= -\frac{1}{2}\pi\sin(\frac{1}{4}\pi t) \cdot -\frac{1}{8}\pi^2\cos(\frac{1}{4}\pi t) + \frac{1}{2}\pi\cos(\frac{1}{4}\pi t) \cdot -\frac{1}{8}\pi^2\sin(\frac{1}{4}\pi t) \\
&= \frac{1}{16}\pi^3\sin(\frac{1}{4}\pi t)\cos(\frac{1}{4}\pi t) - \frac{1}{16}\pi^3\sin(\frac{1}{4}\pi t)\cos(\frac{1}{4}\pi t) \\
&= 0
\end{aligned}$$

Dus $\vec{v}_P(t)$ en $\vec{a}_P(t)$ staan in ieder punt van de cirkel loodrecht op elkaar.

b Uit de bewegingsvergelijkingen van Q volgt $y_Q(t) = x_Q(t)$.

Als P zich in A of B bevindt, geldt $2\sin(\frac{1}{4}\pi t) = 2\cos(\frac{1}{4}\pi t)$

$$\begin{aligned}
\frac{\sin(\frac{1}{4}\pi t)}{\cos(\frac{1}{4}\pi t)} &= 1 \\
\tan(\frac{1}{4}\pi t) &= 1 \\
\frac{1}{4}\pi t &= \frac{1}{4}\pi + k \cdot \pi \\
t &= 1 + k \cdot 4
\end{aligned}$$

$$x_P(1) = 2\cos(\frac{1}{4}\pi) = \sqrt{2}$$

$$x_Q(1) = \sqrt{2} \text{ geeft } \sqrt{2}\sin(c) = \sqrt{2}$$

$$\sin(c) = 1$$

$$c = \frac{1}{2}\pi + k \cdot 2\pi$$

Dus voor $c = \frac{1}{2}\pi$.

c Voor $t = 5$ bevindt P zich voor het eerst in het punt $B(-\sqrt{2}, -\sqrt{2})$.

Voor $t = 5$ geldt $x_Q(t) = \sqrt{2}\sin(5c)$ en $y_Q(t) = \sqrt{2}\sin(5c)$.

$$x_Q(1) = -\sqrt{2} \text{ geeft } \sqrt{2}\sin(5c) = -\sqrt{2}$$

$$\sin(5c) = -1$$

$$5c = \frac{3}{2}\pi + k \cdot 2\pi$$

$$c = \frac{3}{10}\pi + k \cdot \frac{2}{5}\pi$$

Dus voor $0 < c < \frac{3}{10}\pi$.

a $y(t) = 0$ geeft $\cos(3t) = 0$

$$3t = \frac{1}{2}\pi + k \cdot \pi$$

$$t = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$$

$$t \text{ in } [0, \pi] \text{ geeft } t = \frac{1}{6}\pi \vee t = \frac{1}{2}\pi \vee t = \frac{5}{6}\pi$$

$$x(\frac{1}{6}\pi) = \cos(\frac{1}{2}\pi) = \frac{1}{2}$$

$$x(\frac{1}{2}\pi) = \cos(\pi) = -1$$

$$x(\frac{5}{6}\pi) = \cos(\frac{5}{2}\pi) = \frac{1}{2}$$

$$\begin{cases} x(t) = \cos(2t) \\ y(t) = \cos(3t) \end{cases} \text{ geeft } \begin{cases} x'(t) = -2 \sin(2t) \\ y'(t) = -3 \sin(3t) \end{cases} \text{ en dus } \vec{v}(t) = \begin{pmatrix} -2 \sin(2t) \\ -3 \sin(3t) \end{pmatrix}.$$

$$\vec{v}(\frac{1}{6}\pi) = \begin{pmatrix} -2 \sin(\frac{1}{3}\pi) \\ -3 \sin(\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$$

$$\vec{v}(\frac{5}{6}\pi) = \begin{pmatrix} -2 \sin(\frac{5}{3}\pi) \\ -3 \sin(\frac{5}{2}\pi) \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$$

$$\cos(\varphi) = \frac{\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \right|} = \frac{1 \cdot 1 + \sqrt{3} \cdot -\sqrt{3}}{\sqrt{4} \cdot \sqrt{4}} = -\frac{1}{2} \text{ geeft } \varphi = 60^\circ$$

De gevraagde hoek is 60° .**b** $x(t) = 0$ geeft $\cos(2t) = 0$

$$2t = \frac{1}{2}\pi + k \cdot \pi$$

$$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

$$t \text{ in } [0, \pi] \text{ geeft } t = \frac{1}{4}\pi \vee t = \frac{3}{4}\pi$$

$$y(\frac{1}{4}\pi) = \cos(\frac{3}{4}\pi) = -\frac{1}{2}\sqrt{2}$$

$$y(\frac{3}{4}\pi) = \cos(2\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2}$$

$$\vec{v}(\frac{3}{4}\pi) = \begin{pmatrix} -2 \sin(1\frac{1}{2}\pi) \\ -3 \sin(2\frac{1}{4}\pi) \end{pmatrix} = \begin{pmatrix} 2 \\ -1\frac{1}{2}\sqrt{2} \end{pmatrix}$$

$$k: y = ax + b \text{ met } a = \frac{-1\frac{1}{2}\sqrt{2}}{2} = -\frac{3}{4}\sqrt{2} \text{ en } k \text{ door } (0, \frac{1}{2}\sqrt{2}).$$

$$\text{Dus } k: y = -\frac{3}{4}\sqrt{2} \cdot x + \frac{1}{2}\sqrt{2}.$$

c De baansnelheid is $v(\frac{1}{2}\pi) = \sqrt{(-2 \sin(\pi))^2 + (-3 \sin(1\frac{1}{2}\pi))^2} = \sqrt{(-2 \cdot 0)^2 + (-3 \cdot -1)^2} = 3$.**d** $d(A, B) = 1$, dus $y(t) = \frac{1}{2}$ omdat de baan symmetrisch is in de x -as.

$$\cos(3t) = \frac{1}{2}$$

$$3t = \frac{1}{3}\pi + k \cdot 2\pi \vee 3t = -\frac{1}{3}\pi + k \cdot 2\pi$$

$$t = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi \vee t = -\frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$$

$$t \text{ in } [0, \pi] \text{ geeft } t = \frac{1}{9}\pi \vee t = \frac{7}{9}\pi \vee t = \frac{5}{9}\pi$$

$$x(\frac{1}{9}\pi) = \cos(\frac{2}{9}\pi) = 0,766...$$

$$x(\frac{7}{9}\pi) = \cos(1\frac{5}{9}\pi) = 0,173...$$

$$x(\frac{5}{9}\pi) = \cos(1\frac{1}{9}\pi) = -0,939...$$

$$\text{Dus } p \approx -0,94 \vee p \approx 0,17 \vee p \approx 0,77.$$

- 5 a** De baan van P is de cirkel met middelpunt O en straal 4.
 De baan van Q is de cirkel met middelpunt O en straal 5.
 De minimale afstand tussen P en Q is $5 - 4 = 1$ voor $5t = 3t + k \cdot 2\pi$

$$2t = k \cdot 2\pi$$

$$t = k \cdot \pi$$

Dus de minimale afstand wordt bereikt voor $t = 0$, $t = \pi$ en $t = 2\pi$.

De maximale afstand tussen P en Q is $5 + 4 = 9$ voor $5t = 3t + \pi + k \cdot 2\pi$

$$2t = \pi + k \cdot 2\pi$$

$$t = \frac{1}{2}\pi + k \cdot \pi$$

Dus de maximale afstand wordt bereikt voor $t = \frac{1}{2}\pi$ en $t = \frac{3}{2}\pi$.

$$\begin{aligned} \mathbf{b} \quad PQ^2 &= (5 \cos(3t) - 4 \cos(5t))^2 + (5 \sin(3t) - 4 \sin(5t))^2 \\ &= 25 \cos^2(3t) - 40 \cos(3t) \cos(5t) + 16 \cos^2(5t) + 25 \sin^2(3t) - 40 \sin(3t) \sin(5t) + 16 \sin^2(5t) \\ &= 25 + 16 - 40(\cos(5t) \cos(3t) + \sin(5t) \sin(3t)) \\ &= 41 - 40 \cos(5t - 3t) \\ &= 41 - 40 \cos(2t) \\ &= 41 - 40(1 - 2 \sin^2(t)) \\ &= 41 - 40 + 80 \sin^2(t) \\ &= 1 + 80 \sin^2(t) \end{aligned}$$

Dus $PQ = \sqrt{1 + 80 \sin^2(t)}$.

$$\begin{aligned} \mathbf{c} \quad PQ = \sqrt{41} \text{ geeft } \sqrt{1 + 80 \sin^2(t)} &= \sqrt{41} \\ 1 + 80 \sin^2(t) &= 41 \\ 80 \sin^2(t) &= 40 \\ \sin^2(t) &= \frac{1}{2} \\ \sin(t) &= \frac{1}{2} \sqrt{2} \vee \sin(t) = -\frac{1}{2} \sqrt{2} \\ t = \frac{1}{4}\pi + k \cdot 2\pi \vee t = \frac{3}{4}\pi + k \cdot 2\pi \vee t &= \frac{5}{4}\pi + k \cdot 2\pi \vee t = \frac{7}{4}\pi + k \cdot 2\pi \\ t \text{ in } [0, 2\pi] \text{ geeft } t &= \frac{1}{4}\pi \vee t = \frac{3}{4}\pi \vee t = \frac{5}{4}\pi \vee t = \frac{7}{4}\pi \end{aligned}$$

- d** Evenwijdig met de y -as, dus $x_P = x_Q$

$$4 \cos(5t) = 5 \cos(3t)$$

 Invoeren van de bijbehorende formules en gebruiken van de optie snijpunt geeft $t \approx 0,76$.

6 a $y_P(t) = 0$ geeft $2 \cos(2t) = 0$

$$\cos(2t) = 0$$

$$2t = \frac{1}{2}\pi + k \cdot \pi$$

$$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$$

 $t \text{ in } [-\frac{1}{2}\pi, \frac{1}{2}\pi] \text{ geeft } t = -\frac{1}{4}\pi \vee t = \frac{1}{4}\pi$

$$x_P(-\frac{1}{4}\pi) = 3 \cos(\frac{1}{4}\pi) = 1\frac{1}{2}\sqrt{2}$$

$$x_P(\frac{1}{4}\pi) = 3 \cos(\frac{3}{4}\pi) = -1\frac{1}{2}\sqrt{2}$$

$$\begin{cases} x_P(t) = 3 \cos(t + \frac{1}{2}\pi) \\ y_P(t) = 2 \cos(2t) \end{cases} \text{ geeft } \begin{cases} x'_P(t) = -3 \sin(t + \frac{1}{2}\pi) \\ y'_P(t) = -4 \sin(2t) \end{cases}, \text{ dus } \vec{v}_P(t) = \begin{pmatrix} -3 \sin(t + \frac{1}{2}\pi) \\ -4 \sin(2t) \end{pmatrix}.$$

$$\vec{v}_P(-\frac{1}{4}\pi) = \begin{pmatrix} -3 \sin(\frac{1}{4}\pi) \\ -4 \sin(-\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2}\sqrt{2} \\ 4 \end{pmatrix}$$

$$\tan(\varphi) = \frac{4}{-1\frac{1}{2}\sqrt{2}} \text{ geeft } \varphi = -62,06\dots^\circ$$

De gevraagde hoek is $62,1^\circ$.

b De baansnelheid is $v_P(-\frac{1}{4}\pi) = \sqrt{(-1\frac{1}{2}\sqrt{2})^2 + 4^2} = \sqrt{4\frac{1}{2} + 16} = \sqrt{\frac{41}{2}} = \frac{1}{2}\sqrt{82}$.

$$\begin{aligned}
 \text{c } rc_k &= 1\frac{1}{3}\sqrt{2} \text{ geeft } \frac{-4\sin(2t)}{-3\sin(t+\frac{1}{2}\pi)} = 1\frac{1}{3}\sqrt{2} \\
 &\frac{\sin(2t)}{\sin(t+\frac{1}{2}\pi)} = \sqrt{2} \\
 &\frac{2\sin(t)\cos(t)}{\cos(t)} = \sqrt{2} \\
 &2\sin(t) = \sqrt{2} \\
 &\sin(t) = \frac{1}{2}\sqrt{2} \\
 &t = \frac{1}{4}\pi + k \cdot 2\pi \vee t = \frac{3}{4}\pi + k \cdot 2\pi \\
 &t \text{ in } [-\frac{1}{2}\pi, \frac{1}{2}\pi] \text{ geeft } t = \frac{1}{4}\pi
 \end{aligned}$$

$t = \frac{1}{4}\pi$ geeft $A(-1\frac{1}{2}\sqrt{2}, 0)$ (zie vraag a)

$$\left. \begin{aligned} y &= 1\frac{1}{3}\sqrt{2} \cdot x + b \\ \text{door } A(-1\frac{1}{2}\sqrt{2}, 0) \end{aligned} \right\} \begin{aligned} 1\frac{1}{3}\sqrt{2} \cdot -1\frac{1}{2}\sqrt{2} + b &= 0 \\ -4 + b &= 0 \\ b &= 4 \end{aligned}$$

Dus $k: y = 1\frac{1}{3}\sqrt{2} \cdot x + 4$.

$$\begin{aligned}
 \text{d } x_P(t) &= 0 \text{ geeft } 3\cos(t+\frac{1}{2}\pi) = 0 \\
 t + \frac{1}{2}\pi &= \frac{1}{2}\pi + k \cdot \pi
 \end{aligned}$$

$$\begin{aligned}
 t &= k \cdot \pi \\
 t \text{ in } [-\frac{1}{2}\pi, \frac{1}{2}\pi] \text{ geeft } t &= 0
 \end{aligned}$$

$y_P(0) = 2\cos(0) = 2$, dus de baan snijdt de y -as in het punt $(0, 2)$.

Dus de formule van de parabool is vermoedelijk van de vorm $y = ax^2 + 2$.

$x_P(\frac{1}{2}\pi) = 3\cos(\pi) = -3$ en $y_P(\frac{1}{2}\pi) = 2\cos(\pi) = -2$, dus

$$\left. \begin{aligned} y &= ax^2 + 2 \\ \text{door } (-3, -2) \end{aligned} \right\} \begin{aligned} a \cdot (-3)^2 + 2 &= -2 \\ 9a &= -4 \\ a &= -\frac{4}{9} \end{aligned}$$

Dus $y = -\frac{4}{9}x^2 + 2$.

Substitutie van $x = 3\cos(t+\frac{1}{2}\pi)$ in $-\frac{4}{9}x^2 + 2$ geeft

$$\begin{aligned}
 -\frac{4}{9}x^2 + 2 &= -\frac{4}{9} \cdot (3\cos(t+\frac{1}{2}\pi))^2 + 2 \\
 &= -\frac{4}{9} \cdot 9\cos^2(t+\frac{1}{2}\pi) + 2 \\
 &= -4\sin^2(t+\pi) + 2 \\
 &= -4(-\sin(t))^2 + 2 \\
 &= 2 - 4\sin^2(t) \\
 &= 2(1 - 2\sin^2(t)) \\
 &= 2\cos(2t) \text{ en dit is gelijk aan } y.
 \end{aligned}$$

Dus de baan van P is een deel van de parabool $y = -\frac{4}{9}x^2 + 2$.

$$\begin{aligned}
 \text{e } \overrightarrow{OQ} &= \overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OP} + \overrightarrow{PO}_R \\
 \overrightarrow{OP} &= \begin{pmatrix} 3\cos(t+\frac{1}{2}\pi) \\ 2\cos(2t) \end{pmatrix}, \text{ dus } \overrightarrow{PO} = \begin{pmatrix} -3\cos(t+\frac{1}{2}\pi) \\ -2\cos(2t) \end{pmatrix} \text{ en } \overrightarrow{PO}_R = \begin{pmatrix} -2\cos(2t) \\ 3\cos(t+\frac{1}{2}\pi) \end{pmatrix} \\
 \overrightarrow{OQ} &= \begin{pmatrix} 3\cos(t+\frac{1}{2}\pi) \\ 2\cos(2t) \end{pmatrix} + \begin{pmatrix} -2\cos(2t) \\ 3\cos(t+\frac{1}{2}\pi) \end{pmatrix} = \begin{pmatrix} 3\cos(t+\frac{1}{2}\pi) - 2\cos(2t) \\ 2\cos(2t) + 3\cos(t+\frac{1}{2}\pi) \end{pmatrix}
 \end{aligned}$$

De bewegingsvergelijkingen van Q zijn $\begin{cases} x_Q(t) = 3\cos(t+\frac{1}{2}\pi) - 2\cos(2t) \\ y_Q(t) = 2\cos(2t) + 3\cos(t+\frac{1}{2}\pi) \end{cases}$

f Q passeert y -as geeft $x_Q(t) = 0$ oftewel $3\cos(t+\frac{1}{2}\pi) - 2\cos(2t) = 0$.

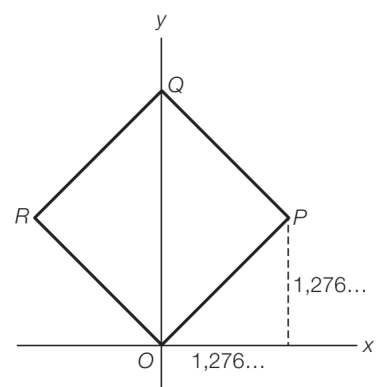
Invoeren van de bijbehorende formule en gebruiken van de optie nulpunt geeft $t = -0,439\dots$

$$x_P(-0,439\dots) = 3\cos(-0,439\dots + \frac{1}{2}\pi) = 1,276\dots \text{ en}$$

$$y_P(-0,439\dots) = 2\cos(2 \cdot -0,439\dots) = 1,276\dots, \text{ dus } P(1,276\dots; 1,276\dots).$$

Zie de figuur hiernaast.

$$O(OPQR) = OP^2 = 1,276\dots^2 + 1,276\dots^2 \approx 3,26.$$



K Voortgezette integraalrekening

K.1 De substitutiemethode

Bladzijde 178

V1

a $f(x) = x^2 + \sin(x)$ geeft $F(x) = \frac{1}{3}x^3 - \cos(x) + c$

b $f(x) = \frac{x^2 - 2}{x^3} = \frac{1}{x} - 2x^{-3}$ geeft $F(x) = \ln|x| + x^{-2} + c = \ln|x| + \frac{1}{x^2} + c$

c $f(x) = 4 \cdot 3^x$ geeft $F(x) = 4 \cdot \frac{3^x}{\ln(3)} + c = \frac{4 \cdot 3^x}{\ln(3)} + c$

d $f(x) = 3e^{x+1}$ geeft $F(x) = 3e^{x+1} + c$

V2

a $f(x) = 4 \sin(\pi x)$ geeft $F(x) = 4 \cdot \frac{1}{\pi} \cdot -\cos(\pi x) + c = -\frac{4 \cos(\pi x)}{\pi} + c$

b $f(x) = \frac{5}{(3x+1)^2} = 5(3x+1)^{-2}$ geeft $F(x) = 5 \cdot \frac{1}{3} \cdot -(3x+1)^{-1} + c = -\frac{5}{3(3x+1)} + c = -\frac{5}{9x+3} + c$

c $f(x) = (\frac{1}{2})^{2x-3}$ geeft $F(x) = \frac{1}{2} \cdot \frac{(\frac{1}{2})^{2x-3}}{\ln(\frac{1}{2})} + c = \frac{(\frac{1}{2})^{2x-2}}{\ln(\frac{1}{2})} + c$

d $f(x) = \frac{4x+1}{2\sqrt{4x+1}} = \frac{1}{2}\sqrt{4x+1} = \frac{1}{2}(4x+1)^{\frac{1}{2}}$ geeft $F(x) = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} (4x+1)^{\frac{1}{2}+1} + c = \frac{1}{12}(4x+1)\sqrt{4x+1} + c$

V3

a $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin(2x) dx = [-\frac{1}{2} \cos(2x)]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = -\frac{1}{2} \cos(\frac{2}{3}\pi) + \frac{1}{2} \cos(\frac{1}{3}\pi) = -\frac{1}{2} \cdot -\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

b $\int_1^4 \frac{6}{2x-1} dx = [6 \cdot \frac{1}{2} \ln|2x-1|]_1^4 = 3 \ln(7) - 3 \ln(1) = 3 \ln(7)$

c $\int_0^4 6e^{\frac{1}{2}x-3} dx = [6 \cdot 2e^{\frac{1}{2}x-3}]_0^4 = 12e^{-1} - 12e^{-3} = \frac{12}{e} - \frac{12}{e^3}$

Bladzijde 179

1

a $f(x) = \sin(x^2 + x)$ geeft $f'(x) = \cos(x^2 + x) \cdot (2x + 1) = (2x + 1) \cos(x^2 + x)$

b $G'(x) = (2x + 1) \cos(x^2 + x)$, dus $G(x) = \sin(x^2 + x) + 3$ is een primitieve van $g(x) = (2x + 1) \cos(x^2 + x)$.

Bladzijde 180

2

a Omdat $\frac{d(x^2 + 1)}{dx} = 2x$ en $6x = 3 \cdot 2x$, lukt het primitiveren van f wel op deze manier.

Omdat $\frac{d(x^3 + 1)}{dx} = 3x^2$ en $6x$ niet geschreven kan worden als het product van een constante en $3x^2$, lukt het primitiveren van h niet op deze manier.

b Omdat $\frac{d(x^4 + 7)}{dx} = 4x^3$ en $10x^3 = 2\frac{1}{2} \cdot 4x^3$, lukt het primitiveren van g wel op deze manier.

Omdat $\frac{d(x^4 + x)}{dx} = 4x^3 + 1$ en $10x^3$ niet geschreven kan worden als het product van een constante en $4x^3 + 1$, lukt het primitiveren van k niet op deze manier.

c $\frac{d(x^4 + x)}{dx} = 4x^3 + 1$
 $10x^3 + a = 2\frac{1}{2}(4x^3 + \frac{2}{5}a)$ moet gelijk zijn aan $2\frac{1}{2}(4x^3 + 1)$, dus $\frac{2}{5}a = 1$ en dit geeft $a = 2\frac{1}{2}$.

3

a $3x^2 dx = d(x^3 + 5)$

b $5x^4 dx = d(x^5 - 3)$

c $\cos(2x) dx = d(\frac{1}{2} \sin(2x) + \pi)$

d $\frac{1}{x} dx = d \ln|x|$

e $(5 - 2x) dx = d(5x - x^2 + 5)$

f $2 \cos(4x) dx = d\frac{1}{2} \sin(4x)$

- 4 a** $F(x) = \int 2x(x^2 + 4)^3 dx = \int (x^2 + 4)^3 \cdot 2x dx = \int (x^2 + 4)^3 d(x^2 + 4) = \int u^3 du = \frac{1}{4}u^4 + c$
 $= \frac{1}{4}(x^2 + 4)^4 + c$
- b** $G(x) = \int 6x\sqrt{x^2 + 1} dx = \int 3 \cdot \sqrt{x^2 + 1} \cdot 2x dx = \int 3\sqrt{x^2 + 1} d(x^2 + 1) = \int 3\sqrt{u} du = \int 3u^{\frac{1}{2}} du = 2u^{\frac{3}{2}} + c$
 $= 2u\sqrt{u} + c = 2(x^2 + 1)\sqrt{x^2 + 1} + c$
- c** $H(x) = \int 6x^2(x^3 - 1)^4 dx = \int 2 \cdot (x^3 - 1)^4 \cdot 3x^2 dx = \int 2(x^3 - 1)^4 d(x^3 - 1) = \int 2u^4 du = \frac{2}{5}u^5 + c$
 $= \frac{2}{5}(x^3 - 1)^5 + c$
- d** $K(x) = \int x \cos(x^2 + 3) dx = \int \frac{1}{2} \cdot \cos(x^2 + 3) \cdot 2x dx = \int \frac{1}{2} \cos(x^2 + 3) d(x^2 + 3) = \int \frac{1}{2} \cos(u) du$
 $= \frac{1}{2} \sin(u) + c = \frac{1}{2} \sin(x^2 + 3) + c$

- 5 a** $F(x) = \int x(3x^2 - 4)^3 dx = \int \frac{1}{6} \cdot (3x^2 - 4)^3 \cdot 6x dx = \int \frac{1}{6} (3x^2 - 4)^3 d(3x^2 - 4) = \int \frac{1}{6} u^3 du = \frac{1}{24}u^4 + c$
 $= \frac{1}{24}(3x^2 - 4)^4 + c$
- b** $G(x) = \int 2x\sqrt{x^2 - 3} dx = \int \sqrt{x^2 - 3} \cdot 2x dx = \int \sqrt{x^2 - 3} d(x^2 - 3) = \int \sqrt{u} du = \int u^{\frac{1}{2}} du = \frac{2}{3}u^{\frac{3}{2}} + c$
 $= \frac{2}{3}u\sqrt{u} + c = \frac{2}{3}(x^2 - 3)\sqrt{x^2 - 3} + c$
- c** $H(x) = \int \frac{6x}{3x^2 + 2} dx = \int \frac{1}{3x^2 + 2} \cdot 6x dx = \int \frac{1}{3x^2 + 2} d(3x^2 + 2) = \int \frac{1}{u} du = \ln|u| + c = \ln|3x^2 + 2| + c$
 $= \ln(3x^2 + 2) + c$
- d** $K(x) = \int 3x^2 \sin(x^3 - 1) dx = \int \sin(x^3 - 1) \cdot 3x^2 dx = \int \sin(x^3 - 1) d(x^3 - 1) = \int \sin(u) du$
 $= -\cos(u) + c = -\cos(x^3 - 1) + c$

- 6 a** $F(x) = \int \frac{\ln(x)}{x} dx = \int \ln(x) \cdot \frac{1}{x} dx = \int \ln(x) d \ln(x) = \int u du = \frac{1}{2}u^2 + c = \frac{1}{2} \ln^2(x) + c$
- b** $G(x) = \int x e^{-x^2} dx = \int -\frac{1}{2} \cdot e^{-x^2} \cdot -2x dx = \int -\frac{1}{2} e^{-x^2} d(-x^2) = \int -\frac{1}{2} e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2} + c$
- c** $H(x) = \int x\sqrt{5 - x^2} dx = \int -\frac{1}{2} \cdot \sqrt{5 - x^2} \cdot -2x dx = \int -\frac{1}{2} \sqrt{5 - x^2} d(5 - x^2) = \int -\frac{1}{2} \sqrt{u} du = \int -\frac{1}{2} u^{\frac{1}{2}} du$
 $= -\frac{1}{3} u^{\frac{3}{2}} + c = -\frac{1}{3} u\sqrt{u} + c = -\frac{1}{3} (5 - x^2)\sqrt{5 - x^2} + c$
- d** $K(x) = \int \frac{x}{\sqrt{x^2 + 1}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + 1}} \cdot 2x dx = \int \frac{1}{2\sqrt{x^2 + 1}} d(x^2 + 1) = \int \frac{1}{2\sqrt{u}} du = \sqrt{u} + c = \sqrt{x^2 + 1} + c$

- 7 a** $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{1}{\cos(x)} \cdot \sin(x)$
- b** $\int \tan(x) dx = \int \frac{1}{\cos(x)} \cdot \sin(x) dx = \int -1 \cdot \frac{1}{\cos(x)} \cdot -\sin(x) dx = \int -\frac{1}{\cos(x)} d \cos(x)$

- 8 a** $\int_0^{\frac{1}{6}\pi} \tan(2x) dx = \left[-\frac{1}{2} \ln|\cos(2x)| \right]_0^{\frac{1}{6}\pi} = -\frac{1}{2} \ln|\cos(\frac{1}{3}\pi)| + \frac{1}{2} \ln|\cos(0)| = -\frac{1}{2} \ln(\frac{1}{2}) + \frac{1}{2} \ln(1)$
 $= -\frac{1}{2} \ln(2^{-1}) + 0 = \frac{1}{2} \ln(2)$
- b** $\int \sin^3(x) dx = \int \sin^2(x) \cdot \sin(x) dx = \int -(1 - \cos^2(x)) \cdot -\sin(x) dx = \int (\cos^2(x) - 1) d \cos(x)$
 $= \int (u^2 - 1) du = \frac{1}{3}u^3 - u + c = \frac{1}{3} \cos^3(x) - \cos(x) + c$
- $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \sin^3(x) dx = \left[\frac{1}{3} \cos^3(x) - \cos(x) \right]_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} = \frac{1}{3} \cdot (\frac{1}{2})^3 - \frac{1}{2} - (\frac{1}{3} \cdot (\frac{1}{2}\sqrt{2})^3 - \frac{1}{2}\sqrt{2}) = \frac{5}{12}\sqrt{2} - \frac{11}{24}$
- c** $\int \frac{\sin(x)}{\cos^3(x)} dx = \int -\frac{1}{\cos^3(x)} \cdot -\sin(x) dx = \int -\frac{1}{\cos^3(x)} d \cos(x) = \int -\frac{1}{u^3} du = \int -u^{-3} du = \frac{1}{2}u^{-2} + c$
 $= \frac{1}{2u^2} + c = \frac{1}{2 \cos^2(x)} + c$
- $\int_0^{\frac{1}{6}\pi} \frac{\sin(x)}{\cos^3(x)} dx = \left[\frac{1}{2 \cos^2(x)} \right]_0^{\frac{1}{6}\pi} = \frac{1}{2 \cdot (\frac{1}{2}\sqrt{3})^2} - \frac{1}{2} = \frac{1}{6}$

- 9 a** $\int \frac{\cos(x)}{\sin^2(x)} dx = \int \frac{1}{\sin^2(x)} \cdot \cos(x) dx = \int \frac{1}{\sin^2(x)} d\sin(x) = \int \frac{1}{u^2} du = \int u^{-2} du = -u^{-1} + c = -\frac{1}{u} + c$
 $= -\frac{1}{\sin(x)} + c$
 $f(x) = \cos(x) - \frac{\cos(x)}{\sin^2(x)}$ geeft $F(x) = \sin(x) + \frac{1}{\sin(x)} + c$
- b** $G(x) = \int \sin^5(x) dx = \int -\sin^4(x) \cdot -\sin(x) dx = \int -(\sin^2(x))^2 d\cos(x) = \int -(1 - \cos^2(x))^2 d\cos(x)$
 $= \int -(1 - u^2)^2 du = \int -(1 - 2u^2 + u^4) du = \int (-u^4 + 2u^2 - 1) du = -\frac{1}{5}u^5 + \frac{2}{3}u^3 - u + c$
 $= -\frac{1}{5}\cos^5(x) + \frac{2}{3}\cos^3(x) - \cos(x) + c$

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- 10 a** $\int \frac{\ln^2(x)}{x} dx = \int \ln^2(x) \cdot \frac{1}{x} dx = \int \ln^2(x) d\ln(x) = \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}\ln^3(x) + c$
 $\int_1^2 \frac{\ln^2(x)}{x} dx = \left[\frac{1}{3}\ln^3(x) \right]_1^2 = \frac{1}{3}\ln^3(2) - \frac{1}{3}\ln^3(1) = \frac{1}{3}\ln^3(2)$
- b** $\int \frac{1}{x \ln(x)} dx = \int \frac{1}{\ln(x)} \cdot \frac{1}{x} dx = \int \frac{1}{\ln(x)} d\ln(x) = \int \frac{1}{u} du = \ln|u| + c = \ln|\ln(x)| + c$
 $\int_e^{e^2} \frac{1}{x \ln(x)} dx = [\ln|\ln(x)|]_e^{e^2} = \ln(2) - \ln(1) = \ln(2)$
- c** $\int \frac{x^2}{e^{2x^3}} dx = \int \frac{1}{6} \cdot \frac{1}{e^{2x^3}} \cdot 6x^2 dx = \int \frac{1}{6} \cdot \frac{1}{e^{2x^3}} d(2x^3) = \int \frac{1}{6} \cdot \frac{1}{e^u} du = \int \frac{1}{6} e^{-u} du = -\frac{1}{6} e^{-u} + c = -\frac{1}{6e^u} + c$
 $= -\frac{1}{6e^{2x^3}} + c$
 $\int_0^1 \frac{x^2}{e^{2x^3}} dx = \left[-\frac{1}{6e^{2x^3}} \right]_0^1 = -\frac{1}{6e^2} + \frac{1}{6} = \frac{e^2 - 1}{6e^2}$
- 11 a** $\sqrt{x+4} = u$
kwadrateren geeft
 $x+4 = u^2$
 $x = u^2 - 4$
Omdat $x = u^2 - 4$ en $\sqrt{x+4} = u$, krijg je $\int x\sqrt{x+4} dx = \int (u^2 - 4) \cdot u d(u^2 - 4)$.
- b** $\int (u^2 - 4) \cdot u d(u^2 - 4) = \int (u^2 - 4) \cdot u \cdot 2u du = \int (2u^4 - 8u^2) du$
- c** $\int x\sqrt{x+4} dx = \int (2u^4 - 8u^2) du = \frac{2}{5}u^5 - \frac{8}{3}u^3 + c = \frac{2}{5}(\sqrt{x+4})^5 - \frac{8}{3}(\sqrt{x+4})^3 + c$
 $\int_0^5 x\sqrt{x+4} dx = \left[\frac{2}{5}(\sqrt{x+4})^5 - \frac{8}{3}(\sqrt{x+4})^3 \right]_0^5 = \frac{2}{5}(\sqrt{9})^5 - \frac{8}{3}(\sqrt{9})^3 - \left(\frac{2}{5}(\sqrt{4})^5 - \frac{8}{3}(\sqrt{4})^3 \right) = 33\frac{11}{15}$
- 12 a** $\sqrt{x+1} = u$
kwadrateren geeft
 $x+1 = u^2$
 $x = u^2 - 1$
 $\int x\sqrt{x+1} dx = \int (u^2 - 1) \cdot u d(u^2 - 1) = \int (u^2 - 1) \cdot u \cdot 2u du = \int (2u^4 - 2u^2) du = \frac{2}{5}u^5 - \frac{2}{3}u^3 + c$
 $= \frac{2}{5}(\sqrt{x+1})^5 - \frac{2}{3}(\sqrt{x+1})^3 + c$
 $\int_0^3 x\sqrt{x+1} dx = \left[\frac{2}{5}(\sqrt{x+1})^5 - \frac{2}{3}(\sqrt{x+1})^3 \right]_0^3 = \frac{2}{5}(\sqrt{4})^5 - \frac{2}{3}(\sqrt{4})^3 - \left(\frac{2}{5}(\sqrt{1})^5 - \frac{2}{3}(\sqrt{1})^3 \right) = 7\frac{11}{15}$
- b** $\sqrt{x+9} = u$
kwadrateren geeft
 $x+9 = u^2$
 $x = u^2 - 9$
 $\int \frac{5x}{\sqrt{x+9}} dx = \int \frac{5(u^2 - 9)}{u} d(u^2 - 9) = \int \frac{5(u^2 - 9)}{u} \cdot 2u du = \int (10u^2 - 90) du = \frac{10}{3}u^3 - 90u + c$
 $= \frac{10}{3}(\sqrt{x+9})^3 - 90\sqrt{x+9} + c$
 $\int_0^7 \frac{5x}{\sqrt{x+9}} dx = \left[\frac{10}{3}(\sqrt{x+9})^3 - 90\sqrt{x+9} \right]_0^7 = \frac{10}{3}(\sqrt{16})^3 - 90\sqrt{16} - \left(\frac{10}{3}(\sqrt{9})^3 - 90\sqrt{9} \right) = 33\frac{1}{3}$

13 a $f(x) = 0$ geeft $2 + \ln(x) = 0$

$$\ln(x) = -2$$

$$x = e^{-2}$$

$$F(x) = \int \frac{2 + \ln(x)}{x} dx = \int (2 + \ln(x)) \cdot \frac{1}{x} dx = \int (2 + \ln(x)) d \ln(x) = \int (2 + u) du = 2u + \frac{1}{2}u^2 + c$$

$$= 2 \ln(x) + \frac{1}{2} \ln^2(x) + c$$

$$O(V) = \int_{e^{-2}}^e f(x) dx = \left[2 \ln(x) + \frac{1}{2} \ln^2(x) \right]_{e^{-2}}^e = 2 \ln(e) + \frac{1}{2} \ln^2(e) - (2 \ln(e^{-2}) + \frac{1}{2} \ln^2(e^{-2}))$$

$$= 2 + \frac{1}{2} - (2 \cdot -2 + \frac{1}{2} \cdot (-2)^2) = 4\frac{1}{2}$$

b $\int_1^p f(x) dx = 6$ geeft $2 \ln(p) + \frac{1}{2} \ln^2(p) - (2 \ln(1) + \frac{1}{2} \ln^2(1)) = 6$

$$\frac{1}{2} \ln^2(p) + 2 \ln(p) - 6 = 0$$

$$\ln^2(p) + 4 \ln(p) - 12 = 0$$

$$(\ln(p) - 2)(\ln(p) + 6) = 0$$

$$\ln(p) = 2 \vee \ln(p) = -6$$

$$p = e^2 \vee p = e^{-6}$$

$p = e^{-6}$ voldoet niet, dus $p = e^2$.

14 a $f(x) = g(x)$ geeft $\frac{4 \ln^2(x)}{x} = \frac{1}{x}$

$$4 \ln^2(x) = 1$$

$$\ln^2(x) = \frac{1}{4}$$

$$\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$$

$$x = e^{\frac{1}{2}} \vee x = e^{-\frac{1}{2}}$$

$$x = \sqrt{e} \vee x = \frac{1}{\sqrt{e}}$$

$$f(x) \leq g(x) \text{ geeft } \frac{1}{\sqrt{e}} \leq x \leq \sqrt{e}$$

b $F(x) = \int \frac{4 \ln^2(x)}{x} dx = \int 4 \ln^2(x) \cdot \frac{1}{x} dx = \int 4 \ln^2(x) d \ln(x) = \int 4u^2 du = \frac{4}{3}u^3 + c = \frac{4}{3} \ln^3(x) + c$

$$g(x) = \frac{1}{x} \text{ geeft } G(x) = \ln|x| + c$$

$$O(V) = \int_{e^{-\frac{1}{2}}}^{e^{\frac{1}{2}}} (g(x) - f(x)) dx = \left[\ln|x| - \frac{4}{3} \ln^3(x) \right]_{e^{-\frac{1}{2}}}^{e^{\frac{1}{2}}} = \ln(e^{\frac{1}{2}}) - \frac{4}{3} \ln^3(e^{\frac{1}{2}}) - (\ln(e^{-\frac{1}{2}}) - \frac{4}{3} \ln^3(e^{-\frac{1}{2}}))$$

$$= \frac{1}{2} - \frac{4}{3} \cdot \left(\frac{1}{2}\right)^3 - \left(-\frac{1}{2} - \frac{4}{3} \cdot \left(-\frac{1}{2}\right)^3\right) = \frac{2}{3}$$

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15 a $f(x) = \frac{3x^2}{x^3 + 4}$ geeft $f'(x) = \frac{(x^3 + 4) \cdot 6x - 3x^2 \cdot 3x^2}{(x^3 + 4)^2} = \frac{6x^4 + 24x - 9x^4}{(x^3 + 4)^2} = \frac{-3x^4 + 24x}{(x^3 + 4)^2}$

$$f'(x) = 0 \text{ geeft } -3x^4 + 24x = 0$$

$$-3x(x^3 - 8) = 0$$

$$x = 0 \vee x^3 = 8$$

$$x = 0 \vee x = 2$$

$$\text{min. is } f(0) = 0$$

$$\text{max. is } f(2) = 1$$

b $F(x) = \int \frac{3x^2}{x^3 + 4} dx = \int \frac{1}{x^3 + 4} \cdot 3x^2 dx = \int \frac{1}{x^3 + 4} d(x^3 + 4) = \int \frac{1}{u} du = \ln|u| + c = \ln|x^3 + 4| + c$

$$\int_0^p f(x) dx = 2 \text{ geeft } \left[\ln|x^3 + 4| \right]_0^p = 2$$

$$\ln(p^3 + 4) - \ln(4) = 2$$

$$\ln\left(\frac{p^3 + 4}{4}\right) = 2$$

$$\frac{p^3 + 4}{4} = e^2$$

$$p^3 + 4 = 4e^2$$

$$p^3 = 4e^2 - 4$$

$$p = \sqrt[3]{4e^2 - 4}$$

16 $\int (f(x))^2 dx = \int \left(\frac{1}{10}\sqrt{3x} \cdot e^{x^2}\right)^2 dx = \int \frac{3x \cdot e^{2x^2}}{100} dx = \int \frac{3}{4} \cdot \frac{e^{2x^2}}{100} \cdot 4x dx = \int \frac{3}{400} e^{2x^2} d(2x^2) = \int \frac{3}{400} e^u du$
 $= \frac{3}{400} e^u + c = \frac{3}{400} e^{2x^2} + c$
 $I(L_p) = \pi \int_0^p (f(x))^2 dx = \pi \left[\frac{3}{400} e^{2x^2} \right]_0^p = \pi \left(\frac{3}{400} e^{2p^2} - \frac{3}{400} \right) = \frac{3}{400} \pi (e^{2p^2} - 1)$
 $I(L_p) = 3 \frac{3}{10} \pi$ geeft $\frac{3}{400} \pi (e^{2p^2} - 1) = 3 \frac{3}{10} \pi$
 $e^{2p^2} - 1 = 440$
 $e^{2p^2} = 441$
 $2p^2 = \ln(441)$
 $p^2 = \frac{1}{2} \ln(441)$
 $p^2 = \ln(21)$
 $p = \sqrt{\ln(21)} \vee p = -\sqrt{\ln(21)}$
 $p = -\sqrt{\ln(21)}$ voldoet niet, dus $p = \sqrt{\ln(21)}$.

C1 **a** $F(x) = \int 4x(x^2 - 3)^3 dx = \int 2 \cdot (x^2 - 3)^3 \cdot 2x dx = \int 2(x^2 - 3)^3 d(x^2 - 3) = \int 2u^3 du = \frac{1}{2} u^4 + c$
 $= \frac{1}{2} (x^2 - 3)^4 + c$
b $G(x) = \int 4x\sqrt{x^2 + 1} dx = \int 2 \cdot \sqrt{x^2 + 1} \cdot 2x dx = \int 2\sqrt{x^2 + 1} d(x^2 + 1) = \int 2\sqrt{u} du = \int 2u^{\frac{1}{2}} du$
 $= \frac{4}{3} u^{\frac{3}{2}} + c = \frac{4}{3} u\sqrt{u} + c = \frac{4}{3} (x^2 + 1)\sqrt{x^2 + 1} + c$

C2 $\int \sin^4(x) \cos(x) dx = \int \sin^4(x) d\sin(x) = \int u^4 du = \frac{1}{5} u^5 + c = \frac{1}{5} \sin^5(x) + c$
 $\int_0^{\frac{1}{2}\pi} \sin^4(x) \cos(x) dx = \left[\frac{1}{5} \sin^5(x) \right]_0^{\frac{1}{2}\pi} = \frac{1}{5} \cdot 1^5 - \frac{1}{5} \cdot 0^5 = \frac{1}{5}$

K.2 Partieel integreren

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17 **a** $k'(x) \neq h(x)$, dus k is geen primitieve van h .
b $(2x + 3) \cos(x) = k'(x) - 2 \sin(x)$ geeft $\int (2x + 3) \cos(x) dx = \int k'(x) dx - \int 2 \sin(x) dx$
 $= k(x) - \int 2 \sin(x) dx$
 $= (2x + 3) \sin(x) - \int 2 \sin(x) dx$
c $H(x) = \int (2x + 3) \cos(x) dx = (2x + 3) \sin(x) - \int 2 \sin(x) dx = (2x + 3) \sin(x) + 2 \cos(x) + c$

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18 **a** $\int \frac{\ln(x)}{\sqrt{x}} dx = \int \ln(x) \cdot \frac{2}{2\sqrt{x}} dx = \ln(x) \cdot 2\sqrt{x} - \int \frac{1}{x} \cdot 2\sqrt{x} dx = 2\sqrt{x} \ln(x) - \int \frac{4}{2\sqrt{x}} dx$
 $= 2\sqrt{x} \ln(x) - 4\sqrt{x} + c = 2\sqrt{x} (\ln(x) - 2) + c$

b $\int x \sin(x) dx = \sin(x) \cdot \frac{1}{2} x^2 - \int \cos(x) \cdot \frac{1}{2} x^2 dx$
 Het wordt op deze manier alleen maar ingewikkelder, dus het lukt zo niet.

19 **a** $F(x) = \int x e^{2x} dx = x \cdot \frac{1}{2} e^{2x} - \int 1 \cdot \frac{1}{2} e^{2x} dx = \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + c$
b $F(x) = \int 2x \cos(x) dx = 2x \cdot \sin(x) - \int 2 \cdot \sin(x) dx = 2x \sin(x) + 2 \cos(x) + c$
c $f(x) = \ln(5x)$ geeft $F(x) = \frac{1}{5} (5x \ln(5x) - 5x) + c = x \ln(5x) - x + c$
d $\int x \ln(x) dx = \ln(x) \cdot \frac{1}{2} x^2 - \int \frac{1}{x} \cdot \frac{1}{2} x^2 dx = \frac{1}{2} x^2 \ln(x) - \int \frac{1}{2} x dx = \frac{1}{2} x^2 \ln(x) - \frac{1}{4} x^2 + c$
 $f(x) = x \ln(x) + x$ geeft $F(x) = \frac{1}{2} x^2 \ln(x) - \frac{1}{4} x^2 + \frac{1}{2} x^2 + c = \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2 + c$

20 $F(x) = \int e^{\sqrt{x}} dx = \int 2\sqrt{x} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx = \int 2\sqrt{x} \cdot e^{\sqrt{x}} d\sqrt{x} = \int 2u e^u du = 2u \cdot e^u - \int 2 \cdot e^u du$
 $= 2u e^u - \int 2 e^u du = 2u e^u - 2 e^u + c = 2\sqrt{x} \cdot e^{\sqrt{x}} - 2 e^{\sqrt{x}} + c$

21 a $\int 2x e^{x+1} dx = 2x \cdot e^{x+1} - \int 2 \cdot e^{x+1} dx = 2x e^{x+1} - \int 2 e^{x+1} dx = 2x e^{x+1} - 2 e^{x+1} + c$

$$\int_0^1 2x e^{x+1} dx = [2x e^{x+1} - 2 e^{x+1}]_0^1 = 2e^2 - 2e^2 - (0 - 2e) = 2e$$

b $\int (3x+1) \sin(x) dx = (3x+1) \cdot -\cos(x) - \int 3 \cdot -\cos(x) dx = -(3x+1) \cos(x) + \int 3 \cos(x) dx$
 $= -(3x+1) \cos(x) + 3 \sin(x) + c$

$$\int_0^\pi (3x+1) \sin(x) dx = [-(3x+1) \cos(x) + 3 \sin(x)]_0^\pi = -(3\pi+1) \cdot -1 + 3 \cdot 0 - (-1 \cdot 1 + 3 \cdot 0) = 3\pi + 2$$

22 a $f(x) = x^2 \ln(x)$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x \ln(x) + x$

$$f'(x) = 0 \text{ geeft } 2x \ln(x) + x = 0$$

$$x(2 \ln(x) + 1) = 0$$

$$x = 0 \vee 2 \ln(x) = -1$$

$$x = 0 \vee \ln(x) = -\frac{1}{2}$$

$$x = 0 \vee x = e^{-\frac{1}{2}}$$

$$x = 0 \text{ voldoet niet, dus } x = e^{-\frac{1}{2}}.$$

$$f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \cdot -\frac{1}{2} = -\frac{1}{2} e^{-1}$$

$$\text{Dus } A(e^{-\frac{1}{2}}, -\frac{1}{2} e^{-1}) \text{ oftewel } A\left(\frac{1}{\sqrt{e}}, -\frac{1}{2e}\right).$$

b $F(x) = \int x^2 \ln(x) dx = \ln(x) \cdot \frac{1}{3} x^3 - \int \frac{1}{x} \cdot \frac{1}{3} x^3 dx = \frac{1}{3} x^3 \ln(x) - \int \frac{1}{3} x^2 dx = \frac{1}{3} x^3 \ln(x) - \frac{1}{9} x^3 + c$

$$O(V) = \int_1^e x^2 \ln(x) dx = \left[\frac{1}{3} x^3 \ln(x) - \frac{1}{9} x^3 \right]_1^e = \frac{1}{3} e^3 \ln(e) - \frac{1}{9} e^3 - \left(\frac{1}{3} \ln(1) - \frac{1}{9} \right) = \frac{2}{9} e^3 + \frac{1}{9}$$

23 a $f(x) = \frac{\ln(x)}{x^2}$ geeft $f'(x) = \frac{x^2 \cdot \frac{1}{x} - \ln(x) \cdot 2x}{x^4} = \frac{x - 2x \ln(x)}{x^4} = \frac{1 - 2 \ln(x)}{x^3}$

$$f'(x) = 0 \text{ geeft } 1 - 2 \ln(x) = 0$$

$$\ln(x) = \frac{1}{2}$$

$$x = e^{\frac{1}{2}}$$

$$\text{max. is } f(e^{\frac{1}{2}}) = \frac{\frac{1}{2}}{(e^{\frac{1}{2}})^2} = \frac{\frac{1}{2}}{e} = \frac{1}{2e}$$

$$\text{Dus } B_f = \left[\leftarrow, \frac{1}{2e} \right].$$

b $F(x) = \int \frac{\ln(x)}{x^2} dx = \int \ln(x) \cdot x^{-2} dx = \ln(x) \cdot -x^{-1} - \int \frac{1}{x} \cdot -x^{-1} dx = -\frac{\ln(x)}{x} + \int x^{-2} dx$

$$= -\frac{\ln(x)}{x} - x^{-1} + c = -\frac{\ln(x)}{x} - \frac{1}{x} + c = -\frac{\ln(x) + 1}{x} + c$$

$$O(V) = \int_1^e \frac{\ln(x)}{x^2} dx = \left[-\frac{\ln(x) + 1}{x} \right]_1^e = -\frac{1+1}{e} + \frac{0+1}{1} = -\frac{2}{e} + 1$$

24 a Partieel integreren geeft $\int x^2 e^x dx = x^2 \cdot e^x - \int 2x \cdot e^x dx = x^2 e^x - \int 2x e^x dx$.

b Partieel integreren geeft $\int 2x e^x dx = 2x \cdot e^x - \int 2 \cdot e^x dx = 2x e^x - \int 2 e^x dx = 2x e^x - 2 e^x + c$.

c $F(x) = \int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - (2x e^x - 2 e^x + c) = x^2 e^x - 2x e^x + 2 e^x + c$
 $= (x^2 - 2x + 2) e^x + c$

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25 a $F(x) = (ax^2 + bx + c) e^x$ geeft $F'(x) = (2ax + b) e^x + (ax^2 + bx + c) e^x = (ax^2 + (2a + b)x + b + c) e^x$

$$\text{Dus } F'(x) = f(x) = (ax^2 + px + q) e^x \text{ met } p = 2a + b \text{ en } q = b + c.$$

b $F(x) = (ax^2 + bx + c) e^x$ geeft $f(x) = (ax^2 + (2a + b)x + b + c) e^x$

$$\left. \begin{array}{l} a = 4 \\ 2a + b = -5 \end{array} \right\} \begin{array}{l} 8 + b = -5 \\ b = -13 \end{array}$$

$$\left. \begin{array}{l} b = -13 \\ b + c = 6 \end{array} \right\} \begin{array}{l} -13 + c = 6 \\ c = 19 \end{array}$$

$$\text{Dus de primitieven van } f(x) = (4x^2 - 5x + 6) e^x \text{ zijn } F(x) = (4x^2 - 13x + 19) e^x + c.$$

26 a $\int e^x \sin(x) dx = \sin(x) \cdot e^x - \int \cos(x) \cdot e^x dx = e^x \sin(x) - (\cos(x) \cdot e^x - \int -\sin(x) \cdot e^x dx)$
 $= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx$
 $\int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx$ geeft $2 \int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x)$
 $\int e^x \sin(x) dx = \frac{1}{2} e^x (\sin(x) - \cos(x))$

$g(x) = e^x \sin(x)$ geeft $G(x) = \frac{1}{2} e^x (\sin(x) - \cos(x)) + c$

b $F(x) = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)$ geeft
 $F'(x) = -2x \cdot \cos(x) - x^2 \cdot -\sin(x) + 2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \sin(x) = x^2 \sin(x)$

27 a $F(x) = \int \frac{1}{4} x^2 \cos(x) dx = \frac{1}{4} x^2 \cdot \sin(x) - \int \frac{1}{2} x \cdot \sin(x) dx$
 $= \frac{1}{4} x^2 \sin(x) - (\frac{1}{2} x \cdot -\cos(x) - \int \frac{1}{2} \cdot -\cos(x) dx) = \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \int \frac{1}{2} \cos(x) dx$
 $= \frac{1}{4} x^2 \sin(x) + \frac{1}{2} x \cos(x) - \frac{1}{2} \sin(x) + c$

b $\int e^{-x} \cos(x) dx = e^{-x} \cdot \sin(x) - \int -e^{-x} \cdot \sin(x) dx = e^{-x} \sin(x) + \int e^{-x} \sin(x) dx$
 $= e^{-x} \sin(x) + e^{-x} \cdot -\cos(x) - \int -e^{-x} \cdot -\cos(x) dx$
 $= e^{-x} \sin(x) - e^{-x} \cos(x) - \int e^{-x} \cos(x) dx$
 $\int e^{-x} \cos(x) dx = e^{-x} \sin(x) - e^{-x} \cos(x) - \int e^{-x} \cos(x) dx$ geeft $2 \int e^{-x} \cos(x) dx = e^{-x} \sin(x) - e^{-x} \cos(x)$
 $\int e^{-x} \cos(x) dx = \frac{1}{2} e^{-x} (\sin(x) - \cos(x))$

$g(x) = e^{-x} \cos(x)$ geeft $G(x) = \frac{1}{2} e^{-x} (\sin(x) - \cos(x)) + c$
c $\int e^{2x} \sin(x) dx = e^{2x} \cdot -\cos(x) - \int 2 e^{2x} \cdot -\cos(x) dx = -e^{2x} \cos(x) + \int 2 e^{2x} \cos(x) dx$
 $= -e^{2x} \cos(x) + 2 e^{2x} \cdot \sin(x) - \int 4 e^{2x} \cdot \sin(x) dx$
 $= 2 e^{2x} \sin(x) - e^{2x} \cos(x) - 4 \int e^{2x} \sin(x) dx$
 $\int e^{2x} \sin(x) dx = 2 e^{2x} \sin(x) - e^{2x} \cos(x) - 4 \int e^{2x} \sin(x) dx$ geeft $5 \int e^{2x} \sin(x) dx = 2 e^{2x} \sin(x) - e^{2x} \cos(x)$
 $\int e^{2x} \sin(x) dx = \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x))$

$h(x) = e^{2x} \sin(x)$ geeft $H(x) = \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + c$

d $K(x) = \int \ln^2(x) \cdot 1 dx = \ln^2(x) \cdot x - \int 2 \ln(x) \cdot \frac{1}{x} \cdot x dx = x \ln^2(x) - \int 2 \ln(x) dx$
 $= x \ln^2(x) - 2(x \ln(x) - x) + c = x \ln^2(x) - 2x \ln(x) + 2x + c$

28 a $F(x) = (ax^2 + bx + c) e^x$ geeft $f(x) = (ax^2 + (2a + b)x + b + c) e^x$
 $a = 1$
 $2a + b = -1$ } $2 + b = -1$
 $b = -3$

$b = -3$
 $b + c = 0$ } $-3 + c = 0$
 $c = 3$

Dus $F(x) = (x^2 - 3x + 3) e^x$ is een primitieve van $f(x) = (x^2 - x) e^x$.

$\int_1^3 (x^2 - x) e^x dx = [(x^2 - 3x + 3) e^x]_1^3 = (9 - 9 + 3) e^3 - (1 - 3 + 3) e = 3e^3 - e$

b $\int x \ln^2(x) dx = \ln^2(x) \cdot \frac{1}{2} x^2 - \int 2 \ln(x) \cdot \frac{1}{x} \cdot \frac{1}{2} x^2 dx = \frac{1}{2} x^2 \ln^2(x) - \int x \ln(x) dx$
 $= \frac{1}{2} x^2 \ln^2(x) - (\ln(x) \cdot \frac{1}{2} x^2 - \int \frac{1}{x} \cdot \frac{1}{2} x^2 dx) = \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \int \frac{1}{2} x dx$
 $= \frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2 + c$
 $\int_1^e x \ln^2(x) dx = [\frac{1}{2} x^2 \ln^2(x) - \frac{1}{2} x^2 \ln(x) + \frac{1}{4} x^2]_1^e$
 $= \frac{1}{2} e^2 \ln^2(e) - \frac{1}{2} e^2 \ln(e) + \frac{1}{4} e^2 - (\frac{1}{2} \ln^2(1) - \frac{1}{2} \ln(1) + \frac{1}{4}) = \frac{1}{4} e^2 - \frac{1}{4}$

29 a $f(x) = e^x \cos(x)$ geeft $f'(x) = e^x \cdot \cos(x) + e^x \cdot -\sin(x) = e^x (\cos(x) - \sin(x))$
 $f'(x) = 0$ geeft $e^x = 0 \vee \sin(x) = \cos(x)$
 $\tan(x) = 1$
 $x = \frac{1}{4} \pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = \frac{1}{4} \pi \vee x = \frac{5}{4} \pi$

$f(\frac{1}{4} \pi) \approx 1,55$ en $f(\frac{5}{4} \pi) \approx -35,89$, dus de toppen zijn $(\frac{1}{4} \pi; 1,55)$ en $(\frac{5}{4} \pi; -35,89)$.

b $f(x) = 0$ geeft $e^x = 0 \vee \cos(x) = 0$

$$x = \frac{1}{2}\pi + k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi$$

$$\begin{aligned} \int e^x \cos(x) dx &= e^x \cdot \sin(x) - \int e^x \cdot \sin(x) dx = e^x \sin(x) - (e^x \cdot -\cos(x) - \int e^x \cdot -\cos(x) dx) \\ &= e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x) dx \end{aligned}$$

$$\begin{aligned} \int e^x \cos(x) dx &= e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x) dx \text{ geeft } 2 \int e^x \cos(x) dx = e^x \sin(x) + e^x \cos(x) \\ \int e^x \cos(x) dx &= \frac{1}{2} e^x (\sin(x) + \cos(x)) \end{aligned}$$

$$\begin{aligned} O(V) &= - \int_{\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^x \cos(x) dx = - \left[\frac{1}{2} e^x (\sin(x) + \cos(x)) \right]_{\frac{1}{2}\pi}^{\frac{1}{2}\pi} = -\frac{1}{2} e^{\frac{1}{2}\pi} (-1 + 0) + \frac{1}{2} e^{\frac{1}{2}\pi} (1 + 0) \\ &= \frac{1}{2} e^{\frac{1}{2}\pi} + \frac{1}{2} e^{\frac{1}{2}\pi} = \frac{1}{2} e^{\frac{1}{2}\pi} (e^{\frac{1}{2}\pi} + 1) \end{aligned}$$

$$\text{Dus } a = \frac{1}{2} \text{ en } b = \frac{1}{2}.$$

30 a $f(x) = 0$ geeft $\ln^2(x) = 0$

$$\ln(x) = 0$$

$$x = 1$$

$$\begin{aligned} F(x) &= \int \frac{\ln^2(x)}{\sqrt{x}} dx = \int 2 \ln^2(x) \cdot \frac{1}{2\sqrt{x}} dx = 2 \ln^2(x) \cdot \sqrt{x} - \int 4 \ln(x) \cdot \frac{1}{x} \cdot \sqrt{x} dx \\ &= 2\sqrt{x} \ln^2(x) - \int 8 \ln(x) \cdot \frac{1}{2\sqrt{x}} dx = 2\sqrt{x} \ln^2(x) - (8 \ln(x) \cdot \sqrt{x} - \int \frac{8}{x} \cdot \sqrt{x} dx) \\ &= 2\sqrt{x} \ln^2(x) - 8\sqrt{x} \ln(x) + \int 16 \cdot \frac{1}{2\sqrt{x}} dx = 2\sqrt{x} \ln^2(x) - 8\sqrt{x} \ln(x) + 16\sqrt{x} + c \end{aligned}$$

$$\begin{aligned} O(V) &= \int_1^e \frac{\ln^2(x)}{\sqrt{x}} dx = [2\sqrt{x} \ln^2(x) - 8\sqrt{x} \ln(x) + 16\sqrt{x}]_1^e \\ &= 2\sqrt{e} \ln^2(e) - 8\sqrt{e} \ln(e) + 16\sqrt{e} - (2 \ln^2(1) - 8 \ln(1) + 16) \\ &= 2\sqrt{e} - 8\sqrt{e} + 16\sqrt{e} - 16 = 10\sqrt{e} - 16 \end{aligned}$$

b $\int (f(x))^2 dx = \int \frac{\ln^4(x)}{x} dx = \int \ln^4(x) \cdot \frac{1}{x} dx = \int \ln^4(x) d \ln(x) = \int u^4 du = \frac{1}{5} u^5 + c = \frac{1}{5} \ln^5(x) + c$

$$I(L) = \pi \int_1^e (f(x))^2 dx = \pi \left[\frac{1}{5} \ln^5(x) \right]_1^e = \pi \left(\frac{1}{5} \ln^5(e) - \frac{1}{5} \ln^5(1) \right) = \frac{1}{5} \pi$$

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C3 a $F(x) = \int 4x e^{3x} dx = 4x \cdot \frac{1}{3} e^{3x} - \int 4 \cdot \frac{1}{3} e^{3x} dx = \frac{4}{3} x e^{3x} - \int \frac{4}{3} e^{3x} dx = \frac{4}{3} x e^{3x} - \frac{4}{9} e^{3x} + c$

$$= \frac{4}{9} e^{3x} (3x - 1) + c$$

b $\int x \cos(x) dx = x \cdot \sin(x) - \int 1 \cdot \sin(x) dx = x \sin(x) + \cos(x) + c$

$$\int_0^{\frac{1}{2}\pi} x \cos(x) dx = [x \sin(x) + \cos(x)]_0^{\frac{1}{2}\pi} = \frac{1}{2}\pi + 0 - (0 + 1) = \frac{1}{2}\pi - 1$$

C4 a $\int (1 - x^2) e^x dx = (1 - x^2) \cdot e^x - \int -2x \cdot e^x dx = (1 - x^2) e^x + \int 2x e^x dx$

$$\begin{aligned} &= (1 - x^2) e^x + 2x \cdot e^x - \int 2 \cdot e^x dx = (1 - x^2) e^x + 2x e^x - 2 e^x + c \\ &= (-x^2 + 2x - 1) e^x + c \end{aligned}$$

$$\int_{-1}^1 (1 - x^2) e^x dx = [(-x^2 + 2x - 1) e^x]_{-1}^1 = (-1 + 2 - 1) e - (-1 - 2 - 1) e^{-1} = \frac{4}{e}$$

Alternatieve uitwerking

$$F(x) = (ax^2 + bx + c) e^x \text{ geeft } f(x) = (ax^2 + (2a + b)x + b + c) e^x$$

$$\left. \begin{array}{l} a = -1 \\ 2a + b = 0 \end{array} \right\} \begin{array}{l} -2 + b = 0 \\ b = 2 \end{array}$$

$$\left. \begin{array}{l} b = 2 \\ b + c = 1 \end{array} \right\} \begin{array}{l} 2 + c = 1 \\ c = -1 \end{array}$$

$$\text{Dus } F(x) = (-x^2 + 2x - 1) e^x \text{ is een primitieve van } f(x) = (1 - x^2) e^x.$$

$$\int_{-1}^1 (1 - x^2) e^x dx = [(-x^2 + 2x - 1) e^x]_{-1}^1 = (-1 + 2 - 1) e - (-1 - 2 - 1) e^{-1} = \frac{4}{e}$$

$$\begin{aligned}
\text{b } \int e^x \cos(2x) dx &= e^x \cdot \frac{1}{2} \sin(2x) - \int e^x \cdot \frac{1}{2} \sin(2x) dx \\
&= \frac{1}{2} e^x \sin(2x) - (e^x \cdot -\frac{1}{4} \cos(2x) - \int e^x \cdot -\frac{1}{4} \cos(2x) dx) \\
&= \frac{1}{2} e^x \sin(2x) + \frac{1}{4} e^x \cos(2x) - \frac{1}{4} \int e^x \cos(2x) dx \\
\int e^x \cos(2x) dx &= \frac{1}{2} e^x \sin(2x) + \frac{1}{4} e^x \cos(2x) - \frac{1}{4} \int e^x \cos(2x) dx \text{ geeft } \frac{5}{4} \int e^x \cos(2x) dx = \frac{1}{2} e^x \sin(2x) + \frac{1}{4} e^x \cos(2x) \\
\int e^x \cos(2x) dx &= \frac{1}{5} e^x (2 \sin(2x) + \cos(2x)) \\
f(x) = e^x \cos(2x) \text{ geeft } F(x) &= \frac{1}{5} e^x (2 \sin(2x) + \cos(2x)) + c
\end{aligned}$$

K.3 Cyclometrische functies

Bladzijde 191

- 31 a** Op het interval $\langle 0, 2\pi \rangle$ zijn er functiewaarden die meerdere keren voorkomen.
b $\langle -\frac{1}{2}\pi, \frac{1}{2}\pi \rangle$ is een zo groot mogelijk open interval waarop f wel een inverse heeft.

Bladzijde 193

- 32 a** Het bereik van $f(x) = \arctan(x)$ is $\langle -\frac{1}{2}\pi, \frac{1}{2}\pi \rangle$.
 $1\frac{1}{6}\pi$ zit niet in het bereik, dus $\arctan(\frac{1}{3}\sqrt{3}) \neq 1\frac{1}{6}\pi$.
b $\sqrt{3} > \frac{1}{2}\pi$, dus de vergelijking $\arctan(x) = \sqrt{3}$ heeft geen oplossing.
c $G(x) = \arctan(x^2 + 1)$ is een primitieve van g .
d Een primitieve van $f(ax + b)$ is $\frac{1}{a} \cdot F(ax + b)$.

33	x	$-\sqrt{3}$	-1	$-\frac{1}{3}\sqrt{3}$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
	$\arctan(x)$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$

- 34 a** $\arctan(x) = \frac{1}{3}\pi$
 $x = \tan(\frac{1}{3}\pi)$
 $x = \sqrt{3}$
b $\arctan(x - 2) = -\frac{1}{4}\pi$
 $x - 2 = \tan(-\frac{1}{4}\pi)$
 $x - 2 = -1$
 $x = 1$
c $\arctan(x^2 - 1) = \frac{1}{4}\pi$
 $x^2 - 1 = \tan(\frac{1}{4}\pi)$
 $x^2 - 1 = 1$
 $x^2 = 2$
 $x = \sqrt{2} \vee x = -\sqrt{2}$
d $\arctan(x) = \frac{2}{3}\pi$
geen oplossing, want $\frac{2}{3}\pi > \frac{1}{2}\pi$
e $\arctan(x) = \sqrt{2}$
 $x = \tan(\sqrt{2})$
 $x \approx 6,334$
f $\arctan(x^2 - 1) = 1$
 $x^2 - 1 = \tan(1)$
 $x^2 = \tan(1) + 1$
 $x = \sqrt{\tan(1) + 1} \vee x = -\sqrt{\tan(1) + 1}$
 $x \approx 1,599 \vee x = -1,599$

- 35 a** $f(x) = 2 \arctan(\frac{1}{2}x)$ geeft $f'(x) = 2 \cdot \frac{1}{(\frac{1}{2}x)^2 + 1} \cdot \frac{1}{2} = \frac{1}{\frac{1}{4}x^2 + 1} = \frac{4}{x^2 + 4}$
b $g(x) = \arctan(x - 2)$ geeft $g'(x) = \frac{1}{(x - 2)^2 + 1} = \frac{1}{x^2 - 4x + 5}$
c $h(x) = \arctan(x^2)$ geeft $h'(x) = \frac{1}{(x^2)^2 + 1} \cdot 2x = \frac{2x}{x^4 + 1}$

$$36 \quad \mathbf{a} \quad \int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{1}{x^2 + 1} dx = [\arctan(x)]_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} = \arctan(\sqrt{3}) - \arctan(-\frac{1}{\sqrt{3}}) = \frac{1}{3}\pi - -\frac{1}{6}\pi = \frac{1}{2}\pi$$

$$\mathbf{b} \quad \int_{-2}^{-1} \frac{1}{(x+1)^2 + 1} dx = [\arctan(x+1)]_{-2}^{-1} = \arctan(0) - \arctan(-1) = 0 - -\frac{1}{4}\pi = \frac{1}{4}\pi$$

$$\mathbf{c} \quad \int_0^{\frac{1}{\sqrt{3}}} \frac{3}{9x^2 + 1} dx = \int_0^{\frac{1}{\sqrt{3}}} \frac{3}{(3x)^2 + 1} dx = \left[3 \cdot \frac{1}{3} \arctan(3x) \right]_0^{\frac{1}{\sqrt{3}}} = [\arctan(3x)]_0^{\frac{1}{\sqrt{3}}} = \arctan(\sqrt{3}) - \arctan(0) \\ = \frac{1}{3}\pi - 0 = \frac{1}{3}\pi$$

Bladzijde 194

$$37 \quad \mathbf{a} \quad \arctan(2x - 1) = \frac{1}{6}\pi$$

$$2x - 1 = \tan\left(\frac{1}{6}\pi\right)$$

$$2x - 1 = \frac{1}{3}\sqrt{3}$$

$$2x = 1 + \frac{1}{3}\sqrt{3}$$

$$x = \frac{1}{2} + \frac{1}{6}\sqrt{3}$$

$$\mathbf{b} \quad \arctan\left(1\frac{1}{2}\pi + \sqrt{x}\right) = \sqrt{2}$$

$$1\frac{1}{2}\pi + \sqrt{x} = \tan(\sqrt{2})$$

$$\sqrt{x} = \tan(\sqrt{2}) - 1\frac{1}{2}\pi$$

$$x = (\tan(\sqrt{2}) - 1\frac{1}{2}\pi)^2$$

$$x \approx 2,63$$

$$\mathbf{c} \quad f(x) = \arctan(2x^2 + 1) \text{ geeft } f'(x) = \frac{1}{(2x^2 + 1)^2 + 1} \cdot 4x = \frac{4x}{4x^4 + 4x^2 + 2} = \frac{2x}{2x^4 + 2x^2 + 1}$$

$$\mathbf{d} \quad \int_{\frac{1}{5}}^{\frac{1}{\sqrt{3}}} \frac{10}{25x^2 + 1} dx = \int_{\frac{1}{5}}^{\frac{1}{\sqrt{3}}} \frac{10}{(5x)^2 + 1} dx = \left[10 \cdot \frac{1}{5} \arctan(5x) \right]_{\frac{1}{5}}^{\frac{1}{\sqrt{3}}} = 2 \arctan(\sqrt{3}) - 2 \arctan(1) \\ = 2 \cdot \frac{1}{3}\pi - 2 \cdot \frac{1}{4}\pi = \frac{1}{6}\pi$$

$$38 \quad \mathbf{a} \quad f(x) = \frac{10}{x^2 + 4} = \frac{2\frac{1}{2}}{\frac{1}{4}x^2 + 1} = \frac{2\frac{1}{2}}{(\frac{1}{2}x)^2 + 1}$$

$$\text{Dus } a = 2\frac{1}{2} \text{ en } b = \frac{1}{2}.$$

$$\mathbf{b} \quad f(x) = \frac{2\frac{1}{2}}{(\frac{1}{2}x)^2 + 1} \text{ geeft } F(x) = 2\frac{1}{2} \cdot 2 \arctan(\frac{1}{2}x) + c = 5 \arctan(\frac{1}{2}x) + c$$

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$$39 \quad \mathbf{a} \quad F(x) = \int \frac{5}{32x^2 + 2} dx = \int \frac{2\frac{1}{2}}{16x^2 + 1} dx = \int \frac{2\frac{1}{2}}{(4x)^2 + 1} dx = 2\frac{1}{2} \cdot \frac{1}{4} \arctan(4x) + c = \frac{5}{8} \arctan(4x) + c$$

$$\mathbf{b} \quad G(x) = \int \frac{5}{x^2 + 6x + 10} dx = \int \frac{5}{(x+3)^2 - 9 + 10} dx = \int \frac{5}{(x+3)^2 + 1} dx = 5 \arctan(x+3) + c$$

a $\int_0^1 \frac{3}{x^2 + 3} dx = \int_0^1 \frac{1}{\frac{1}{3}x^2 + 1} dx = \int_0^1 \frac{1}{\left(\frac{x}{\sqrt{3}}\right)^2 + 1} dx = \left[\sqrt{3} \cdot \arctan\left(\frac{x}{\sqrt{3}}\right) \right]_0^1$
 $= \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan\left(\frac{0}{\sqrt{3}}\right) = \sqrt{3} \cdot \arctan\left(\frac{1}{\sqrt{3}}\right) - \sqrt{3} \cdot \arctan(0)$
 $= \sqrt{3} \cdot \frac{1}{6}\pi - \sqrt{3} \cdot 0 = \frac{1}{6}\pi\sqrt{3}$

b $\int \frac{2x}{x^4 + 1} dx = \int \frac{1}{(x^2)^2 + 1} \cdot 2x dx = \int \frac{1}{(x^2)^2 + 1} dx^2 = \int \frac{1}{u^2 + 1} du = \arctan(u) + c = \arctan(x^2) + c$
 $\int_0^1 \frac{2x}{x^4 + 1} dx = [\arctan(x^2)]_0^1 = \arctan(1) - \arctan(0) = \frac{1}{4}\pi - 0 = \frac{1}{4}\pi$

c $\int_3^6 \frac{5}{x^2 - 6x + 18} dx = \int_3^6 \frac{5}{(x-3)^2 - 9 + 18} dx = \int_3^6 \frac{5}{(x-3)^2 + 9} dx = \int_3^6 \frac{\frac{5}{9}}{\frac{(x-3)^2}{9} + 1} dx = \int_3^6 \frac{\frac{5}{9}}{\left(\frac{x-3}{3}\right)^2 + 1} dx$
 $= \int_3^6 \frac{\frac{5}{9}}{\left(\frac{1}{3}x - 1\right)^2 + 1} dx = \left[\frac{5}{9} \cdot 3 \arctan\left(\frac{1}{3}x - 1\right) \right]_3^6 = \left[\frac{5}{3} \arctan\left(\frac{1}{3}x - 1\right) \right]_3^6$
 $= \frac{5}{3} \arctan(1) - \frac{5}{3} \arctan(0) = \frac{5}{3} \cdot \frac{1}{4}\pi - \frac{5}{3} \cdot 0 = \frac{5}{12}\pi$

d $\int \frac{\arctan(x)}{x^2 + 1} dx = \int \arctan(x) \cdot \frac{1}{x^2 + 1} dx = \int \arctan(x) d\arctan(x) = \int u du = \frac{1}{2}u^2 + c = \frac{1}{2}\arctan^2(x) + c$
 $\int_0^{\sqrt{3}} \frac{\arctan(x)}{x^2 + 1} dx = \left[\frac{1}{2}\arctan^2(x) \right]_0^{\sqrt{3}} = \frac{1}{2}\arctan^2(\sqrt{3}) - \frac{1}{2}\arctan^2(0) = \frac{1}{2} \cdot \left(\frac{1}{3}\pi\right)^2 - \frac{1}{2} \cdot 0^2 = \frac{1}{18}\pi^2$

a $\int_0^{\frac{1}{2}} \frac{2}{4x^2 + 9} dx = \int_0^{\frac{1}{2}} \frac{\frac{2}{9}}{\frac{4}{9}x^2 + 1} dx = \int_0^{\frac{1}{2}} \frac{\frac{2}{9}}{\left(\frac{2}{3}x\right)^2 + 1} dx = \left[\frac{2}{9} \cdot \frac{3}{2} \arctan\left(\frac{2}{3}x\right) \right]_0^{\frac{1}{2}} = \left[\frac{1}{3} \arctan\left(\frac{2}{3}x\right) \right]_0^{\frac{1}{2}}$
 $= \frac{1}{3} \arctan(1) - \frac{1}{3} \arctan(0) = \frac{1}{3} \cdot \frac{1}{4}\pi - \frac{1}{3} \cdot 0 = \frac{1}{12}\pi$

b $\int \frac{e^x}{e^{2x} + 1} dx = \int \frac{1}{(e^x)^2 + 1} \cdot e^x dx = \int \frac{1}{(e^x)^2 + 1} de^x = \int \frac{1}{u^2 + 1} du = \arctan(u) + c = \arctan(e^x) + c$
 $\int_0^{\frac{1}{2}\ln(3)} \frac{e^x}{e^{2x} + 1} dx = [\arctan(e^x)]_0^{\frac{1}{2}\ln(3)} = \arctan(\sqrt{3}) - \arctan(1) = \frac{1}{3}\pi - \frac{1}{4}\pi = \frac{1}{12}\pi$

c $\int_0^1 \frac{1}{4x^2 - 4x + 2} dx = \int_0^1 \frac{1}{(2x-1)^2 - 1 + 2} dx = \int_0^1 \frac{1}{(2x-1)^2 + 1} dx = \left[\frac{1}{2} \arctan(2x-1) \right]_0^1$
 $= \frac{1}{2} \arctan(1) - \frac{1}{2} \arctan(-1) = \frac{1}{2} \cdot \frac{1}{4}\pi - \frac{1}{2} \cdot -\frac{1}{4}\pi = \frac{1}{4}\pi$

d $\int \frac{\sin(x)}{\cos^2(x) + 1} dx = \int -\frac{1}{\cos^2(x) + 1} \cdot -\sin(x) dx = \int -\frac{1}{\cos^2(x) + 1} d\cos(x) = \int -\frac{1}{u^2 + 1} du$
 $= -\arctan(u) + c = -\arctan(\cos(x)) + c$
 $\int_0^{\frac{1}{2}\pi} \frac{\sin(x)}{\cos^2(x) + 1} dx = [-\arctan(\cos(x))]_0^{\frac{1}{2}\pi} = -\arctan(0) + \arctan(1) = 0 + \frac{1}{4}\pi = \frac{1}{4}\pi$

a $f(x) = \frac{10}{x^2 - 8x + 17} = 10(x^2 - 8x + 17)^{-1}$ geeft
 $f'(x) = -10(x^2 - 8x + 17)^{-2} \cdot (2x - 8) = \frac{-20x + 80}{(x^2 - 8x + 17)^2}$
 $f'(x) = 0$ geeft $-20x + 80 = 0$
 $-20x = -80$
 $x = 4$
max. is $f(4) = \frac{10}{16 - 32 + 17} = 10$

b $F(x) = \int \frac{10}{x^2 - 8x + 17} dx = \int \frac{10}{(x-4)^2 - 16 + 17} dx = \int \frac{10}{(x-4)^2 + 1} dx = 10 \arctan(x-4) + c$
 $O(V) = \int_4^5 f(x) dx = [10 \arctan(x-4)]_4^5 = 10 \arctan(1) - 10 \arctan(0) = 10 \cdot \frac{1}{4}\pi - 10 \cdot 0 = 2\frac{1}{2}\pi$

$$\text{c } O(W_p) = \int_4^p f(x) dx = [10 \arctan(x-4)]_4^p = 10 \arctan(x-p) - 10 \arctan(0) = 10 \arctan(x-p)$$

$$O(W_p) = 3\frac{1}{3}\pi \text{ geeft } 10 \arctan(p-4) = 3\frac{1}{3}\pi$$

$$\arctan(p-4) = \frac{1}{3}\pi$$

$$p-4 = \tan(\frac{1}{3}\pi)$$

$$p-4 = \sqrt{3}$$

$$p = 4 + \sqrt{3}$$

$$\text{43 } F(x) = \int \frac{4}{x \ln^2(x) + x} dx = \int \frac{4}{\ln^2(x) + 1} \cdot \frac{1}{x} dx = \int \frac{4}{\ln^2(x) + 1} d \ln(x) = \int \frac{4}{u^2 + 1} du = 4 \arctan(u) + c$$

$$= 4 \arctan(\ln(x)) + c$$

$$O(V_p) = \int_1^p f(x) dx = [4 \arctan(\ln(x))]_1^p = 4 \arctan(\ln(p)) - 4 \arctan(0) = 4 \arctan(\ln(p))$$

$$O(V_p) = \pi \text{ geeft } 4 \arctan(\ln(p)) = \pi$$

$$\arctan(\ln(p)) = \frac{1}{4}\pi$$

$$\ln(p) = \tan(\frac{1}{4}\pi)$$

$$\ln(p) = 1$$

$$p = e$$

44 a Op het interval $[0, \pi]$ zijn er functiewaarden die meerdere keren voorkomen.

b $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ is een zo groot mogelijk gesloten interval waarop f wel een inverse heeft.

Bladzijde 197

45	x	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
	$\arcsin(x)$	$-\frac{1}{2}\pi$	$-\frac{1}{3}\pi$	$-\frac{1}{4}\pi$	$-\frac{1}{6}\pi$	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$

$$\text{46 a } \arcsin(x) = \frac{1}{2}\pi$$

$$x = \sin(\frac{1}{2}\pi)$$

$$x = 1$$

$$\text{b } \arcsin(x) = -\frac{1}{6}\pi$$

$$x = \sin(-\frac{1}{6}\pi)$$

$$x = -\frac{1}{2}$$

$$\text{c } \arcsin(x) = 2$$

geen oplossing, want $2 > \frac{1}{2}\pi$

$$\text{d } 3 \arcsin(x - \sqrt{3}) = \pi$$

$$\arcsin(x - \sqrt{3}) = \frac{1}{3}\pi$$

$$x - \sqrt{3} = \sin(\frac{1}{3}\pi)$$

$$x - \sqrt{3} = \frac{1}{2}\sqrt{3}$$

$$x = 1\frac{1}{2}\sqrt{3}$$

$$\text{47 a } f(x) = \arcsin(\frac{1}{3}x) \text{ geeft } f'(x) = \frac{1}{\sqrt{1 - (\frac{1}{3}x)^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{1 - \frac{1}{9}x^2}} = \frac{1}{\sqrt{9 - x^2}}$$

$$\text{b } g(x) = \arcsin(x^2) \text{ geeft } g'(x) = \frac{1}{\sqrt{1 - x^4}} \cdot 2x = \frac{2x}{\sqrt{1 - x^4}}$$

48

$$\text{a} \quad \int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-9x^2}} dx = \int_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} \frac{1}{\sqrt{1-(3x)^2}} dx = \left[\frac{1}{3} \arcsin(3x) \right]_{\frac{1}{6}}^{\frac{1}{6}\sqrt{3}} = \frac{1}{3} \arcsin\left(\frac{1}{2}\sqrt{3}\right) - \frac{1}{3} \arcsin\left(\frac{1}{2}\right) \\ = \frac{1}{3} \cdot \frac{1}{3}\pi - \frac{1}{3} \cdot \frac{1}{6}\pi = \frac{1}{18}\pi$$

$$\text{b} \quad \int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} \frac{1}{\sqrt{9-x^2}} dx = \int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} \frac{1}{3\sqrt{1-\frac{1}{9}x^2}} dx = \int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} \frac{\frac{1}{3}}{\sqrt{1-(\frac{1}{3}x)^2}} dx = \left[\frac{1}{3} \cdot 3 \arcsin\left(\frac{1}{3}x\right) \right]_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} \\ = \left[\arcsin\left(\frac{1}{3}x\right) \right]_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}} = \arcsin\left(\frac{1}{2}\sqrt{2}\right) - \arcsin\left(-\frac{1}{2}\sqrt{2}\right) = \frac{1}{4}\pi - \left(-\frac{1}{4}\pi\right) = \frac{1}{2}\pi$$

$$\text{c} \quad \int \frac{x}{\sqrt{1-x^4}} dx = \int \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x dx = \int \frac{\frac{1}{2}}{\sqrt{1-(x^2)^2}} dx^2 = \int \frac{\frac{1}{2}}{\sqrt{1-u^2}} du = \frac{1}{2} \arcsin(u) + c \\ = \frac{1}{2} \arcsin(x^2) + c$$

$$\int_0^{\frac{1}{2}\sqrt{2}} \frac{x}{\sqrt{1-x^4}} dx = \left[\frac{1}{2} \arcsin(x^2) \right]_0^{\frac{1}{2}\sqrt{2}} = \frac{1}{2} \arcsin\left(\frac{1}{2}\right) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - \frac{1}{2} \cdot 0 = \frac{1}{12}\pi$$

$$\text{d} \quad \int \frac{x}{\sqrt{4-x^4}} dx = \int \frac{x}{2\sqrt{1-\frac{1}{4}x^4}} dx = \int \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{2}x^2)^2}} \cdot x dx = \int \frac{\frac{1}{2}}{\sqrt{1-(\frac{1}{2}x^2)^2}} d\left(\frac{1}{2}x^2\right) = \int \frac{\frac{1}{2}}{\sqrt{1-u^2}} du \\ = \frac{1}{2} \arcsin(u) + c = \frac{1}{2} \arcsin\left(\frac{1}{2}x^2\right) + c$$

$$\int_0^1 \frac{x}{\sqrt{4-x^4}} dx = \left[\frac{1}{2} \arcsin\left(\frac{1}{2}x^2\right) \right]_0^1 = \frac{1}{2} \arcsin\left(\frac{1}{2}\right) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - \frac{1}{2} \cdot 0 = \frac{1}{12}\pi$$

49

$$\text{a} \quad F(x) = \int \frac{6}{\sqrt{1-(2x-1)^2}} dx = 6 \cdot \frac{1}{2} \arcsin(2x-1) + c = 3 \arcsin(2x-1) + c$$

$$\text{b} \quad G(x) = \int \frac{12}{\sqrt{16-(x+1)^2}} dx = \int \frac{12}{4\sqrt{1-\frac{1}{16}(x+1)^2}} dx = \int \frac{3}{\sqrt{1-(\frac{1}{4}x+\frac{1}{4})^2}} dx = 3 \cdot 4 \arcsin\left(\frac{1}{4}x+\frac{1}{4}\right) + c \\ = 12 \arcsin\left(\frac{1}{4}x+\frac{1}{4}\right) + c$$

50

$$\text{a} \quad F(x) = \int \arctan(x) dx = \int \arctan(x) \cdot 1 dx = \arctan(x) \cdot x - \int \frac{1}{x^2+1} \cdot x dx \\ = x \arctan(x) - \int \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x dx = x \arctan(x) - \int \frac{1}{2} \cdot \frac{1}{x^2+1} d(x^2+1) \\ = x \arctan(x) - \int \frac{1}{2} \cdot \frac{1}{u} du = x \arctan(x) - \frac{1}{2} \ln|u| + c = x \arctan(x) - \frac{1}{2} \ln(x^2+1) + c$$

$$\text{b} \quad G(x) = \int \arcsin(x) dx = \int \arcsin(x) \cdot 1 dx = \arcsin(x) \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x dx \\ = x \arcsin(x) - \int -\frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot -2x dx = x \arcsin(x) + \int \frac{1}{2\sqrt{1-x^2}} d(1-x^2) \\ = x \arcsin(x) + \int \frac{1}{2\sqrt{u}} du = x \arcsin(x) + \sqrt{u} + c = x \arcsin(x) + \sqrt{1-x^2} + c$$

51

$$\text{a} \quad F(x) = \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx = \int \arcsin(x) \cdot \frac{1}{\sqrt{1-x^2}} dx = \int \arcsin(x) d \arcsin(x) = \int u du = \frac{1}{2} u^2 + c \\ = \frac{1}{2} \arcsin^2(x) + c$$

$$\text{b} \quad G(x) = \int \frac{1}{x\sqrt{1-\ln^2(x)}} dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} \cdot \frac{1}{x} dx = \int \frac{1}{\sqrt{1-\ln^2(x)}} d \ln(x) = \int \frac{1}{\sqrt{1-u^2}} du \\ = \arcsin(u) + c = \arcsin(\ln(x)) + c$$

a $25 - x^4 > 0$

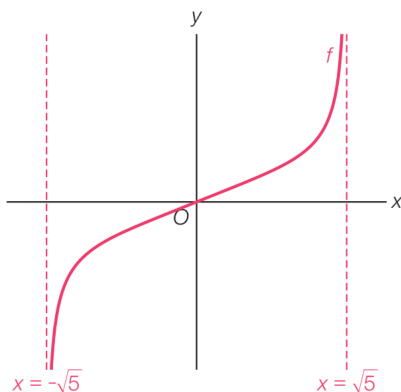
$$x^4 < 25$$

$$-5 < x^2 < 5$$

$$x^2 < 5$$

$$-\sqrt{5} < x < \sqrt{5}$$

$$\text{Dus } D_f = \langle -\sqrt{5}, \sqrt{5} \rangle.$$



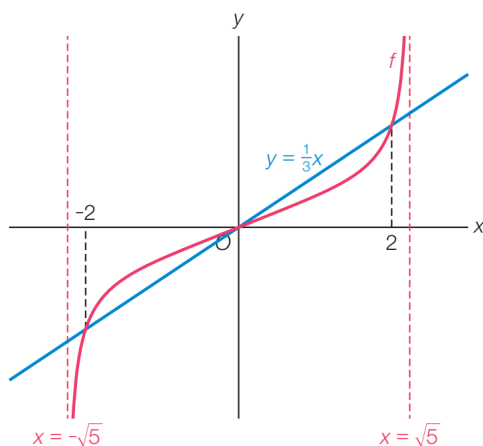
b $f(x) = \frac{1}{3}x$ geeft $\frac{x}{\sqrt{25 - x^4}} = \frac{x}{3}$

$$x = 0 \vee \sqrt{25 - x^4} = 3$$

$$x = 0 \vee 25 - x^4 = 9$$

$$x = 0 \vee x^4 = 16$$

$$x = 0 \vee x = 2 \vee x = -2$$



$$f(x) \leq \frac{1}{3}x \text{ geeft } -\sqrt{5} < x \leq -2 \vee 0 \leq x \leq 2$$

$$\begin{aligned} \text{c } F(x) &= \int \frac{x}{\sqrt{25 - x^4}} dx = \int \frac{x}{5\sqrt{1 - \frac{1}{25}x^4}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{1 - (\frac{1}{5}x)^2}} \cdot \frac{2}{5}x dx = \int \frac{\frac{1}{2}}{\sqrt{1 - (\frac{1}{5}x)^2}} d(\frac{1}{5}x^2) \\ &= \int \frac{\frac{1}{2}}{\sqrt{1 - u^2}} du = \frac{1}{2} \arcsin(u) + c = \frac{1}{2} \arcsin(\frac{1}{5}x^2) + c \\ O(V) &= \int_0^{\frac{1}{2}\sqrt{10}} f(x) dx = \left[\frac{1}{2} \arcsin(\frac{1}{5}x^2) \right]_0^{\frac{1}{2}\sqrt{10}} = \frac{1}{2} \arcsin(\frac{1}{2}) - \frac{1}{2} \arcsin(0) = \frac{1}{2} \cdot \frac{1}{6}\pi - \frac{1}{2} \cdot 0 = \frac{1}{12}\pi \end{aligned}$$

a $\arctan(x) = -1$

$$x = \tan(-1)$$

$$x \approx -1,56$$

b $f(x) = \arctan(x^3)$ geeft $f'(x) = \frac{1}{x^6 + 1} \cdot 3x^2 = \frac{3x^2}{x^6 + 1}$

$$\begin{aligned} \text{c } \int_{-2}^{-1} \frac{3}{x^2 + 4x + 5} dx &= \int_{-2}^{-1} \frac{3}{(x+2)^2 - 4 + 5} dx = \int_{-2}^{-1} \frac{3}{(x+2)^2 + 1} dx = [3 \arctan(x+2)]_{-2}^{-1} \\ &= 3 \arctan(1) - 3 \arctan(0) = 3 \cdot \frac{1}{4}\pi - 3 \cdot 0 = \frac{3}{4}\pi \end{aligned}$$

a $\arcsin(x) = \frac{1}{4}\pi$

$$x = \sin\left(\frac{1}{4}\pi\right)$$

$$x = \frac{1}{2}\sqrt{2}$$

b $f(x) = \arcsin(2x)$ geeft $f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$

c $\int_{-2\frac{1}{2}}^{2\frac{1}{2}} \frac{10}{\sqrt{25-x^2}} dx = \int_{-2\frac{1}{2}}^{2\frac{1}{2}} \frac{10}{5\sqrt{1-\frac{1}{25}x^2}} dx = \int_{-2\frac{1}{2}}^{2\frac{1}{2}} \frac{2}{\sqrt{1-(\frac{1}{5}x)^2}} dx = \left[2 \cdot 5 \arcsin\left(\frac{1}{5}x\right)\right]_{-2\frac{1}{2}}^{2\frac{1}{2}} = \left[10 \arcsin\left(\frac{1}{5}x\right)\right]_{-2\frac{1}{2}}^{2\frac{1}{2}}$

$$= 10 \arcsin\left(\frac{1}{2}\right) - 10 \arcsin\left(-\frac{1}{2}\right) = 10 \cdot \frac{1}{6}\pi - 10 \cdot -\frac{1}{6}\pi = 3\frac{1}{3}\pi$$

K.4 Breuksplitsen

Bladzijde 200

53 a $g(x) = 2 + \frac{3}{x+1} = \frac{2(x+1)+3}{x+1} = \frac{2x+5}{x+1} = f(x)$

b $f(x) = 2 + \frac{3}{x+1}$ geeft $F(x) = 2x + 3 \ln|x+1| + c$

Bladzijde 201

54 Door zo te beginnen houd je alsnog de term $-2x$ in de teller, dus dit is niet handiger.

55 a $f(x) = \frac{2x+1}{x+1} = \frac{2(x+1)-2+1}{x+1} = 2 - \frac{1}{x+1}$

Dus $F(x) = 2x - \ln|x+1| + c$.

b $f(x) = \frac{x+1}{2x+1} = \frac{\frac{1}{2}(2x+1) - \frac{1}{2} + 1}{2x+1} = \frac{1}{2} + \frac{\frac{1}{2}}{2x+1}$

Dus $F(x) = \frac{1}{2}x + \frac{1}{4} \ln|2x+1| + c$.

56 a $\int \frac{x^2-2x+3}{x-1} dx = \int \frac{(x-1)^2-1+3}{x-1} dx = \int \left(x-1 + \frac{2}{x-1}\right) dx = \frac{1}{2}x^2 - x + 2 \ln|x-1| + c$

$$\int_2^3 \frac{x^2-2x+3}{x-1} dx = \left[\frac{1}{2}x^2 - x + 2 \ln|x-1|\right]_2^3 = \frac{1}{2} \cdot 3^2 - 3 + 2 \ln(2) - \left(\frac{1}{2} \cdot 2^2 - 2 + 2 \ln(1)\right) = 1\frac{1}{2} + 2 \ln(2)$$

b $\int \frac{-2x^2-x}{x+1} dx = \int \frac{-2x(x+1)+2x-x}{x+1} dx = \int \left(-2x + \frac{x}{x+1}\right) dx = \int \left(-2x + \frac{x+1-1}{x+1}\right) dx$

$$= \int \left(-2x + 1 - \frac{1}{x+1}\right) dx = -x^2 + x - \ln|x+1| + c$$

$$\int_{-4}^{-2} \frac{-2x^2-x}{x+1} dx = \left[-x^2 + x - \ln|x+1|\right]_{-4}^{-2} = -(-2)^2 - 2 - \ln(1) - (-(-4)^2 - 4 - \ln(3)) = 14 + \ln(3)$$

57 a $f(x) = \frac{3-4x}{2x+1} = \frac{3-2(2x+1)+2}{2x+1} = -2 + \frac{5}{2x+1}$

Dus $F(x) = -2x + 2\frac{1}{2} \ln|2x+1| + c$.

$$g(x) = \frac{6x-1}{1-2x} = \frac{-3(1-2x)+3-1}{1-2x} = -3 + \frac{2}{1-2x}$$

Dus $G(x) = -3x - \ln|1-2x| + c$.

b $\int \frac{2x^2-5x}{2x-1} dx = \int \frac{x(2x-1)+x-5x}{2x-1} dx = \int \left(x - \frac{4x}{2x-1}\right) dx = \int \left(x - \frac{2(2x-1)+2}{2x-1}\right) dx$

$$= \int \left(x - 2 - \frac{2}{2x-1}\right) dx = \frac{1}{2}x^2 - 2x - \ln|2x-1| + c$$

$$\int_3^6 \frac{2x^2-5x}{2x-1} dx = \left[\frac{1}{2}x^2 - 2x - \ln|2x-1|\right]_3^6 = \frac{1}{2} \cdot 6^2 - 2 \cdot 6 - \ln(11) - \left(\frac{1}{2} \cdot 3^2 - 2 \cdot 3 - \ln(5)\right) = 7\frac{1}{2} + \ln\left(\frac{5}{11}\right)$$

58 a $f(x) = \frac{x^3 + x}{x + 1}$ geeft

$$f'(x) = \frac{(x+1) \cdot (3x^2 + 1) - (x^3 + x) \cdot 1}{(x+1)^2} = \frac{3x^3 + x + 3x^2 + 1 - x^3 - x}{(x+1)^2} = \frac{2x^3 + 3x^2 + 1}{(x+1)^2}$$

$f'(0) = 1$, dus $k: y = x$.

$f(x) = x$ geeft $\frac{x^3 + x}{x + 1} = x$

$$\begin{aligned} x^3 + x &= x(x + 1) \\ x^3 + x &= x^2 + x \\ x^3 - x^2 &= 0 \\ x^2(x - 1) &= 0 \\ x &= 0 \vee x = 1 \end{aligned}$$

$f(1) = 1$, dus $A(1, 1)$.

b $f(x) = \frac{x^3 + x}{x + 1} = \frac{x^2(x + 1) - x^2 + x}{x + 1} = x^2 + \frac{-x^2 + x}{x + 1} = x^2 + \frac{-x(x + 1) + x + x}{x + 1} = x^2 - x + \frac{2x}{x + 1}$

$$= x^2 - x + \frac{2(x + 1) - 2}{x + 1} = x^2 - x + 2 - \frac{2}{x + 1}$$

$$O(V) = \int_0^2 f(x) dx = \int_0^2 \left(x^2 - x + 2 - \frac{2}{x + 1} \right) dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x - 2\ln|x + 1| \right]_0^2$$

$$= \frac{8}{3} - 2 + 4 - 2\ln(3) - (0 - 0 + 0 - 2\ln(1)) = 4\frac{2}{3} - 2\ln(3)$$

- 59 a** $f(x)$ is voor elke x gedefinieerd, want de noemer $x^2 + 4x + 5$ heeft geen nulpunten.
 $x^2 + 4x + 4 = (x + 2)^2$ heeft één nulpunt, namelijk -2 , dus de grafiek van g heeft één verticale asymptoot.
 $x^2 + 4x + 3 = (x + 1)(x + 3)$ heeft twee nulpunten, namelijk -1 en -3 , dus de grafiek van h heeft twee verticale asymptoten.

b $x^2 + 4x + 5 = (x + 2)^2 - 4 + 5 = (x + 2)^2 + 1$

c $\int \frac{2x + 4}{x^2 + 4x + 5} dx = \int \frac{1}{x^2 + 4x + 5} \cdot (2x + 4) dx = \int \frac{1}{x^2 + 4x + 5} d(x^2 + 4x + 5) = \int \frac{1}{u} du = \ln|u| + c$

$$= \ln|x^2 + 4x + 5| + c$$

$$\int \frac{a}{(x + 2)^2 + 1} dx = a \arctan(x + 2) + c$$

d $\int \frac{a}{x + 2} dx = a \ln|x + 2| + c$

$$\int \frac{b}{(x + 2)^2} dx = \int b(x + 2)^{-2} dx = -b(x + 2)^{-1} + c = \frac{-b}{x + 2} + c$$

e $\int \frac{a}{x + 1} dx = a \ln|x + 1| + c$

$$\int \frac{b}{x + 3} dx = b \ln|x + 3| + c$$

60 a $p(x) = x^2 - 3x + 5$, $q(x) = 1$, $l(x) = x + 1$, $b = -4$ en $c = 4$

b Er geldt $u = x^2 + 4x + 5$, dus $u > 0$, oftewel $|u| = u$ voor iedere waarde van x .

61 a $f(x) = \frac{x^2 + x}{x^2 + 1} = \frac{x^2 + 1 - 1 + x}{x^2 + 1} = 1 + \frac{x - 1}{x^2 + 1} = 1 + \frac{\frac{1}{2} \cdot 2x}{x^2 + 1} - \frac{1}{x^2 + 1}$

$$\int \frac{\frac{1}{2} \cdot 2x}{x^2 + 1} dx = \int \frac{1}{2} \cdot \frac{1}{x^2 + 1} \cdot 2x dx = \int \frac{1}{2} \cdot \frac{1}{x^2 + 1} d(x^2 + 1) = \int \frac{1}{2} \cdot \frac{1}{u} du = \frac{1}{2} \ln(u) + c = \frac{1}{2} \ln(x^2 + 1) + c$$

Dus $F(x) = x + \frac{1}{2} \ln(x^2 + 1) - \arctan(x) + c$.

b $f(x) = \frac{x^2 - 6x + 8}{x^2 - 6x + 9} = \frac{x^2 - 6x + 9 - 1}{x^2 - 6x + 9} = 1 - \frac{1}{(x - 3)^2} = 1 - (x - 3)^{-2}$

Dus $F(x) = x + (x - 3)^{-1} + c = x + \frac{1}{x - 3} + c$.

$$\begin{aligned} \text{c } f(x) &= \frac{x^3}{x^2+2x-3} = \frac{x(x^2+2x-3)-2x^2+3x}{x^2+2x-3} = x + \frac{-2x^2+3x}{x^2+2x-3} = x + \frac{-2(x^2+2x-3)+4x-6+3x}{x^2+2x-3} \\ &= x-2 + \frac{7x-6}{x^2+2x-3} = x-2 + \frac{7x-6}{(x-1)(x+3)} = x-2 + \frac{a}{x-1} + \frac{b}{x+3} = x-2 + \frac{a(x+3)+b(x-1)}{(x-1)(x+3)} \\ &= x-2 + \frac{ax+3a+bx-b}{(x-1)(x+3)} = x-2 + \frac{(a+b)x+3a-b}{(x-1)(x+3)} \end{aligned}$$

$$\begin{cases} a+b=7 \\ 3a-b=-6 \end{cases} +$$

$$4a=1$$

$$a=\frac{1}{4}$$

$$a=\frac{1}{4} \text{ geeft } \frac{1}{4}+b=7$$

$$b=6\frac{3}{4}$$

$$\text{Dus } f(x) = x-2 + \frac{\frac{1}{4}}{x-1} + \frac{6\frac{3}{4}}{x+3}.$$

$$\text{Dus } F(x) = \frac{1}{2}x^2 - 2x + \frac{1}{4}\ln|x-1| + 6\frac{3}{4}\ln|x+3| + c.$$

62 a $f(x) = \frac{x^3+x}{x^2+1} = \frac{x(x^2+1)}{x^2+1} = x$ geeft $F(x) = \frac{1}{2}x^2 + c$

$$\begin{aligned} \text{b } f(x) &= \frac{x^3+x}{x^2-6x+9} = \frac{x(x^2-6x+9)+6x^2-9x+x}{x^2-6x+9} = x + \frac{6x^2-8x}{x^2-6x+9} \\ &= x + \frac{6(x^2-6x+9)+36x-54-8x}{x^2-6x+9} = x + 6 + \frac{28x-54}{x^2-6x+9} = x + 6 + \frac{28(x-3)+84-54}{(x-3)^2} \\ &= x + 6 + \frac{28}{x-3} + \frac{30}{(x-3)^2} = x + 6 + \frac{28}{x-3} + 30(x-3)^{-2} \end{aligned}$$

$$\text{Dus } F(x) = \frac{1}{2}x^2 + 6x + 28\ln|x-3| - 30(x-3)^{-1} + c = \frac{1}{2}x^2 + 6x + 28\ln|x-3| - \frac{30}{x-3} + c.$$

$$\begin{aligned} \text{c } f(x) &= \frac{2x^3+1}{2x^2+x} = \frac{x(2x^2+x)-x^2+1}{2x^2+x} = x + \frac{-x^2+1}{2x^2+x} = x + \frac{-\frac{1}{2}(2x^2+x)+\frac{1}{2}x+1}{2x^2+x} = x - \frac{1}{2} + \frac{\frac{1}{2}x+1}{2x^2+x} \\ &= x - \frac{1}{2} + \frac{\frac{1}{2}x+1}{x(2x+1)} = x - \frac{1}{2} + \frac{a}{x} + \frac{b}{2x+1} = x - \frac{1}{2} + \frac{a(2x+1)+bx}{x(2x+1)} = x - \frac{1}{2} + \frac{2ax+a+bx}{x(2x+1)} \\ &= x - \frac{1}{2} + \frac{(2a+b)x+a}{x(2x+1)} \end{aligned}$$

$$\begin{cases} 2a+b=\frac{1}{2} \\ a=1 \end{cases} \left. \begin{array}{l} 2+b=\frac{1}{2} \\ b=-1\frac{1}{2} \end{array} \right\}$$

$$\text{Dus } f(x) = x - \frac{1}{2} + \frac{1}{x} - \frac{1\frac{1}{2}}{2x+1}.$$

$$\text{Dus } F(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \ln|x| - \frac{3}{4}\ln|2x+1| + c.$$

63 a $\frac{x^3}{x^2+4} = \frac{x(x^2+4)-4x}{x^2+4} = x - \frac{4x}{x^2+4} = x - 2 \cdot \frac{2x}{x^2+4}$

$$\int \frac{2x}{x^2+4} dx = \int \frac{1}{x^2+4} \cdot 2x dx = \int \frac{1}{x^2+4} d(x^2+4) = \int \frac{1}{u} du = \ln(u) + c = \ln(x^2+4) + c$$

$$\int_0^2 \frac{x^3}{x^2+4} dx = \int_0^2 \left(x - 2 \cdot \frac{2x}{x^2+4} \right) dx = \left[\frac{1}{2}x^2 - 2\ln(x^2+4) \right]_0^2 = \frac{1}{2} \cdot 2^2 - 2\ln(8) - (0 - 2\ln(4)) = 2 - 2\ln(2)$$

$$\begin{aligned}
\text{b } \frac{x^3}{x^2+4x+4} &= \frac{x(x^2+4x+4)-4x^2-4x}{x^2+4x+4} = x + \frac{-4(x^2+4x+4)+16x+16-4x}{x^2+4x+4} \\
&= x-4 + \frac{12x+16}{x^2+4x+4} = x-4 + \frac{12x+16}{(x+2)^2} = x-4 + \frac{12(x+2)-24+16}{(x+2)^2} \\
&= x-4 + \frac{12}{x+2} - \frac{8}{(x+2)^2} = x-4 + \frac{12}{x+2} - 8(x+2)^{-2} \\
\int_0^8 \frac{x^3}{x^2+4x+4} dx &= \int_0^8 \left(x-4 + \frac{12}{x+2} - 8(x+2)^{-2} \right) dx = \left[\frac{1}{2}x^2 - 4x + 12 \ln|x+2| + 8(x+2)^{-1} \right]_0^8 \\
&= \left[\frac{1}{2}x^2 - 4x + 12 \ln|x+2| + \frac{8}{x+2} \right]_0^8 \\
&= \frac{1}{2} \cdot 8^2 - 4 \cdot 8 + 12 \ln(10) + \frac{4}{5} - (0 - 0 + 12 \ln(2) + 4) = 12 \ln(5) - 3\frac{1}{5}
\end{aligned}$$

$$\begin{aligned}
\text{c } \frac{4x-8}{x^2-4x-5} &= \frac{4x-8}{(x+1)(x-5)} = \frac{a}{x+1} + \frac{b}{x-5} = \frac{a(x-5)+b(x+1)}{(x+1)(x-5)} = \frac{ax-5a+bx+b}{(x+1)(x-5)} \\
&= \frac{(a+b)x-5a+b}{(x+1)(x-5)} \\
\begin{cases} a+b=4 \\ -5a+b=-8 \end{cases} & \quad \begin{aligned} 6a &= 12 \\ a &= 2 \\ a=2 \text{ geeft } 2+b &= 4 \\ b &= 2 \end{aligned}
\end{aligned}$$

$$\begin{aligned}
\int_0^2 \frac{4x-8}{x^2-4x-5} dx &= \int_0^2 \left(\frac{2}{x+1} + \frac{2}{x-5} \right) dx = [2 \ln|x+1| + 2 \ln|x-5|]_0^2 \\
&= 2 \ln(3) + 2 \ln(3) - (2 \ln(1) + 2 \ln(5)) = 4 \ln(3) - 2 \ln(5)
\end{aligned}$$

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$$\begin{aligned}
\text{a } \frac{x^4+1}{x^2+1} &= \frac{x^2(x^2+1)-x^2+1}{x^2+1} = x^2 + \frac{-x^2+1}{x^2+1} = x^2 + \frac{-(x^2+1)+1+1}{x^2+1} = x^2 - 1 + \frac{2}{x^2+1} \\
\int_0^1 \frac{x^4+1}{x^2+1} dx &= \int_0^1 \left(x^2 - 1 + \frac{2}{x^2+1} \right) dx = \left[\frac{1}{3}x^3 - x + 2 \arctan(x) \right]_0^1 \\
&= \frac{1}{3} - 1 + 2 \arctan(1) - (0 - 0 + 2 \arctan(0)) = -\frac{2}{3} + 2 \cdot \frac{1}{4}\pi = -\frac{2}{3} + \frac{1}{2}\pi
\end{aligned}$$

$$\begin{aligned}
\text{b } \frac{x^4+1}{x^2+2x+1} &= \frac{x^2(x^2+2x+1)-2x^3-x^2+1}{x^2+2x+1} = x^2 + \frac{-2x^3-x^2+1}{x^2+2x+1} \\
&= x^2 + \frac{-2x(x^2+2x+1)+4x^2+2x-x^2+1}{x^2+2x+1} = x^2 - 2x + \frac{3x^2+2x+1}{x^2+2x+1} \\
&= x^2 - 2x + \frac{3(x^2+2x+1)-6x-3+2x+1}{x^2+2x+1} = x^2 - 2x + 3 + \frac{-4x-2}{x^2+2x+1} \\
&= x^2 - 2x + 3 + \frac{-4x-2}{(x+1)^2} = x^2 - 2x + 3 + \frac{-4(x+1)+4-2}{(x+1)^2} \\
&= x^2 - 2x + 3 - \frac{4}{x+1} + \frac{2}{(x+1)^2} = x^2 - 2x + 3 - \frac{4}{x+1} + 2(x+1)^{-2} \\
\int_0^1 \frac{x^4+1}{x^2+2x+1} dx &= \int_0^1 \left(x^2 - 2x + 3 - \frac{4}{x+1} + 2(x+1)^{-2} \right) dx \\
&= \left[\frac{1}{3}x^3 - x^2 + 3x - 4 \ln|x+1| - 2(x+1)^{-1} \right]_0^1 = \left[\frac{1}{3}x^3 - x^2 + 3x - 4 \ln|x+1| - \frac{2}{x+1} \right]_0^1 \\
&= \frac{1}{3} - 1 + 3 - 4 \ln(2) - 1 - (0 - 0 + 0 - 4 \ln(1) - 2) = 3\frac{1}{3} - 4 \ln(2)
\end{aligned}$$

$$\begin{aligned}
\text{c } \frac{x^4}{x^2+4x+3} &= \frac{x^2(x^2+4x+3)-4x^3-3x^2}{x^2+4x+3} = x^2 + \frac{-4x^3-3x^2}{x^2+4x+3} \\
&= x^2 + \frac{-4x(x^2+4x+3)+16x^2+12x-3x^2}{x^2+4x+3} = x^2 - 4x + \frac{13x^2+12x}{x^2+4x+3} \\
&= x^2 - 4x + \frac{13(x^2+4x+3)-52x-39+12x}{x^2+4x+3} = x^2 - 4x + 13 + \frac{-40x-39}{x^2+4x+3} \\
&= x^2 - 4x + 13 + \frac{-40x-39}{(x+1)(x+3)} = x^2 - 4x + 13 + \frac{a}{x+1} + \frac{b}{x+3} \\
&= x^2 - 4x + 13 + \frac{a(x+3)+b(x+1)}{(x+1)(x+3)} = x^2 - 4x + 13 + \frac{ax+3a+bx+b}{(x+1)(x+3)} \\
&= x^2 - 4x + 13 + \frac{(a+b)x+3a+b}{(x+1)(x+3)}
\end{aligned}$$

$$\begin{cases} a+b=-40 \\ 3a+b=-39 \end{cases} \quad \begin{array}{l} -2a=-1 \\ a=\frac{1}{2} \end{array}$$

$$\begin{aligned} a = \frac{1}{2} \text{ geeft } \frac{1}{2} + b &= -40 \\ b &= -40\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \int_0^2 \frac{x^4}{x^2+4x+3} dx &= \int_0^2 \left(x^2 - 4x + 13 + \frac{\frac{1}{2}}{x+1} - \frac{40\frac{1}{2}}{x+3} \right) dx \\ &= \left[\frac{1}{3}x^3 - 2x^2 + 13x + \frac{1}{2}\ln|x+1| - 40\frac{1}{2}\ln|x+3| \right]_0^2 \\ &= \frac{1}{3} \cdot 2^3 - 2 \cdot 2^2 + 13 \cdot 2 + \frac{1}{2}\ln(3) - 40\frac{1}{2}\ln(5) - (0 - 0 + 0 + \frac{1}{2}\ln(1) - 40\frac{1}{2}\ln(3)) \\ &= 20\frac{2}{3} + 41\ln(3) - 40\frac{1}{2}\ln(5) \end{aligned}$$

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$$\begin{aligned} f(x) &= \frac{2x-5}{x^2-5x+6} = \frac{2x-5}{(x-2)(x-3)} = \frac{a}{x-2} + \frac{b}{x-3} = \frac{a(x-3)+b(x-2)}{(x-2)(x-3)} = \frac{ax-3a+bx-2b}{(x-2)(x-3)} \\ &= \frac{(a+b)x-3a-2b}{(x-2)(x-3)} \\ \begin{cases} a+b=2 \\ -3a-2b=-5 \end{cases} \Big| \begin{array}{l} 2 \\ 1 \end{array} &\text{ geeft } \begin{cases} 2a+2b=4 \\ -3a-2b=-5 \end{cases} + \\ &\quad \begin{array}{l} -a=-1 \\ a=1 \end{array} \end{aligned}$$

$$\begin{aligned} a = 1 \text{ geeft } 1 + b &= 2 \\ b &= 1 \end{aligned}$$

$$\text{Dus } f(x) = \frac{1}{x-2} + \frac{1}{x-3}.$$

Dus met breuksplitsen krijg je $F(x) = \ln|x-2| + \ln|x-3| + c$.

Zonder breuksplitsen krijg je

$$\begin{aligned} F(x) &= \int \frac{2x-5}{x^2-5x+6} dx = \int \frac{1}{x^2-5x+6} \cdot (2x-5) dx = \int \frac{1}{x^2-5x+6} d(x^2-5x+6) = \int \frac{1}{u} du = \ln|u| + c \\ &= \ln|x^2-5x+6| + c. \end{aligned}$$

$$\ln|x-2| + \ln|x-3| = \ln|(x-2)(x-3)| = \ln|x^2-5x+6|$$

Dus de primitieven komen op hetzelfde neer.

66 a Als $x > \frac{1}{2}$ dan is $10x + 5 > 10$ en $4x^2 - 4x + 1 > 0$, dus dan is $f(x) > 0$.

$$f(x) = \frac{10x + 5}{4x^2 - 4x + 1} \text{ geeft}$$

$$f'(x) = \frac{(4x^2 - 4x + 1) \cdot 10 - (10x + 5) \cdot (8x - 4)}{(4x^2 - 4x + 1)^2} = \frac{40x^2 - 40x + 10 - (80x^2 - 40x + 20)}{(2x - 1)^4}$$

$$= \frac{40x^2 - 40x + 10 - 80x^2 + 40x - 10}{(2x - 1)^4} = \frac{-40x^2 - 40x + 30}{(2x - 1)^4}$$

$$f'(x) = 0 \text{ geeft } -40x^2 - 40x + 30 = 0$$

$$4x^2 + 4x - 3 = 0$$

$$D = 4^2 - 4 \cdot 4 \cdot (-3) = 64, \text{ dus } \sqrt{D} = 8$$

$$x = \frac{-4 \pm 8}{8} = \frac{1}{2} \vee x = \frac{-4 - 8}{8} = -1\frac{1}{2}$$

$x = \frac{1}{2}$ voldoet niet, dus $x = -1\frac{1}{2}$.

$$\text{min. is } f(-1\frac{1}{2}) = -\frac{5}{8}$$

$$\text{Dus } B_f = [-\frac{5}{8}, \rightarrow).$$

$$\text{b } f(x) = \frac{10x + 5}{4x^2 - 4x + 1} = \frac{10x + 5}{(2x - 1)^2} = \frac{5(2x - 1) + 5 + 5}{(2x - 1)^2} = \frac{5}{2x - 1} + \frac{10}{(2x - 1)^2} = \frac{5}{2x - 1} + 10(2x - 1)^{-2}$$

$$O(V) = \int_1^3 f(x) dx = \int_1^3 \left(\frac{5}{2x - 1} + 10(2x - 1)^{-2} \right) dx = \left[2\frac{1}{2} \ln|2x - 1| - 5(2x - 1)^{-1} \right]_1^3$$

$$= \left[2\frac{1}{2} \ln|2x - 1| - \frac{5}{2x - 1} \right]_1^3 = 2\frac{1}{2} \ln(5) - 1 - (2\frac{1}{2} \ln(1) - 5) = 4 + 2\frac{1}{2} \ln(5)$$

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67 a $f(x) = \frac{x^2 + 4}{x^2 + 5x + 4}$ geeft

$$f'(x) = \frac{(x^2 + 5x + 4) \cdot 2x - (x^2 + 4) \cdot (2x + 5)}{(x^2 + 5x + 4)^2} = \frac{2x^3 + 10x^2 + 8x - (2x^3 + 5x^2 + 8x + 20)}{(x^2 + 5x + 4)^2}$$

$$= \frac{2x^3 + 10x^2 + 8x - 2x^3 - 5x^2 - 8x - 20}{(x^2 + 5x + 4)^2} = \frac{5x^2 - 20}{(x^2 + 5x + 4)^2}$$

$$f'(x) = 0 \text{ geeft } 5x^2 - 20 = 0$$

$$5x^2 = 20$$

$$x^2 = 4$$

$$x = 2 \vee x = -2$$

$$\text{max. is } f(-2) = -4$$

$$\text{min. is } f(2) = \frac{4}{9}$$

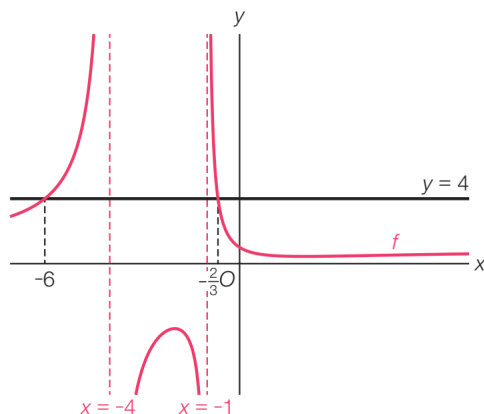
$$\text{b } f(x) = 4 \text{ geeft } \frac{x^2 + 4}{x^2 + 5x + 4} = 4$$

$$4x^2 + 20x + 16 = x^2 + 4$$

$$3x^2 + 20x + 12 = 0$$

$$D = 20^2 - 4 \cdot 3 \cdot 12 = 256, \text{ dus } \sqrt{D} = 16$$

$$x = \frac{-20 \pm 16}{6} = -\frac{2}{3} \vee x = \frac{-20 - 16}{6} = -6$$



$$f(x) \leq 4 \text{ geeft } x \leq -6 \vee -4 < x < -1 \vee x \geq -\frac{2}{3}$$

$$\begin{aligned} \text{c } f(x) &= \frac{x^2 + 4}{x^2 + 5x + 4} = \frac{x^2 + 5x + 4 - 5x}{x^2 + 5x + 4} = 1 - \frac{5x}{x^2 + 5x + 4} = 1 - \frac{5x}{(x+1)(x+4)} = 1 - \left(\frac{a}{x+1} + \frac{b}{x+4} \right) \\ &= 1 - \frac{a(x+4) + b(x+1)}{(x+1)(x+4)} = 1 - \frac{ax + 4a + bx + b}{(x+1)(x+4)} = 1 - \frac{(a+b)x + 4a + b}{(x+1)(x+4)} \end{aligned}$$

$$\begin{cases} a + b = 5 \\ 4a + b = 0 \end{cases}$$

$$-3a = 5$$

$$a = -1\frac{2}{3}$$

$$a = -1\frac{2}{3} \text{ geeft } -1\frac{2}{3} + b = 5$$

$$b = 6\frac{2}{3}$$

$$\text{Dus } f(x) = 1 + \frac{1\frac{2}{3}}{x+1} - \frac{6\frac{2}{3}}{x+4}.$$

$$\begin{aligned} O(V) &= \int_0^6 f(x) dx = \int_0^6 \left(1 + \frac{1\frac{2}{3}}{x+1} - \frac{6\frac{2}{3}}{x+4} \right) dx = [x + 1\frac{2}{3} \ln|x+1| - 6\frac{2}{3} \ln|x+4|]_0^6 \\ &= 6 + 1\frac{2}{3} \ln(7) - 6\frac{2}{3} \ln(10) - (0 + 1\frac{2}{3} \ln(1) - 6\frac{2}{3} \ln(4)) = 6 + 1\frac{2}{3} \ln(7) + 6\frac{2}{3} \ln(\frac{2}{5}) \end{aligned}$$

68 a $f(x) = \frac{x}{x+1} = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$

$$O(V) = \int_0^3 f(x) dx = \int_0^3 \left(1 - \frac{1}{x+1} \right) dx = [x - \ln|x+1|]_0^3 = 3 - \ln(4) - (0 - \ln(1)) = 3 - \ln(4)$$

$$\begin{aligned} \text{b } I(L) &= \pi \int_0^3 (f(x))^2 dx = \pi \int_0^3 \left(1 - \frac{1}{x+1} \right)^2 dx = \pi \int_0^3 \left(1 - \frac{2}{x+1} + \frac{1}{(x+1)^2} \right) dx \\ &= \pi \int_0^3 \left(1 - \frac{2}{x+1} + (x+1)^{-2} \right) dx = \pi [x - 2 \ln|x+1| - (x+1)^{-1}]_0^3 = \pi \left[x - 2 \ln|x+1| - \frac{1}{x+1} \right]_0^3 \\ &= \pi(3 - 2 \ln(4) - \frac{1}{4} - (0 - 2 \ln(1) - 1)) = \pi(3\frac{3}{4} - 2 \ln(4)) \end{aligned}$$

69 a $f(x) = \ln(x^2 + 1)$ geeft $f'(x) = \frac{1}{x^2 + 1} \cdot 2x = \frac{2x}{x^2 + 1}$

$$f'(x) = \frac{2}{5} \text{ geeft } \frac{2x}{x^2 + 1} = \frac{2}{5}$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$D = 10^2 - 4 \cdot 3 \cdot 3 = 64, \text{ dus } \sqrt{D} = 8$$

$$x = \frac{10 \pm 8}{6} = 3 \vee x = \frac{10 - 8}{6} = \frac{1}{3}$$

$$f(3) = \ln(10) \text{ en } f(\frac{1}{3}) = \ln(1\frac{1}{9})$$

Dus de raakpunten zijn $(3, \ln(10))$ en $(\frac{1}{3}, \ln(1\frac{1}{9}))$.

b $f(x) = \ln(2)$ geeft $\ln(x^2 + 1) = \ln(2)$

$$x^2 + 1 = 2$$

$$x^2 = 1$$

$$x = 1 \vee x = -1$$

$$\begin{aligned} \int f(x) dx &= \int \ln(x^2 + 1) dx = \int \ln(x^2 + 1) \cdot 1 dx = \ln(x^2 + 1) \cdot x - \int \frac{1}{x^2 + 1} \cdot 2x \cdot x dx \\ &= x \ln(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} dx = x \ln(x^2 + 1) - \int \frac{2(x^2 + 1) - 2}{x^2 + 1} dx \\ &= x \ln(x^2 + 1) - \int \left(2 - \frac{2}{x^2 + 1} \right) dx = x \ln(x^2 + 1) - (2x - 2 \arctan(x)) + c \\ &= x \ln(x^2 + 1) - 2x + 2 \arctan(x) + c \end{aligned}$$

$$\begin{aligned} O(V) &= \int_{-1}^1 (\ln(2) - f(x)) dx = [x \ln(2) - (x \ln(x^2 + 1) - 2x + 2 \arctan(x))]_{-1}^1 \\ &= [x \ln(2) - x \ln(x^2 + 1) + 2x - 2 \arctan(x)]_{-1}^1 \\ &= \ln(2) - \ln(2) + 2 - 2 \arctan(1) - (-\ln(2) + \ln(2) - 2 - 2 \arctan(-1)) \\ &= 2 - 2 \cdot \frac{1}{4}\pi + 2 + 2 \cdot -\frac{1}{4}\pi = 4 - \pi \end{aligned}$$

C7

$$\mathbf{a} \quad f(x) = \frac{2x+1}{x-1} = \frac{2(x-1)+2+1}{x-1} = 2 + \frac{3}{x-1}$$

$$\text{Dus } F(x) = 2x + 3 \ln|x-1| + c.$$

$$\mathbf{b} \quad f(x) = \frac{x^2+6x+1}{2x+4} = \frac{\frac{1}{2}x(2x+4) - 2x + 6x + 1}{2x+4} = \frac{1}{2}x + \frac{4x+1}{2x+4} = \frac{1}{2}x + \frac{2(2x+4) - 8 + 1}{2x+4}$$

$$= \frac{1}{2}x + 2 - \frac{7}{2x+4}$$

$$\text{Dus } F(x) = \frac{1}{4}x^2 + 2x - 3\frac{1}{2} \ln|2x+4| + c.$$

C8

$$\mathbf{a} \quad f(x) = \frac{2x+4}{x^2+6x+10} = \frac{2x+6-2}{x^2+6x+10} = \frac{2x+6}{x^2+6x+10} - \frac{2}{x^2+6x+10}$$

$$\int \frac{2x+6}{x^2+6x+10} dx = \int \frac{1}{x^2+6x+10} \cdot (2x+6) dx = \int \frac{1}{x^2+6x+10} d(x^2+6x+10) = \int \frac{1}{u} du = \ln(u) + c_1$$

$$= \ln(x^2+6x+10) + c_1$$

$$\int \frac{2}{x^2+6x+10} dx = \int \frac{2}{(x+3)^2+1} dx = 2 \arctan(x+3) + c_2$$

$$\text{Dus } F(x) = \ln(x^2+6x+10) - 2 \arctan(x+3) + c.$$

$$\mathbf{b} \quad f(x) = \frac{2x+5}{x^2+2x+1} = \frac{2x+5}{(x+1)^2} = \frac{2(x+1)+3}{(x+1)^2} = \frac{2}{x+1} + \frac{3}{(x+1)^2} = \frac{2}{x+1} + 3(x+1)^{-2}$$

$$\text{Dus } F(x) = 2 \ln|x+1| - 3(x+1)^{-1} + c = 2 \ln|x+1| - \frac{3}{x+1} + c.$$

$$\mathbf{c} \quad f(x) = \frac{-6x+14}{x^2+2x-3} = \frac{-6x+14}{(x-1)(x+3)} = \frac{a}{x-1} + \frac{b}{x+3} = \frac{a(x+3)+b(x-1)}{(x-1)(x+3)} = \frac{ax+3a+bx-b}{(x-1)(x+3)}$$

$$= \frac{(a+b)x+3a-b}{(x-1)(x+3)}$$

$$\begin{cases} a+b=-6 \\ 3a-b=14 \end{cases} +$$

$$4a=8$$

$$a=2$$

$$a=2 \text{ geeft } 2+b=-6$$

$$b=-8$$

$$\text{Dus } f(x) = \frac{2}{x-1} - \frac{8}{x+3}.$$

$$\text{Dus } F(x) = 2 \ln|x-1| - 8 \ln|x+3| + c.$$

K.5 Integralen bij parameterkrommen

70

$$\mathbf{a} \quad \int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) = -(F(a) - F(b)) = -[F(x)]_b^a = -\int_b^a f(x) dx$$

$$\mathbf{b} \quad \int_a^b f(x) dx + \int_b^c f(x) dx = [F(x)]_a^b + [F(x)]_b^c = F(b) - F(a) + F(c) - F(b) = F(c) - F(a)$$

$$= [F(x)]_a^c = \int_a^c f(x) dx$$

71

$\mathbf{a}$ Bij A hoort $t=0$, bij B hoort $t=2$ en bij C hoort $t=4$, dus

$$O(V) = \int_{x_A}^{x_B} y dx - \int_{x_C}^{x_B} y dx = \int_{t=0}^{t=2} y dx - \int_{t=4}^{t=2} y dx.$$

$$\text{Herleiden geeft } O(V) = \int_{t=0}^{t=2} y dx - \int_{t=4}^{t=2} y dx = \int_{t=0}^{t=2} y dx + \int_{t=2}^{t=4} y dx = \int_{t=0}^{t=4} y dx.$$

$$\mathbf{b} \quad O(V) = \int_{t=0}^{t=4} (-\frac{1}{2}t^2 + 2) d(-\frac{1}{2}t^2 + 2t) = \int_0^4 (-\frac{1}{2}t^2 + 2) \cdot (-t + 2) dt = \int_0^4 (\frac{1}{2}t^3 - t^2 - 2t + 4) dt$$

$$= [\frac{1}{8}t^4 - \frac{1}{3}t^3 - t^2 + 4t]_0^4 = \frac{1}{8} \cdot 4^4 - \frac{1}{3} \cdot 4^3 - 4^2 + 4 \cdot 4 - 0 = 10\frac{2}{3}$$

c Er geldt $O(V) = \int_{y_C}^{y_A} x \, dy = \int_{t=4}^{t=0} x \, dy$, dus deze integraal geeft ook de oppervlakte van V .

$$O(V) = \int_{t=4}^{t=0} x \, dy = \int_{t=4}^{t=0} (-\frac{1}{2}t^2 + 2t) d(-\frac{1}{2}t^2 + 2) = \int_4^0 (-\frac{1}{2}t^2 + 2t) \cdot -t \, dt = \int_4^0 (\frac{1}{2}t^3 - 2t^2) \, dt$$

$$= [\frac{1}{8}t^4 - \frac{2}{3}t^3]_4^0 = \frac{1}{8} \cdot 0^4 - \frac{2}{3} \cdot 0^3 - (\frac{1}{8} \cdot 4^4 - \frac{2}{3} \cdot 4^3) = 10\frac{2}{3}$$

Bladzijde 209

72 a Van $t = 2$ naar $t = -2$ gaat K 'van beneden naar boven' en geldt $x \leq 0$, dus $-\int_{t=2}^{t=-2} x \, dy$ geeft de oppervlakte van het gedeelte van V dat zich links van de y -as bevindt.

Van $t = 2$ naar $t = 4$ gaat K 'van beneden naar boven' en geldt $x \geq 0$, dus $\int_{t=2}^{t=4} x \, dy$ geeft de oppervlakte van het gedeelte van V dat zich rechts van de y -as bevindt.

$$\text{Dus } O(V) = -\int_{t=2}^{t=-2} x \, dy + \int_{t=2}^{t=4} x \, dy.$$

b Van $t = 2$ naar $t = -2$ gaat K 'van beneden naar boven' en geldt $x \leq 0$, dus $-\int_{t=2}^{t=-2} x \, dy$ geeft de oppervlakte van het gedeelte van V dat zich links van de y -as bevindt.

Van $t = 2$ naar $t = 4$ gaat K 'van links naar rechts' en geldt $y \leq 0$, dus $-\int_{t=2}^{t=4} y \, dx$ geeft de oppervlakte van het gedeelte van V dat zich rechts van de y -as bevindt.

$$\text{Dus } O(V) = -\int_{t=2}^{t=-2} x \, dy - \int_{t=2}^{t=4} y \, dx.$$

Deze manier is niet handig, omdat je deze vorm niet kunt herleiden tot één integraal.

Bladzijde 210

73 $I(L) = \pi \int_{t=2}^{t=-2} x^2 \, dy = \pi \int_{t=2}^{t=-2} (\frac{1}{2}t^2 - 2)^2 d(\frac{1}{2}t^2 - 2t) = \pi \int_2^{-2} (\frac{1}{2}t^2 - 2)^2 \cdot (t - 2) \, dt$

$$= \pi \int_2^{-2} (\frac{1}{4}t^4 - 2t^2 + 4)(t - 2) \, dt = \pi \int_2^{-2} (\frac{1}{4}t^5 - \frac{1}{2}t^4 - 2t^3 + 4t^2 + 4t - 8) \, dt$$

$$= \pi [\frac{1}{24}t^6 - \frac{1}{10}t^5 - \frac{1}{2}t^4 + \frac{4}{3}t^3 + 2t^2 - 8t]_2^{-2}$$

$$= \pi (\frac{64}{24} + \frac{32}{10} - 8 - 10\frac{2}{3} + 8 + 16 - (\frac{64}{24} - \frac{32}{10} - 8 + 10\frac{2}{3} + 8 - 16)) = 17\frac{1}{15}\pi$$

74 $y(t) = 0$ geeft $t^2 - 2t = 0$
 $t(t - 2) = 0$
 $t = 0 \vee t = 2$

$t = 0$ geeft $(0, 0)$

$t = 2$ geeft $(6, 0)$

$$O(V) = -\int_{t=0}^{t=2} y \, dx = \int_{t=2}^{t=0} y \, dx = \int_2^0 (t^2 - 2t) d(t^3 - t) = \int_2^0 (t^2 - 2t) \cdot (3t^2 - 1) \, dt = \int_2^0 (3t^4 - t^2 - 6t^3 + 2t) \, dt$$

$$= [\frac{3}{5}t^5 - \frac{1}{3}t^3 - \frac{1}{2}t^4 + t^2]_2^0 = 0 - (\frac{3}{5} \cdot 2^5 - \frac{1}{3} \cdot 2^3 - \frac{1}{2} \cdot 2^4 + 2^2) = 3\frac{7}{15}$$

75 $x(t) = 0$ geeft $-t^2 + 2t = 0$
 $t(-t + 2) = 0$
 $t = 0 \vee t = 2$

$t = 0$ geeft $(0, 2)$

$t = 2$ geeft $(0, -\frac{2}{3})$

$$y(t) = 0 \text{ geeft } \frac{2}{3}t^2 - 2\frac{2}{3}t + 2 = 0$$

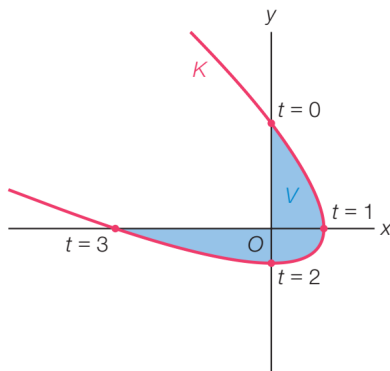
$$t^2 - 4t + 3 = 0$$

$$(t - 1)(t - 3) = 0$$

$$t = 1 \vee t = 3$$

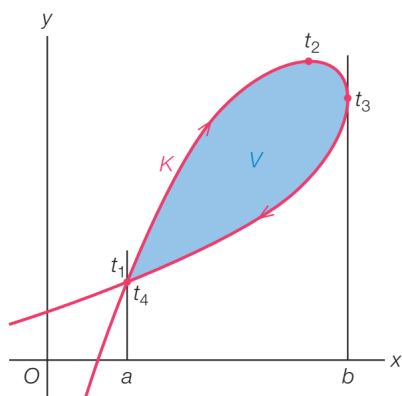
$t = 1$ geeft $(1, 0)$

$t = 3$ geeft $(-3, 0)$



$$\begin{aligned}
 O(V) &= - \int_{t=3}^{t=1} y \, dx + \int_{t=0}^{t=1} y \, dx = \int_{t=1}^{t=3} y \, dx + \int_{t=0}^{t=1} y \, dx = \int_{t=0}^{t=3} y \, dx = \int_{t=0}^{t=3} \left(\frac{2}{3}t^2 - \frac{8}{3}t + 2 \right) d(-t^2 + 2t) \\
 &= \int_0^3 \left(\frac{2}{3}t^2 - \frac{8}{3}t + 2 \right) \cdot (-2t + 2) \, dt = \int_0^3 \left(-\frac{4}{3}t^3 + \frac{20}{3}t^2 - \frac{28}{3}t + 4 \right) dt = \left[-\frac{1}{3}t^4 + \frac{20}{9}t^3 - \frac{14}{3}t^2 + 4t \right]_0^3 \\
 &= -\frac{1}{3} \cdot 3^4 + \frac{20}{9} \cdot 3^3 - \frac{14}{3} \cdot 3^2 + 4 \cdot 3 - 0 = 3
 \end{aligned}$$

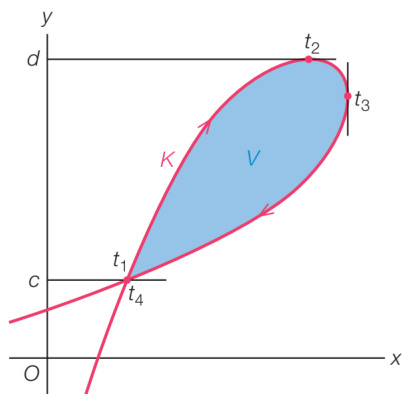
- 76 a** Teken de lijn $x = a$ door het punt waar K zichzelf snijdt en de lijn $x = b$ die K raakt in het punt waarvoor $t = t_3$.



Noem V_1 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het bovenste deel van K , en V_2 het vlakdeel ingesloten door de x -as, de lijnen $x = a$ en $x = b$ en het onderste deel van K .

$$\text{Dan geldt } O(V) = O(V_1) - O(V_2) = \int_{t=t_1}^{t=t_3} y \, dx - \int_{t=t_4}^{t=t_3} y \, dx = \int_{t=t_1}^{t=t_3} y \, dx + \int_{t=t_3}^{t=t_4} y \, dx = \int_{t=t_1}^{t=t_4} y \, dx.$$

- b** Teken de lijn $y = c$ door het punt waar K zichzelf snijdt en de lijn $y = d$ die K raakt in het punt waarvoor $t = t_2$.



Noem V_3 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het rechter deel van K , en V_4 het vlakdeel ingesloten door de y -as, de lijnen $y = c$ en $y = d$ en het linker deel van K .

$$\text{Dan geldt } O(V) = O(V_3) - O(V_4) = \int_{t=t_4}^{t=t_2} x \, dy - \int_{t=t_1}^{t=t_2} x \, dy = \int_{t=t_4}^{t=t_2} x \, dy + \int_{t=t_2}^{t=t_1} x \, dy = \int_{t=t_4}^{t=t_1} x \, dy = - \int_{t=t_1}^{t=t_4} x \, dy.$$

77 $\int_{t=-1}^{t=2} y \, dx = \int_{t=-1}^{t=2} (t^3 - 3t) \, d(t^2 - t) = \int_{-1}^2 (t^3 - 3t) \cdot (2t - 1) \, dt = \int_{-1}^2 (2t^4 - t^3 - 6t^2 + 3t) \, dt$
 $= \left[\frac{2}{5} t^5 - \frac{1}{4} t^4 - 2t^3 + \frac{3}{2} t^2 \right]_{-1}^2 = \frac{2}{5} \cdot 2^5 - \frac{1}{4} \cdot 2^4 - 2 \cdot 2^3 + \frac{3}{2} \cdot 2^2 - \left(-\frac{2}{5} - \frac{1}{4} + 2 + \frac{3}{2} \right) = -4 \frac{1}{20}$
 Dus $O(V) = 4 \frac{1}{20}$.

78 $x(t) = 0$ geeft $-t^2 + 6t = 0$
 $-t(t - 6) = 0$
 $t = 0 \vee t = 6$
 $\int_{t=0}^{t=6} x \, dy = \int_{t=0}^{t=6} (-t^2 + 6t) \, d(-\frac{1}{3}t^3 + 2t^2) = \int_0^6 (-t^2 + 6t) \cdot (-t^2 + 4t) \, dt = \int_0^6 (t^4 - 10t^3 + 24t^2) \, dt$
 $= \left[\frac{1}{5} t^5 - 2\frac{1}{2} t^4 + 8t^3 \right]_0^6 = \frac{1}{5} \cdot 6^5 - 2\frac{1}{2} \cdot 6^4 + 8 \cdot 6^3 - 0 = 43 \frac{1}{5}$
 Dus $O(V) = 43 \frac{1}{5}$.

79 $y(t) = 8$ geeft $t^2 + 2t = 8$
 $t^2 + 2t - 8 = 0$
 $(t - 2)(t + 4) = 0$
 $t = 2 \vee t = -4$
 $\int_{t=-4}^{t=2} y \, dx = \int_{t=-4}^{t=2} (t^2 + 2t) \, d(-\frac{1}{3}t^3 + 4t) = \int_{-4}^2 (t^2 + 2t) \cdot (-t^2 + 4) \, dt = \int_{-4}^2 (-t^4 + 4t^2 - 2t^3 + 8t) \, dt$
 $= \left[-\frac{1}{5} t^5 + \frac{4}{3} t^3 - \frac{1}{2} t^4 + 4t^2 \right]_{-4}^2$
 $= -\frac{1}{5} \cdot 2^5 + \frac{4}{3} \cdot 2^3 - \frac{1}{2} \cdot 2^4 + 4 \cdot 2^2 - \left(-\frac{1}{5} \cdot (-4)^5 + \frac{4}{3} \cdot (-4)^3 - \frac{1}{2} \cdot (-4)^4 + 4 \cdot (-4)^2 \right) = -43 \frac{1}{5}$
 Dus $O(V) = 43 \frac{1}{5}$.

80 a $y(t) = -a$ geeft $t^2 - (a + 1)t = -a$
 $t^2 - (a + 1)t + a = 0$
 $(t - a)(t - 1) = 0$
 $t = a \vee t = 1$

$$x(a) = a + \frac{a}{a} = a + 1$$

$$x(1) = 1 + \frac{a}{1} = a + 1$$

Dus de kromme snijdt zichzelf in het punt $(a + 1, -a)$.

b $\int_{t=1}^{t=6} x \, dy = \int_{t=1}^{t=6} \left(t + \frac{6}{t} \right) \, d(t^2 - 7t) = \int_1^6 \left(t + \frac{6}{t} \right) \cdot (2t - 7) \, dt = \int_1^6 \left(2t^2 - 7t + 12 - \frac{42}{t} \right) \, dt$
 $= \left[\frac{2}{3} t^3 - 3\frac{1}{2} t^2 + 12t - 42 \ln|t| \right]_1^6 = \frac{2}{3} \cdot 6^3 - 3\frac{1}{2} \cdot 6^2 + 12 \cdot 6 - 42 \ln(6) - \left(\frac{2}{3} - 3\frac{1}{2} + 12 - 42 \ln(1) \right)$
 $= 80 \frac{5}{6} - 42 \ln(6)$
 Dus $O(V_6) = 80 \frac{5}{6} - 42 \ln(6)$.

81 a $x(t) = 0$ geeft $2 \sin(t) = 0$
 $t = k \cdot \pi$
 $t = -\pi$ geeft $(0, \frac{1}{2}\pi^2)$
 $t = 0$ geeft $(0, 0)$
 $t = \pi$ geeft $(0, \frac{1}{2}\pi^2)$

$$O(V) = 2 \int_{t=0}^{t=\pi} x \, dy = 2 \int_{t=0}^{t=\pi} 2 \sin(t) \, d(\frac{1}{2}t^2) = 2 \int_0^\pi 2 \sin(t) \cdot t \, dt = 4 \int_0^\pi t \sin(t) \, dt$$

 $\int t \sin(t) \, dt = t \cdot -\cos(t) - \int 1 \cdot -\cos(t) \, dt = -t \cos(t) + \int \cos(t) \, dt = -t \cos(t) + \sin(t) + c$
 Dus $O(V) = 4[-t \cos(t) + \sin(t)]_0^\pi = 4(-\pi \cdot -1 + 0 - (0 + 0)) = 4\pi$.

$$\begin{aligned}
 \text{b } I(L) &= \pi \int_{t=0}^{t=\pi} x^2 dy = \pi \int_{t=0}^{t=\pi} 4 \sin^2(t) d\left(\frac{1}{2}t^2\right) = \pi \int_0^\pi 4\left(\frac{1}{2} - \frac{1}{2}\cos(2t)\right) \cdot t dt = \pi \int_0^\pi (2t - 2t\cos(2t)) dt \\
 &= 2\pi \int_0^\pi (t - t\cos(2t)) dt = 2\pi \left(\int_0^\pi t dt - \int_0^\pi t\cos(2t) dt \right) \\
 \int t\cos(2t) dt &= t \cdot \frac{1}{2}\sin(2t) - \int 1 \cdot \frac{1}{2}\sin(2t) dt = \frac{1}{2}t\sin(2t) - \int \frac{1}{2}\sin(2t) dt = \frac{1}{2}t\sin(2t) + \frac{1}{4}\cos(2t) + c \\
 \text{Dus } I(L) &= 2\pi \left[\frac{1}{2}t^2 - \frac{1}{2}t\sin(2t) - \frac{1}{4}\cos(2t) \right]_0^\pi = 2\pi \left(\frac{1}{2}\pi^2 - \frac{1}{2}\pi \cdot 0 - \frac{1}{4} - (0 - 0 - \frac{1}{4}) \right) = \pi^3.
 \end{aligned}$$

Bladzijde 213

C9 **a** $O(V) = \int_{t=0}^{t=1} x dy - \int_{t=0}^{t=-2} x dy = \int_{t=0}^{t=1} x dy + \int_{t=-2}^{t=0} x dy = \int_{t=-2}^{t=1} x dy = \int_{t=-2}^{t=1} (t^2 + 2t) d(t^2 - 1) = \int_{-2}^1 (t^2 + 2t) \cdot 2t dt$

$$= \int_{-2}^1 (2t^3 + 4t^2) dt = \left[\frac{1}{2}t^4 + \frac{4}{3}t^3 \right]_{-2}^1 = \frac{1}{2} + \frac{4}{3} - \left(\frac{1}{2} \cdot (-2)^4 + \frac{4}{3} \cdot (-2)^3 \right) = 4\frac{1}{2}$$

b $I(L) = \pi \int_{t=0}^{t=-2} x^2 dy = \pi \int_{t=0}^{t=-2} (t^2 + 2t)^2 d(t^2 - 1) = \pi \int_0^{-2} (t^2 + 2t)^2 \cdot 2t dt = \pi \int_0^{-2} 2t(t^4 + 4t^3 + 4t^2) dt$

$$= \pi \int_0^{-2} (2t^5 + 8t^4 + 8t^3) dt = \pi \left[\frac{2}{6}t^6 + \frac{8}{5}t^5 + 2t^4 \right]_0^{-2} = \pi \left(\frac{1}{3} \cdot (-2)^6 + \frac{8}{5} \cdot (-2)^5 + 2 \cdot (-2)^4 - 0 \right)$$

$$= 2\frac{2}{15}\pi$$

C10 $x(t) = 8$ geeft $t^2 - 2t = 8$

$$t^2 - 2t - 8 = 0$$

$$(t + 2)(t - 4) = 0$$

$$t = -2 \vee t = 4$$

$$\begin{aligned}
 \int_{t=-2}^{t=4} x dy &= \int_{t=-2}^{t=4} (t^2 - 2t) d\left(\frac{2}{3}t^3 - 8t\right) = \int_{-2}^4 (t^2 - 2t) \cdot (2t^2 - 8) dt = \int_{-2}^4 (2t^4 - 8t^2 - 4t^3 + 16t) dt \\
 &= \left[\frac{2}{5}t^5 - \frac{8}{3}t^3 - t^4 + 8t^2 \right]_{-2}^4 = \frac{2}{5} \cdot 4^5 - \frac{8}{3} \cdot 4^3 - 4^4 + 8 \cdot 4^2 - \left(\frac{2}{5} \cdot (-2)^5 - \frac{8}{3} \cdot (-2)^3 - (-2)^4 + 8 \cdot (-2)^2 \right) \\
 &= 86\frac{2}{5} \\
 \text{Dus } O(V) &= 86\frac{2}{5}.
 \end{aligned}$$

Kennis- en vaardigheidentest

Bladzijde 214

1 **a** $F(x) = \int 3x\sqrt{x^2 + 2} dx = \int \frac{1}{2} \cdot \sqrt{x^2 + 2} \cdot 2x dx = \int \frac{1}{2} \sqrt{x^2 + 2} d(x^2 + 2) = \int \frac{1}{2} \sqrt{u} du = \int \frac{1}{2} u^{\frac{1}{2}} du$

$$= u^{\frac{3}{2}} + c = u\sqrt{u} + c = (x^2 + 2)\sqrt{x^2 + 2} + c$$

b $G(x) = \int \frac{x}{e^{x^2+1}} dx = \int \frac{1}{2} \cdot \frac{1}{e^{x^2+1}} \cdot 2x dx = \int \frac{1}{2} \cdot \frac{1}{e^{x^2+1}} d(x^2 + 1) = \int \frac{1}{2} \cdot \frac{1}{e^u} du = \int \frac{1}{2} e^{-u} du = -\frac{1}{2} e^{-u} + c$

$$= -\frac{1}{2} e^{-(x^2+1)} + c = -\frac{1}{2e^{x^2+1}} + c$$

c $H(x) = \int \cos^5(x) dx = \int \cos^4(x) \cdot \cos(x) dx = \int (\cos^2(x))^2 d\sin(x) = \int (1 - \sin^2(x))^2 d\sin(x)$

$$= \int (1 - u^2)^2 du = \int (1 - 2u^2 + u^4) du = \int (u^4 - 2u^2 + 1) du = \frac{1}{5}u^5 - \frac{2}{3}u^3 + u + c$$

$$= \frac{1}{5}\sin^5(x) - \frac{2}{3}\sin^3(x) + \sin(x) + c$$

2 **a** $\int \frac{\sqrt{\ln(x)}}{x} dx = \int \sqrt{\ln(x)} \cdot \frac{1}{x} dx = \int \sqrt{\ln(x)} d\ln(x) = \int \sqrt{u} du = \int u^{\frac{1}{2}} du = \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{3}u\sqrt{u} + c$

$$= \frac{2}{3}\ln(x)\sqrt{\ln(x)} + c$$

$$\int_1^e \frac{\sqrt{\ln(x)}}{x} dx = \left[\frac{2}{3}\ln(x)\sqrt{\ln(x)} \right]_1^e = \frac{2}{3} \cdot 1 \cdot 1 - \frac{2}{3} \cdot 0 \cdot 0 = \frac{2}{3}$$

b $\int \cos(x) \sin^3(x) dx = \int \sin^3(x) \cdot \cos(x) dx = \int \sin^3(x) d\sin(x) = \int u^3 du = \frac{1}{4}u^4 + c = \frac{1}{4}\sin^4(x) + c$

$$\int_0^{\frac{1}{2}\pi} \cos(x) \sin^3(x) dx = \left[\frac{1}{4}\sin^4(x) \right]_0^{\frac{1}{2}\pi} = \frac{1}{4} \cdot 1^4 - \frac{1}{4} \cdot 0^4 = \frac{1}{4}$$

$$\begin{aligned} \text{c} \quad \int \frac{6x}{x^2+1} dx &= \int \frac{3}{x^2+1} \cdot 2x dx = \int \frac{3}{x^2+1} d(x^2+1) = \int \frac{3}{u} du = 3 \ln|u| + c = 3 \ln|x^2+1| + c \\ \int_1^3 \frac{6x}{x^2+1} dx &= [3 \ln|x^2+1|]_1^3 = 3 \ln(10) - 3 \ln(2) = 3 \ln(5) \end{aligned}$$

3 $f(x) = 0$ geeft $x\sqrt{9-x^2} = 0$

$$x = 0 \vee x^2 = 9$$

$$x = 0 \vee x = 3 \vee x = -3$$

$$\begin{aligned} F(x) &= \int x\sqrt{9-x^2} dx = \int -\frac{1}{2} \cdot \sqrt{9-x^2} \cdot -2x dx = \int -\frac{1}{2} \sqrt{9-x^2} d(9-x^2) = \int -\frac{1}{2} \sqrt{u} du = \int -\frac{1}{2} u^{\frac{1}{2}} du \\ &= -\frac{1}{3} u^{\frac{3}{2}} + c = -\frac{1}{3} u \sqrt{u} + c = -\frac{1}{3} (9-x^2) \sqrt{9-x^2} + c \end{aligned}$$

$$O(V) = \int_0^3 f(x) dx = \left[-\frac{1}{3} (9-x^2) \sqrt{9-x^2} \right]_0^3 = -\frac{1}{3} \cdot 0 \cdot 0 - \left(-\frac{1}{3} \cdot 9 \cdot 3 \right) = 9$$

4 a $F(x) = \int 4x \sin(2x) dx = 4x \cdot -\frac{1}{2} \cos(2x) - \int 4 \cdot -\frac{1}{2} \cos(2x) dx = -2x \cos(2x) + \int 2 \cos(2x) dx$
 $= -2x \cos(2x) + 2 \cdot \frac{1}{2} \sin(2x) + c = \sin(2x) - 2x \cos(2x) + c$

b $G(x) = \int x^3 \ln(x) dx = \ln(x) \cdot \frac{1}{4} x^4 - \int \frac{1}{4} x^4 \cdot \frac{1}{x} dx = \frac{1}{4} x^4 \ln(x) - \int \frac{1}{4} x^3 dx = \frac{1}{4} x^4 \ln(x) - \frac{1}{16} x^4 + c$

5 a $f(x) = (2x - x^2) e^x$ geeft $f'(x) = (2 - 2x) \cdot e^x + (2x - x^2) \cdot e^x = (2 - x^2) e^x$

$$f'(x) = 0 \text{ geeft } (2 - x^2) e^x = 0$$

$$2 - x^2 = 0$$

$$x^2 = 2$$

$$x = \sqrt{2} \vee x = -\sqrt{2}$$

$$\text{min. is } f(-\sqrt{2}) = (-2\sqrt{2} - 2) e^{-\sqrt{2}}$$

$$\text{max. is } f(\sqrt{2}) = (2\sqrt{2} - 2) e^{\sqrt{2}}$$

b $f(x) = 0$ geeft $2x - x^2 = 0$

$$x(2 - x) = 0$$

$$x = 0 \vee x = 2$$

$$\begin{aligned} F(x) &= \int (2x - x^2) e^x dx = (2x - x^2) \cdot e^x - \int (2 - 2x) \cdot e^x dx = (2x - x^2) e^x + \int (2x - 2) e^x dx \\ &= (2x - x^2) e^x + (2x - 2) \cdot e^x - \int 2 \cdot e^x dx = (2x - x^2) e^x + (2x - 2) e^x - 2 e^x + c \\ &= (-x^2 + 4x - 4) e^x + c \end{aligned}$$

$$O(V) = \int_0^2 f(x) dx = [(-x^2 + 4x - 4) e^x]_0^2 = (-4 + 8 - 4) e^2 - (0 + 0 - 4) = 4$$

Alternatieve uitwerking

$$f(x) = 0 \text{ geeft } 2x - x^2 = 0$$

$$x(2 - x) = 0$$

$$x = 0 \vee x = 2$$

$$F(x) = (ax^2 + bx + c) e^x \text{ geeft } f(x) = (ax^2 + (2a + b)x + b + c) e^x$$

$$\left. \begin{aligned} a &= -1 \\ 2a + b &= 2 \end{aligned} \right\} \begin{aligned} -2 + b &= 2 \\ b &= 4 \end{aligned}$$

$$\left. \begin{aligned} b &= 4 \\ b + c &= 0 \end{aligned} \right\} \begin{aligned} 4 + c &= 0 \\ c &= -4 \end{aligned}$$

Dus $F(x) = (-x^2 + 4x - 4) e^x$ is een primitieve van $f(x) = (2x - x^2) e^x$.

$$O(V) = \int_0^2 f(x) dx = [(-x^2 + 4x - 4) e^x]_0^2 = (-4 + 8 - 4) e^2 - (0 + 0 - 4) = 4$$

6 a $F(x) = \int 4x^2 \cos(2x) dx = 4x^2 \cdot \frac{1}{2} \sin(2x) - \int 8x \cdot \frac{1}{2} \sin(2x) dx$
 $= 2x^2 \sin(2x) - (8x \cdot -\frac{1}{4} \cos(2x) - \int 8 \cdot -\frac{1}{4} \cos(2x) dx)$
 $= 2x^2 \sin(2x) + 2x \cos(2x) - \int 2 \cos(2x) dx = 2x^2 \sin(2x) + 2x \cos(2x) - \sin(2x) + c$

$$\begin{aligned}
\text{b } \int e^x \sin(2x) dx &= e^x \cdot -\frac{1}{2} \cos(2x) - \int e^x \cdot -\frac{1}{2} \cos(2x) dx = -\frac{1}{2} e^x \cos(2x) + \int \frac{1}{2} e^x \cos(2x) dx \\
&= -\frac{1}{2} e^x \cos(2x) + \frac{1}{2} e^x \cdot \frac{1}{2} \sin(2x) - \int \frac{1}{2} e^x \cdot \frac{1}{2} \sin(2x) dx \\
&= \frac{1}{4} e^x \sin(2x) - \frac{1}{2} e^x \cos(2x) - \frac{1}{4} \int e^x \sin(2x) dx \\
\int e^x \sin(2x) dx &= \frac{1}{4} e^x \sin(2x) - \frac{1}{2} e^x \cos(2x) - \frac{1}{4} \int e^x \sin(2x) dx \text{ geeft } \frac{5}{4} \int e^x \sin(2x) dx = \frac{1}{4} e^x \sin(2x) - \frac{1}{2} e^x \cos(2x) \\
&\int e^x \sin(2x) dx = \frac{1}{5} e^x (\sin(2x) - 2 \cos(2x)) \\
g(x) &= e^x \sin(2x) \text{ geeft } G(x) = \frac{1}{5} e^x (\sin(2x) - 2 \cos(2x)) + c
\end{aligned}$$

$$\begin{aligned}
\text{7 a } -\frac{1}{2} \arctan(4x+1) &= \frac{1}{8} \pi \\
\arctan(4x+1) &= -\frac{1}{4} \pi \\
4x+1 &= \tan(-\frac{1}{4} \pi) \\
4x+1 &= -1 \\
4x &= -2 \\
x &= -\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{b } 6 \arcsin(x^2) &= \pi \\
\arcsin(x^2) &= \frac{1}{6} \pi \\
x^2 &= \sin(\frac{1}{6} \pi) \\
x^2 &= \frac{1}{2} \\
x &= \sqrt{\frac{1}{2}} = \frac{1}{2} \sqrt{2} \vee x = -\frac{1}{2} \sqrt{2}
\end{aligned}$$

$$\text{8 a } f(x) = 4 \arctan(\pi x) \text{ geeft } f'(x) = 4 \cdot \frac{1}{(\pi x)^2 + 1} \cdot \pi = \frac{4\pi}{\pi^2 x^2 + 1}$$

$$\text{b } g(x) = 5 \arcsin(2x^3) \text{ geeft } g'(x) = 5 \cdot \frac{1}{\sqrt{1-(2x^3)^2}} \cdot 6x^2 = \frac{30x^2}{\sqrt{1-4x^6}}$$

Bladzijde 215

$$\begin{aligned}
\text{9 a } \int_0^{2\sqrt{3}} \frac{4}{x^2+4} dx &= \int_0^{2\sqrt{3}} \frac{1}{\frac{1}{4}x^2+1} dx = \int_0^{2\sqrt{3}} \frac{1}{(\frac{1}{2}x)^2+1} dx = [2 \arctan(\frac{1}{2}x)]_0^{2\sqrt{3}} = 2 \arctan(\sqrt{3}) - 2 \arctan(0) \\
&= 2 \cdot \frac{1}{3} \pi - 2 \cdot 0 = \frac{2}{3} \pi
\end{aligned}$$

$$\begin{aligned}
\text{b } \int_1^2 \frac{1}{x^2-2x+2} dx &= \int_1^2 \frac{1}{(x-1)^2-1+2} dx = \int_1^2 \frac{1}{(x-1)^2+1} dx = [\arctan(x-1)]_1^2 = \arctan(1) - \arctan(0) \\
&= \frac{1}{4} \pi - 0 = \frac{1}{4} \pi
\end{aligned}$$

$$\begin{aligned}
\text{c } \int \frac{\arctan^2(x)}{x^2+1} dx &= \int \arctan^2(x) \cdot \frac{1}{x^2+1} dx = \int \arctan^2(x) d \arctan(x) = \int u^2 du = \frac{1}{3} u^3 + c \\
&= \frac{1}{3} \arctan^3(x) + c
\end{aligned}$$

$$\int_0^1 \frac{\arctan^2(x)}{x^2+1} dx = \left[\frac{1}{3} \arctan^3(x) \right]_0^1 = \frac{1}{3} \arctan^3(1) - \frac{1}{3} \arctan^3(0) = \frac{1}{3} \cdot \left(\frac{1}{4} \pi \right)^3 - \frac{1}{3} \cdot 0^3 = \frac{1}{192} \pi^3$$

$$\begin{aligned}
\text{d } \int \frac{1}{\sqrt{1-4x^2}} dx &= \int \frac{1}{2} \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{1-(2x)^2}} d(2x) = \int \frac{1}{2} \cdot \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \arcsin(u) + c \\
&= \frac{1}{2} \arcsin(2x) + c
\end{aligned}$$

$$\int_{-\frac{1}{4}}^{\frac{1}{4}} \frac{1}{\sqrt{1-4x^2}} dx = \left[\frac{1}{2} \arcsin(2x) \right]_{-\frac{1}{4}}^{\frac{1}{4}} = \frac{1}{2} \arcsin\left(\frac{1}{2}\right) - \frac{1}{2} \arcsin\left(-\frac{1}{2}\right) = \frac{1}{2} \cdot \frac{1}{6} \pi - \frac{1}{2} \cdot -\frac{1}{6} \pi = \frac{1}{6} \pi$$

$$\begin{aligned}
\text{e } \int \frac{5x^4}{\sqrt{1-x^{10}}} dx &= \int \frac{1}{\sqrt{1-(x^5)^2}} \cdot 5x^4 dx = \int \frac{1}{\sqrt{1-(x^5)^2}} dx^5 = \int \frac{1}{\sqrt{1-u^2}} du = \arcsin(u) + c \\
&= \arcsin(x^5) + c
\end{aligned}$$

$$\int_{-1}^1 \frac{5x^4}{\sqrt{1-x^{10}}} dx = [\arcsin(x^5)]_{-1}^1 = \arcsin(1) - \arcsin(-1) = \frac{1}{2} \pi - -\frac{1}{2} \pi = \pi$$

$$\begin{aligned}
\text{f } \int \frac{e^x}{\sqrt{1-e^{2x}}} dx &= \int \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x dx = \int \frac{1}{\sqrt{1-(e^x)^2}} de^x = \int \frac{1}{\sqrt{1-u^2}} du = \arcsin(u) + c \\
&= \arcsin(e^x) + c
\end{aligned}$$

$$\int_{-\ln(2)}^{\ln(\frac{1}{2}\sqrt{3})} \frac{e^x}{\sqrt{1-e^{2x}}} dx = [\arcsin(e^x)]_{-\ln(2)}^{\ln(\frac{1}{2}\sqrt{3})} = \arcsin\left(\frac{1}{2}\sqrt{3}\right) - \arcsin\left(\frac{1}{2}\right) = \frac{1}{3} \pi - \frac{1}{6} \pi = \frac{1}{6} \pi$$

10 a $f(x) = \frac{1}{2}$ geeft $\frac{1}{x^2 - 4x + 5} = \frac{1}{2}$
 $x^2 - 4x + 5 = 2$
 $x^2 - 4x + 3 = 0$
 $(x-1)(x-3) = 0$
 $x = 1 \vee x = 3$

$$F(x) = \int \frac{1}{x^2 - 4x + 5} dx = \int \frac{1}{(x-2)^2 - 4 + 5} dx = \int \frac{1}{(x-2)^2 + 1} dx = \arctan(x-2) + c$$

$$O(V) = \int_1^3 f(x) dx - 2 \cdot \frac{1}{2} = [\arctan(x-2)]_1^3 - 1 = \arctan(1) - \arctan(-1) - 1 = \frac{1}{4}\pi - -\frac{1}{4}\pi - 1 = \frac{1}{2}\pi - 1$$

b $O(W_p) = \int_2^p f(x) dx = [\arctan(x-2)]_2^p = \arctan(p-2) - \arctan(0) = \arctan(p-2)$

$$O(W_p) = \frac{1}{3}\pi \text{ geeft } \arctan(p-2) = \frac{1}{3}\pi$$

$$p-2 = \tan\left(\frac{1}{3}\pi\right)$$

$$p-2 = \sqrt{3}$$

$$p = 2 + \sqrt{3}$$

11 a $f(x) = \frac{3x+4}{x+4} = \frac{3(x+4)-12+4}{x+4} = 3 - \frac{8}{x+4}$

$$\text{Dus } F(x) = 3x - 8 \ln|x+4| + c.$$

b $g(x) = \frac{x^3 - x + 1}{x+1} = \frac{x^2(x+1) - x^2 - x + 1}{x+1} = x^2 + \frac{-x^2 - x + 1}{x+1} = x^2 + \frac{-x(x+1) + 1}{x+1} = x^2 - x + \frac{1}{x+1}$

$$\text{Dus } G(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + \ln|x+1| + c.$$

12 a $f(x) = \frac{x^2 - 3x}{x+1}$ geeft

$$f'(x) = \frac{(x+1) \cdot (2x-3) - (x^2-3x) \cdot 1}{(x+1)^2} = \frac{2x^2 - 3x + 2x - 3 - x^2 + 3x}{(x+1)^2} = \frac{x^2 + 2x - 3}{(x+1)^2}$$

$$f'(x) = 0 \text{ geeft } x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1 \vee x = -3$$

$$x = -3 \text{ voldoet niet, dus } x = 1.$$

$$\text{min. is } f(1) = -1$$

b $f(x) = 0$ geeft $x^2 - 3x = 0$

$$x(x-3) = 0$$

$$x = 0 \vee x = 3$$

$$f(x) = \frac{x^2 - 3x}{x+1} = \frac{x(x+1) - x - 3x}{x+1} = x - \frac{4x}{x+1} = x - \frac{4(x+1) - 4}{x+1} = x - 4 + \frac{4}{x+1}$$

$$O(V) = -\int_0^3 f(x) dx = -\int_0^3 \left(x - 4 + \frac{4}{x+1}\right) dx = -\left[\frac{1}{2}x^2 - 4x + 4 \ln|x+1|\right]_0^3$$

$$= -\left(\frac{1}{2} \cdot 3^2 - 4 \cdot 3 + 4 \ln(4) - (0 - 0 + 4 \ln(1))\right) = 7\frac{1}{2} - 4 \ln(4)$$

13 a $f(x) = \frac{2x+3}{x^2-6x+10} = \frac{2x-6+9}{x^2-6x+10} = \frac{2x-6}{x^2-6x+10} + \frac{9}{x^2-6x+10}$

$$\int \frac{2x-6}{x^2-6x+10} dx = \int \frac{1}{x^2-6x+10} \cdot (2x-6) dx = \int \frac{1}{x^2-6x+10} d(x^2-6x+10) = \int \frac{1}{u} du = \ln(u) + c_1$$

$$= \ln(x^2-6x+10) + c_1$$

$$\int \frac{9}{x^2-6x+10} dx = \int \frac{9}{(x-3)^2+1} dx = 9 \arctan(x-3) + c_2$$

$$\text{Dus } F(x) = \ln(x^2-6x+10) + 9 \arctan(x-3) + c.$$

$$\begin{aligned} \text{b } g(x) &= \frac{x^2 + 4x}{x^2 - 4x + 4} = \frac{x^2 - 4x + 4 + 8x - 4}{x^2 - 4x + 4} = 1 + \frac{8x - 4}{(x-2)^2} = 1 + \frac{8(x-2) + 16 - 4}{(x-2)^2} \\ &= 1 + \frac{8}{x-2} + \frac{12}{(x-2)^2} = 1 + \frac{8}{x-2} + 12(x-2)^{-2} \end{aligned}$$

$$\text{Dus } G(x) = x + 8 \ln|x-2| - 12(x-2)^{-1} + c = x + 8 \ln|x-2| - \frac{12}{x-2} + c.$$

$$\begin{aligned} \text{c } h(x) &= \frac{6}{x^2 - 1} = \frac{6}{(x-1)(x+1)} = \frac{a}{x-1} + \frac{b}{x+1} = \frac{a(x+1) + b(x-1)}{(x-1)(x+1)} \\ &= \frac{ax + a + bx - b}{(x-1)(x+1)} = \frac{(a+b)x + a-b}{(x-1)(x+1)} \end{aligned}$$

$$\begin{cases} a+b=0 \\ a-b=6 \end{cases} + \frac{2a}{2a} = 6 +$$

$$a=3$$

$$a=3 \text{ geeft } 3+b=0$$

$$b=-3$$

$$\text{Dus } h(x) = \frac{3}{x-1} - \frac{3}{x+1}.$$

$$\text{Dus } H(x) = 3 \ln|x-1| - 3 \ln|x+1| + c.$$

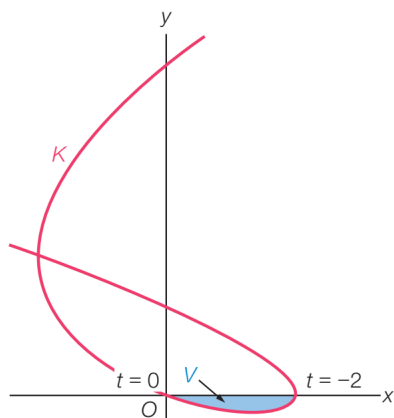
14 a $y(t) = 0$ geeft $t^2 + 2t = 0$

$$t(t+2) = 0$$

$$t = 0 \vee t = -2$$

$$t = 0 \text{ geeft } (0, 0)$$

$$t = -2 \text{ geeft } (4, 0)$$



$$O(V) = - \int_{t=0}^{t=-2} y \, dx = - \int_{t=0}^{t=-2} (t^2 + 2t) \, d\left(\frac{1}{4}t^3 - 3t\right) = - \int_0^{-2} (t^2 + 2t) \cdot \left(\frac{3}{4}t^2 - 3\right) dt$$

$$= - \int_0^{-2} \left(\frac{3}{4}t^4 - 3t^2 + 1\frac{1}{2}t^3 - 6t\right) dt = - \left[\frac{3}{20}t^5 - t^3 + \frac{3}{8}t^4 - 3t^2\right]_0^{-2}$$

$$= - \left(\frac{3}{20} \cdot (-2)^5 - (-2)^3 + \frac{3}{8} \cdot (-2)^4 - 3 \cdot (-2)^2 - 0\right) = 2\frac{4}{5}$$

$$\text{b } I(L) = \pi \int_{t=0}^{t=-2} y^2 \, dx = \pi \int_{t=0}^{t=-2} (t^2 + 2t)^2 \, d\left(\frac{1}{4}t^3 - 3t\right) = \pi \int_0^{-2} (t^4 + 4t^3 + 4t^2) \cdot \left(\frac{3}{4}t^2 - 3\right) dt$$

$$= \pi \int_0^{-2} \left(\frac{3}{4}t^6 - 3t^4 + 3t^5 - 12t^3 + 3t^4 - 12t^2\right) dt = \pi \int_0^{-2} \left(\frac{3}{4}t^6 + 3t^5 - 12t^3 - 12t^2\right) dt$$

$$= \pi \left[\frac{3}{28}t^7 + \frac{1}{2}t^6 - 3t^4 - 4t^3\right]_0^{-2} = \pi \left(\frac{3}{28} \cdot (-2)^7 + \frac{1}{2} \cdot (-2)^6 - 3 \cdot (-2)^4 - 4 \cdot (-2)^3 - 0\right) = 2\frac{2}{7}\pi$$

c $y(t) = 8$ geeft $t^2 + 2t = 8$

$$t^2 + 2t - 8 = 0$$

$$(t-2)(t+4) = 0$$

$$t = 2 \vee t = -4$$

$$\int_{t=-4}^{t=2} y \, dx = \left[\frac{3}{20}t^5 - t^3 + \frac{3}{8}t^4 - 3t^2\right]_{-4}^2$$

$$= \frac{3}{20} \cdot 2^5 - 2^3 + \frac{3}{8} \cdot 2^4 - 3 \cdot 2^2 - \left(\frac{3}{20} \cdot (-4)^5 - (-4)^3 + \frac{3}{8} \cdot (-4)^4 - 3 \cdot (-4)^2\right) = 32\frac{2}{5}$$

$$\text{Dus } O(W) = 32\frac{2}{5}.$$

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$$\mathbf{a} \quad f(x) = \frac{x+2}{x^2-4x+8} = \frac{\frac{1}{2}(2x-4) + 2 + 2}{x^2-4x+8} = \frac{\frac{1}{2}(2x-4)}{x^2-4x+8} + \frac{4}{x^2-4x+8}$$

$$\int \frac{\frac{1}{2}(2x-4)}{x^2-4x+8} dx = \int \frac{1}{2} \cdot \frac{1}{x^2-4x+8} \cdot (2x-4) dx = \int \frac{1}{2} \cdot \frac{1}{x^2-4x+8} d(x^2-4x+8) = \int \frac{1}{2} \cdot \frac{1}{u} du$$

$$= \frac{1}{2} \ln(u) + c_1 = \frac{1}{2} \ln(x^2-4x+8) + c_1$$

$$\int \frac{4}{x^2-4x+8} dx = \int \frac{4}{(x-2)^2+4} dx = \int \frac{1}{\frac{(x-2)^2}{4}+1} dx = \int \frac{1}{\left(\frac{x-2}{2}\right)^2+1} dx = \int \frac{1}{\left(\frac{1}{2}x-1\right)^2+1} dx$$

$$= 2 \arctan\left(\frac{1}{2}x-1\right) + c_2$$

Dus $F(x) = \frac{1}{2} \ln(x^2-4x+8) + 2 \arctan\left(\frac{1}{2}x-1\right) + c$.

$$\mathbf{b} \quad F(x) = \int x \tan(x^2) dx = \int \frac{1}{2} \tan(x^2) \cdot 2x dx = \int \frac{1}{2} \tan(x^2) dx^2 = \int \frac{1}{2} \tan(u) du = -\frac{1}{2} \ln|\cos(u)| + c$$

$$= -\frac{1}{2} \ln|\cos(x^2)| + c$$

$$\mathbf{c} \quad F(x) = \int \cos(x) \cdot \sqrt[3]{1+\sin(x)} dx = \int \sqrt[3]{1+\sin(x)} \cdot \cos(x) dx = \int \sqrt[3]{1+\sin(x)} d\sin(x) = \int \sqrt[3]{1+u} du$$

$$= \int (1+u)^{\frac{1}{3}} du = \frac{3}{4}(1+u)^{\frac{4}{3}} + c = \frac{3}{4}(1+u) \cdot \sqrt[3]{1+u} + c = \frac{3}{4}(1+\sin(x)) \cdot \sqrt[3]{1+\sin(x)} + c$$

$$\mathbf{d} \quad f(x) = \frac{2x+1}{\sqrt{16-x^2}} = \frac{2x}{\sqrt{16-x^2}} + \frac{1}{\sqrt{16-x^2}}$$

$$\int \frac{2x}{\sqrt{16-x^2}} dx = \int -\frac{1}{\sqrt{16-x^2}} \cdot -2x dx = \int -\frac{1}{\sqrt{16-x^2}} d(16-x^2) = \int -\frac{1}{\sqrt{u}} du = -2\sqrt{u} + c = -2\sqrt{16-x^2} + c$$

$$\int \frac{1}{\sqrt{16-x^2}} dx = \int \frac{1}{4\sqrt{1-\frac{1}{16}x^2}} dx = \int \frac{\frac{1}{4}}{\sqrt{1-(\frac{1}{4}x)^2}} dx = \frac{1}{4} \cdot 4 \arcsin\left(\frac{1}{4}x\right) + c = \arcsin\left(\frac{1}{4}x\right) + c$$

Dus $F(x) = \arcsin\left(\frac{1}{4}x\right) - 2\sqrt{16-x^2} + c$.

$$\mathbf{e} \quad F(x) = \int (2x+4) \cos(2x) dx = (2x+4) \cdot \frac{1}{2} \sin(2x) - \int 2 \cdot \frac{1}{2} \sin(2x) dx = (x+2) \sin(2x) - \int \sin(2x) dx$$

$$= (x+2) \sin(2x) + \frac{1}{2} \cos(2x) + c$$

$$\mathbf{f} \quad F(x) = \int \frac{2x+4}{\sqrt{-x^2-4x-3}} dx = \int -\frac{1}{\sqrt{-x^2-4x-3}} \cdot (-2x-4) dx = \int -\frac{1}{\sqrt{-x^2-4x-3}} d(-x^2-4x-3)$$

$$= \int -\frac{1}{\sqrt{u}} du = -2\sqrt{u} + c = -2\sqrt{-x^2-4x-3} + c$$

$$\mathbf{g} \quad f(x) = \frac{x^3}{x^2+6x+8} = \frac{x(x^2+6x+8)-6x^2-8x}{x^2+6x+8} = x + \frac{-6x^2-8x}{x^2+6x+8}$$

$$= x + \frac{-6(x^2+6x+8)+36x+48-8x}{x^2+6x+8} = x - 6 + \frac{28x+48}{x^2+6x+8}$$

$$= x - 6 + \frac{28x+48}{(x+2)(x+4)} = x - 6 + \frac{a}{x+2} + \frac{b}{x+4} = x - 6 + \frac{a(x+4)+b(x+2)}{(x+2)(x+4)}$$

$$= x - 6 + \frac{ax+4a+bx+2b}{(x+2)(x+4)} = x - 6 + \frac{(a+b)x+4a+2b}{(x+2)(x+4)}$$

$$\begin{cases} a+b=28 \\ 4a+2b=48 \end{cases} \begin{matrix} |2 \\ |1 \end{matrix} \text{ geeft } \begin{cases} 2a+2b=56 \\ 4a+2b=48 \end{cases}$$

$$\begin{matrix} -2a=8 \\ a=-4 \end{matrix}$$

$a = -4$ geeft $-4 + b = 28$
 $b = 32$

Dus $f(x) = x - 6 - \frac{4}{x+2} + \frac{32}{x+4}$.

Dus $F(x) = \frac{1}{2}x^2 - 6x - 4 \ln|x+2| + 32 \ln|x+4| + c$.

$$\mathbf{h} \quad f(x) = \frac{x+1}{(2x-1)^2} = \frac{\frac{1}{2}(2x-1) + \frac{1}{2} + 1}{(2x-1)^2} = \frac{1}{2(2x-1)} + \frac{\frac{3}{2}}{(2x-1)^2} = \frac{1}{4x-2} + \frac{3}{2(2x-1)^2}$$

Dus $F(x) = \frac{1}{4} \ln|4x-2| - \frac{3}{4(2x-1)} + c = \frac{1}{4} \ln|4x-2| - \frac{3}{8x-4} + c$.

- 2 a** $f(x) = 4 \sin^2(x) \cos(x)$ geeft $f'(x) = 8 \sin(x) \cdot \cos(x) \cdot \cos(x) + 4 \sin^2(x) \cdot -\sin(x) = 8 \sin(x) \cos^2(x) - 4 \sin^3(x)$
 $f(x) = 0$ geeft $4 \sin^2(x) \cos(x) = 0$
 $\sin(x) = 0 \vee \cos(x) = 0$
 $x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot \pi$
 x in $[0, 2\pi]$ geeft $x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{1}{2}\pi \vee x = \frac{3}{2}\pi$
 $f'(0) = f'(2\pi) = 8 \cdot 0 \cdot 1^2 - 4 \cdot 0 = 0$
 $f'(\pi) = 8 \cdot 0 \cdot (-1)^2 - 4 \cdot 0 = 0$
Dus de grafiek raakt de x -as in $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.
b $F(x) = \int 4 \sin^2(x) \cos(x) dx = \int 4 \sin^2(x) d \sin(x) = \int 4u^2 du = \frac{4}{3}u^3 + c = \frac{4}{3}\sin^3(x) + c$
 $O(V) = \int_0^{\frac{1}{2}\pi} f(x) dx = \left[\frac{4}{3}\sin^3(x) \right]_0^{\frac{1}{2}\pi} = \frac{4}{3} \cdot 1 - \frac{4}{3} \cdot 0 = 1\frac{1}{3}$

- 3 a** $f(x) = \frac{2}{3}$ geeft $\frac{2x}{x^2 - 4} = \frac{2}{3}$
 $2x^2 - 8 = 6x$
 $2x^2 - 6x - 8 = 0$
 $x^2 - 3x - 4 = 0$
 $(x + 1)(x - 4) = 0$
 $x = -1 \vee x = 4$
 $F(x) = \int \frac{2x}{x^2 - 4} dx = \int \frac{1}{x^2 - 4} \cdot 2x dx = \int \frac{1}{x^2 - 4} \cdot dx^2 = \int \frac{1}{u - 4} du = \ln|u - 4| + c = \ln|x^2 - 4| + c$
 $O(V) = \int_{-1}^0 \left(\frac{2}{3} - \frac{2x}{x^2 - 4} \right) dx = \left[\frac{2}{3}x - \ln|x^2 - 4| \right]_{-1}^0 = 0 - \ln(4) - \left(-\frac{2}{3} - \ln(3) \right) = \frac{2}{3} + \ln\left(\frac{3}{4}\right)$
b $O(W) = \int_p^{p+4} \frac{2x}{x^2 - 4} dx = [\ln|x^2 - 4|]_p^{p+4} = \ln((p+4)^2 - 4) - \ln(p^2 - 4)$
 $= \ln(p^2 + 8p + 12) - \ln(p^2 - 4) = \ln\left(\frac{p^2 + 8p + 12}{p^2 - 4}\right)$
 $O(W) = \ln(2)$ geeft $\frac{p^2 + 8p + 12}{p^2 - 4} = 2$
 $p^2 + 8p + 12 = 2p^2 - 8$
 $-p^2 + 8p + 20 = 0$
 $p^2 - 8p - 20 = 0$
 $(p + 2)(p - 10) = 0$
 $p = -2 \vee p = 10$
 $p = -2$ voldoet niet, dus $p = 10$.

- 4 a** $f(x) = x + e^{-x}$ geeft $f'(x) = 1 - e^{-x}$
 $f'(x) = 0$ geeft $1 - e^{-x} = 0$
 $e^{-x} = 1$
 $-x = 0$
 $x = 0$
min. is $f(0) = 1$
 $B_f = [1, \rightarrow)$
b $O(V_p) = \int_{-p}^p (x + e^{-x}) dx = \left[\frac{1}{2}x^2 - e^{-x} \right]_{-p}^p = \frac{1}{2}p^2 - e^{-p} - \left(\frac{1}{2}p^2 - e^p \right) = e^p - e^{-p}$
 $O(V_p) = 6$ geeft $e^p - e^{-p} = 6$
 $e^p - 6 - e^{-p} = 0$
 $(e^p)^2 - 6e^p - 1 = 0$
Stel $e^p = u$.
 $u^2 - 6u - 1 = 0$
 $D = (-6)^2 - 4 \cdot 1 \cdot -1 = 40$, dus $\sqrt{D} = 2\sqrt{10}$
 $u = \frac{6 + 2\sqrt{10}}{2} \vee u = \frac{6 - 2\sqrt{10}}{2}$
 $u = 3 + \sqrt{10} \vee u = 3 - \sqrt{10}$
 $e^p = 3 + \sqrt{10} \vee e^p = 3 - \sqrt{10}$
 $p = \ln(3 + \sqrt{10})$

$$\begin{aligned} \text{c } I(L) &= \pi \int_{-1}^0 (f(x))^2 dx = \pi \int_{-1}^0 (x + e^{-x})^2 dx = \pi \int_{-1}^0 (x^2 + 2xe^{-x} + e^{-2x}) dx \\ \int 2xe^{-x} dx &= 2x \cdot -e^{-x} - \int 2 \cdot -e^{-x} dx = -2xe^{-x} + \int 2e^{-x} dx = -2xe^{-x} - 2e^{-x} + c \\ I(L) &= \pi \left[\frac{1}{3}x^3 - 2xe^{-x} - 2e^{-x} - \frac{1}{2}e^{-2x} \right]_{-1}^0 = \pi(0 - 0 - 2 - \frac{1}{2} - (-\frac{1}{3} + 2e - 2e - \frac{1}{2}e^2)) = (\frac{1}{2}e^2 - 2\frac{1}{6})\pi \end{aligned}$$

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5

$$\begin{aligned} \text{a } f_{-1}(x) &= \ln^2(x) - \ln(x) \\ f_{-1}(x) &= 0 \text{ geeft } \ln^2(x) - \ln(x) = 0 \\ \ln(x)(\ln(x) - 1) &= 0 \\ \ln(x) &= 0 \vee \ln(x) = 1 \\ x &= 1 \vee x = e \end{aligned}$$

$$\int \ln^2(x) dx = \int \ln^2(x) \cdot 1 dx = \ln^2(x) \cdot x - \int 2 \ln(x) \cdot \frac{1}{x} \cdot x dx = x \ln^2(x) - \int 2 \ln(x) dx$$

$$= x \ln^2(x) - 2(x \ln(x) - x) + c = x \ln^2(x) - 2x \ln(x) + 2x + c$$

$$\text{Dus } F_{-1}(x) = x \ln^2(x) - 2x \ln(x) + 2x - (x \ln(x) - x) + c = x \ln^2(x) - 3x \ln(x) + 3x + c.$$

$$O(V_{-1}) = - \int_1^e (\ln^2(x) - \ln(x)) dx = - [x \ln^2(x) - 3x \ln(x) + 3x]_1^e = -(e - 3e + 3e - (0 - 0 + 3)) = 3 - e$$

$$\text{b } f_p(x) = \ln^2(x) + p \ln(x) \text{ geeft } f'_p(x) = 2 \ln(x) \cdot \frac{1}{x} + p \cdot \frac{1}{x} = \frac{2 \ln(x) + p}{x}$$

$$f'_p(x) = 0 \text{ geeft } 2 \ln(x) + p = 0$$

$$2 \ln(x) = -p$$

$$\ln(x) = -\frac{1}{2}p$$

$$\text{Invullen van } \ln(x) = -\frac{1}{2}p \text{ in } f_p(x) = \ln^2(x) + p \ln(x) \text{ geeft } f_p(x) = (-\frac{1}{2}p)^2 + p \cdot -\frac{1}{2}p = \frac{1}{4}p^2 - \frac{1}{2}p^2 = -\frac{1}{4}p^2.$$

Omdat $-\frac{1}{4}p^2 < 0$ voor elke $p \neq 0$ ligt de top van de grafiek van f_p voor elke $p \neq 0$ onder de x -as.

$$\begin{aligned} \text{c } f_p(x) &= 0 \text{ geeft } \ln^2(x) + p \ln(x) = 0 \\ \ln(x)(\ln(x) + p) &= 0 \\ \ln(x) &= 0 \vee \ln(x) = -p \\ x &= 1 \vee x = e^{-p} \end{aligned}$$

$$\begin{aligned} f_p(x) &= \ln^2(x) + p \ln(x) \text{ geeft } F_p(x) = x \ln^2(x) - 2x \ln(x) + 2x + p(x \ln(x) - x) + c \\ &= x \ln^2(x) - 2x \ln(x) + 2x + px \ln(x) - px + c \end{aligned}$$

Als $p < 0$ geldt $e^{-p} > 1$.

$$\begin{aligned} O(V_p) &= - \int_1^{e^{-p}} (\ln^2(x) + p \ln(x)) dx = - [x \ln^2(x) - 2x \ln(x) + 2x + px \ln(x) - px]_1^{e^{-p}} \\ &= -(e^{-p} \cdot (-p)^2 - 2e^{-p} \cdot -p + 2e^{-p} + p e^{-p} \cdot -p - p e^{-p} - (0 - 0 + 2 + 0 - p)) \\ &= -(p + 2)e^{-p} - p + 2 \end{aligned}$$

$$O(V_p) = 4 \text{ geeft } -(p + 2)e^{-p} - p + 2 = 4$$

$$-(p + 2)e^{-p} = p + 2$$

$$p + 2 = 0 \vee -e^{-p} = 1$$

$$p = -2 \vee e^{-p} = -1$$

$$p = -2$$

6

$$\text{a } f(x) = \frac{6x}{x^4 + 1} \text{ geeft } f'(x) = \frac{(x^4 + 1) \cdot 6 - 6x \cdot 4x^3}{(x^4 + 1)^2} = \frac{6x^4 + 6 - 24x^4}{(x^4 + 1)^2} = \frac{6 - 18x^4}{(x^4 + 1)^2}$$

$$f'(x) = 0 \text{ geeft } 6 - 18x^4 = 0$$

$$18x^4 = 6$$

$$x^4 = \frac{1}{3}$$

$$x = \sqrt[4]{\frac{1}{3}} \vee x = -\sqrt[4]{\frac{1}{3}}$$

$$\text{max. is } f(\sqrt[4]{\frac{1}{3}}) = 4\frac{1}{2} \cdot \sqrt[4]{\frac{1}{3}}$$

$$\begin{aligned} \text{b } f'(x) &= \frac{6 - 18x^4}{(x^4 + 1)^2} \text{ geeft} \\ f''(x) &= \frac{(x^4 + 1)^2 \cdot -72x^3 - (6 - 18x^4) \cdot 2(x^4 + 1) \cdot 4x^3}{(x^4 + 1)^4} = \frac{-72x^3(x^4 + 1) - 8x^3(6 - 18x^4)}{(x^4 + 1)^3} \\ &= \frac{-72x^7 - 72x^3 - 48x^3 + 144x^7}{(x^4 + 1)^3} = \frac{72x^7 - 120x^3}{(x^4 + 1)^3} \end{aligned}$$

$$\begin{aligned} f''(x) = 0 \text{ geeft } 72x^7 - 120x^3 &= 0 \\ 24x^3(3x^4 - 5) &= 0 \\ x = 0 \vee 3x^4 &= 5 \\ x = 0 \vee x^4 &= \frac{5}{3} \\ x = 0 \vee x &= \sqrt[4]{\frac{5}{3}} \vee x = -\sqrt[4]{\frac{5}{3}} \end{aligned}$$

$$f'(-\sqrt[4]{\frac{5}{3}}) = -3\frac{3}{8}, f'(\sqrt[4]{\frac{5}{3}}) = -3\frac{3}{8} \text{ en } f'(0) = 6$$

Dus de helling van de grafiek van f is maximaal 6.

$$\text{c } f(-x) = \frac{6 \cdot -x}{(-x)^4 + 1} = -\frac{6x}{x^4 + 1} = -f(x)$$

Dus de grafiek van f is symmetrisch in de oorsprong.

$$\begin{aligned} \text{d } f(x) = 3x \text{ geeft } \frac{6x}{x^4 + 1} &= 3x \\ 3x^5 + 3x &= 6x \\ 3x^5 - 3x &= 0 \\ 3x(x^4 - 1) &= 0 \\ x = 0 \vee x^4 &= 1 \\ x = 0 \vee x &= 1 \vee x = -1 \end{aligned}$$

$$\begin{aligned} F(x) &= \int \frac{6x}{x^4 + 1} dx = \int \frac{3}{(x^2)^2 + 1} \cdot 2x dx = \int \frac{3}{(x^2)^2 + 1} dx^2 = \int \frac{3}{u^2 + 1} du = 3 \arctan(u) + c \\ &= 3 \arctan(x^2) + c \end{aligned}$$

$$\begin{aligned} \int_0^1 (f(x) - 3x) dx &= \left[3 \arctan(x^2) - \frac{1}{2}x^2 \right]_0^1 = 3 \arctan(1) - \frac{1}{2} - (3 \arctan(0) - 0) = 3 \cdot \frac{1}{4}\pi - \frac{1}{2} - (0 - 0) \\ &= \frac{3}{4}\pi - \frac{1}{2} \end{aligned}$$

Dus de oppervlakte van de twee vlakdelen samen is $2(\frac{3}{4}\pi - \frac{1}{2}) = \frac{1}{2}\pi - 3$.

7

$$\begin{aligned} \text{a } f(x) &= \frac{x^2 + 2}{2x^2 - 8x} \text{ geeft} \\ f'(x) &= \frac{(2x^2 - 8x) \cdot 2x - (x^2 + 2) \cdot (4x - 8)}{(2x^2 - 8x)^2} = \frac{4x^3 - 16x^2 - 4x^3 + 8x^2 - 8x + 16}{(2x^2 - 8x)^2} = \frac{-8x^2 - 8x + 16}{(2x^2 - 8x)^2} \end{aligned}$$

$$\begin{aligned} f'(x) = 0 \text{ geeft } -8x^2 - 8x + 16 &= 0 \\ x^2 + x - 2 &= 0 \\ (x - 1)(x + 2) &= 0 \\ x = 1 \vee x &= -2 \end{aligned}$$

$$\text{min. is } f(-2) = \frac{1}{4}$$

$$\text{max. is } f(1) = -\frac{1}{2}$$

$$\begin{aligned} \text{b } f(x) &= \frac{1}{2} \text{ geeft } \frac{x^2 + 2}{2x^2 - 8x} = \frac{1}{2} \\ 2x^2 + 4 &= 2x^2 - 8x \\ 8x &= -4 \\ x &= -\frac{1}{2} \end{aligned}$$

$$2x^2 - 8x = 0$$

$$2x(x - 4) = 0$$

$$x = 0 \vee x = 4$$

Dus de lijnen $x = 0$ en $x = 4$ zijn de verticale asymptoten van de grafiek van f .

$$f(x) \geq \frac{1}{2} \text{ geeft } -\frac{1}{2} \leq x < 0 \vee x > 4$$

$$\begin{aligned} \text{c } f(x) &= \frac{x^2 + 2}{2x^2 - 8x} = \frac{\frac{1}{2}(2x^2 - 8x) + 4x + 2}{2x^2 - 8x} = \frac{1}{2} + \frac{4x + 2}{2x^2 - 8x} = \frac{1}{2} + \frac{2x + 1}{x^2 - 4x} = \frac{1}{2} + \frac{2x + 1}{x(x - 4)} \\ &= \frac{1}{2} + \frac{a}{x} + \frac{b}{x - 4} = \frac{1}{2} + \frac{a(x - 4) + bx}{x(x - 4)} = \frac{1}{2} + \frac{ax - 4a + bx}{x(x - 4)} = \frac{1}{2} + \frac{(a + b)x - 4a}{x(x - 4)} \end{aligned}$$

Uit $a + b = 2$ en $-4a = 1$ volgt $a = -\frac{1}{4}$ en $b = 2\frac{1}{4}$.

$$\text{Dus } f(x) = \frac{1}{2} - \frac{1}{4x} + \frac{2\frac{1}{4}}{x - 4}.$$

$$\text{Dus } F(x) = \frac{1}{2}x - \frac{1}{4}\ln|x| + 2\frac{1}{4}\ln|x - 4| + c.$$

$$\begin{aligned} O(V) &= -\int_1^3 f(x) dx = -\left[\frac{1}{2}x - \frac{1}{4}\ln|x| + 2\frac{1}{4}\ln|x - 4|\right]_1^3 = -(1\frac{1}{2} - \frac{1}{4}\ln(3) + 2\frac{1}{4}\ln(1) - (\frac{1}{2} - \frac{1}{4}\ln(1) + 2\frac{1}{4}\ln(3))) \\ &= 2\frac{1}{2}\ln(3) - 1 \end{aligned}$$

8 a $x'(t) = -t^2 + 1$
 $x'(t) = 0$ geeft $t^2 = 1$
 $t = 1 \vee t = -1$

$$x(1) = \frac{2}{3} \text{ en } y(1) = 0$$

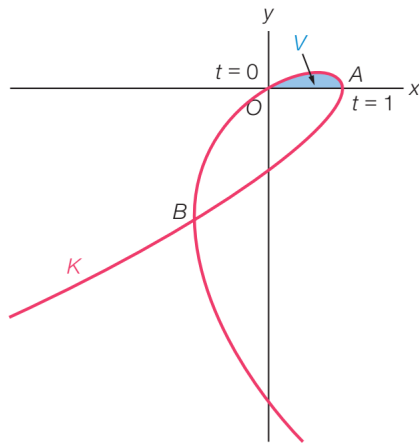
$$x(-1) = -\frac{2}{3} \text{ en } y(-1) = -4$$

$$\text{Dus } B(-\frac{2}{3}, -4).$$

b $y(t) = 0$ geeft $-2t^2 + 2t = 0$
 $-2t(t - 1) = 0$
 $t = 0 \vee t = 1$

$$t = 0 \text{ geeft } (0, 0)$$

$$t = 1 \text{ geeft } (\frac{2}{3}, 0)$$



$$\begin{aligned} I(L) &= \pi \int_{t=0}^{t=1} y^2 dx = \pi \int_{t=0}^{t=1} (-2t^2 + 2t)^2 d(-\frac{1}{3}t^3 + t) = \pi \int_0^1 (-2t^2 + 2t)^2 \cdot (-t^2 + 1) dt \\ &= \pi \int_0^1 (4t^4 - 8t^3 + 4t^2)(-t^2 + 1) dt = \pi \int_0^1 (-4t^6 + 4t^4 + 8t^5 - 8t^3 - 4t^4 + 4t^2) dt \\ &= \pi \int_0^1 (-4t^6 + 8t^5 - 8t^3 + 4t^2) dt = \pi \left[-\frac{4}{7}t^7 + \frac{4}{3}t^6 - 2t^4 + \frac{4}{3}t^3\right]_0^1 \\ &= \pi(-\frac{4}{7} + \frac{4}{3} - 2 + \frac{4}{3} - 0) = \frac{2}{21}\pi \end{aligned}$$

c $y(t) = -4$ geeft $-2t^2 + 2t = -4$
 $-2t^2 + 2t + 4 = 0$
 $t^2 - t - 2 = 0$
 $(t + 1)(t - 2) = 0$
 $t = -1 \vee t = 2$

$$\begin{aligned} \int_{t=-1}^{t=2} y dx &= \int_{t=-1}^{t=2} (-2t^2 + 2t) d(-\frac{1}{3}t^3 + t) = \int_{-1}^2 (-2t^2 + 2t) \cdot (-t^2 + 1) dt = \int_{-1}^2 (2t^4 - 2t^2 - 2t^3 + 2t) dt \\ &= \left[\frac{2}{5}t^5 - \frac{2}{3}t^3 - \frac{1}{2}t^4 + t^2\right]_{-1}^2 = \frac{2}{5} \cdot 2^5 - \frac{2}{3} \cdot 2^3 - \frac{1}{2} \cdot 2^4 + 2^2 - (-\frac{2}{5} + \frac{2}{3} - \frac{1}{2} + 1) = 2\frac{7}{10} \end{aligned}$$

$$\text{Dus } O(W) = 2\frac{7}{10}.$$

Verantwoording

Beeld

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